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THEORY AND DESIGN OF RECOIL SYSTEMS AND GUN CARRIAGES

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By order of the Secretary of ~~War~~.
DEFENSE

C. C. WILLIAMS,
MAJOR GENERAL, CHIEF OF ORDNANCE.

F O R E W O R D

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PREFACE

Although strictly artillery design may be considered a highly specialized branch of machine design, there are so many features that differentiate this work from ordinary machine design, it has been felt that a volume covering the specialized points is of fundamental importance in order that our designing engineers may have in a readily accessible from reference data covering the general subject and in particular those features of modern development not now covered in published works. Such is the purpose of this volume.

Artillery design may be subdivided into the design of cannon, and the design of gun carriages and recoil systems. During the late war the extensive introduction of self propelled gun mounts, such as caterpillar vehicles, has introduced automotive problems in the design of these types of gun mounts in addition to the ordinary consideration affecting design of gun mounts. Further in the design of artillery we have three important aspects, - (1) the technical and theoretical considerations of a design, (2) the fabrication, standardization and production features, and (3) the service and field requirements to be fulfilled. All three aspects are equally important and a successful design results only from a balanced consideration of the three.

This discussion has been written under the auspices of Colonel G. F. Jenks, Chief of the Artillery Division, Ordnance Department, U. S. A. and of Colonel J. B. Rose, Chief of the Mobile Gun Carriage Section of that Division. Effort has been made to arrange systematically in a form for reference the great quantity of engineering data in the files of the office. In order to develop and analyze this data, it has

been necessary to introduce a considerable number of original discussions and deductions.

The work is an attempt to cover only the technical aspect of the design of gun carriages and recoil systems. The fabrication and field service phases, though of course inherently coordinate in a design are subjects of such complexity and broadness that they require for their full appreciation a separate treatment. These aspects have therefore necessarily been entirely omitted, except in so far as they are directly connected with the technical features involved.

Acknowledgement and thanks are especially due to Colonel J. B. Rose, who has proof read the complete work in the view of bringing the data into conformity with the practice and standards of the Ordnance Department. It should be stated, however, that this has been done only to the degree which was found possible without destroying original conclusions and discussions or without alteration of the system of nomenclature used. The latter is in partial but not complete agreement with the most general practice. Further acknowledgement and thanks for suggestions on the various parts of the work are due to: -

Mr. D. A. Gurney, Ordnance Engineer, Mobile Gun Carriage Section, Artillery Division.

Prof. E. V. Huntington, Professor of Mathematics and Mechanics, Harvard University

Professor C. E. Fuller, Professor of Applied Theoretical Mechanics, Massachusetts Institute of Technology.

Professor G. Lanza, Professor Emeritus in charge of Mechanical Engineering Department, Massachusetts Institute of Technology.

Acknowledgement of assistance on the computation work is due to Mr. E. V. B. Thomas, Mr. Kasargian and Mr. McVey of the Artillery Division, also to Messrs. Murray H. Resni Coff and O. L. Garver for preparing this data for publication.

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T A B L E O F C O N T E N T S

C H A P T E R I

TYPES AND PRINCIPAL ELEMENTS OF CANNON AND CARRIAGES.

INTRODUCTION: The fundamental principles of gun carriage design are entirely the same as those of engine and machine design, and it is the object of this volume merely to bring out the specific application of these principles to the design of gun carriages.

A gun carriage is a machine exercising primarily the following functions:

(1) To provide a fixed firing platform which dissipates the energy given to the recoiling parts in reaction to the energy imparted to the projectile and powder gases.

(2) To return the recoiling parts to their initial position for further firing.

(3) To provide the mechanism for elevating the gun for different ranges and angles of site, and for traversing the gun for changes in the direction of fire.

The effect of allowing the gun and a part of the carriage to recoil is to reduce many times the stresses in the carriage and to maintain its equilibrium. A properly designed recoil system will give reactions consistent with the strength and stability of the carriage, and a smoothness of action which is essential for long service and accuracy. The success of one design over another is due to perfection of many details, which insures smooth action and long service and to a judicious compromise between many opposing conditions and requirements.

To approach the study of carriage design, it is necessary to know the elements of interior ballistics and the characteristics of guns for meeting different

ballistic conditions in so far as these affect the form of carriage and determine the forces acting upon it. These subjects will, therefore, be briefly considered, but a complete discussion must be obtained from works treating them specifically.

CANNON From one view point a cannon may be considered as a tube of proper thickness for strength, having a chamber in the rear of somewhat larger diameter which contains the powder charge. The powder charge is inserted by opening a breech block in the rear end of the cannon. This breech block necessarily must withstand the maximum powder pressure over its cross section and a powerful locking device is therefore needed. The details of this mechanism are complicated, but need not be considered in carriage design, except in special cases where the breech mechanism is operated during counter recoil. The design of the rifling grooves and capacity of powder chamber will be considered later.

The elements of a gun are shown in figure (1). "A" is the powder chamber, "B" the rifled portion of the bore, "C" the breech block, "D" the gun lug for the attachment of piston rods, which restrain the gun in recoil.

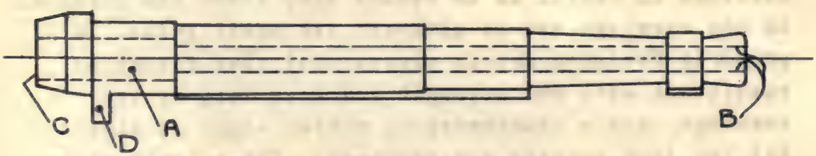

Fig. 1

The caliber of a gun is the diameter of the bore and is expressed usually in millimeters or inches. Speaking very roughly, small guns range from 37 m/m

to 75 m/m, and are suitable for mounting on aeroplanes or for use with infantry. Light field guns range from 75 m/m to 105 m/m. Ordinary medium artillery ranges from 105 m/m to 8 inches. Heavy artillery ranges from 8 inches upward. The above classification refers to mobile field materiel only.

HOWITZERS AND GUNS Carriages are designed for either howitzers or guns. Howitzers are for high angle fire, the striking angle being generally above 25 degrees. They have a medium or low muzzle velocity. A gun is designed for range and, therefore, has a high muzzle velocity.

The angle of elevation of howitzers is usually between 20° and 70° and the muzzle velocities from 400 to 1800 feet per second. The angle of elevation of a gun is usually from minus 5 degrees to plus 45 degrees with muzzle velocities ranging from 1700 to 3000 feet per second. The angular velocity of the projectile is also considerably higher for a gun than for a howitzer. In modern practice the line of demarcation between guns, howitzers and mortars has become somewhat less distinct, and we may consider all of them as cannon which decrease in power in the order named and generally for use at elevations which increase in the order named.

Against aircraft, firing is at elevations from 0 to 80 degrees, but the muzzle velocity is high; hence, the pieces used in such work are properly classified as "guns".

The traversing limitations of a gun and howitzer may be the same or different but do not enter in the differentiation between a gun and howitzer.

The muzzle velocity of howitzers being lower than that of guns, it is possible with the same total weight of materiel to fire a much heavier projectile.

RECOILING PARTS The recoiling parts consist of the gun together with the various parts attached to it and recoiling with it. We have two methods of arrangement of recoiling parts:

(1) the piston rods with their pistons attached to the gun lug and recoiling with the gun.

(2) the pistons and their rods held stationary.

So far as the recoil mechanism is concerned we are only concerned with the relative motion between the rods and pistons and their cylinders.

The greater part of our guns in the service translate in recoil directly along the axis of the bore, others as on certain Barbette mounts and double recoil systems have a translation in addition to that along the axis of the bore. Guns on Disappearing carriages and special other types have rotation in addition to translation.

In ordinary recoil systems the center of gravity of the recoiling parts is usually located slightly below the axis of the bore. This insures a positive jump (muzzle up) during the powder pressure period. If the center of gravity of the recoiling parts is greatly below the axis of the bore considerable stresses are brought upon the elevating rack and pinion, due to the fact that the powder pressure causes an excessive turning effect about the trunnions the amount depending also upon the location of these. For this reason when the cylinders recoil with the gun, extra weight is very often introduced on the top of the gun. This, of course, raises the center of gravity of the recoiling parts nearer the axis of the bore.

The recoiling parts are constrained to recoil parallel with the axis of the bore by gun clips engaging in guides in a fixed cradle or by the gun itself sliding in a fixed cylindrical sleeve. Due to the fact that the braking forces developed in the

cylinders are usually considerably below the axis of the bore during recoil, considerable pinching action takes place at the front and rear clip contact with the guides. This causes somewhat greater friction than would be obtained by mere sliding friction.

The clips attached to the recoiling parts, or rather to the gun itself, which in turn engage in the guides of the cradle, are usually either continuous or three to four in number. In order to maintain a constant friction throughout the recoil, clips should be evenly spaced along the gun and the front clip should engage in the guides before the rear clip leaves the guides. When the gun recoils in a sleeve or cylinder which is a part of the cradle, it is cometimes possible to distribute the various pistons and cylinders symmetrically about the axis of the bore. As we shall see, this decreases the friction during the recoil and counter recoil.

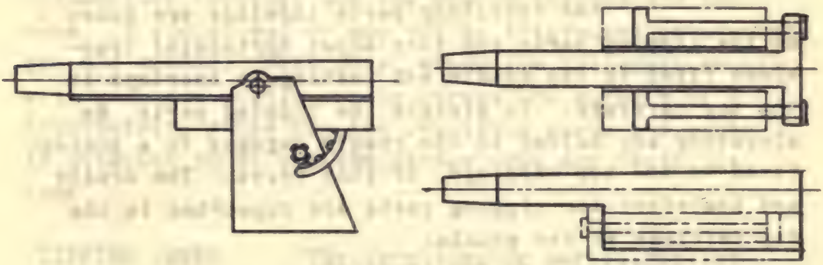

Fig. 2

Figure (2) shows the recoiling mass where the pistons and their rods recoil with the gun. Below in figure (3) is shown a recoiling mass consisting of the cylinders grouped together in a single forging in a so-called slide or sleigh, and rigidly attached to the gun.

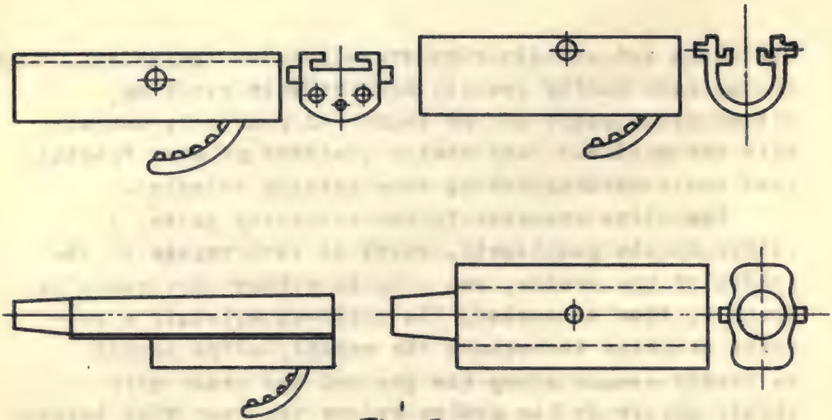

Fig. 3

THE CRADLE. The cradle serves as a constraining member for the sliding of the gun to the rear in recoil and as a support for elevating the gun. The cradle and the recoiling parts together are known as the tipping parts and turn about horizontal trunnions fixed to the cradle and resting in bearings in the top carriage. To elevate the tipping parts, an elevating arc bolted to the cradle engages in a pinion fixed to the top carriage, or vice versa. The cradle and therefore the tipping parts are supported in the top carriage at two points:

(1) at the trunnions

and

(2) at the tooth contact of the elevating arc and pinion.

See figure (4).

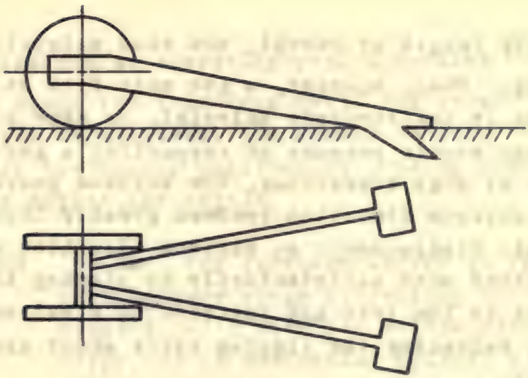

Fig. 4

When the cylinders do not recoil they are in turn an integral part of the cradle, and therefore, the recuperator forgings and the cradle are one and the same. A sleigh may or may not be interposed between the gun and cradle. With guns, where the cylinders recoil with gun, the cradle merely serves the purpose of a constraining guide for the recoiling parts and rigidly attached to it are the piston rods and their pistons.

TIPPING PARTS The term "tipping parts" applies to those parts of a carriage which move in the process of elevating the gun. In order to rapidly elevate the gun, it is considered very important that the tipping parts are nicely balanced about the trunnions. Thus the center of gravity of the tipping parts must be located at the trunnions. As the height of the trunnions and axis of the bore are governed by stability at horizontal elevation, clearance in traveling and accessibility for loading, the length of recoil at maximum elevation becomes limited. If a minimum elevation of about 20 degrees is allowed for a howitzer, we might raise the trunnions, thereby in-

crease the length of recoil, and thus maintain stability. When, however, a gun must fire at high elevation as in antiaircraft materiel, or when a carriage serves the double purpose of supporting a gun or a howitzer at high elevations, the maximum possible recoil at maximum elevation becomes greatly limited. The recoil displacement at maximum elevation may be increased most satisfactorily by placing the trunnions to the rear and introducing a balancing gear for balancing the tipping parts about the trunnions.

The balancing gear usually consists of an oscillating spring or pneumatic cylinder, the trunnions of which rest in bearings in the top carriage, the end of the piston rod being attached to the cradle. Since it is difficult to obtain perfect balance by this method throughout the elevation, the maximum unbalanced moment in the process of elevation should be considered in the design of the elevating gear mechanism. A method by which exact balance can be maintained throughout the elevation is obtained by use of a cam and chain connecting the cradle with the spring or pneumatic cylinder. In this case the cam is fixed to the cradle and the spring cylinder to the top carriage. However, due to the variation in trunnion friction and other similar factors the former method is probably better since a very close approximation in balance throughout the elevation can be obtained.

The reaction on the elevating arc and the trunnion reaction are modified by the introduction of the balancing gear, though ordinarily where the weight of tipping parts is relatively small as compared with the recoil reaction the effect of the balancing gear on the reactions may be neglected.

When it is desired to use an independent line of sight, a rocker is introduced between the elevating pinion and cradle. The rocker, when moving, is a part of the tipping parts. In the process of

elevating the gun an elevating pinion rotates the rocker about the trunnions until the proper line of sight is obtained; the cradle is then brought into its proper position by gearing connecting the rocker and cradle.

TOP CARRIAGE The top carriage serves as an intermediary piece connecting the tipping parts with the bottom carriage, or in semi-fixed mounts, with the bottom platform. The top carriage is supported at its bottom by a vertical pintle block and circular traversing clips. At the top it supports the tipping parts on its trunnion bearings and elevating pinion bearing. The top carriage together with the tipping parts are known as the traversing parts. To traverse the gun, the top carriage with the tipping parts are rotated in a horizontal plane about the pintle block by a circular traversing rack and pinion or worm gear.

In certain types of field artillery the top carriage is an integral part of the trail, in which case traversing is obtained with respect to the wheels and axle by moving the trail along the axle and about the spade point as a pivot. Traverse by this method is naturally very limited as compared to traverse with a rotating top carriage. All stationary mounts or field platform mounts have a separate top carriage which serves this specific function of traversing about the vertical pintle support. In very large carriages the top carriage is supported by a circular ring of horizontal rollers, the pintle bearing merely serving as a constraining pivot. In certain types where the bottom carriage itself is traversed, the top carriage is used for translation only. It is then supported on rollers moving along an inclined or horizontal plane and the braking is affected by a recoil cylinder in the top carriage which

connects the top carriage with the bottom carriage through the piston rods.

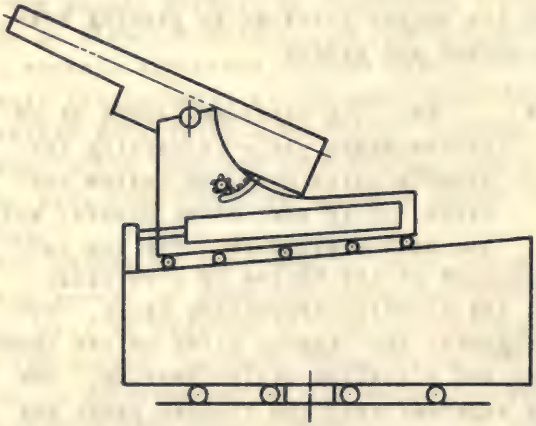

Fig. 5

Top carriages may be roughly classified into:

(1) the ordinary type of side frames connected at the front or rear by cross beams or transoms which contain the pivot bearing.

(2) pivot yoke type used on small mobile mounts

and

(3) trail carriages.

The ordinary side frame type of top carriage is extensively used on the stationary mounts on mobile platform mounts and even on trail supported carriages. The pivot yoke type is especially useful when split trails are introduced, since it supports the equalizer bar for balancing the distribution of the load between the two trails.

TRAIL AND SPADE With mobile field artillery it is customary to use a trail and spade for the double purpose of preventing a backward motion of the carriage on firing of the gun, and of giving sufficient stability to the carriage in order that the wheels may not leave the ground. We have two classifications of trails, - (1) the single or box trail, (2) the split trail. With a single trail it is necessary to have a large U-shaped aperture or fork arrangement at the forward end in order to elevate, load and traverse the gun without interference. When split trails are used we have two separate single trails which may turn at the wheel ends about the axle. It is customary with the split trails to introduce an equalizing mechanism which connects the two trails and distributes the load between the trails on firing.

The spade and float support the trail and are designed to take up the horizontal and vertical reactions at the rear end. In the design of the spade and floats it is important that the unit bearing pressure be held to a low value. This should not be more than about 30 lb. per sq. in. for the float and 40 lb. per sq. in. for the spade.

For wide traverse of the gun it is necessary to lift the spade from the ground and turn the carriage to the desired line of fire. For this reason the static load on the spades should not exceed about 100 lbs. for light carriages. Thus in a preliminary lay-out of the carriage, it is necessary to locate the center of gravity of the total system in battery very close to the axle in order that the static load under the float does not exceed the desired amount. This inherently makes the counter recoil stability in battery very small especially at horizontal recoil and requires considerable care in the design of a counter recoil

system.

At horizontal elevation the carriage is usually designed with a very small margin of stability. Therefore, in firing the vertical load on the float practically equals the weight of the total system. The bending moment in the trail gradually increases from the spade toward the wheel axle. We have maximum bending moment at the attachment of the trail to the top carriage or wheel axle.

PLATFORM MOUNTS. With fixed mounts and heavier types of field artillery it is customary to support the traversing parts on a platform, that is, the top carriage rests upon a platform which serves as a bottom carriage. When a platform bottom carriage is used, it must be either bolted to a concrete foundation as in fixed mounts or else it must have a vertical projection similar to a spade on a field carriage to take up the horizontal reaction in firing. Further, the bearing surfaces of this platform must be sufficient to prevent overturning of the carriage firing at low angles of elevation or change in level in firing at any elevation. That is, the center of pressure of the reaction of the earth must be within the middle third of the length or diameter of the platform in the line of fire. Since platform mounts vary considerably in construction of detail no attempt will be made to catalogue the various types used.

With fixed mounts the bottom carriage or platform is usually secured to a concrete foundation by a distribution of bolts along a circular flange; and since with fixed mounts all round traverse is possible, each bolt should be designed for maximum tension.

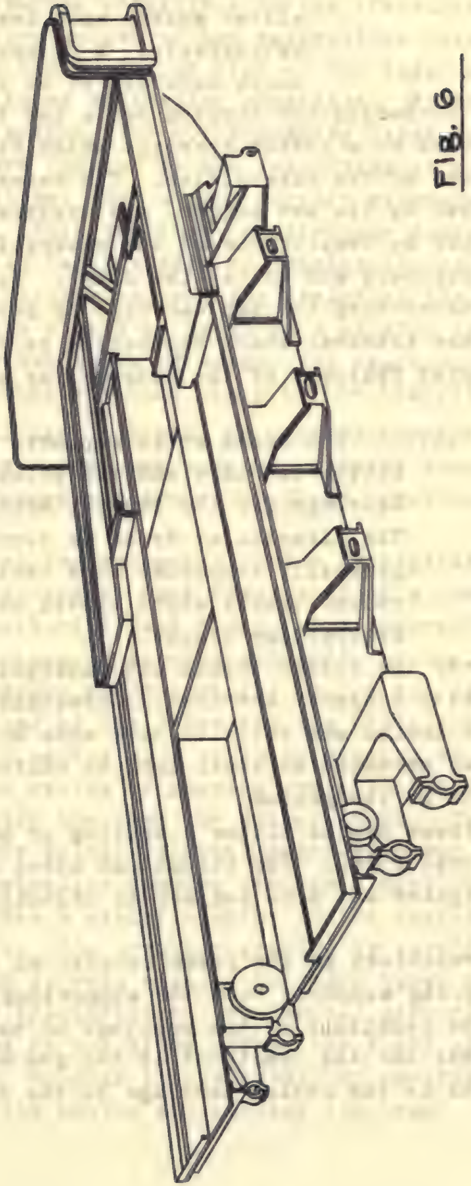

FIG. 6

CATERPILLAR MOUNTS. To increase mobility during the World War, caterpillar mounts were developed extensively. A caterpillar mount consists of an ordinary gun mount including the tipping parts and top carriage mounted on a bottom carriage which fits within the frame of the caterpillar. The caterpillar is propelled by its own engine, and traverse can be readily made by keeping one of the caterpillar tracks stationary and moving the other. For more delicate traversing the top carriage is provided with limited traverse about the bottom carriage. The essential features of the caterpillar proper are:

(1) The frame which supports the bottom carriage and the principal bearings for the driving mechanism. The caterpillar frame in turn is generally supported on a series of roller trucks which travel on the caterpillar tracks.

Between the roller trucks and caterpillar frame, spring supports are usually provided, and the roller trucks are built to have more or less up and down movement at their ends to conform with the contour of the ground.

The frame may be either a casting or built up of structural steel. The structural steel frame is perhaps lighter but more subject to objectional deflections.

The reactions on the frame consist of the various spring supports from the supporting roller trucks, the reactions of the bearings of the running gear and the reactions of the gun mount transmitted by the bottom carriage to the frame on firing.

The frame of a caterpillar is subjected to a complicated system of stresses. Due to various possible loading conditions during traveling such as the entire weight of the caterpillar being carried in the center or else at the ends, we have different types of loading reactions. Further a wrenching action with corresponding large transverse stresses are induced by the supporting reactions being on either side at the further extremities of the track. This requires considerable lateral bracing. In fact outside of fabrication and construction considerations, the design of the caterpillar frame should be based on a careful analysis of the various types of supporting reaction combinations that may take place in the traveling of the caterpillar. It will be usually found that the traveling stresses are somewhat greater than the firing stresses and are often of an opposite character.

The driving mechanism of the caterpillar consists of two tracks each consisting of a continuous track or belt of linked shoes. The caterpillar track is driven by sprockets usually at the rear end. The drive shaft contains at one end the track sprocket, and at the other end the drive sprocket gear, which meshes by a suitable gearing to a clutch, the system of gearing and clutch being symmetrically the same for either track. The clutches are driven by bevel gears or other forms of reduction gearing through a gear box, and sometimes a master clutch, to the engine crank shaft. The traction gearing is straight forward and is very similar to other types of drive gear transmission. Mechanical steering is obtained by operating either the right or left track, holding it stationary or sometimes reversing the motion and running the track backward.

Electric drive caterpillar mounts are in two (2) units and possess certain advantages; first, the transmission can be greatly reduced in either unit by the use of compact motors and gearing; second, the units can be made similar and the mobility thereby increased; third, a better design of gun mount is possible due to less limitations on clearance and other corresponding factors. The electric drive consists of the gun mount unit and the power plant unit. The power plant unit supplies power for driving itself, as well as the gun mount; fourth, the caterpillar is braked in traveling by suitable band brakes in the transmission. When, however, the gun is fired, it is necessary to brake the caterpillar from running back. The braking and torque being usually in an opposite direction and necessarily of a large value as compared with the traveling braking; it is usually customary to introduce a band brake on the final drive shaft and thus eliminate the stresses in the transmission during firing. The braking should be designed to produce a traction reaction equal to approximately 80 percent of the total caterpillar. Fifth, in a design of caterpillar mounts, stability is of prime importance due to the limited wheel base and necessity of maintaining as light a mount as possible. Stability may be increased by the use of outriggers attached to the caterpillar body. To decrease the overturning reaction of the recoil on firing and thus increase the stability, double recoil systems have been successfully introduced on larger caterpillar guns. A double recoil system consists of an ordinary recoil system between the gun and cradle of the top carriage and a lower recoil system between the top carriage and frame. The top carriage is designed to roll up an inclined plane of sufficient elevation to bring the recoiling masses into battery and the caterpillar lies in a

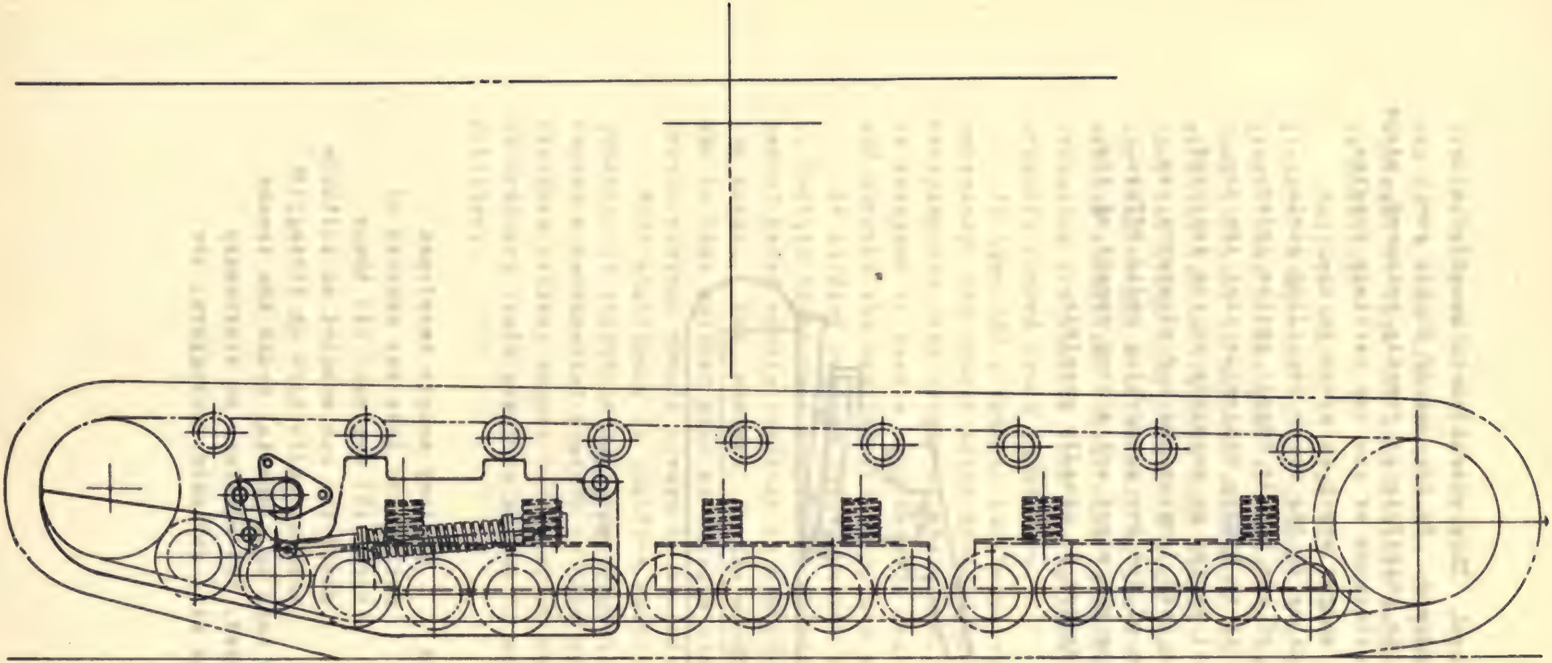

Fig. 7

horizontal plane. This elevation is usually at from 6 to 7 degrees. By the use of double recoil systems the stability is greatly enhanced, since the inertia resistance of the top carriage creates a stabilizing moment which is added to the inertia resistance of the upper recoiling parts. In the design of the double recoil system caterpillar mount, it is highly desirable that the top carriage recoil as far as possible up the inclined plane. Due to less limitations and clearance, an electric drive of the two supporting units offers a very suitable gun mount and a long recoil of the lower recoil system is usually possible.

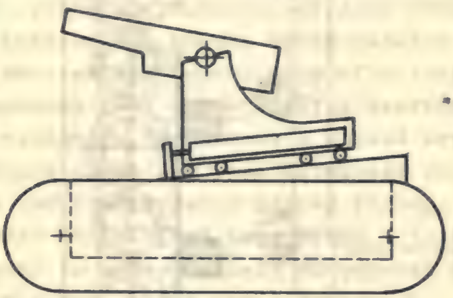

Fig. 8

RAILWAY MOUNTS Railway mounts developed during the late war consist of three (3) systems: (1) these where the car mounted on suitable tracks, rolls back on firing; (2) those sliding back on a special track the trucks being disengaged, (3) platform or stationary railway mounts with suitable outriggers, the

trucks being entirely disengaged. In types (1) and (2) a very limited traverse is possible, whereas, in type (3) considerable amount of traverse is possible.

Railway mounts of type (1), rest upon suitable girders, supported by the trucks at either end. The girder must be designed to carry the maximum firing load stresses at maximum elevation, as well as stresses due to the dead load weights. The trucks take the supporting reactions from the girders of the dead weight load as well as the firing load at maximum elevation. Great care is needed in distributing the loading from the various axles by properly formed truck equalizers.

In type (2) a special built-up track is necessary, the trucks being disengaged merely carrying the dead weight of the mount. The mount is designed to have a considerable bearing surface, and thereby the bearing pressures are greatly reduced.

In sliding railway types, recoil systems have in certain types been completely eliminated, the recoil being merely resisted by the friction of the track. Due, however, to the enormous stresses due to high caliber guns at maximum elevation, recoil systems should always be introduced.

With stationary or platform mounts the question of stabilizers of corresponding outriggers become a fundamental feature in this type of design. Platform railway mounts have similar characteristics as ordinary field platform mounts in mobile artillery.

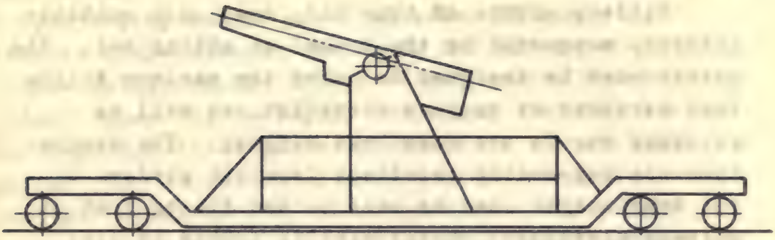

Fig. 9

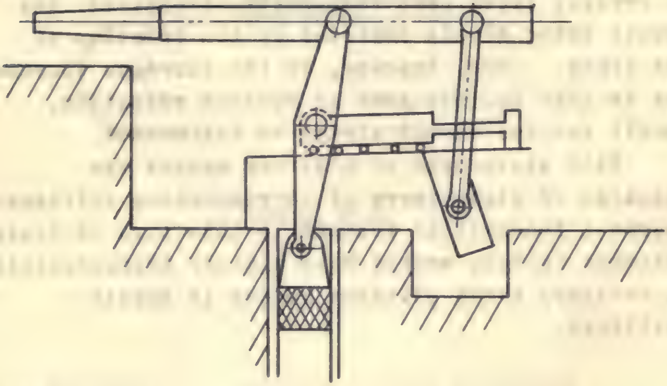

Fig. 10

CHAPTER II

DYNAMICS OF INTERIOR BALLISTICS AS AFFECTING RECOIL DESIGN.

The object of interior ballistics is partly to derive expressions for the acceleration and velocity of the projectile during the travel in the bore, and the corresponding pressures on the base of the shell and breech in terms of the powder loading, the form of powder grain, the initial volume of powder chamber in the gun, and other variables upon which the velocity and pressure depend. In the design of the recoil mechanism as well as the carriage for its maximum stresses, it is very important to know the accelerations, velocities, and pressures in the gun to a considerable degree of accuracy throughout the time the powder gases act.

In the study of interior ballistics, it is convenient to divide the powder pressure interval into two periods:

(1) The interior period while the shot travels up the bore to the muzzle.

(2) The after effect period while the powder gases expand after the shot has left the muzzle.

During the interior period, we have considerable combustion of the charge and corresponding gas evolved in the powder chamber before the shot has left its initial position in the breech end of the bore, the temperature rising and the pressure reaching a value sufficient to force the projectile into the rifling groove and to overcome initial frictions, usually a considerable fraction of the powder pressure obtained. The projectile then moves up the bore followed by further combustion and expansion of the gases evolved from the combustion of the powder. The combustion exceeds the expansion up

to the time of maximum powder pressure which is reached after a travel up the bore roughly from 1/6 to 1/3 the length of the bore depending greatly on the type of cannon, charge, etc.

The energy of combustion is expended:

(a) In Kinetic Energy of translation of the projectile.

(b) In Kinetic Energy of translation of the recoiling mass (assuming the recoiling mass free).

(c) In the Kinetic Energy of the charge itself.

(d) In the work on the rifling and in friction.

(e) In the angular energy given to the projectile.

(f) In dissipated heat.

The last three are very small as compared with (a), (b) and (c). Further (b) and (c) are small as compared with (a).

"Ingalls" states that about 83% of the total energy of the work of expansion goes into the Kinetic Energy of translation of the shot. the remainder 17% going into the forms b, c, d, e and f.

The rate of combustion depends upon the form and size of the grain, it being an observed fact that powder burns in layers always parallel to the initial surface. Further the rate of combustion is a function of the actual pressure generated, varying as some power of the pressure. The value used for this exponent is one of the most tentative features in the whole subject of interior ballistics.

DYNAMIC RELATIONSHIPS IN INTERIOR BALLISTICS.

Let m = the mass of the projectile.

w = the weight of the projectile.

$\bar{m}$ = the mass of the charge.

$\bar{w}$ = the weight of the charge.
m_r = the mass of the recoiling parts.
w_r = the weight of the recoiling parts.
u = the travel up the bore.
x = the absolute displacement of the shot in the bore.
X = the corresponding displacement of the recoiling parts.
v = the absolute velocity of the shot in the bore.
v_o = the muzzle velocity of the shot.
V = the free velocity of the recoiling parts (absolute).
P_b = the total pressure on the breech.
P = the total pressure on the base of the shot.
p_b = the intensity of pressure on the breech (lbs. per sq. in.).
p = the intensity of pressure on the base of the shot (sq. in.).
f = the component of the rifling reaction parallel to the axis of the bore.

Then,

$$P - f = m \frac{dv}{dt}$$, for the motion of the projectile (1)

and $$P_b - f = m_r \frac{dV}{dt}$$, for the motion of the recoiling mass in free recoil (2)

and further assuming the charge to expand in parallel laminae with the successive laminae having velocities as a linear function of the end velocities, we have,

$$P_b - P = \frac{\bar{m}}{2} \left(\frac{dv}{dt} - \frac{dV}{dt}\right) \qquad (3)$$

where $$\frac{\frac{dv}{dt} - \frac{dV}{dt}}{2}$$ = the mean acceleration

of the powder.

Combining (1), (2) and (3)

$$(m_r + \frac{\bar{m}}{2})\frac{dV}{dt} = (m + \frac{\bar{m}}{2})\frac{dv}{dt} \tag{4}$$

Integrating successively,

$$(m_r + \frac{m}{2})\,V = (m + \frac{\bar{m}}{2})v \tag{5}$$

$$(m_r + \frac{\bar{m}}{2})\,X = (m + \frac{\bar{m}}{2})x \tag{6}$$

The absolute displacement of the shot in the bore is connected with the travel (u) up the bore by the following relation:

$$x = u - X$$

since the positive value of X is assumed opposite to x.

Substituting in (6), we have,

$$X = \frac{(m + \frac{\bar{m}}{2})\,u}{m_r + m + \bar{m}} \tag{7}$$

which gives the relation of free recoil to the travel of the shot up the bore.

Obviously (5), (6) and (7) may be written immediately from the principle of "linear momentum" (that is, the total momentum of the system remains constant unless acted on by external forces) and the principle that the center of gravity remains fixed unless acted upon by external forces. In free recoil the exterior forces are nil.

The pressure on the breech exceeds that on the shot by the inertia resistance offered by the mass of the powder gases,

Now $$\frac{p_b - f}{p - f} = \frac{m_r dV}{m dv} = \frac{m_r}{m}\left(\frac{m + \frac{\bar{m}}{2}}{m_r + \frac{\bar{m}}{2}}\right)$$

Neglecting $\frac{\bar{m}}{2}$ as small compared with m_r,

$$\frac{p_b - f}{p - f} = \frac{(m + \frac{\bar{m}}{2})}{m} .$$

hence
$$p_b = p\,\frac{m + \frac{\bar{m}}{2}}{m} - \frac{f\bar{m}}{2m}$$

Since the rifling reaction expecially during the movement of the shot up the bore is roughly 2 per cent or less of the value of p, we may entirely neglect the term $f\,\frac{\bar{m}}{2m}$

in the above expression, which simplifies to
$$p_b = \frac{m + \frac{\bar{m}}{2}}{m}\,p \qquad (8)$$

From a series of experiments conducted by the United States Navy the value

$$\frac{m + \frac{\bar{m}}{2m}}{m} = 1.12 \text{ a constant, approx.}$$

hence
$$p_b = 1.12\,p \text{ approx.} \qquad (9)$$

It is to be noted that the acceleration of the powder is very likely somewhat different from the assumption upon which (8) was derived, but nevertheless equations (8) and (9) give a good approximation of the increase of breech pressure over that at the base of the projectile. During the "forcing in of the rifling" before the commencement of motion of the shot, obviously $p_b = p$.

According to the previous assumptions the pressure varies progressively, decreasing from its maximum value at the breech block to a slightly smaller value at the base of the projectile.

Therefore, if we let p_m be the average or mean instantaneous pressure or rather the pressure in the powder chamber and bore, we have,

$$P_m = \frac{P_b + P}{2}$$

In terms of the total pressure at the base of the projectile,

$$P_m = \frac{m_r \frac{dV}{dt} + m\frac{dv}{dt}}{2} = \left(\frac{2m + \frac{\bar{m}}{2}}{2}\right)\frac{dv}{dt}$$

but $\frac{dv}{dt} = \frac{P}{m}$

hence

$$\frac{p_b + P}{2} = p_m = P\left(1 + \frac{\bar{m}}{4m}\right) = P\left(1 + \frac{\bar{w}}{4w}\right) \quad (10)$$

or in terms of the total breech pressure

$$P_m = \frac{m_r \frac{dV}{dt} + \frac{mm_r}{m + \frac{\bar{m}}{2}} \frac{dV}{dt}}{2} = \frac{P_m}{2}\left(1 + \frac{m}{m + \frac{\bar{m}}{2}}\right)$$

or (11)

$$P_m = P_b \frac{m + \frac{\bar{m}}{4}}{m + \frac{\bar{m}}{2}} = P_b \frac{w + \frac{\bar{w}}{4}}{w + \frac{\bar{w}}{2}}$$

EQUIVALENT -----
MASS OF
PROJECTILE

The rifling grooves in the gun come in contact with the copper rifling band on the projectile and angular motion is transmitted to the projectile in addition to the translatory motion. The object of the angular motion is to give the projectile a gyroscopic effect maintaining, with a combination of the air reaction, the axis of the projectile parallel to the tangent of the trajectory and further making an oblong projectile possible with greater ballistic efficiency.

Let

P = the reaction of the powder on the base of the shell.

m = the mass of the projectile.

f = the total rifling reaction normal to the rifling groove.

uf = the friction component of the rifling reaction tangent to the rifling groove.

θ = the angle of pitch of the rifling, (i. e. the angle the rifling makes with the axis of the bore).

p = the pitch of the rifling

d = the diameter of the bore

k = the radius of gyration of the projectile..

x = the displacement of the projectile up the bore from its initial position.

ϕ = the corresponding angular displacement twist of the projectile.

Then we have,

$$P - f(\sin\theta + u\cos\theta) = m\frac{d^2x}{dt^2} \qquad (12)$$

$$f(\cos\theta - u\sin\theta)\frac{d}{2} = mk^2\frac{d^2\theta}{dt^2} \qquad (13)$$

Further since, the number of complete turns or revolutions of the projectile in its linear displacement x or its angular displacement Ø, is

$$\frac{x}{p} \text{ or } \frac{\phi}{2\pi}$$

we have

$$\frac{d^2\phi}{dt^2} = \frac{2\pi}{p}\frac{d^2x}{dt^2} \qquad (14)$$

In terms of the angle of pitch of the rifling,

$$\frac{d}{2}\phi = x \tan\theta \qquad \text{or } \frac{\pi}{p} = \frac{\tan\theta}{d}$$

hence

$$\frac{d^2\phi}{dt^2} = \frac{2}{d}\tan\theta\,\frac{d^2x}{dt^2} \qquad (15)$$

Substituting (14) or (15) in equation (13) we have

$$f = \frac{mk^2\, 4\,\frac{\pi}{p}\,\frac{d^2x}{dt^2}}{(\cos\theta - u\sin\theta)d} = \frac{mk^2 4\tan\theta\,\frac{d^2x}{dt^2}}{(\cos\theta - u\sin\theta)d^2} \qquad (16)$$

which shows the reaction f is always proportional to the linear acceleration of the projectile. Therefore, the friction uf , is also proportional to the linear acceleration.

Substituting (16) in (12), we have

$$P = [1 + (\frac{\sin\theta + u\cos\theta}{\cos\theta - u\sin\theta})\frac{4\pi k^2}{dp}]\, m\frac{d^2x}{dt^2} \qquad (17)$$

or in terms of the rifling angle,

$$P = [1 + (\frac{\sin\theta + u\cos\theta}{\cos\theta - u\sin\theta})\frac{4k^2\tan\theta}{d^2}]\, m\frac{d^2x}{dt^2} \qquad (18)$$

which shows that the powder reaction P is also directly proportional to the linear acceleration of the projectile. Evidently the equivalent mass of the projectile, is

$$m'' = [1 + (\frac{\sin\theta + u\cos\theta}{\cos\theta - u\sin\theta})\ \frac{4\pi k^2}{dp}]\ m$$

$$= [1 + (\frac{\sin\theta + u\cos\theta}{\cos\theta - u\sin\theta})\ \frac{4\tan\theta\ k^2}{d^2}]\ m \qquad (19)$$

Hence the rifling reaction and friction due to rifling are directly proportional to the powder reaction, that is the pressure on the rifling grooves always varies at any instant directly with the powder reaction.

Thus we have the relationship that rifling friction behaves exactly like an additional mass; that is, it has an inertia effect since it is proportional to the acceleration.

The true equivalent mass due to the linear and angular inertia of the projectile alone, can be obtained by assuming the rifling friction zero, (i.e., putting $u = o$)

$$m' = (1\ \frac{4\ \pi^2 k^2}{p^2})\ m$$

$$= (1\ \frac{4k^2\ \tan^2\theta}{d^2})\ m \qquad (20)$$

The true equivalent mass may be readily checked by a consideration of the total energy of the projectile, that is,

$$\tfrac{1}{2}\ m'v^2 = \tfrac{1}{2}\ mv^2 + \tfrac{1}{2}\ Iw^2$$

and $w = \dfrac{2\pi v}{p} = \dfrac{2v\tan\theta}{d}$

and $I = mk^2$ where k = radius of gyration about its longitudinal axis.

hence

$$m' = \left(1 + \frac{4\pi^2 k^2}{p^2}\right) m = \left(1 + \frac{4k^2 \tan^2 \theta}{d^2}\right) m$$

EQUIVALENT MASS OF POWDER CHARGE For a differential layer of the powder charge at the base of the projectile, its velocity evidently is equal to that of the projectile while for a differential layer at the breech, the velocity is equal to that of the gun. For intermediate layers, we must assume some law of variation of velocities, between the two end limits. For simplicity and probably a fairly close approximation, we will assume for the various laminae, a linear variation of velocity between the end limits. Further since the velocity of the gun is small as compared with that of the projectile, in virtue of the approximation of the whole analysis, we are entirely justified in assuming the recoil velocity entirely negligible.

If,

Velocity of projectile = v (ft. sec.)

Distance between breech and base of projectile = x (ft.)

Velocity of any intermediate lamina ---- = v' (ft/sec.)

Distance from breech to lamina ----------- = x' (ft.)

Then

$$v' = \frac{x'}{u} v \quad \text{(ft.sec.)}$$

If we assume the density of the powder is uniform through the distance x, so that the weight of the lamina is $\frac{w}{x} dx$, then the kinetic

energy of the lamina is

$$\frac{\bar{w}}{x} \frac{v'^2}{2g} dx' \text{ or } \frac{\bar{w}}{x^3} \frac{v^2}{2g} x'^2 dx'$$

and the Kinetic energy for the total charge becomes,

$$\text{K. E. of } \bar{w} = \frac{\bar{w}}{x^3} \frac{v^2}{2g} \int_0^x x'^2 dx'$$

$$= \frac{1}{2} \left(\frac{\bar{w}}{3g}\right) v^2 \quad (21)$$

That is, the equivalent mass, when dealing with the energy equation, is 1/3 the mass of the total charge.

It is important to note that when dealing with momentum, the momentum for the total charge becomes, on the same assumption

$$\frac{\bar{w}}{gx} \int_0^u \frac{x'}{x} v\, dx' = \frac{\bar{w}}{2g} v \quad (22)$$

that is the equivalent mass from the moment or aspect is 1/2 the mass of the total charge.

EQUIVALENT MASS OF THE RECOILING PARTS

It is convenient in deriving the energy equation, to express the Kinetic Energy of the recoiling parts in terms of the velocity of the projectile.

Neglecting $\frac{\bar{m}}{2}$ as small as compared with m_r and, neglecting the recoil brake reaction as small, we have, by the principle of linear momentum,

$$m_r V = \left(m + \frac{\bar{m}}{2}\right) v \text{ (approx.)}$$

hence

$$V = \frac{(m + \frac{\bar{m}}{2})}{m_r} v$$

The recoil Energy, becomes,

$$\tfrac{1}{2} m_r V^2 = \tfrac{1}{2} \frac{(m + \frac{\bar{m}}{2})^2}{m_r} v^2 \qquad (23)$$

and therefore the equivalent mass of the recoiling parts, in terms of the velocity of the projectile, becomes,

$$\frac{(m + \frac{\bar{m}}{2})^2}{m_r}$$

ENERGY EQUATION The mechanical work expended by the gases of the powder charge in the bore is equal to the external work exerted on the projectile and gun, plus the Kinetic Energy given to the gases themselves, plus the heat energy lost in radiation through the walls of the gun.

If

$\bar{W}$ = the Potential Energy of the Gases at any instant.

P_b = the total reaction exerted on the breech of the gun.

P = the total reaction exerted on the base of the projectile.

X = the displacement of the gun measured in the direction of its movement.

x = the displacement of the projectile measured in the direction of its motion.

E = the Kinetic Energy of the powder charge.

Q = the loss of heat due to radiation.

J = the mechanical equivalent of heat = $778 \frac{\text{ft. lb.}}{\text{B. T. U.}}$

Then for the energy equation of the powder gases, we have,

$$-P_b\, dX - P\, dx = d(E + W) + JdQ \qquad (1)$$

hence

$$-dW = P_b\, dX + P\, dx + dE + JdQ \qquad (2)$$

that is the loss of the potential energy of the gases, due to a differential expansion goes into mechanical work $(P_b dX + P\, dx + dE)$ and radiation JdQ.

Further by (19), (23) and (21),

$$P\, dx = d[\tfrac{1}{2}(m'' v^2)]$$

$$P_b dX = d[\tfrac{1}{2} \frac{(m + \frac{\bar{m}}{2})^2}{m_r} v^2]$$

$$dE = d[\tfrac{1}{2} (\frac{\bar{m}}{3} v^2)]$$

so that

$$- dW = d[\tfrac{1}{2}(m'' + \frac{(m + \frac{\bar{m}}{2})^2}{m_r} + \frac{\bar{m}}{3}) v^2] + \frac{dg}{J} \qquad (3)$$

Further, in terms, of a hypothetical mean pressure P_m(over the cross section of the bore) equation (3) may be expressed in terms of the travel up the bore u, (i.e. the relative displacement between the gun and projectile).

$$- dW = P_m\, du + JdQ \qquad (4)$$

where

$$P_m\, du = d[\tfrac{1}{2}(m'' + \frac{(m + \frac{\bar{m}}{2})^2}{m_r} + \frac{\bar{m}}{3}) v^2] \qquad (5)$$

and

$$P_m = [m'' + \frac{(m + \frac{\bar{m}}{2})^2}{m_r} + \frac{\bar{m}}{3}] v \frac{dv}{du} \qquad (6)$$

where $v \frac{dv}{du}$ = the acceleration of the projectile up the bore of the gun, approximately since

$$v \frac{dv}{dV} = \frac{dx}{df} \frac{dv}{dx + dX} = \frac{dv}{df} \frac{1}{1 + \frac{dX}{dx}}$$, and dX is compared with dx, and

m'' = the equivalent mass of the projectile which takes care of its angular acceleration as well as the rifling friction, see equation (19).

EXPANSION OF POWDER GASES

It will be assumed, that the expansion of the gases due to the combustion of the powder charge obeys the law of a perfect gas. Hence, we have,

$$PV = RwT$$

where p = the Intensity of Pressure exerted by the gas ----- lbs/sq. ft.

V = the volume of the gas. (cu. ft.)

w = the weight of gas (lb.)

R = a coefficient (ft. lbs. per lb. gas.)

T = absolute temperature reached.

Further, with a perfect gas, the internal energy of the molecules of the gas is entirely in a Kinetic or Vibratory form, and therefore, is directly proportional to the temperature.

Hence, we have,

$$dQ = cwd\,T \text{ and } c = \frac{dQ}{dT}$$

where dQ = the heat required to raise the gas for a change of temperature dT.

c = Specific heat or the heat required to raise one lb. of gas one degree of temperature at the temperature considered.

We are concerned especially with the expansion of a gas at constant pressure or at constant volume or a combination of the two.

Hence, $dQ = wC_p\ dT = C_p \frac{pdV}{R}$ at constant pressure.

$dQ = wC_v\ dT = \frac{C_v\ V\ dp}{R}$ at constant volume.

If the volume and pressure vary together, then, we have the sum of the partial variations, above, that is,

$$dQ = \frac{1}{Rw}\left(C_p\ p\ dV + C_v\ V\ dp\right)$$

but since, $dT = \frac{1}{R}\left(p\ dV + V\ dp\right)$

we have, $dQ = C_v w dT + \frac{C_p - C_v}{R} p\ dV$

and

$$Q = wC_v \int dT + \frac{C_p - C_v}{R} \int p\ dV$$

This relation can be interpreted, physically immediately, since the internal energy being entirely of a kinetic or vibratory form, must be proportional to the change in temperature <u>at constant volume</u> otherwise additional heat must be added for the external work. Hence $wC_v \int dT$ measures the molecular kinetic energy. Considering an expansion <u>at constant pressure</u>, the total heat required is,

$$Q = V \frac{1}{J} pdV$$

where U = the internal energy = wC_v dT.

Since the heat is added at constant pressure, we also have,

$pdV = wR\ dT$

But the heat added at constant pressure is,

$Q = wC_p\ dT$

hence, substituting in the total heat equation,

$$wC_p\ dT = wC_v\ dT\ \ \frac{1}{J} w\ R\ dT$$

and

$$C_p - C_v = \frac{R}{J} \text{ or } J = \frac{R}{C_p - C_v}$$

If now the specific heats C_v and C_p are assumed constant for the range of temperatures during the expansion of the powder gases, we have,

$$Q = wC_v(T - T_1) + \frac{C_p - C_v}{R}W \qquad (7)$$

where

W is the external work performed.

T_1 is the initial temperature.

Neglecting the loss of heat by radiation as small, we have practically an adiabatic expansion in the bore of a gun; that is,

$$wC_v dT + \frac{C_p - C_v}{R} p \, d \, V = 0$$

Since $p = \frac{wRT}{V}$, dividing by T, we have

$$C_v \frac{dT}{T} + (C_p - C_v)\frac{dV}{V} = 0$$

and if we let

$$\frac{C_p}{C_v} = n, \text{ then } \frac{dT}{T} + (n-1)\frac{dV}{V} = 0$$

and

$$\int_{I_1}^{I} \frac{dT}{T} = (n-1)\int_{V}^{V_1} dV$$

Therefore,

$$\frac{T}{T_1} = \left(\frac{V_1}{V}\right)^{n-1}$$

Now for an adiabatic expansion, $Q = 0$ and therefore, (eq.7) becomes,

$$W = \frac{RwC_v(T_1 - T)}{C_p - C_v} = \frac{R}{n-1}(T_1 - T)$$

hence

$$W = wC_v \, J \, T_1 \left[1 - \left(\frac{V_1}{V}\right)^{n-1}\right] \qquad (8)$$

This equation is in convenient form since it is in terms of the initial and final volume in the bore.

The equivalent length of the powder chamber of the gun in terms of the area of the bore, becomes,

$$u_o = \frac{V_1}{0.785\ d^2} \text{ so that } \frac{V_1}{V} = \frac{u_o}{u_o + u}$$

In terms of the displacement up the bore the work of the adiabatic expansion of the gases, becomes,

$$W = wC_v\ J\ T_1 \left[1 - \left(\frac{u_o}{u_o + u} \right)^{n - 1} \right] \quad \text{(ft - lbs)} \quad (9)$$

$$\left.\begin{aligned} W &= C_v J w T_1 \left[1 - \left(\frac{u_o}{u_o + u} \right)^{n - 1} \right] \quad \text{(ft - lbs.)} \\ &= \frac{wRT_1}{n - 1} \left[1 - \left(\frac{u_o}{u_o + u} \right)^{n - 1} \right] \quad (\text{"} \quad \text{"}) \end{aligned}\right\} (10)$$

Here

w = weight of gases (lbs.)

W = external work performed during the (adiabatic) expansion (ft. lbs.)

C_v = specific heat for constant volume (B. T. U. per lb. per deg.)

C_p = specific heat for constant pressure (B. T. U. per lb. per deg.)

$N = \frac{C_p}{C_v}$

T_1 = initial temperature (degrees)

V_1 = initial volume (i.e. volume of powder chamber)

$u_o = \frac{V_1}{0.7854 d^2}$ where $0.7854 d^2$ = area of bore

u = displacement of projectile up the bore

J = mechanical equivalent of heat (= 778 ft. lb. per B. T. U.)

$R = (C_p - C_v)J$ = the gas constant (ft. per degree)

From this equation we may deduce the differential equations of velocity and powder pressure for the movement of the projectile up the bore, provided we know the manner of burning of the powder gases, etc.

The energy equation therefore becomes:-

$$2 \frac{wRT_1}{n - 1} \left(1 - \frac{u_o}{u_o + u}\right)^{n - 1} = m'' \frac{m \frac{m}{2}}{m_r} \frac{m}{3} v^2 \frac{dg}{J} \quad (11)$$

From which we may determine v in terms of u. The factor $\frac{dg}{J}$ which represents the loss due to heat radiation must be determined by experiment.

Based on the energy equation (or its derivation, the force equation of the motion of the projectile in terms of the displacement up the bore) various interior ballistic formulae have been derived differing in the method assumed as to the combustion and expansion of the charge. The formulae of Ingalls and Hugoniot have been used by our Ordnance from time to time especially in ballistic calculations. In recoil design, however, rough approximations are sufficient since the manner of combustion has small effect on the recoil. The formula of Leduc is sufficiently condensed with sufficient approximation to be admirably suited for recoil design.

TORQUE REACTION OF THE PROJECTILE

It is important in the design of traversing gear for guns shooting at high angles of elevation to compute the average torque reaction of the projectile upon the gun.

Let

ω = ang. vel. of projectile at any point in the bore (rad/sec)

v = linear velocity in bore along X axis (ft/sec)
r = radius of bore or of projectile (ft)
P_b = powder pressure at base of projectile (lbs)
f = normal reaction of rifling groove (lbs)
T = torque on projectile (lb. ft.)
θ = angle of rifling grooves with XX'
$I = mk^2$ = moment of Inertia of Projectile
$\varnothing$ = angle turned by projectile

Then

$$\omega = \frac{v \tan \theta}{r}$$

$$\frac{d\omega}{dt} = \frac{\tan \theta}{r} \frac{dv}{dt}$$

$$\frac{d^2\varnothing}{dt^2} = \frac{\tan \theta}{r} \frac{d^2x}{dt^2} \quad (21)$$

hence

$$T = mk^2 \frac{d\varnothing}{dt^2} = \frac{mk^2}{r} \tan \varnothing \frac{d^2x}{dt^2} \quad (12)$$

but

$$\frac{d^2x}{dt^2} = \frac{P_b}{m} \quad \text{(approx.)}$$

hence $T = \frac{r}{2} \tan \varnothing \; P_b$ (lb ft.) (13)

where $\tan \varnothing = \dfrac{2\pi r}{p}$

p being the pitch of the rifling at the point considered.

From equation (23) we see that the torque is proportional to the powder reaction on the projectile, and the "slope" of the rifling grooves, the steeper the grooves heing the greater the torque reaction with a given powder pressure.

Further, if the rifling pitch is made constant throughout the greater part of the bore, the torque

varies as the powder pressure curve along the bore and therefore is a maximum at the beginning of the travel of the projectile.

For the average torque, we have,

$$T_{av} = mk^2 \left(\frac{d^2\emptyset}{dt^2}\right)_{av}$$

The moment of inertia of the projectile may be roughly evaluated by assuming a solid cylindrical projectile:-

If C = mean or equivalent length
D = density or weight per cu. ft.

then

$$mk^2 = \frac{DC}{g}\int_0^r 2\pi r^3\,dr = \frac{DC}{g}.2\pi\frac{r^4}{4} = \frac{DC}{g}\frac{\pi r^4}{2}$$

but $m = \frac{DC}{g}\pi r^2$

hence $\frac{DC}{g}\pi r^2 k^2 = \frac{DC}{g}\,\frac{\pi r^4}{2}$ and $k^2 = \frac{r^2}{2}$;

$k = 0.707\ r$

Further, it is customary to designate the "rifling" as a "twist" of 1 turn in "g" calibers.

Therefore, if we let,

Twist = 1 turn in "g" calibers

Time of travel in bore (approx.) = $t = 3/2\,\frac{u}{v}$

Radius of gyration of projectile = $k = 0.7\ r$

Number of rev. per sec. = n

Then,

$$n\,g\,2\,r = v$$

and

$$\frac{d\emptyset}{dt} = 2\pi n = \frac{\pi v}{gr}$$

) at the muzzle

Therefore, since $T_w t = mk\left(\frac{d\emptyset}{dt}\right)_m$,

$$T\,\frac{3u}{2v} = m \times 0.49\ r^2\,\frac{\pi v}{gr}$$

and

$$T = 1.05\,\frac{mv^2}{ug}r \qquad (24)$$

which gives the average torque reaction on the gun due to the angular acceleration of the projectile.

Since the slope of the rifling grooves is small, we may roughly assume, that,

$$fr = T \quad \text{and} \quad f = 1.05 \frac{mv^2}{ug} \qquad (25)$$

which gives the mean pressure on the rifling band.

It is of interest to compare the maximum torque reaction to the average torque reaction in an actual gun.

Type of Gun: 240 m/m Howitzer
Muzzle Velocity: 1700 ft/sec.
Weight of Projectile: 356 lbs.
Max. Powder Pressure: 32000 lbs/sq.in.
Rifling = 1 turn in 20 cals.
Travel up bore = 13.33 ft.

Then, for the max. powder reaction, we have,

$$Pb_{max.} = 32000 \times 0.7854 \times \overline{9.45}^{2} = 2,242,000 \text{ lbs.}$$

and for the rifling slope,

$$\tan \theta = \frac{2\pi r}{20 \times 2r} = \frac{\pi}{20} = 0.157$$

hence, for the max. torque reaction,

$$T_{max} = \frac{9.45}{48} \times 0.157 \times 2,242,000 = 69,500 \text{ lbs. ft.}$$

where as for average torque, we have,

$$T_{av} = \frac{1.05 \times 356 \times \overline{1700}^{2} \times 9.45}{13.33 \times 32.2 \times 20 \times 24} = 49600 \text{ lb. ft.}$$

Therefore, the ratio of max. torque to the average torque becomes,

$$\frac{T_{max}}{T_{av}} = \frac{69500}{49600} = 1.4$$

Due to the short time action of the travel up the bore, the effect on the traversing gear depends upon the average torque rather than the maximum.

M U Z Z L E   B R A K E

GENERAL DESCRIPTION The muzzle brake consists of curved vanes secured to the end of the muzzle upon which a portion of the powder gases are deflected in the second part of the powder period after the projectile has left the muzzle. The gases are deflected somewhat to the rear, and we have a forward reaction due to the change of momentum of the gases, which materially checks the recoil. The design and best arrangement of vanes requires a considerable experimental investigation and it is merely proposed here to outline certain general limitations based on an elemantary theory.

ELEMANTARY THEORY If it were possible to calculate the mass of gas discharged through the vanes, as well as the mean extrance and exit velocities, the reaction on the vanes could be determined. But the method is complicated, since the powder pressure after the shot has left the muzzle falls off according to a complicated function of the time, and, considering the variable volume of gas, this makes it difficult to approximate the mass of the gas as a function of the time. Further the amount discharged through the vanes depends upon the initial mean muzzle velocity of the gases, the caliber of the bore and the entranee areas to the vanes, as well as the variation of muzzle velocity of the gases against time. We see therefore to approximate roughly the problem from a theoretical point of view would require an elaborate analysis combined with a long and eleborate experimental research.

Let

w = weight of projectile (lbs)

$\tilde{w}$ = weight of total charge (lbs)

v = muzzle velocity ft/sec.

V_o = velocity of recoiling parts when the projectile leaves the muzzle ft/sec.

t_o = time for shot to reach muzzle.

T = total time of powder period.

P_b = total pressure on breech due to powder gases (lbs)

W_r = weight of reeoiling parts

v_w = mean velocity of gases after free expansion (ft/sec.)

$\int_{t_o}^{T} R dt$ = impulsive reaction on vanes (lbs)

C_w = ratio of $\dfrac{\text{charge through vanes}}{\text{total charge}}$

Then, without vanes, we have,

$$\int_0^T P_b\, dt = \frac{W_r}{g} V_m$$

$$\text{but} \int_0^T P_b\, dt = \int_0^{t_o} P_b\, dt + \int_{t_o}^T P_b dt$$

Total powder reaction on the Gun during the Powder period.

$$\text{Now} \int_0^{t_o} P_b\, dt = \frac{(w + \frac{\bar{w}}{2})\, v}{g}$$

and since the powder charge has a mean velocity = $\frac{v}{2}$ when the projectile leaves the bore,

$$\int_{t_o}^T P_b\, dt = \frac{\bar{w}}{g}\left(v_w - \frac{v}{2} \right)$$

$$\text{hence} \int_0^T P_b\, dt = \frac{w}{g}\, v + \frac{\bar{w}}{g} v_w$$

$$\text{therefore, } W_r V_m = wv + \bar{w}\, v_w$$

as we should expect from the principle of the conservation of momentum.

With the muzzle brake acting, the total reaction

during the second period $(T - t_o)$, on the gun becomes,

$$\int_{t_o}^{T} P_b \, dt - \int_{t_o}^{T} R \, dt$$

and therefore the momentum imparted to the gun becomes

$$\int_{t_o}^{T} P_b \, dt - \int_{t_o}^{T} R \, dt = \frac{W_r}{g} (V_m - V_o)$$

If it were possible to deflect the total charge entirely backward and maintaining the same expansion, then, for the total reaction on the gun during the expansion period of the powder gases, we have,

$$\int_{t_o}^{T} R \, dt - \int_{t_o}^{T} P_b \, dt = \frac{\bar{w}}{g} (v_w + \frac{v}{2})$$

since the change in velocity $= v_w + \frac{v}{2}$ ft/sec.

Therefore, the momentum given to the gun during the powder period becomes,

$$\frac{(w + \frac{\bar{w}}{2})}{g} v - \frac{\bar{w}}{g} (v_w + \frac{v}{2}) = \frac{wv - \bar{w} v_w}{g}$$

hence

$$W_r V_r = w v - \bar{w} v_w$$

which gives the impulse imparted if we had muzzle brake, with the same expansion backward as forward through the vanes.

With the same expansion to the rear the maximum possible recoil energy that can be absorbed with a muzzle brake, becomes,

$$\text{Max. Energy absorbed} = \frac{1}{2g W_r} [(wv + \bar{w} v_w)^2 - (wv - \bar{w} v_w)^2]$$

$$A_{ab} = \frac{2 w \bar{w} v v_w}{g W_r}$$

and the maximum possible percentage of the total recoil energy absorbed by the ideal muzzle brake becomes,

$$\frac{A_{ab}}{A} = \frac{4\, w\, \bar{w}\, v\, v_m}{(wv + \bar{w}\, v_w)^2}$$

Since $w\, v_w$ is always less than wv, we see that even with an ideal muzzle brake and complete expansion, it is impossible to completely check the recoil energy, unless greater expansion is obtained to the rear than forward.

The total gas reaction on the gun due to the combined expansion and deflection of the gases, is represented by the impulsive reaction,

$\int_{t_o}^{T} R\, dt - \int_{t_o}^{T} P_b\, dt$ in a forward direction (i.e. towards the muzzle)

If we have complete expansion of the gases, before entrance into the muzzle vanes, then,

$$\int_{t_o}^{T} P_b\, dt = \frac{\bar{w}}{g}\left(v_w - \frac{v}{2}\right)$$

and if now the total gases are deflected entirely back, then

$$\int_{t_o}^{T} R\, dt = \frac{2\, \bar{w}\, v_w}{g}$$

and as before,

$$\int_{t_o}^{T} R\, dt - \int_{t_o}^{T} P_b\, dt = \frac{\bar{w}}{g}\left(v_w + \frac{v}{2}\right)$$

If, however, the gases are accelerated to a mean velocity v' before entrance into the vanes, then

$$\int_{t_o}^{T} P_b\, dt = \frac{\bar{w}}{g}\left(v' - \frac{v}{2}\right)$$

and further expansion takes place through the vanes to the maximum value v_w to the rear, we have

$$\int_{t_o}^{T} R\, dt = \frac{\bar{w}}{g}\left(v' + v_w\right)$$

since $v' + v_w$ is the change in velocity.

As before the total impulsive reaction on the gun, becomes,

$$\int_{t_o}^{T} R\,dt - \int_{t_o}^{T} P_b\,dt = \frac{\bar{w}}{g}\left(v_w + \frac{v}{2}\right)$$

Without the vanes, the reaction on the gun breech becomes,

$$\int_{t_o}^{T} P_b\,dt = \frac{\bar{w}}{g}\left(v_w - \frac{v}{2}\right)$$

and with the vanes the reaction on the breech is probably different and modified to,

$$\int_{t_o}^{T} P_b\,dt = \bar{w}\left(v' - \frac{v}{2}\right)$$

since some expansion probably takes place within the vanes, themselves.

Now as to the actual reaction obtained, the ideal brake differs from actual conditions, essentially in the following points:-

(1) Only a part of the total charge can be deflected through the vanes.

(2) The entrance velocity can only be a component of the actual muzzle velocity of the gases.

(3) Only a partial expansion of the gases can take place before entrance into the vanes.

(4) The exit velocity can not, for practical considerations, be entirely to the rear, 30° from the rear, being likely the maximum angle that the gases can be deflected.

(5) Only a very small expansion can take place through the vanes themselves; of the gases passing through the vanes the total expansion is small.

In consideration of (1) (unless the vanes are extended a considerable way out) the higher the muzzle

velocity the less the total charge passing through the vanes. It has been found experimentally that it is useless to add more than a given column of vanes, further addition of vanes having very little effect on the reaction. Further the first one or two vanes nearest the muzzle, are subjected to an intensity of pressure practically equal to that of the gases at the muzzle. Further development of the muzzle brake should be directed in obtaining greater expansion to the rear by a suitable combination of vanes, curvatures of same, etc.

LEDUC'S FORMULA The empirical formula established by Leduc is especially serviceable and sufficiently accurate for a predetermination of the reaction of the powder, during the powder period and its effect on the recoil.

Leduc's formula, assumes that the velocity curve of the projectile during its travel up the bore follows that of an equilateral hyperbola, with parameters a and b, that is,

If v = the velocity of the projectile at any point in the bore (ft/sec)

u = the corresponding travel up the bore (ft)

a and b being parameters of the hyperbola,

then

$$v = \frac{au}{b + u} \quad \text{(ft/sec)}$$

where a and b must be determined by the elemantary principles of Interior Ballistics.

<u>Determination of the parameters a and b:-</u>

When u is made infinite, that is $u = \alpha$

and $$v = \frac{a}{\frac{b}{\alpha} + 1} = a$$

then $a = v$

a is therefore determined by considering the expansion in a gun of an infinite length.

If n = the ratio of the heat capacities ($\frac{C_p}{C_v}$ =1.41) and for an adiabatic expansion $pV^n = k$,

Then the work of an expansion from, initial Volume V_1 to final Volume V_2, becomes,

$W = \int_{V_1}^{V_2} p\, d\, V$, but $p = \frac{k}{V^n}$ where k and n are constants,

$$= k \int_{V_1}^{V_2} \frac{dV}{V^n} = \frac{k}{n - 1} \left(\frac{1}{V_1^{\,n-1}} - \frac{1}{V_2^{\,n-1}}\right) \text{(ft/sec)}$$

Now when V_2 becomes infinite

$$W = \frac{k}{n - 1} \frac{1}{V_1^{\,n-1}} \quad \text{(ft.lbs.)}$$

Since, 1 lb. of water = 27.68 cu. in. for unit density,

$$W = E = \frac{k}{n - 1} \frac{1}{27.68^{n-1}}$$ Work for an Infinite Expansion at Unit Density.

If now

$$\Delta = \frac{\text{Weight of given volume of powder gas}}{\text{Weight of same volume of water}}$$

and if

V_c = the given volume of the chamber (cu. in.)

V_{1a} = the volume of 1 lb. of gas (cu. in.)

then

$$\Delta = \frac{\frac{V_c}{V_{1a}}}{\frac{V_c}{27.68}} = \frac{27.68}{V_{1a}} \quad \text{per lb. of powder gas.}$$

hence the specific volume of the gas, becomes,

$$V_{1a} = \frac{27.68}{\Delta}$$

Therefore, the work of expansion of 1 lbs. of the gas to α becomes,

$$W = \frac{k}{n - 1} \frac{\Delta^{n-1}}{27.68^{n-1}} = E \Delta^{n-1}$$

Since the gas evolved is proportional to the weight of the charge $\bar{w}$, and $a = v$ for an infinite expansion in the bore, we have

$$\frac{wa^2}{2g} = \bar{w} E \Delta^{n-1}$$, for a complete expansion of $\bar{w}$ lbs. of powder gas,

hence $a = \sqrt{2gE} \left(\frac{\bar{w}}{w}\right)^{\frac{1}{2}} \Delta^{\frac{n-1}{2}}$

Now E has a value = 653 ft. tons roughly, and by experiment $\frac{n-1}{2} = 1/12$ (approx.) Taking into account the various losses, it has been further found experimentally that $\sqrt{2gE} = 6823$ for ordinary good powder.

Therefore, the parameter "a" becomes,

$$a = 6823 \left(\frac{\bar{w}}{w}\right)^{\frac{1}{2}} \Delta^{\frac{1}{12}} \qquad (1)$$

Now $\Delta = \frac{27.68}{V_{1_a}}$

but with a powder chamber V_c, loaded with w lbs. of powder, the specific volume of 1 lbs. of powder evidently becomes,

$$V_{1_a} = \frac{V_c}{w}$$ assuming complete combustion of the charge,

hence

$$\Delta = \frac{27.68\ \bar{w}}{V_c} \qquad (2)$$

that is the density of loading may be defined as the ratio of the weight of the charge to the weight of a

volume of water sufficient to fill the powder chamber.

Hence the parameter a becomes,

$$a = 6823 \left(\frac{\bar{w}}{w}\right)^{\frac{1}{2}} \left(\frac{27.68\ \bar{w}}{V_c}\right)^{\frac{1}{12}}$$

To evaluate the parameter b, we must consider the acceleration of the projectile, and the reaction of the powder gases on its base, during its travel up the bore.

The acceleration up the bore, becomes,

$$v \frac{dv}{dn} = v \left[\frac{(b+u)a - av}{(b+u)^2}\right] = \frac{a^2 bu}{(b+u)^3} \quad (3)$$

since

$$v = \frac{av}{b+u}$$ from Leduc's formula, hence the pressure against the projectile, for a displacement u, becomes,

$$P = \frac{w}{g} \frac{a^2 bu}{(b+u)^2} \qquad \text{(lbs)}$$

Further the maximum pressure occurs, when $\frac{dp}{du} = 0$ i.e. when,

$$\frac{d\left(\frac{u}{(b+u)^3}\right)}{du} = 0$$

Now

$$\frac{d[u(b+u)^{-3}]}{du} = -3u(b+u)^{-4} + (b+u)^{-3}$$

$$= \frac{-30}{(b+u)^4} + \frac{1}{(b+u)^3}$$

hence

$$\frac{-3u + b + u}{(b+u)^4} = 0 \qquad \text{and } u = \frac{b}{2} \quad \text{(ft)}$$

that is the maximum pressure in the bore occurs at a displacement equal to one half the parameter b or the parameter b = twice the displacement of the maximum powder reaction in the bore. We have, therefore, substituting $v = \frac{b}{2}$ in (3)

that $P_{max} = \frac{4}{27}\frac{\bar{w}}{g}\frac{a^2}{b}$ (lbs) (4)

The mean powder reaction on the base of the projectile during its travel up the bore, becomes,

$$P_e = \frac{wv_o^2}{2g\ u_o} \qquad \text{(lbs)} \quad (5)$$

where

v_o = the muzzle velocity ft/sec.

u_o = the total travel up the bore (ft)

The pressure against the projectile when the shot is about to leave the muzzle, becomes,

$$P_o = \frac{w}{g}\ \frac{a^2bu_o}{(b+u_o)^3} \qquad \text{(lbs)} \ (6)$$

Hence to determine the parameter "b" we have the following equations:-

$$P_m = \frac{4}{27}\ \frac{w}{g}\ \frac{a^2}{b}$$

$$P_o = \frac{w}{g}\ \frac{a^2bu_o}{(b+u_o)^3}$$

where P_m v_o and u_o are known.

$$P_e = \frac{wv_o^2}{2gu_o}$$

$$v = \frac{au}{b+u}$$

and P_o P_e, a and b are unknown.

Hence a solution is possible:- If A_d = Area of bore and P_m = a given property of the powder used

P_m = 30,000 to 33,000 lbs/sq.in. usually.

Then $P_m = p_m A$ for the max. powder reaction.

Substituting

$$v_o = \frac{au_o}{b + u_o}, \text{ we have,}$$

$$P_e = \frac{w}{2g}\ \frac{a^2u_o}{(b+u_o)^2} \quad \text{(lbs) hence } a^2 = \frac{2gP_e(b+u_o)^2}{w\ u_o}$$

and

$$P_m = \frac{4}{27}\ \frac{w}{g}\ \frac{a^2}{b} \quad \text{(lbs) hence } a^2 = \frac{27\ g\ b\ P_m}{4\ w}$$

Equating, we have,

$$\frac{2\ P_e(b + u_o)^2}{w\ u_o} = \frac{27\ b\ P_m}{4w}$$

hence

$$b^2 + 2\ b\ u_o + u^2{}_o = \frac{27\ b\ u_o}{8}\ \frac{P_m}{P_e}$$

$$b^2 + (2 - \frac{27}{8}\ \frac{P_m}{P_e})\ v\ b + u^2{}_o = 0$$

Solving, we have,

$$b = \frac{-(2 - \frac{27}{8}\frac{P_m}{P_e})\ u_o \pm \sqrt{(2 - \frac{27}{8}\ \frac{P_m}{P_e})^2\ v^2{}_o - 4u^2{}_o}}{2}$$

$$= u_o\ [(\frac{27}{16}\ \frac{P_m}{P_e} - 1) \pm \sqrt{(\ 1 - \frac{27}{16}\ \frac{P_m}{P_e}\ )^2 - 1}\ ]$$

(ft.) (7)

which determines the parameter b, in terms of the travel up the bore, the given maximum powder reaction and the mean powder reaction, being determined from the muzzle velocity and travel up the bore.

To completely determine the velocity, powder force, and time against the travel up the bore, we have

$$v = \frac{au}{b + u} \quad \text{(ft/sec)}$$

$$P = \frac{w}{g} \frac{a^2bu}{(b+u)^3} \quad \text{(lbs)}$$

and the corresponding time of travel, becomes,

$$t = \int \frac{du}{v} = \int \frac{(b+u)}{(au)} du$$

$$= \frac{b}{a} \log_e u + \frac{1}{a} u + \text{Constant}$$

Now when $u = 0$, $t = 0$ and $\log_e u = -\infty$, and the constant cannot be evaluated without making some assumption. Since the initial powder reaction required to force the projectile into the rifling grooves is large and the displacement $u = \frac{b}{2}$, to Max. powder pressure is small, we can reasonably assume the powder reaction constant and equal to the maximum powder reaction during the initial travel $u = \frac{b}{2}$. Hence assuming the maximum powder reaction to be reached at the beginning of the travel of the shot up the bore, and to remain constant up to $u = \frac{b}{2}$, then

$$\frac{b}{2} = \frac{1}{2} \frac{P_{max}}{\frac{w}{g}} t_m^2$$ and substituting $P_{max.}$ from

(4), we have

$$b = \frac{4}{27} \frac{a^2}{b} t_m^2$$

hence $t_m = \sqrt{\frac{27}{4}} \left(\frac{b}{a}\right)$ (8)

Sunstituting in the previous time equation, we have,

$$\sqrt{\frac{27}{4}} \left(\frac{b}{a}\right) = \frac{b}{a} \log \frac{b}{2} + \frac{b}{2a} + \text{constant}$$

and

$$\text{Constant} = \frac{b}{2a}\left[(\sqrt{27} - 1) - 2 \log_e \frac{b}{2}\right]$$

$$= \frac{b}{2}\left(2.098 \log_e \frac{b}{a}\right)$$

therefore

$$t = \frac{b}{a}\left(\log_e \frac{2u}{b} + \frac{u}{b} + 2.098\right)$$

$$= \frac{b}{a}\left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right) \text{ (approx.)} \quad (9)$$

The powder reaction on the breech during the travel up the bore is somewhat greater than at the base of the projectrle due to the inertia resistance of the powder gases and charge. It has been shown previously that the breech pressure is augmented over that at the base of the projectile by either of the two following formulae:-

$$P_b = \frac{w + \frac{w}{2}}{w} \; p \text{ (lbs)}$$

or

$$P_b = 1.12 \, P \quad \text{(lbs)}$$

The former is based on a theoretical assumption, and gives an idea as to the change in the pressure drop from the breech to the projectile with different ratios of powder charge to weight of projectile. The latter is entirely empirical and it appears that the ratio of the weight of the charge to that of the projectile has no effect on changing the ratio of the breech pressure to that at the base of the projectile. Unfortunately the latter empirical value is somewhat limited especially for extreme ratio of the projectile weights but is, however, reasonably accurate for ordinary calculations. The former is more or less in error due to the assumptions made, but it gives the

characteristics for extreme ratios. Therefore, for extreme ratios of charge to projectile weights, the former formula should be used, while with ordinary ratios, the latter should be used.

Recapitulation of the Various Formulae

Originating from LEDUC'S Formula -

Let

v = Velocity of projectile up bore (ft/sec)
u = Travel up bore (ft)
v_o = Muzzle velocity (ft/sec)
u_o = Total travel up bore (ft)
t = Time of travel up bore (sec)
t_o = Time of total travel up bore (sec)
$\bar{w}$ = Weight of powder charge (lbs)
w = Weight of projectile (lbs)
V_c = Volume of powder chamber (cu.in.)
Δ = Density of loading
P = Powder reaction on base of projectile (lbs)
P_b = Powder reaction on base of breech (lbs)
P_m = Max. Powder reaction on projectile (lbs)
P_e = Mean Powder reaction on projectile (lbs)
A_d = Area of the bore (sq.in.)
p_m = Max. given powder pressure (lbs/sq.in.)
(from 30,000 to 33,000 lbs/sq.in.)

Given :- P_m v_o, V_c, w $\bar{w}$ and u_o

To eveluate: — v, P and t

$$\text{then, } \Delta = \frac{27.68\, w}{V_c} \quad (1)$$

$$a = 6823 \left(\frac{W}{w}\right)^{\frac{1}{2}} \Delta^{1/12} \quad (2)$$

$$P_m = p_m A_d \quad \text{(lbs)} \quad (3)$$

$$P_e = \frac{w v_o^2}{2 g u_o} \quad \text{(lbs)} \quad (4)$$

$$b = u_o\left[\left(\frac{27}{16}\frac{P_m}{P_e} - 1\right) \pm \sqrt{\left(1 - \frac{27}{16}\frac{P_m}{P_e}\right)^2 - 1}\right] \quad \text{(ft)} \quad (5)$$

$$v = \frac{au}{b + u} \quad \text{(ft/sec)} \quad (6)$$

$$P = \frac{w}{g} \frac{a^2 b u}{(b+u)^3} \quad \text{(lbs)} \quad (7)$$

$$P_b = 1.12 \frac{w}{g} \frac{a^2 b u}{(b+u)^3} \quad \text{(lbs)} \quad (8)$$

$$P_{ob} = 1.12 \frac{w}{g} \frac{a^2 b u_o}{(b+u_o)^3} \quad \text{(lbs)} \quad (9)$$

$$t = \frac{b}{a}\left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right) \quad \text{(sec)} \quad (10)$$

$$t_o = \frac{b}{a}\left(2.3 \log \frac{2u_o}{b} + \frac{u_o}{b} + 2\right) \quad \text{(sec)}$$

$$= \frac{3}{2} \frac{u_o}{v_o} \quad \text{(approx.) (sec)} \quad (11)$$

DYNAMICS OF RECOIL DURING THE TRAVEL OF THE SHOT UP BORE

The velocity and displacement of the recoiling mass with respect to the powder charge and projectile is obtained by the principle of linear momentum.

Assuming, one half the charge to move forward with the projectile and the other half to move backward with the recoiling parts, we have,

$$\left(w_r + \frac{\bar{w}}{2} \right) V_f = \left(w + \frac{\bar{w}}{2} \right) v$$

and $$\left(w_r = \frac{\bar{w}}{2} \right) X_f = \left(w + \frac{\bar{w}}{2} \right) x$$

Now the absolute displacement of the shot in the bore is related to the travel up the bore u, by the equation

$$x = u - X$$

Hence, we have,

$$V_f = \frac{w + \frac{\bar{w}}{2}}{w_r + \frac{\bar{w}}{2}} v \quad \text{(ft/sec)}$$

and

$$X_f = \frac{\left(w + \frac{\bar{w}}{2}\right) u}{w_r + w + \bar{w}} \quad \text{(ft)}$$

Since $\bar{w}$ and w are small as compared with w_r, we have for a sufficient approximation

$$V_f = \frac{w + \frac{\bar{w}}{2}}{w_r} v \quad \text{(ft.sec)}$$

$$X_f = \frac{\left(w + \frac{\bar{w}}{2}\right)}{w_r} u \quad \text{ft.}$$

The equation of velocity displacement and time of free recoil during the travel up the bore, becomes,

$$V_f = \frac{(w + \frac{\bar{w}}{2})}{(w_r)} \left(\frac{au}{b + u}\right) \quad \text{(ft/sec)}$$

$$X_f = \left(\frac{w + \frac{\bar{w}}{2}}{w_r}\right) u \quad \text{(ft)}$$

$$t = \frac{b}{a} \left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right) \quad \text{(sec)}$$

With constrained recoil, assuming a recoil reaction X we have,

$$P_b - K = m_r \frac{dV}{dt} \quad \text{(lbs)}$$

hence $\int_0^t \frac{P_b dt}{m_r} - \frac{Kt}{m_r} = V$ but $\int_0^t \frac{P_b\, dt}{m_r} = V_f$

therefore,

$$V_f - \frac{Kt}{m_r} = V \text{ (ft/sec)}$$

$$X_f - \frac{Kt^2}{2m_r} = X \text{ (ft)}$$

$$t = \frac{b}{a} \left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right. \quad \text{(sec)}$$

In the several equations, it will be noted, that the common parameter is the time of travel up the bore in the gun. Hence if for various values of u, we obtain correspondingly values of time, the free velocity and displacement is obtained for the given time and the corresponding effect of the recoil brake during this time is deducted from the velocity and displacement respectively. Further it has been tacidly assumed that the powder reaction with constrained recoil is the same as with free recoil at the same time interval. This, however, is not strictly true since the

powder reaction is somewhat modified due to the slightly different motion of the gun with constrained and free recoil respectively. The effect, however, is entirely negligible as compared with the magnitude of the reaction and other factors involved, even with the most refined measurements and analysis.

EXPANSION OF THE GASES AFTER THE SHOT HAS LEFT THE BORE AND ITS EFFECT ON THE RECOIL

The manner of the expansion of the powder gases after the projectile has left the bore is very difficult to calculate, and various assumptions based on empirical data have been formulated, for calculations during this period.

The following theory though imperfect gives an idea as to the manner of the expansion of the powder gases in the "After effect Period".

(1) The momentum imparted to the gun during this period evidently equals the momentum imparted to the powder gases:

$$m_r(V_f - V_{fo}) = \bar{m}(v_w - \frac{v}{2})$$

where V_f = maximum free velocity of the recoiling parts. (ft/sec)

V_{fo} = free velocity of recoil when the shot leaves the bore (ft/sec)

v_w = mean velocity of the powder gases attained (ft/sec)

$\bar{m}$ = mass of powder charge (lbs)

v = muzzle velocity of projectile (ft/sec)

Since

$$m_r V_{fo} = (m + \frac{\bar{m}}{2})v \quad \text{we have} \quad m_r V_f = mv + \bar{m}\, v_w$$

In other words, the maximum free momentum obtained by

the gun, equals the sum of the total momentum of the projectile and the total momentum of the powder charge.

It is important to note that the momentum relations are very nearly true provided we are able to calculate v_w the mean velocity of the powder gases and can neglect the small effect of the air pressures exerted on the gases.

(2) We have the following energy relations due to the expansion of the gases:

(a) Initially the gases have a Kinetic Energy = $\frac{1}{2}\left(\frac{\bar{m}}{3}\right)v^2$

(b) The work of expansion of the gases in expanding from the pressure in the bore when the shot leaves the gun (i.e. the muzzle pressure) to the atmospheric pressure, becomes

$$W_e = \int_{V_o}^{V_a} p\,dV$$

where V_o = volume of powder chamber + volume of the bore of the gun.
V_a = volume of gases at atmospheric pressure.

(c) The final Kinetic Energy of the gases may be approximately assumed equal to: $\frac{1}{2}\bar{m}v_w^2$.

It is to be noted that the final Kinetic Energy of the gases is difficult to calculate due to the divergence or cone effect produced when the gases expand into the atmosphere The total Kinetic Energy equals

the sum of the Kinetic Energy of the center of gravity of the gases plus the relative Kinetic Energy of the gases relative to the center of gravity.

From a series of experimental tests conducted by the Navy on the velocity of free recoil with guns of various caliber it has been ascertained that the momentum effect of the powder gases is equivalent to the weight of the charge times a constant velocity of 4700 ft/sec.

Assuming the divergence of the spreading of the gases to be similar at all muzzle velocities, it is possible to estimate the divergence factor and then in guns of very high muzzle velocities we may calculate the maximum free velocity by multiplying the work of expansion by the divergence constant and the solving for the mean velocity of the gases.

The pressure of the gases rapidly falls to the atmospheric value or approximately this value, before the divergence of spread of the gases is appreciable, hence the maximum Kinetic Energy of the gases will be attained at approximately atmospheric pressure.

The change in Kinetic Energy of the powder cases therefore, becomes,

$$\frac{1}{2}\,\overline{m}\,v_w^2 - \frac{1}{2}\,\frac{\overline{m}}{3}\,v^2 = \text{change in Kinetic Energy, and}$$

the work done on the gases, equals the work done by the external pressures p_o and p_a and the work of expansion pdV. Hence,

$$p_oV_o - p_aV_a + \int_{V_o}^{V_a} pdV = \text{total work done.}$$

To allow for the relative Kinetic Energy due to the spreading of the gases, we may multiply the work done on the gases by a constant, and then equate this value to the changes of the translatory Kinetic Energy of the guns.

$$K(p_o V_p - p_a V_a + \int_{V_o}^{V_a} pdV) = \tfrac{1}{2} \bar{m} v_w^2 - \tfrac{1}{2} \frac{\bar{m}}{3} v^2$$

where K = the divergence constant to allow for the spreading of the gases at the muzzle. Now the work of expansion, becomes,

$$W_e = \int_{V_o}^{V_a} p\, d\, V = \frac{p_o V_o - p_a V_a}{k - 1}$$

where the expansion exponent k = 1.3 approx. Hence the total work done on the gases, becomes,

$$p_o V_o - p_a V_a + \frac{p_o V_o - p_a V_a}{k - 1} = \frac{k}{k - 1} (p_o V_o - p_a V_a)$$

Further, since $p_o V_o^k = p_a V_a^k$, we have,

$$\frac{k}{k - 1} (p_o V_o - p_a V_a) = \frac{k}{k - 1} p_o V_o [1 - (\frac{p_a}{p_o})^{\frac{k - 1}{k}}]$$

Hence the energy expression reduces to the convenient form,

$$K[\frac{k}{k - 1} p_o V_o (1 - (\frac{p_a}{p_o})^{\frac{k - 1}{k}})] = \tfrac{1}{2} \frac{\bar{m}}{3} v^2$$

from which knowing p_o, V_o, p_a $\bar{m}$ and v enables us to immediately calculate v_w, the mean free velocity of the powder gases.

To evaluate the dispersion constant, to take care of the relative Kinetic Energy of the gases after expansion, the ballistic data of the 155 m/m Filloux gun has been chosen, since assuming a mean velocity of the gases 4700 ft/sec., calculated and experimental results were found to check very closely.

Weight of powder charge $\bar{w}$ = 26 (lbs)
Volume of powder chamber S = 1334 (cu.in.)
Total length of bore u = 186 (in.)
Muzzle velocity v = 2410 (ft/sec)
Area of bore A_b = 29.2 (sq.in.)
Weight of projectile = 96.1 (lbs)
Max. powder pressure p_m = 35300 (lbs/sq.in.)
Mean Powder pressure =

$$\frac{wv^2}{644\ uA_b} = p_e \quad \text{-------} \quad 19200\ \text{(lbs/sq.in.)}$$

Twice Abscissa of maximum pressure

$$e = u[(\frac{27}{16}\frac{p_m}{p_e} - 1) \pm \sqrt{(1 - \frac{27}{16}\frac{p_m}{p_e})^2 - 1}] = 57.38$$

Muzzle pressure when shot leaves muzzle

$$p_o = \frac{27}{4} e^2 \frac{u}{(e+u)^3} p_m = \frac{27}{4} \quad \frac{2}{57.38} \times \frac{185.68 \times 35300}{(57.38 + 185.68)^3} =$$

10140 lbs/sq.in.

we have then,

$$K\ 32.16\ [\frac{1.3}{0.3} \times 10140 \times 144\ V_o[1 - (\frac{14.7}{10140})^{\frac{0.3}{1.3}}]$$

$$= \frac{1}{2} \times 26 \times \frac{2}{4700} - \frac{1}{2} \times \frac{26}{3} \times \frac{2}{2410}$$

Solving, we have,

$$156 \times 10^6\ K\ V_o = (287 - 25)\ 10^6 = 262 \times 10^6$$

Now $V_o = \dfrac{1334 + 186 \times 29.2}{1728} = 3.915$ cu. ft.

Hence $K = \dfrac{262}{611} = 0.430$

Hence the energy of translation is but 43% of the total Kinetic Energy of the gases after complete expansion. Therefore with guns of numeral ballistic relations, we may estimate the mean translatory velocity of the gases after complete expansion, by the formula:

$$v_w = \sqrt{\frac{v^2}{3} + \frac{0.86}{\bar{w}}\left[\frac{k}{k-1} p_o V_o\left(1 - \left(\frac{p_a}{p_o}\right)^{\frac{k-1}{k}}\right)\right]}$$

where v = muzzle velocity (ft.see)

$\bar{w}$ = weight of powder charge (lbs)

b = 1.3 approx.

p_a = atmospheric pressure = 2116 (lbs/sq.ft.)

p_o = muzzle pressure of powder gases (lbs/sq.ft.)

VALLIERS HYPOTHESIS

The hypothesis of Vallier assumes, that during the "after effect Period" in the powder period of the recoil, that the powder reaction on the gun falls off proportional to the time. That is,

If P_{ob} = the total breech reaction of the powder gases, when the projectile leaves the muzzle (lbs)

t_o = time of travel of the projectile to the muzzle (sec)

t_1 = total powder period (sec)

P_b = powder reaction on breech (lbs)

t = corresponding time (sec)

then

$P_b = P_{ob} - C(t - t_o)$ <u>VALLIERS HYPOTHESIS</u>

where

$$C = \frac{P_{ob} - P_b}{t - t_o} = \frac{P_{ob}}{t_1 - t_o}$$

Now the momentum imparted to the recoiling parts by the gases during the after effect period, becomes,

$$\therefore \int_{t_o}^{t_1} P_b \, dt = m_r(V_{f'} - V_{fo})$$

where $V_{f'}$ = Max. free velocity of recoil at end of powder period.

V_{fo} = Free velocity of recoil when the shot leaves the muzzle.

$$\int_{t_o}^{t_1} [P_{ob} \frac{P_{ob}}{t_1 - t_o} (t - t_o)] dt = m_r(V_{f'} - V_{fo})$$

Integrating, we have,

$$\frac{P_{ob}(t_1 - t_o)}{2} = m_r(V_{f'} - V_o)$$

hence $t_1 - t_o = \dfrac{2m_r(V_{f'} - V_{fo})}{P_{ob}}$ (sec)

and

$$C = \frac{P_{ob}^2}{2(V_{f'} - V_{fo})m_r}$$

Therefore the powder reaction during the after effect period, becomes,

$$P_b = P_{ob} - \frac{P_{ob}^2 (t - t_o)}{2(V_{f'} - V_{fo})m_r} \quad \text{(lbs)}$$

RECAPITULATION OF PRINCIPLE FORMULAS OF INTERIOR BALLISTICS PERTAINING TO RECOIL DESIGN.

The velocity and displacement of the recoiling parts during the travel of the projectile up the bore have the following relations with the velocity of the

projectile up the bore and the relative displacement of the projectile in the bore.

If m = mass of projectile $\left(\dfrac{\text{Weight in lbs.}}{32.16}\right)$

$\bar{m}$ = mass of powder charge "

m_r = mass of recoiling parts "

v = velocity of projectile (ft/sec)

u = displacement of projectile in the bore from its breech position

V = velocity of recoiling parts (ft/sec)

X = free displacement of recoiling parts (ft)

then

$$V = \frac{(m + \frac{\bar{m}}{2})v}{m_r + \frac{\bar{m}}{2}} = \frac{m + \frac{\bar{m}}{2}}{m_r} v \quad \text{Approx. (ft/sec)}$$

$$X = \frac{(m + \frac{\bar{m}}{2})u}{m_r + m + \bar{m}} = \frac{m + \frac{\bar{m}}{2}}{m_r} u \quad \text{Approx. (ft)}$$

The pressure on the breech, in terms of the pressure on the base of the projectile, becomes

If

P_b = breech pressure (total) (lbs)

P = pressure at base of projectile (total) (lbs)

$$P_b = \frac{m + \frac{\bar{m}}{2}}{m} P = 1.12\ P \text{ approx.} \quad \text{(lbs)}$$

The mean pressure in the bore, becomes,

$$P_m = P_b \left(\frac{m + \frac{\bar{m}}{4}}{m + \frac{\bar{m}}{2}}\right) \quad \text{(lbs)}$$

For building up the energy equation, we are concerned with the various equivalent masses of the moving elements that the powder reacts on in terms of the major mass of the projectile.

The equivalent mass of the projectile, becomes,

if k = radius of gyration about its longitudinal axis (ft)

p = pitch of the rifling "

θ = pitch angle of the rifling

$$m = (1 + \frac{4\pi^2 k^2}{p^2})m = (1 + \frac{4k^2 \tan^2\theta}{d})\ m \quad (\frac{lbs}{g})$$

If we include the effect of the friction of the rifling we have,

$$m'' = [1 + (\frac{\sin\theta + u\cos\theta}{\cos\theta - u\sin\theta})\ \frac{4\tan\theta\ k^2}{d^2}]\ m \quad (\frac{lbs}{g})$$

The equivalent mass of the powder charge,

for the energy equation $= \frac{\bar{m}}{3} \quad (\frac{lbs}{g})$

for the momentum equation $= \frac{\bar{m}}{2}$ "

The equivalent mass of recoiling parts become,

$$\frac{(m + \frac{\bar{m}}{2})^2}{m_r} \quad (\frac{lbs}{g})$$

The differential equation for the motion of the projectile up the bore becomes in terms of the mean pressure in the bore Pm and the relative displacement u,

$$Pm = [m'' + \frac{(m + \frac{\bar{m}}{2})^2}{m_r} + \frac{\bar{m}}{3}]\ v\frac{dv}{du}$$

If W = the potential energy of the gases at any instant, we have further,

$$- dW = p_m\, du + JdQ_r$$

where Q_r = heat lost by radiation. The energy equation for the expansion of the gases, becomes,

$$\bar{W} = \frac{wRT_1}{n-1}\left[1 - \left(\frac{U_o}{U_o+u}\right)^{n-1}\right] \quad \text{(ft/lbs)}$$

where $\bar{W} = (p_m\, du + J)\, dQ_r$ (ft/sec)(Adiabaticexpansion)

w = weight of gases (lbs)
c_v= specific heat for constant volume (B.T. U. per lbs. per degree)
c_p= specific heat for constant pressure (B. T. U. per lb. per degree)

$$n = \frac{C_p}{c_v}$$

T_1 = Initial temperature (degrees)

V_1 = Initial volume (i.e. volume of powder chamber)

$$U_o = \frac{V_1}{0.7854d^2}$$ where $0.7854d^2$ = area of bore.

u = displacement of projectile up the bore.
J = mechanical equivalent of heat = (778 ft. lbs. per B. T. U.)
$R = (C_p - C_v) J$ = the gas constant (ft. per degree)

The torque reaction of the projectile in traveling up the bore becomes,

$$T = 1.05 \frac{mv^2}{ug} r \quad \text{(lbs.ft)}$$

where r = radius of the bore (ft)

v = muzzle velocity (ft/sec)
g = number of calibers per revolution

LEDUC'S FORMULA Leduc's formula gives results sufficiently accurate for recoil design. The formulas derived from it are compact and sufficiently short to be readily used in ordinary practical design. These formulas have been used in the development of the various recoil formulas in the subsequent chapters. For recoil or gun design: let

v = velocity of projectile up bore (ft/sec)
u = travel up bore (ft)
v_o= muzzle velocity (ft/sec)
u_o= total travel up bore (ft)
t = time of travel up bore (sec)
t_o= time of total travel up bore "
$\bar{w}$ = weight of powder charge (lbs)
w = " of projectile "
V_c =Volume of powder charge (cu. in.)
Δ = Density of loading
P = powder reaction on base of projectile (lbs)
P_b= " reaction on base of breech "
P_{ob}=Pressure on the projectile when the shot leaves the muzzle (lbs)
P_m =Maximum powder reaction on projectile (lbs)
P_e= Mean powder reaction on projectile "
A_d= area of bore (sq.in.)
P_m= Maximum given powder pressure from 25000 to 33000 lbs/sq.in. (lbs/sq.in.)

Given,

P_m, v_o, V_c, w, $\bar{w}$ and u_o

To evaluate:- v, P and t, then,

(1) $$\Delta = \frac{27.68\ \bar{w}}{V_o}$$

(2) $$a = 6823\ (\frac{\bar{w}}{w})^{\frac{1}{2}}\ \Delta^{1/12}$$

(3) $$P_m = p_m A d \quad \text{(lbs)}$$

(4) $$P_e = \frac{w v_o^2}{2gU_o} \quad \text{"} \quad g = 32.16\ \text{ft/sec-2}$$

(5) $$b = U_o[(\frac{27}{16}\frac{P_m}{P_e} - 1) \pm \sqrt{(1 - \frac{27}{16}\frac{P_m}{P_e})^2 - 1}\,]\ \text{ft.}$$

(6) $$v = \frac{au}{b+u} \quad \text{(ft/sec)}$$

(7) $$P = \frac{w}{g}\frac{a^2bu}{(b+u)^3} \quad \text{(lbs)}$$

(8) $$p_b = 1.12\ \frac{w}{g}\frac{a^2bu}{(b+u)^3} \quad \text{(lbs)}$$

(9) $$P_{oo} = 1.12\ \frac{w}{g}\frac{a^2bu_o}{(b+u_o)^3} \quad \text{(lbs)}$$

(10) $$t = \frac{b}{a}\ (2.3\ \log\frac{2u}{b} + \frac{u}{b} + 2) \quad \text{(sec)}$$

$$t_o = \frac{b}{a}\ (2.3\ \log\frac{2U_o}{b} + \frac{U_o}{b} + 2) \quad \text{(sec)}$$

(11) $$t_o = \frac{3}{2}\frac{U_o}{v_o} \quad \text{approx.}$$

The equations of velocity, displacement and time of free recoil during the travel up the bore, becomes,

$$V_f = \left(\frac{w + \frac{\bar{w}}{2}}{w_r}\right)\left(\frac{au}{b + u}\right) \qquad \text{(ft/sec)}$$

$$X_f = \left(\frac{w + \frac{\bar{w}}{2}}{w_r}\right) u \qquad \text{(ft)}$$

$$t = \frac{b}{a}\left(2.3 \log \frac{20}{b} + \frac{u}{b} + 2\right) \qquad \text{(sec)}$$

With constrained recoil, assuming a recoil reaction $\bar{K}$

$$V = V_f - \frac{\bar{K}t}{m_r} \qquad \text{(ft/sec)}$$

$$X = X_f - \frac{\bar{K}t^2}{2m_r} \qquad \text{(ft)}$$

$$t = \frac{b}{a}\left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right) \qquad \text{(sec)}$$

Theexpansion of the gases after the projectile leaves the bore causes an additional recoil effect. The hypothesis of Tallier assumes the powder reaction to fall off proportionally with the time. On this assumption:

If

V_{f_1} = the velocity of free recoil at the end of the powder period.

V_{f_0} = the velocity of free recoil when the shot leaves the muzzle.

t_1 and t_0 the corresponding terms $\frac{P}{n_b}$
then,

$$t_1 - t_0 = \frac{2m_r(V_{f_1} - V_{f_0})}{P_{ob}} \qquad \text{(sec)}$$

$$P_0 = P_{ob} - \frac{P^2_{ob}\, t - t_0}{2(V_{f_1} - V_{f_0})m_r} \qquad \text{(lbs)}$$

THE PARABOLIC TRAJECTORY The nucleus of exterior ballistics is the differential equations of the parabolic path of a shot projected in a vacuum. These equations then may be modified for air resistance and gyroscopic deflections due to the angular momentum of the projectile and air reaction:

Let x and y be the horizontal and vertical coordinates of the trajectory.

m = the mass of the projectile.
V_o = the muzzle velocity.
t = the time of flight.
$Ø'$ = the angle of elevation from the horizontal of the axis of the bore.
$Ø$ = angle of elevation of the departure of the projectile from the muzzle.
e = the increment angle or "jump" to the elastic deformation of the carriage and the movement of the gun in a direction not along the axis of the bore. $Ø' - Ø$
r = angle of sight.
O_a = line of sight.
L = range to given target.
L_n = horizontal range corresponding.
m = striking angle from horizontal.
m' = angle of fall from line of sight.

The differential equations of motion give:

$$m \frac{d^2x}{dt^2} = 0 \qquad m = \frac{d^2y}{dt^2} = -mg$$

Integrating successively, we have,

$$\frac{dx}{dt} = V_o \cos Ø \qquad \frac{dy}{dt} = -gt + V_o \sin Ø$$

and

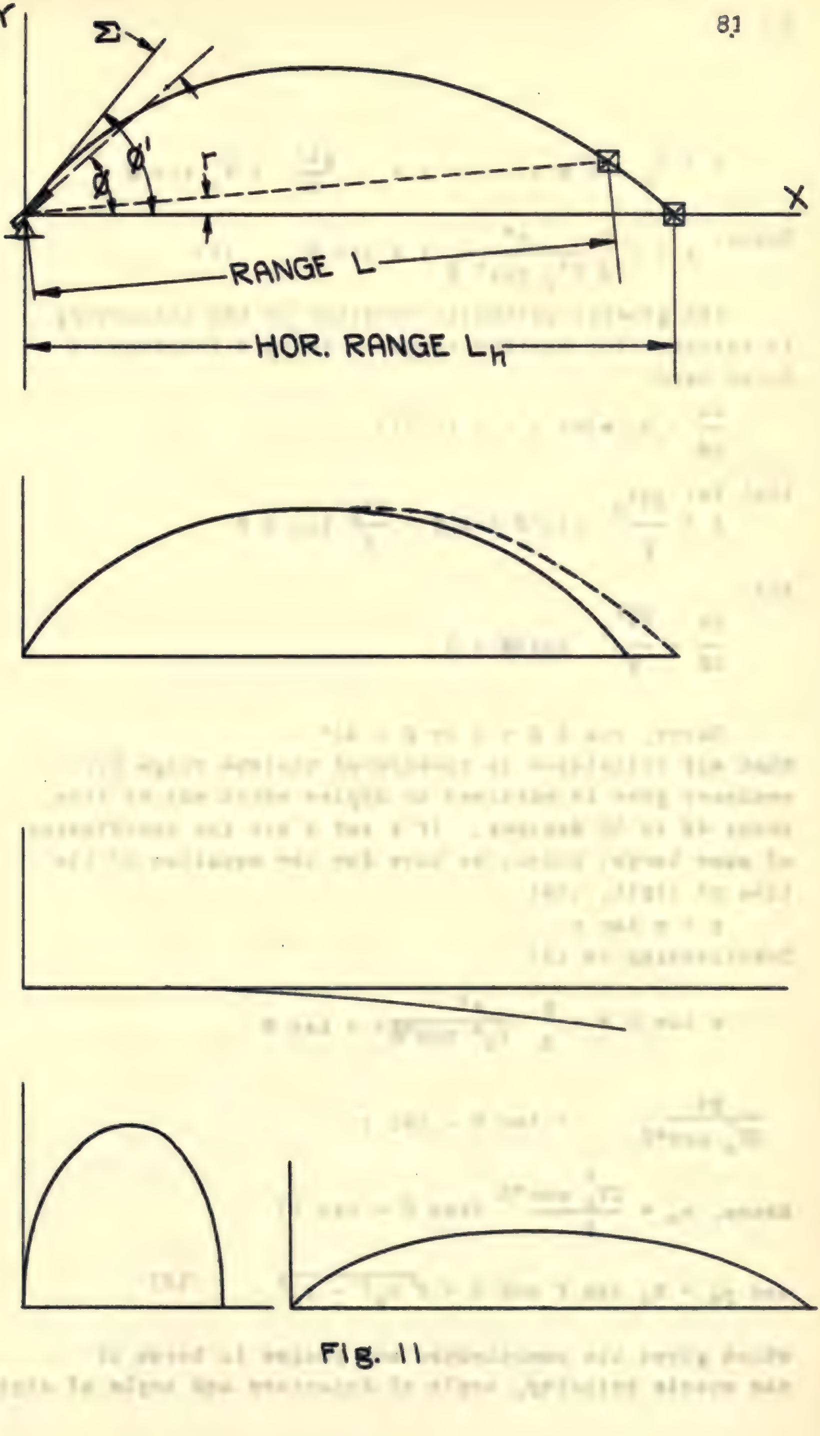

Fig. 11

$$x = V_o \cos \emptyset\, t \qquad y = -\frac{gt^2}{2} + V_o \sin \emptyset\, t$$

Hence
$$y = -\frac{g}{2}\,\frac{x^2}{V^2{}_o \cos^2 \emptyset} + x \tan \emptyset \qquad (1)$$

The general parabolic equation of the trajectory in vacuum. For maximum range, x being a function of $\emptyset$, we have

$$\frac{dx}{d\emptyset} = 0, \text{ when } y = 0 \text{ in } (1)$$

that is,
$$x = \frac{2V^2{}_o}{g} \sin \emptyset \cos \emptyset = \frac{V^2{}_o}{g} \sin 2\emptyset$$

and
$$\frac{dx}{d\emptyset} = \frac{2V^2{}_o}{g} \cos 2\emptyset = 0$$

Hence, $\cos 2\emptyset = 0$ or $\emptyset = 45°$
When air resistance is considered maximum range for ordinary guns is obtained at angles which may be from about 42 to 55 degrees. If x and y are the coordinates of some target point, we have for the equation of the line of sight, that

$y = x \tan r$

Substituting in (1)

$$x \tan r = -\frac{g}{2}\,\frac{x^2}{V_o{}^2 \cos^2\emptyset} + x \tan \emptyset$$

$$\frac{gx}{2V_p^2 \cos^2\emptyset} = \tan \emptyset - \tan r$$

Hence, $x_a = \dfrac{2V_o^2 \cos^2\emptyset}{g} (\tan \emptyset - \tan r)$

and $y_a = x_a \tan r$ and $L = \sqrt{x_a{}^2 - y_a{}^2}$ (2)

which gives the coordinates and ranges in terms of the muzzle velocity, angle of departure and angle of sight.

CHAPTER III

EXTERNAL REACTIONS ON A CARRIAGE DURING RECOIL AND COUNTER RECOIL - STABILITY - JUMP.

EXTERNAL REACTION The external reaction during recoil may be divided into two primary periods; that during which the force of powder pressure on the recoiling parts exceeds the restraining force or accelerating period and the retardation period. Again the period of powder pressure may be divided into the period of the shot traveling up the bore to the muzzle and the after effect period of the powder gases expanding to atmospheric pressure.

Considering the gun, recoiling masses and carriage as one sustem, the external forces are:

(1) The pressure of the powder gases along the axis of the bore = P

(2) The torque reaction due to rifling = T

(3) The weight of the recoiling parts = W_r

(4) The weight of the stationary parts= W_a

(5) The balancing reactions exerted by the ground or platform on the carriage mount.

If we sum these forces up into X and Y components and let M_r equal the mass of the recoiling parts, we have, noting the mass x acceleration of the stationary parts of the system is nil, (that)

$$X = M_r \frac{d^2x_r}{dt^2} \qquad (1)$$

$$Y = M_r \frac{d^2y_r}{dt^2} \qquad (2)$$

If further, we assume our coordinates along and normal to the axis of recoil, we have

$$X = M_r \frac{d^2x_r}{dt^2} \qquad (1')$$

$$Y = 0 \qquad (2')$$

Equation (1) may be written:

$$\Sigma X - M_r \frac{d^2x_r}{dt^2} = 0$$

Hence, by the use of D'Alemberts' principle regarding the inertia effect, that is, mass x acceleration reversed, as an equilibrating force, we reduce the forces to a system of forces in equilibrium.

Thus by including the inertia effect of the recoiling parts as an additional external force, the problem is reduced to one of statics.

This greatly simplifies the procedure of accurately and quickly obtaining certain overall effects in stability and the principal reactions throughout a carriage.

EXTERNAL EFFECTS DURING RECOIL — Considering now the external reactions upon the total system, (gun, recoiling parts, and carriage proper) including the inertia of the recoiling masses, we have the given forces as shown in figure (1), where

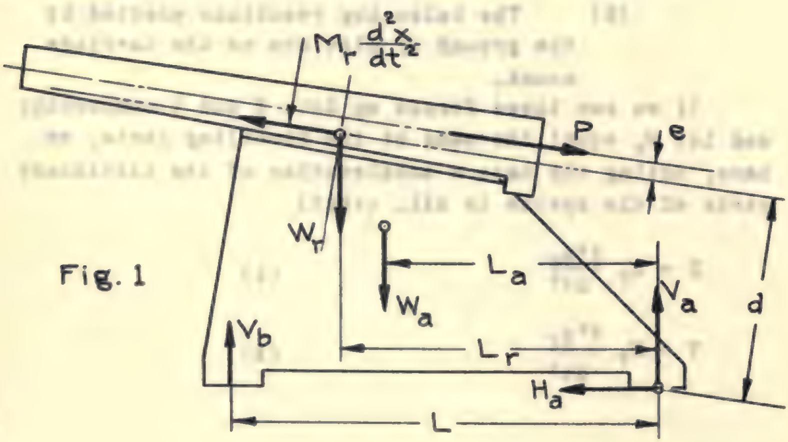

Fig. 1

P = Total Powder Pressure along axis of bore at any instant of Powder Pressure Period.

W_r = Weight of recoiling masses "M_r".

W_a = Weight of carriage proper (includes stationary part of tipping parts)

$M_r \frac{d^2x}{dt^2}$ = Inertia force of recoiling masses

H_a and V_a = Horizontal and Vertical components or equivalent float reaction

V_b = Front Pintle reaction - Horizontal component assumed zero in order that the reaction may be determinate.

B = Braking force, resultant hydraulic and recuperator reaction including stuffing box frictions.

R = Guide frictions = $R_1 + R_2$ in diagram.

K_a = Total resistance to recoil for recoiling masses equals $B + R - W_r \sin \emptyset$ at any instant during powder pressure period.

K_r = Total resistance to recoil during any instant after $P = 0$.

During the powder pressure period, we have for moments about A, see figure (1)

$$P(d + e) - \left(M_r \frac{d^2x}{dt^2}\right)d - W_r L_r - W_a L_a + V_b L = 0$$

hence

$$\left[P - \left(M_r \frac{d^2x}{dt^2}\right)\right]d + Pe - W_r L_r - W_a L_a + V_b L = 0 \tag{3}$$

In like manner we have after the powder pressure ceases

$$\left(M_r \frac{d^2x}{dt^2}\right) d - W_r L_r - W_a L_a + V_b L = 0 \tag{4}$$

Now considering the external reactions on the recoiling parts alone, during the powder pressure period, we have figure (2)

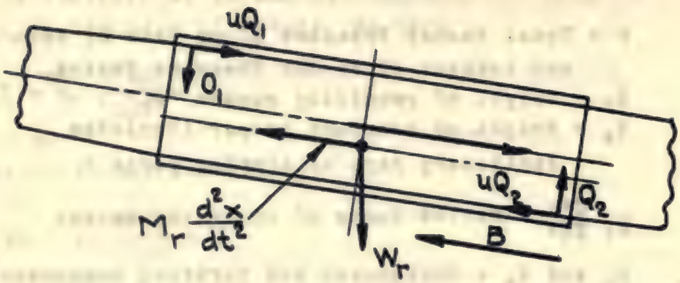

Fig. 2

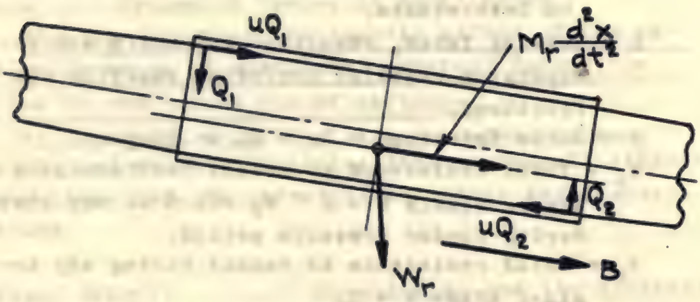

Fig. 3.

$$P - B - R + W_r \sin \emptyset = M_r \frac{d^2x}{dt^2}$$

hence

$$P - M_r \frac{d^2x}{dt^2} = B + R - W_r \sin \emptyset = K_a \qquad (5)$$

and

when $P = 0$ figure (3)

$$B + R - W_r \sin \emptyset = M_r \frac{d^2x}{dt^2} = K_r \qquad (5^1)$$

Substituting (5) and (5^1) in (3) and (4) respectively, we have

$$K_a d + P_e - W_r L_r - W_a L_a + V_b L = 0 \quad (6)$$

$$K_r d - W_r L_r - W_a L_a + V_b L = 0 \quad (7)$$

Thus the external effect during the powder pressure period is always at every instant equal to the total resistance to recoil, that is, the sum of the total braking and guide friction, minus the weight component and a powder pressure couple Pe dependent upon the actual total powder force.

In general, e is very small and usually for a first approximation the powder pressure couple can be neglected.

Further, for constant resistance to recoil

$$K_r = K_a = K = B + R - W_r \sin \emptyset \quad (8)$$

which is the average external effect during recoil on the total system.

As shown in Chapter VI on the "Dynamics of Recoil"

$$K = \frac{\frac{1}{2} M_r V^2{}_f}{b - E + V_f T} \quad (9)$$

where V_f = the maximum free velocity of the recoiling parts, that is

$$V_f = \frac{\bar{W}4700 + Wv_o}{W_r} \quad (10)$$

$\bar{W}$ = Weight of powder charge, W = Weight of shot and W_r = Weight of recoiling parts

v_o = Muzzle velocity of shot

u = Travel up the bore in inches

b = Length of recoil in feet

d = Diameter of bore in inches

E = Unconstrained displacement of recoiling parts during powder pressure period.

T = Total time of powder pressure period.

In equation (9) note that $E = K_1 V_f T$ and
$T = K_2 \frac{u}{v_o}$

Substituting these values in (9) and solving for a wide range of artillery material and thus evaluating the variables as a function of the diameter of bore, muzzle velocity and travel up bore, Mr. C. Bethel has given the very valuable and serviceable formulae, and accurate to one percent.

$$K = \frac{M_r V_f^2}{2} \quad \frac{1}{b + (.096+.0003\ d)\frac{uV_f}{v_o}} \qquad (11)$$

This formula holds only for constant resistance to recoil.

It is important to note that the "total braking" sometimes called "the total pull" is not in general equal to the resistance to recoil, but is the total resistance to recoil plus the weight component, that is

$$B+R=\Sigma P_a+\Sigma P_h+\Sigma R_s+\Sigma R_g = K+W_r \sin \emptyset \qquad (12)$$

where ΣP_a = Total recuperator reaction
ΣP_h = " hydraulic reaction
ΣR_s = " stuffing box friction
ΣR_g = Guide friction

$$K = \frac{1}{2} M_r V_f^2 \quad \frac{1}{(b - E + V_f T)}$$

To obtain the external reactions on the carriage mount, it is convenient to know d in the previous moment formulae about A, in terms of the height of the trunnions and the distance between the trunnions and a line through the center of gravity of the recoiling parts and parallel to the axis of the bore.

Let H_t = height of trunnions above the ground

s = distance from trunnion axis to line through center of gravity of recoiling parts and parallel to bore.

d = moment arm of K about A hor.

l = horizontal distance between reactions A and B.

c = " " from A to center line of trunnions.

As the gun elevates, we have two cases:

(1) When the line of action K passes above A, see figure (4)

(2) When the line of action K passes below A, see fig. (5)

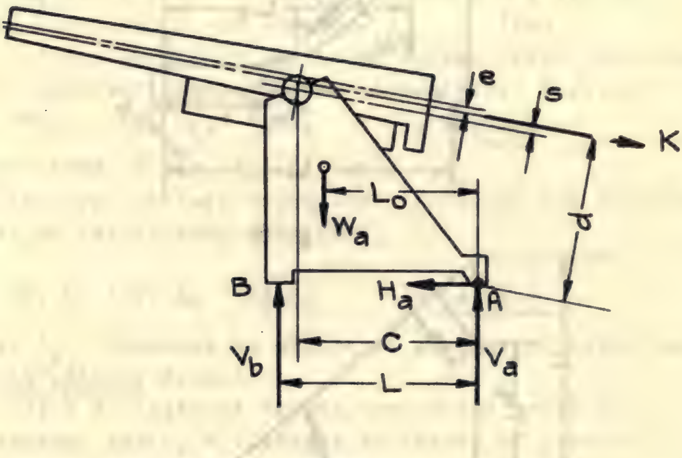

Fig. 4

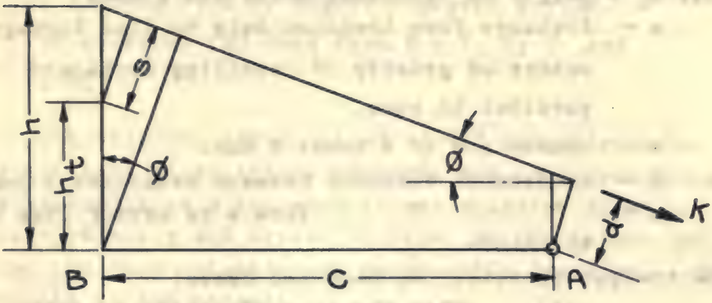

Fig 4'

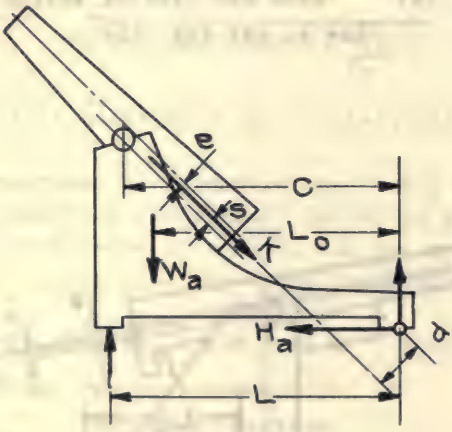

Fig. 5

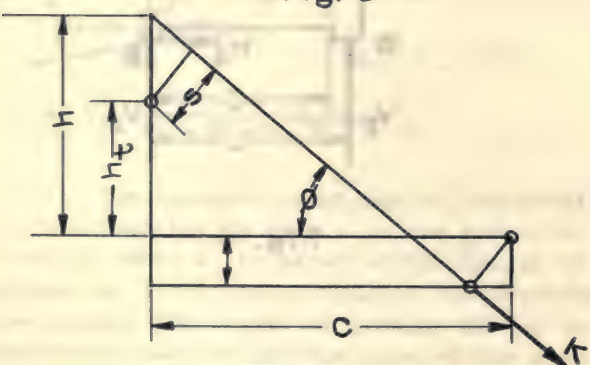

Fig. 5'

For case (1), we note that

$$h' - (d \sin \emptyset + c) \tan \emptyset = d \cos \emptyset$$

but

$$h' = H_t + \frac{s}{\cos \emptyset}$$

and

$$H + \frac{s}{\cos \emptyset} - \frac{d \sin^2 \emptyset}{\cos \emptyset} - c \tan \emptyset = d \cos \emptyset$$

$$h_t \cos \emptyset + s - d \sin^2 \emptyset - c \sin \emptyset = d \cos^2 \emptyset$$

hence

$$d = h_t \cos \emptyset + s - c \sin \emptyset \qquad (13)$$

For case (2), we note that

$$h' + d \cos \emptyset = (c - d \sin \emptyset) \tan \emptyset$$

but

$$h' = h_t + \frac{s}{\cos \emptyset}$$

and

$$h_t + \frac{s}{\cos \emptyset} + d \cos \emptyset = c \frac{\sin \emptyset}{\cos \emptyset} - d \frac{\sin \emptyset}{\cos \emptyset}$$

$$h_t \cos \emptyset + s + d \cos^2 \emptyset = c \sin \emptyset - d \sin^2 \emptyset$$

hence $d = c \sin \emptyset - h_t \cos \emptyset - s \qquad (14)$

If W = weight of the total system (gun, recoiling parts and carriage), we have for moments about A

$W_s L_s = W_r L_r + W_a L_a$ <u>In battery</u>

or in terms of the tipping parts = W_t
and the top carriage alone (not including the stationary parts of the tipping parts = W_a'

$W_s L_s = W_t L_t + W_a' L_a'$ <u>In battery</u>

where L_s = distance to center of gravity of total system <u>in battery</u> from A

If b = length of recoil, and $\emptyset$ the angle of elevation, and L_s' = distance to center of gravity of system out of battery, we have

$$W_s L_s' = W_r (L_r - b \cos \emptyset) + W_a L_a$$
$$= W_r L_r + W_a L_a - W_r b \cos \emptyset$$

hence $W_s L_s' = W_s L_s - W_r b \cos \emptyset$ <u>Out of battery</u>

Hence the external reactions at A and B on the carriage mount become in terms of the resistance to recoil, powder pressure, height of trunnions and distance between trunnions and line through center of gravity of recoiling parts parallel to axis of bore,

For low angles of elevation,

Taking moments about A, we have,

$$V_b\ L + Kd + Pe - W_s\ L_s + W_r(x \cos \phi) = 0$$

hence

$$V_b = \frac{W_s\ L_s - W_r(x \cos \phi) - Kd - Pe}{L}$$

Pe disappearing for a finite value of x or in other words, when pe is used $W_r\ x \cos \phi$ may be neglected. And since $V_a = W_s + K \sin \phi - V_b$ or directly from moments about B, noting that moment arm of K becomes $d' = d + L \sin \phi = h_t \cos \phi + (L-c)\sin \phi + S$

we have,

$$V_a = \frac{W_s(L-L_s) + W_r\ x \cos \phi + K(d+L \sin \phi) - Pe}{L}$$

and as before Pe disappearing unless x is very small. Obviously $H_a = K \cos \phi$ and is in no way directly effected by the powder force.

For high angles of elevation, the moment arm Kd reverses, and d and d' become respectively,

$$d = c \sin \phi - h_t \cos \phi - S$$

and

$$d' = L \sin \phi - d \qquad \text{See(fig.5)}$$
$$= (L-c) \sin \phi + h_t \cos \phi + S$$

Now taking moments about A and B respectively

$$V_b = \frac{W_s\ L_s - W_r\ x \cos \phi + Kd - Pe}{L}$$

and

$$V_a = \frac{W_s(L-L_s) + W_r\ x \cos \phi + K(L \sin \phi - d) + Pe}{L}$$

and $H_a = K \cos \phi$.

For design use, the external reaction formulae may be conveniently grouped.

IN BATTERY: for low angles of elevation:

$$\left\{\begin{aligned} V_b &= \frac{W_s L_s - K(h_t \cos \phi + S - c \sin \phi) - Pe}{L} \\ V_a &= \frac{W_s(L-L_s) + K(h_t \cos \phi + (L-c)\sin \phi + S) + Pe}{L} \\ H_a &= K \cos \phi \end{aligned}\right\} \quad (15)$$

for high angles of elevation:

$$\left\{\begin{aligned} V_b &= \frac{W_s L_s + K(c \sin \phi - h_t \cos \phi - S) - Pe}{L} \\ V_a &= \frac{W_s(L-L_s) + K[(L-c)\sin \phi + h_t \cos \phi + S] + Pe}{L} \\ H_a &= K \cos \phi \end{aligned}\right\} \quad (16)$$

OUT OF BATTERY: for low angles of elevation:

$$\left\{\begin{aligned} V_b &= \frac{W_s L_s - W_r b \cos \phi - K(h_t \cos \phi + S - c \sin \phi)}{L} \\ V_a &= \frac{W_s(L-L_s) + W_r b \cos \phi + K(h_t \cos \phi + (L-c)\sin \phi + S)}{L} \\ H_a &= K \cos \phi \end{aligned}\right\} \quad (17)$$

<u>for high angles of elevation:</u>

$$\left.\begin{aligned}
V_b &= \frac{W_s L_s - W_r\, b \cos \varnothing + K(c \sin \varnothing - h_t \cos \varnothing - S}{L} \\
V_a &= \frac{W_s (L-L_s) + W_r b \cos\varnothing + K(h_t \cos \varnothing + (L-c)\sin\varnothing + S}{L} \\
H_a &= K \cos \varnothing
\end{aligned}\right\} \quad (18)$$

These formulae are immediately applicable to platform mounts traversing about a pintle bearing as well as field carriages.

In platform mounts, the horizontal reaction of the platform on the mount is usually taken at the pintle bearing which is usually located in the front or muzzle end of the mount. Hence in place of H_a we have $H_b = K \cos \varnothing$. The reactions V_b and V_a remain the same, V_a now being the reaction of the platform on the traversing rollers of the mount. Very often V_b is divided into two equal vertical components at the two ends of the traversing arc of the mount, and in such a case L is the horizontal distance in the projection of a vertical plane containing the axis of the bore from the pintle reaction to the traversing reaction, that is, if L is the actual distance from the pintle to the other end of the traversing arc, and θ is the spread of the arc, then

$$L = L' \cos \frac{\theta}{2}$$

In a field carriage, for a first approximation we may assume the horizontal and vertical reaction to be at the contact of spade and ground. These reactions are obviously H_a and V_a of the previous formulae and V_b is the vertical reaction of the ground on the wheels, and L the distance from the wheel contact to the spade contact with the ground. For split trails,

V_a and H_a are obviously equally divided and if the gun is traversed, a horizontal reaction normal to the plane of H_a and V_a is introduced; however, this reaction will not be considered until later, that is, the gun will be assumed at zero traverse.

A closer approximation to actual conditions in a field carriage is to regard H as acting at a vertical distance g from the ground line, usually when from 1/2 to 2/3 the vertical depth of the spade in the ground.

The equations then will have an additional moment:

$H_a g = K \cos \emptyset\, g$,

which is substracted from the moments of the numerator in the expression for V_b and added to the moments in the expression for V_a. The general equations for field carriages are then,

for low angles of elevation:

$$V_b = \frac{W_s L_s - W_r\, x \cos \emptyset - K(d + g \cos \emptyset) - Pe}{L}$$

$$V_a = \frac{W_s(L-L_s) + W_r\, x \cos \emptyset + K(L \sin \emptyset - d + g \cos\emptyset) + pe}{L}$$

$$H_a = K \cos \emptyset$$

$$d = h_t \cos \emptyset + S - c \sin \emptyset$$

and for high angles of elevation:

$$V_b = \frac{W_s L_s - W_r\, x \cos \emptyset + K(d - g \cos \emptyset) - Pe}{L}$$

$$V_a = \frac{W_s(L-L_s) + W_r x \cos \emptyset + K(L \sin\emptyset - d + g \cos\emptyset) + Pe}{L}$$

$$H_a = K \cos \emptyset$$

$$d = c \sin\emptyset - h_t \cos\emptyset - S$$

where Pe disappears if $W_r\, x \cos \emptyset$ is used or vice versa.

Another class of mounts in which the previous formulae are not applicable, are known as pedestal or pivot mounts used on Barbette Coast mountings and for naval guns, as well. These mounts are attached to the foundation by bolts on a circular base usually equally spaced around the circumference.

With such mounts the question of stability is of no consideration. The reaction between the foundation and mount and the distribution of the tension in the bolts, may be obtained approximately by considering the base of the mount as absolutely rigid. Then on firing, the front bolts become the most extended, the deflections and corresponding stress being proportional to the distances measured from the back end of the base along the trace of the intersection of vertical plane, containing the axis of the bore with a horizontal plane, to the perpendicular chord connecting any two front bolts.

Thus if L_o, L, etc. are the lengths from the base end to the perpendicular chord connecting a set of two bolts, and if j_o, j, etc. are the deflections of the bolts, we have $j_o : j_1 : j_2 : --- = L_o : L_1 : L_2$ ----

Now if the bolts are of equal strength, the tensions are proportional to the deflections, that is

$T_o : T_1 : T_2$ ---- $= j_o : j_1 : j_2$ ---- $= L_o : L_1 : L_2$----

that is $T_o = C\,L_o$, $\quad T_1 = C\,L_1$, Q C T C :

Hence the moment about the back end holding the pedestal down, becomes,

$$C\,L_o^2 + 2\,C\,L_1^2 + 2\,C\,L_2^2 + \text{-----}\;C\,L_n^2 = \Sigma M$$

Considering now the gun and mount together we have,

$$K\,d - W_s L_s - W_r \times \cos \varnothing = \Sigma M$$

hence
$$C = \frac{Kd - (W_s L_s - W_r \times \cos \varnothing)}{L_o^2 + 2L_1^2 + 2L_1^2 \text{----} L_n^2}$$

and the maximum tension to which the bolt at the farther end is subjected, becomes,

$$T_o = \frac{[K\,d - (W_s L_s - W_r\,x \cos \emptyset)]L_o}{L_o^2 + 2\,L_1^2 + 2L_2^2 \text{-----} L_n^2}$$

If the gun traverses 360° every bolt should be designed for the maximum tension, T_o.

The same method may be applied to various other combinations for holding a gun down on its foundation.

BENDING IN THE TRAIL AND CARRIAGE

In considering the strength of a carriage body, the reactions at the trail, V_a and H_a, subject the total carriage to a bending stress.

This is of special consideration in field carriages of the trail type. The reaction V_a causes bending while H_a decreases the bending. Hence for maximum bending we should examine the conditions for maximum V_a and minimum H_a.

Now,

$$V_a = \frac{W_s(L-L_s) + K[(L-c)\sin \emptyset + h_t\cos \emptyset + s] + Pe}{L}$$

$$H_a = K \cos \emptyset$$

where

L = horizontal distance between wheel contact and spade contact with ground (in)

c = horizontal distance from spade to vertical plane through trunnions (in)

h_t = height of trunnions from ground. (in)

s = distance from trunnion to line parallel to axis of bore and through center of gravity of recoiling parts (in)

L_s = horizontal distance to center of gravity of total system, recoiling parts in battery (in)

P_e = powder pressure couple (in/lbs)

K = total resistance to recoil (lbs)

With a field carriage, since the trunnion position is very close to the wheel contact with the ground,

$(L-c)\sin\emptyset$ is always very small compared with $h_t \cos\emptyset$, hence, we have approx.

$$V_a = \frac{W_s(1-l_s) + K(h_t \cos\emptyset + s) + P_e}{L}$$

If L_x = distance from trial contact with ground to any section in the carriage body or trail

h_y = the height of the section from the ground we have, for the bending moment at section xy,

$M_{xy} = V_aL_x - H\,h_y$

Substituting the value for V_a and neglecting s being small, we have

$$M_{xy} = [W_s(L-L_s) + K\,h_t \cos\emptyset + P_e]\frac{L_x}{L} - K\cos\emptyset\, h_y$$

$$= W_sL_x(1-\frac{L_s}{L}) + K\cos\emptyset\,(\frac{L_x}{L}h_t - h_y) + P_e\frac{L_x}{L}$$

BENDING IN TRAIL & CARRIAGE

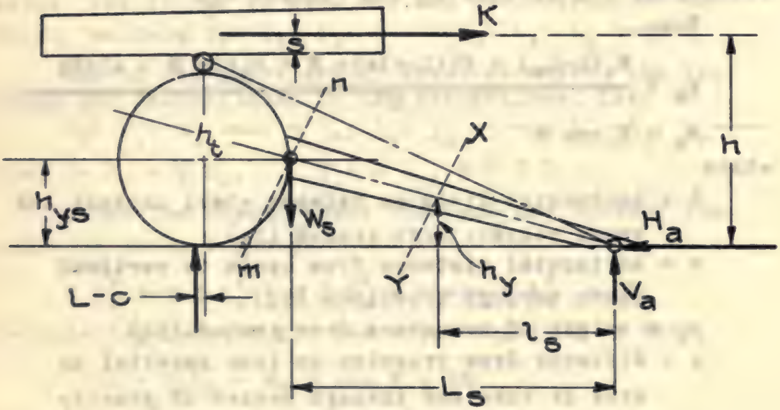

Fig. 6

Now from fig.(6) it is evident $\frac{L_x}{L}h_t$ is always greater than h_y, hence for maximum bending moment we must have $\cos\emptyset = 1$, that is $\emptyset = 0$. Hence the maximum bending moment occurs at horizontal elevation.

At horizontal elevation, $h_t \cos \phi + s = h$

hence
$$V_a = \frac{W_s(L-L_s) + K\,h + P_e}{L}$$

but we also have critical stability at horizontal elevation, that is $K\,h + P_e - W_s L_s = 0$ (approx.)
therefore, $V_a = W_s$ (approx.)
that is in virtue of the mount being just stable at horizontal elevation, or in practice approximately so, the vertical reaction at the spade equals the weight of the entire system, gun and carriage together.

Further $H_a = K = \dfrac{W_s L_s - P_e}{h}$ (lbs)

and the bending moment at section xy in the trail, becomes,

$$M_{xy} = W_s L_x - (W_s L_s - P_e)\frac{h_y}{h} \quad \text{(in lbs)}$$

neglecting P_e as usually small compared with $W_s L_s$, we have,

$$M_{xy} = W_s(L_x - L_s \frac{h_y}{h}) \quad \text{(in lbs)}$$

For the maximum bending moment in the trail, we consider the section at the attachment of the trail to the carriage, then,

$L_x = L_s$ approx. and therefore, the maximum B. M. becomes,

$$M_{xy} = W_s L_s \left(\frac{h - h_{ys}}{h}\right) \quad \text{(in lbs)}$$

a most useful formula in a prelinary carriage layout.

It is important to note that if the recoil varies the above formula and analysis do not hold. When, however, the recoil varies on elevation the maximum bending moment in the trail is obtained at the minimum elevation where the short recoil commences, that is, when $\cos \phi$ is a maximum for the minimum recoil.

If b_s = the short recoil at maximum elevation, then,

K_s = maximum total resistance to recoil, then, we have,

$$M_{xy} = W_s L_x (1 - \frac{L_s}{L}) + K \cos \emptyset \, (\frac{L_x}{L} h_t - h_y) + P_e \frac{L_x}{L}$$

where L_x = distance from trail contact with ground to any distance in the carriage body or trail.

h_y = the height of the section from the ground.

P_e = maximum powder pressure couple.

EXTERNAL REACTIONS DURING COUNTER RECOIL

Counter Recoil may be divided into two periods, the accelerating and the retardation period so far as the external effects on the mount are concerned.

During the accelerating period, the external reactions on the recoiling parts alone, are the elastic reaction of the recuperator in the direction of motion, the guide and stuffing box frictions and a hydraulic resistance during the whole or part of the accelerating period, together with the component of the weight of the recoiling parts parallel to the guides, opposing the motion of counter recoil.

Hence, if

x = the displacement from beginning of counter recoil of the recoiling parts with respect to guides.

K'_a = the resultant accelerating force of counter recoil.

K'_r = the resultant retarding force of counter recoil.

F_x = the recuperator reaction for displacement x from beginning of counter recoil.

R = the total friction.

H'_x = the hydraulic resistance, if any, of throttling through recoil orifices or counter recoil buffer.

Then, during the accelerating period

$$m\frac{d^2x}{dt^2} = F_x - R - H_x' - W_r \sin \emptyset = K_a'$$

and for the subsequent retardation

$$-m\frac{d^2x}{dt^2} = R + H_x + W_r \sin \emptyset - F_x = K_r'$$

Considering now the external forces on the total system (recoiling parts together with mount) the braking resistance for the recoiling parts then become internal reactions, and considering inertia as an equilibrating force, we have, as before the following external forces,

$K_a' = m\frac{d^2x}{dt^2}$ The inertia resistance during acceleration which is opposite to C'recoil.

$K_r' = -m\frac{d^2x}{dt^2}$ The inertia resistance during retardation which is in the direction of C'recoil.

W_r = Wt. of recoiling parts

W_a = Wt. of carriage proper

H_a and V_a Horizontal and vertical reactions of spade and float

V_b = Front Pintle reaction - horizontal component assumed zero as before.

During the accelerating period, obviously, $K_a' < K_r$ that is,

$$F_x - R - H_x' - W_r \sin \emptyset < F_x + R + H_x - W_r \sin \emptyset$$

hence, so far as stability and the balancing reactions exerted by the ground or platform on the carriage mount are concerned, the external effect during the acceleration period of counter recoil

need not be considered.

If now, the inertia resistance is considered as an equilibrating force, we have

$$K_r' \ (d+L \sin \emptyset)-W_r[(L-L_r)+b-x \cos \emptyset]- W_a(L-L_a)+V_aL= 0$$

Let $d' = d + L \sin \emptyset = h_t \cos \emptyset + (L- c) \sin \emptyset + S$

Hence the limitation for counter recoil stability, noting that $W_r(L-L_r) + W_a(L-L_a) = W_s(L-L_a)$ becomes

$$K_r d' \overset{<}{=} W_s \ (L-L_s) + W_r(b-x) \cos \emptyset$$

For a constant marginal counter recoil stability moment G' this equation becomes

$$K_r d' = [G' + W_s(L-L_s) + W_r \ b \cos \emptyset] - W_r \cos \emptyset \ x$$

and the stability slope for a constant marginal counter .ecoil stability is evidently

$$M' = - \frac{W_r \cos \emptyset}{d}$$

that is decreasing as the recoiling masses move into battery. Minimum stability is evidently in battery position and $\emptyset = 0$, that is

$$K_r \overset{<}{=} \frac{W_s(L - L_s)}{h}$$

where $h = d'$ for $\emptyset = 0$

In ordinary field carriages, the weight of the system in battery is very close to the wheel axle or contact of ground and wheel, consequently $(L-L_s)$ is very small.

Therefore counter recoil stability is the primary limitation in the design of a counter recoil system.

STABILITY

The question of stability for field carriages is of fundamental importance, it being a primary limitation imposed on the design of a recoil system. If a gun carriage is to be stable, then

$$Kd - W_aL_a - W_r(l_r - x \cos \emptyset) \overset{<}{=} 0$$

If we have a constant marginal moment G, that is an excess stability, we have

$$Kd - W_a L_a - W_r (l_r - x \cos \emptyset) = G$$

$$K = \frac{-G + W_s L_s - x \bar{W}_r \cos \emptyset}{d}$$

$$= A - m x$$

where $A = \dfrac{-G + W_s L_s}{d}$; $m = \dfrac{W_r \cos \emptyset}{d}$

Thus the resistance to recoil to conform with a constant margin of stability decreases in the recoil proportionally to the distance recoiled from battery.

In battery, the resistance to recoil,

$$K_b = A = \frac{-G + W_s L_s}{d}$$

and out of battery, the resistance to recoil becomes, where b = total length of recoil

$$= A - m b = \frac{-G + W_s L_s}{d} - \frac{W_r b \cos \emptyset}{d}$$

consequently, fot a constant margin of stability,

$$K_b - K = \frac{W_r b \cos \emptyset}{d}$$

From this we obtain the equations of resistance to recoil for constant stability against displacement,

$$K_x = K_b - \frac{W_r \cos \emptyset}{d} x, \text{ where } G_b = \frac{-G + W_s L_s}{d}$$

In our Ordnance Department, $K_x = K_o$ = a constant during the powder pressure period.

Thus if E represents the corresponding length of recoil, then for a constant stability moment G,

$$K_o = \frac{-G + W_s L_s}{d} - \frac{E W_r \cos \emptyset}{d} = A - m E$$

and $K_x = K_o - \dfrac{W_r \cos \emptyset}{d} (x - E)$

Obviously the stability "slope" or space rate of change of resistance to recoil for constant margin of stability, is

$$m = \frac{W_r \cos \emptyset}{d}$$

where

$d = h_t \cos \emptyset + s - c \sin \emptyset$

As the gun elevates, $W_r \cos \emptyset$ remains finite, while d decreases to zero at the elevation $\emptyset$, where the line of action of the resistance to recoil passes through the spade point.

Thus the stability slope "m" thereby increases to an infinite value at that same elevation.

But it is important to note that the resistance to recoil out of battery is finite and increases considerably as "d" decreases so far as it is limited by stability.

Obviously in design it is inconsistent to make the slope of the space rate of change of resistance to recoil consistent with the stability slope as the gun elevates, since the stability becomes sufficiently increased to allow a large resistance to recoil to be used.

We may, therefore, cause the slope to vary arbitrarily as a linear function from a maximum value at an arbitrary low angle of elevation, say some value from 0° to 5°, to zero at the angle of elevation where the resistance to recoil passes through the spade.

Thus if,

$\emptyset^o$ = the initial angle or lower angle of elevation from which the slope is to decrease arbitrarily.

$\emptyset_1$ = the angle of elevation corresponding to where the resistance to recoil passes through the spade.

d_o = moment arm of resistance to recoil about spade point for angle $\emptyset_o$

m = stability slope for any angle of elevation $\emptyset$

$$m_o = \frac{W_r \cos \emptyset}{d_o} = \text{stability slope at lower angle of elevation.}$$

then,

$$m = m_o - k\,(\emptyset - \emptyset_o)$$

At angle of elevation $\emptyset_1$, $m = 0$ hence

$$m_o = k(\emptyset_1 - \emptyset_o) \text{ or } k = \frac{m_o}{\emptyset - \emptyset_o}$$

hence

$$m = m_o - \left(\frac{m_o}{\emptyset_1' - \emptyset_o}\right)(\emptyset - \emptyset_o)$$ or substituting for m_o,

$$m = \frac{W_r \cos \emptyset_o}{d_o}\left[1 - \left(\frac{\emptyset - \emptyset_o}{\emptyset_1 - \emptyset_o}\right)\right] = \frac{W_r \cos \emptyset_o}{d_o}\left[\frac{\emptyset_1 - \emptyset}{\emptyset_1 - \emptyset_o}\right]$$

Thus the variation of the space rate of change of resistance to recoil may be divided into two periods,

(1) from 0° to $\emptyset_o^\circ$

$$m = \frac{W_r \cos \emptyset}{d}$$ which is parallel to the stability slope

(2) from $\emptyset_o^\circ$ to $\emptyset_1^\circ$

$$m = \frac{W_r \cos \emptyset_o}{d_o}\left[\frac{\emptyset_1 - \emptyset}{\emptyset_1 - \emptyset_o}\right]$$ where the slope is arbitrary.

A graph of the variation of the space rate of change of the resistance to recoil against elevation conforming to the assumption (1) and (2).

If there is always to be an excess stability couple G we have from the previous discussion, fixed limitations for the resistance to recoil in and out of battery.

Thus, from 0° to $\emptyset_o^\circ$

$$K_b = \frac{-G + W_s L_s}{d} \;;\quad k = \frac{-G + W_s L_s}{d} - \frac{W_r\, b \cos \phi}{d}$$

where throughout recoil G is a constant marginal stability couple, and from ϕ_0° to ϕ_1° –

$$k = \frac{-G + W_s L_s}{d} - \frac{W_r\, b \cos \phi}{d}$$

the length of recoil being as before shortened as the gun elevates but if the stability marginal movement is never to be decreased for any part of the recoil below G, since the stability slope and space rate of resistance to recoil increase and decrease respectively as ϕ increases from ϕ_0° to ϕ_1° it is obvious that the minimum stability is in the position of out of battery.

Therefore the resistance to recoil in battery is the resistance to recoil out of battery with a marginal moment G of actual stability, augmented by m b.

That is,

$$K = \frac{-G + W_s L_s}{d} - \frac{W_r b \cos \phi}{d} + \frac{W_r b \cos \phi_0}{d_0} \left[\frac{\phi_1 - \phi}{\phi_1 - \phi_0}\right]$$

$$= A - W_r b \left(\frac{\cos \phi}{d} - \frac{\cos \phi_0}{d_0} \left[\frac{\phi_1 - \phi}{\phi_1 - \phi_0}\right]\right)$$

LENGTH OF RECOIL CONSISTENT WITH STABILITY OF MOUNT

Obviously the overturning force, that is the resistance to recoil, is a function of the length of recoil varying roughly inversely as the length of recoil. Hence as the gun elevates the stability increases and the recoil may therefore be shortened.

In a preliminary design it is desirable to know the length of recoil as limited by stability, from 0° or the lowest elevation wherein stability is desired to the elevation ϕ_0° where the stability slope

is made to change arbitrarily.

Let C_s = the constant of stability = $\dfrac{\text{Overturning moment}}{\text{Stabilizing moment}}$ where the overturning moment = $K_r d$ and the stabilizing moment = $W_s L_s - W_r\ b \cos \emptyset$

We may consider the limiting recoil at various elevations:

(1) with a constant resistance to recoil as would occur in certain types of recoil systems.

(2) with a variable resistance to recoil using a stability slope as outlined in the previous paragraph.

<u>For a constant resistance to recoil : = K :</u>

The critical position of stability is obviously with the gun at the end of recoil out of battery.

Then

$$K = \frac{C_s(W_s L_s - W_r\ b \cos \emptyset)}{d}$$

but

$$K = \frac{\frac{1}{2} m_r V_f^2}{b - E + V_f T}$$ See "DYNAMICS OF RECOIL". Chap. VI.

Where E = displacement during powder period in free recoil.

T = total time of free recoil.

V_f = Max. free velocity of recoil.

hence

$$\frac{\frac{1}{2} m_r V_f^2}{b - E + V_f T} = \frac{C_s(W_s L_s - W_r\ b \cos \emptyset)}{d}$$

The above equation reduces to the quadratic form

$Ab^2 + Bb + C = 0$ and its solution is,

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Where $A = W_r \cos \emptyset$

$$B = W_r \cos \emptyset \, (V_f T - E) - W_s L_s$$

$$C = W_s L_s (V_f T - E) + \frac{M_r V_f^2}{2} \frac{d}{C_s}$$

For rough estimates, especially where the length of recoil is comparatively long, we may assume,

$$\frac{\frac{1}{2} m_r V_f^2}{b} = \frac{C_s (W_s L_s - W_r \, b \cos \emptyset)}{d}$$

Where

$$b = \frac{-B \pm \sqrt{B^2 - 4\,AC}}{2A} \quad \text{and } A = \frac{C_s \, W_r \cos \emptyset}{d}$$

$$B = \frac{C_s \, W_s \, L_s}{d}$$

$$C = \tfrac{1}{2} \, m_r \, V_f^2$$

<u>For Variable Resistance to Recoil:</u>

The resistance to recoil is assumed constant during the powder pressure period and thence to decrease uniformly with a stability slope as given in previous article. Therefore, from the end of the powder pressure period to the end of recoil, the stability factor remains constant from 0° to $\emptyset_0^\circ$

(i.e. to where the stability slope is made to change arbitrarily).

At the end of the powder period. (See Dynamics of Recoil):

$$Kd = C_s \left(W_s L_s - W_r \left(E - \frac{KT^2}{2m_r}\right) \cos \emptyset\right)$$

hence

$$K = \frac{C_s (W_s L_s - W_r E \cos \emptyset)}{d - \dfrac{C_s W_r T^2}{2m_r} \cos \emptyset}$$

Now the resistance to recoil out of battery at the end of recoil, becomes,

$$K - m\left(b - E + \frac{KT^2}{2m_r}\right) \text{ (See Dynamics of Recoil)}$$

hence by the equation of energy

$$\left[2K - m\left(b - E + \frac{KT^2}{2m_r}\right)\right]\left(b - E + \frac{KT^2}{2m_r}\right) = M_r\left(V_f - \frac{KT}{m_r}\right)^2$$

Expanding and simplifying, we have the quadratic form: $Ab^2 + Bb + C = 0$

$$\text{where } b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\text{and } A = m = C_s \frac{\bar{W}_r \cos \text{Ø}}{d} \quad \text{from } 0^\circ \text{ to } \text{Ø}_0^\circ$$

$$B = \frac{mKT^2}{m_r} - 2K - 2mE$$

$$C = \left[2E - \frac{mET^2}{m_r} - 2V_fT\right) K + \frac{K^2mT^4}{4m_r^2} + mE + m_r V_f^2]$$

$$\text{and } K = \frac{C_s(W_s L_s - W_r E \cos \text{Ø})}{d - C_s \dfrac{W_r T^2}{2m_r} \cos \text{Ø}}$$

From Ø_0 to Ø_1 degrees, the stability slope is made to change arbitrarily, decreasing proportionally with the elevation from the stability slope at Ø_0 to zero slope at Ø_1° where the line of action of the resistance to recoil passes through the spade point. The critical stability is obviously at the end of recoil, and the resistance to recoil in battery (K) is the resistance to recoil out of battery (k) augmented by the product of the length of recoil from the end of the powder period to the end of recoil multiplied by the arbitrary stability slope (m).

From the energy equation, we have,

$$(K + k)\ (b - E + \frac{KT^2}{2m_r}) = m\ (V_f - \frac{KT}{m_r})$$

now $K = k + m(b-E+ \frac{KT^2}{2m_r})$

hence

$$K = \frac{k+m(b-E)}{1 - \frac{mT^2}{2m_r}}$$ = the constant resistance to recoil during the powder period,

and

$$k = \frac{C_s(W_sL_s - W_r\ b\ \cos\ \emptyset)}{d}$$ = the resistance to recoil at the end of recoil.

Substituting these values in the enery equation, we obtain a quadratic equation in "b". A sufficient approximation and simplification can be made, by noting that

$$E - \frac{KT^2}{2m_r} = 0.9\ E$$ approximately and

$$V_f - \frac{KT}{m_r} = 0.9\ V_f$$ approximately

Therefore, $(K + k)\ (b- 0.9E) = 0.81\ m_r\ V_f^2$

and $K = k + m(b- 0.9E)$

$$= \frac{C_s(W_sL_s - W_r\ b\ \cos\ \emptyset)}{d} + m(b-0.9E)$$

substituting in the energy equation, we have,

$$\frac{2C_s}{d}\ (W_sL_s - W_r\ b\ \cos\ \emptyset)+m(b- 0.9E)(b- 0.9E) = 0.81\ m_rV_f^2$$

Reducing and simplifying, we have the quadratic solution,

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where $A = m - \frac{2C_s}{d} W_r \cos \emptyset$

$$B = \frac{2C_s}{d}(W_s L_s + 0.9E\ W_r \cos \emptyset) - 1.8\ mE$$

$$C = 0.81(mE^2 - m_r V_f^2) - \frac{1.8C_s}{d} W_s L_s E$$

and

$$m = \frac{W_r \cos \emptyset}{d} \quad \text{from } 0^\circ \text{ to } \emptyset_o^\circ$$

$$m = \frac{W_r \cos \emptyset_o}{d_o} \left(\frac{\emptyset_1 - \emptyset}{\emptyset_1 - \emptyset_o}\right) \quad \text{from } \emptyset_o^\circ \text{ to } \emptyset_1^\circ$$

For a close approximation and when the resistance to recoil is not constant during the powder period, if

K = the resistance to recoil in battery

k = the resistance to recoil out of battery,

we have,

$$\left(\frac{K + k}{2}\right)b = \frac{m_r V_f^2}{2} \quad \text{(approximately)}$$

but $K = k + mb$

and $k = \frac{C_s}{d} (W_s L_s - W_r b \cos \emptyset)$

Substituting, we have

$$\left[\frac{2C_s}{d} (W_s L_s - W_r\ b \cos \emptyset) + mb\right] b = m_r V_f^2$$

and the value b, becomes,

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = m - \frac{2C_s}{d} W_r \cos \emptyset$$

$$B = \frac{2C_s}{d} W_s L_s$$

$$C = - m_r V_f^2$$

$$m = \frac{W_r \cos \phi}{d}$$ or any arbitrary slope as desired.

The above formula is sufficiently exact for a preliminary layout with a variable recoil and resistance to recoil provided the margin of stability is chosen fairly large, that is when a low factor of stability is taken.

JUMP OF A FIELD CARRIAGE When the overturning moment exceeds the stabilizing moment, we have unstability and an induced angular rotation about the spade point. After the recoil period, the gun carriage is returned to the ground by the moment of the weights of the system. This phenomena is known as the jump of the carriage.

For the condition of unstability, we have:

$$K\, d - \bar{w}_s l_s + \bar{w}_r \cos > 0$$

where as before,

K = total resistance to recoil

$\bar{w}_s$ = weight of entire gun carriage including gun

l_s = distance from spade contact with ground to center of gravity of total system in the battery position.

$\bar{w}_r$ = weight of the recoiling parts

x = movement in the recoil of the gun.

To analyse the motion of the system, consider

(a) the recoil or accelerating period.

(b) the retardation or return period.

The recoil period may be subdivided into the

powder period and the pure recoil period. During the recoil period the gun and gun carriage are given an angular velocity which reaches its maximum at the end of recoil. During the retardation the angular velocity is gradually decreased to zero, but with increased angular displacement, the maximum angular displacement occuring when the angular velocity reaches its zero value. Further change in angular velocity results in a negative velocity and a corresponding angular return of the mount to its initial position.

The acceleration during the recoil period is not constant, even with constant resistance to recoil, due to the fact that the moment of inertia and the moment of the weights of the recoiling parts about the spade point varies in the relative recoil of the gun. Therefore, the angular acceleration is not constant during the accelerating period. Likewise during the return of the recoiling parts into battery. Further the effect of the relative counter recoil modifies the return angular motion.

Consider the reaction and configuration of the recoiling parts and carriage mount respectively. See figure (7).

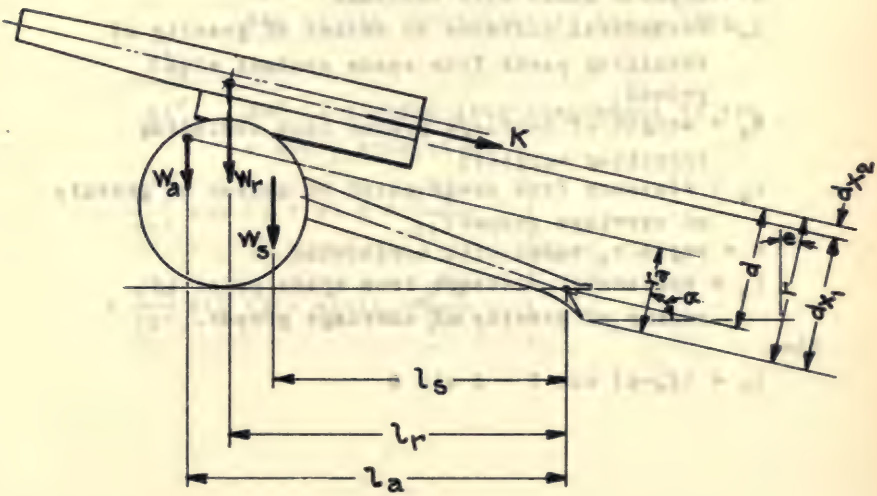

Fig. 7

Let X and Y = the components of the reaction between the recoiling parts and carriage mount, parallel and normal to the guides respectively.

M = the couple exerted between same.

I_a = the moment of inertia of carriage mount about the spade point.

I_r = moment of inertia about the center of gravity of the recoiling parts.

d_{x_1} = perpendicular distance from spade point to line of action of X.

d_{x_2} = perpendicular distance from X to center of gravity of recoiling parts.

$d = d_{x_1} + d_{x_2}$ = perpendicular distance to line parallel to guides and through center of gravity of recoiling parts from the spade constant with ground.

θ = angle made by d with the vertical

ϕ = angle of elevation of the gun (in battery)

x = distance recoiled by gun from battery position

x_0 = distance from "d" to center of gravity of recoiling parts in battery measured in direction of X axis of perpendicular to line d.

r = distance from spade point to center of gravity of recoiling parts.

e = angle r makes with vertical

l_r = horizontal distance to center of gravity of recoiling parts from spade contact with ground.

W_a = weight of carriage proper (not including recoiling weights)

r_a = distance from spade point to center of gravity of carriage proper.

α = angle r_a makes with horizontal

l_a = horizontal distance from spade point to center of gravity of carriage proper.

Then

$$l_r = (x_0 - x)\cos\theta - d\sin\theta$$

$$l_a = r_a \cos (\theta + \alpha - \emptyset)$$

where in battery $\theta = \emptyset$ and for any other angular position during the jump of the carriage,

$\theta = \emptyset + B$ $\quad B$ = a variable angle during the jump.
For the angular motion about the spade point,

For the carriage mount, without the recoiling parts,

$$Xd_{x_1} - Y(x_o - x) + m - \bar{w}_a l_a = I_a \frac{d^2\theta}{d + 2} \qquad (1)$$

and for the recoiling parts,

$$Xd_{x_2} - M - I_r \frac{d^2\theta}{dt^2} = 0 \qquad (2)$$

adding (1) and (2), we have,

$$Xd - Y(x_o - x) - \bar{w}_a l_a = (I_a + I_r) \frac{d^2\theta}{dt^2} \qquad (3)$$

Since the recoiling parts are constrained to rotate with the carriage mount, they partake an angular acceleration about the spade point combined with a relative acceleration along the guides.

The acceleration of tne recoiling parts is divided into:

(1) The tangential acceleration of the recoiling parts about the spade point; due to the constraint in the guides,

$r \dfrac{d^2\theta}{dt^2}$ and is divided into components in the x and y direction

$$r \frac{d^2\theta}{dt^2} \cos (\theta + e) = d \frac{d^2\theta}{dt^2}$$

$$r \frac{d^2 e}{dt^2} \sin (\theta + e) = (x_o - x) \frac{d^2\theta}{dt^2}$$

(2) The centripetal acceleration of the recoiling parts about the spade point due to the constraint in the guides,

$$rw^2 = r\left(\frac{d\theta}{dt}\right)^2$$ and divided into components in the x and y direction.

$$r\left(\frac{d\theta}{dt}\right)^2 \sin(\theta + e) = (x_o - x)\left(\frac{d\theta}{dt}\right)^2$$

$$r\left(\frac{d\,\theta}{dt}\right)^2 \cos(\theta + e) = d\left(\frac{d\,\theta}{dt}\right)^2$$

(3) The relative acceleration of the recoiling parts

$$\frac{d^2x}{dt^2} = \frac{dv_r}{dt}$$ along the x axis

(4) The relative complimentary centripetal acceleration due to the combined angular and relative motion of the recoiling parts:

$$2\,v_r\,\frac{d\theta}{dt} = 2\,\frac{dx}{dt}\,\frac{d\theta}{dt}$$

(5) The angular acceleration of the recoiling parts which obviously equals the angular acceleration about the spade point, that is

$$\frac{d^2\theta}{dt^2}$$

* * * * * * * *

For the motion of the recoiling parts along the x axis, we have

$$P_b - m_r\frac{dv_r}{dt} - m_r d\,\frac{d^2\theta}{dt^2} + \bar{w}_r \sin\theta - m_r(x_o - x)\left(\frac{d\theta}{dt}\right) - X = 0 \qquad (4)$$

For the motion of the recoiling parts normal to the guides,

$$Y - m_r(x_o-x)\frac{d^2\theta}{dt^2} - \bar{W}_r \cos\theta + 2\, m_r v_r \frac{d\theta}{dt} + m_r d\left(\frac{d\theta}{d\,t}\right)^2 = 0 \qquad (5)$$

Substituting (4) and (5) in (3) we have,

$$\left(P_b - m_r\frac{dv_r}{dt}\right)d - m_r(d^2+(x_o-x)^2\frac{d^2\theta}{dt^2} - \bar{w}_r l_r - \bar{w}_a l_a + 2m_r v_r(x_o-x)\frac{d\theta}{dt} - (I_a+I_r)\frac{d^2\theta}{dt^2} = 0 \qquad (6)$$

where

$$\left.\begin{aligned} l_r &= (x_o-x)\cos\theta - d\sin\theta \\ l_a &= r_a\cos(\theta+\alpha-\theta) \end{aligned}\right\} \text{functions of the variable angle } \theta$$

From equations (4) and (6), we have θ as a function of t. An exact solution of these differential equations is complicated and therefore an approximate solution must be resorted to.

APPROXIMATE SOLUTION OF THE JUMP OF A FIELD CARRIAGE.

The static equation of recoil, that is the equation of motion of the recoiling parts upon the carriage is stationary, becomes,

$$P_b - m_r\frac{d^2x}{dt^2} + W_r\sin\theta - X_s = 0$$

and the equation of motion of the recoiling parts along the guides when the carriage jumps, becomes,

$$P_b - m_r\frac{d^2x}{dt^2} + \bar{W}_r\sin\theta - X - m_r d\frac{d^2\theta}{dt^2} - m_r(x_o-x)\left(\frac{d\,\theta}{dt}\right)^2 = 0$$

Now the term $m_r(x_o-x)(\frac{d\theta}{dt})^2$ is small and may be neglected, but on the other hand the term $m_r d \frac{d^2\theta}{dt^2}$ may be considerable. Furthermore the braking X and X_s may differ considerably as well. The term

$$P_b-m_r \frac{d^2x}{dt^2} = K_s \quad \underline{\text{in static recoil.}}$$

whereas with the jump of a carriage

$$P_b- m_r \frac{d^2x}{dt^2} = cK_s \text{ where } c = 0.9 \text{ approx.}$$

During the pure recoil on retardation period of the recoiling parts, we have

$$m_r\frac{d^2x}{dt^2} = K_s \quad \underline{\text{in static recoil.}}$$

whereas when the carriage jumps,

$$m_r \frac{d^2x}{dt^2} = cK_s \text{ where } c = 0.7 \text{ to } 0.9$$

Considering the moment equation for the movement of the total mount about the spade point, we have,

$$(P_b-m_r \frac{d^2x}{dt^2})d-[m_r(d^2+(x_o-x)^2)+I_r+I_a]\frac{d^2\theta}{dt^2} +2m_r v_r(x_o-x)$$

$$\frac{d\theta}{dt} - \bar{w}_r l_r - \bar{w}_a l_a = 0$$

where $l_r=(x_o-x) \cos \theta - d \sin \theta$

$$l_a=r_a \cos (\theta + \alpha - \emptyset)$$

The term $2m_r v_r(x_o-x)\frac{d\theta}{dt}$ is always small and may be neglected.

If b = length of recoil, for the average during the recoil,

let

$$x_o-x = x_o- \frac{b}{2}$$

Then, we have, for an approximate solution,

$$cKd-[m_r(d^2+(x_o-\frac{b}{2})^2 + I_r + I_a]\frac{d^2\theta}{dt^2} - \bar{w}_s l_s = 0 \quad (7)$$

where $\bar{w}_s l_s = \bar{w}_a l_a + \bar{w}_r l_r$ $\quad c = 0.8$ to 0.9

and $\bar{w}_s = \bar{w}_a + \bar{w}_r$ and approximately, if the jump is small,

$$l_r = (x_o - \frac{b}{2}) \cos \phi - d \sin\phi$$

$$l_a = r_a \cos \alpha$$

ϕ = angle of elevation of the gun
b = length of recoil
d = perpendicular distance of line parallel to guides and through center of gravity of recoiling parts from spade contact with ground.
x_o = the perpendicular distance from d to the center of gravity of the recoiling parts
r_a = distance from spade point to center of gravity of carriage proper
α = angle r_a makes with horizontal

Hence for the angular acceleration,

$$\frac{d^2\theta}{dt^2} = \frac{cKd - \bar{w}_s l_s}{m_r(d^2+x_o-\frac{b}{2})^2+I_r+I_a} \quad \frac{\text{rad}}{\text{sec}^2}$$

If we assume a constant acceleration, we have, for the angular velocity attained at the end of recoil,

$$(\frac{d\theta}{dt^2})_1 \quad \frac{(cKd - \bar{w}_s l_s)t_1}{m_r(d^2+(x_o-\frac{b}{2})^2+I_r+I_a} \quad \frac{\text{rad}}{\text{sec}}$$

where t_1 = the time of recoil we have approximately,

$$t_1 = \frac{m_r V}{cK_s} + t_p$$

where $V = 0.9 \quad \dfrac{wv + \bar{w}\ 4700}{W_r}$

w = weight of projectile (lbs)

$\bar{w}$ = weight of charge (lbs)

$\bar{w}_s$ = weight of recoiling parts (lbs)

c = 0.9 approx. and t_p = the total powder period obtained by the methods of interior ballistics.

The angular displacement during the first period of the jump, becomes,

$$\theta_1 = \frac{1}{2}\,\frac{(cKd - w_s l_s)t_1^2}{m_r[d^2 + (x_o - \frac{b}{2})^2] + I_r + I_a} \qquad \text{radius.}$$

During the second period of the jump we have, the angular velocity decreasing but the angular displacement still increasing: then

$$\frac{d^2\theta}{dt^2} = \frac{-\bar{w}_s l_s}{m_r(d^2 + (x_o - \frac{b}{2})^2 + I_r + I_a} \qquad \frac{\text{rad}}{\text{sec}^2}$$

Integrating, we have

$$\left(\frac{d\theta}{dt}\right) = \frac{-\bar{w}_s l_s\ t}{m_r(d^2 (x_o - \frac{b}{2})^2) + I_r + I_a} + \left(\frac{d\theta_1}{d\,t}\right)_1$$

and for the angular displacement,

$$\theta = \frac{1}{2}\,\frac{-\bar{w}_s l_s\ t}{m_r(d^2 + (x_o - \frac{b}{2})^2) + I_r + I_a} + \left(\frac{d\theta}{dt}\right) t + \theta_1$$

To determine the time of jump required to attain the maximum angular displacement, we have the angular velocity reduced to zero, whence,

$$\left(\frac{d\theta_1}{dt}\right)_1 = \frac{\bar{w}_s l_s\ t_2}{m_r(d^2 + (x_o - \frac{b}{2})^2) + I_a + I_r}$$

from which we may determine t_2. Therefore, the

maximum angular displacement, becomes,

$$\theta_2 = \frac{1}{2} \frac{(\bar{w}_s l_s)_2 \, t_2^2}{m_r(d^2+(x_o-\frac{b}{2})+I_a+I_r} + \left(\frac{d\theta_1}{dt}\right)_1 t_2 + \theta_1$$

The effect of counter recoil is to increase t_2 and decrease the negative moment $(-\bar{w}_s l_s)$.

RECAPITULATION OF FORMULAS:

EXTERNAL EFFECTS AND STABILITY.

Resistance to recoil:

$$K = \frac{\frac{1}{2} m_r V_f^2}{b-E+V_f r}$$ (lbs) constant resistance throughout recoil.

m_r = mass of recoiling parts $= \frac{w_r}{g}$ (lbs)

$g = 32.16$ ft/sec.^{2}

b = length of recoil (ft)

E = free recoil displacement during powder period (ft)

T = time of free recoil (sec)

V_f = Max. velocity of free recoil ft/sec.

BETHEL'S FORMULA

$$K = \frac{m_r V_f^2}{2} \frac{1}{b+(.096+.0003d)\frac{M V_f}{V_o}}$$ (lbs) Constant resistance.

M = travel up bore (inches)

V_o = muzzle velocity (ft/sec)

d = diam. of bore (inches)

Assuming a gun carriage to be supported by a hinge joint at the rear (A) and a vertical support

in the front (B) we have the following equations for the reactions of the supports:

Let

H_a = horizontal component at rear hinge support or spade of carriage. (lbs)

V_a = vertical component at rear hinge support (lbs)

V_b = reaction of front support assumed vertical (lbs)

L = horizontal distance between carriage supports (in)

h_t = height of trunnion above support (in)

s = perpendicular distance from center of gravity to recoiling parts to line of action of the resistance to recoil (in)

c = horizontal distance from rear support to trunnion (in)

K = total resistance to recoil (lbs)

$Ø$ = angle of elevation of gun

g = vertical distance from ground to horizontal component of resultant spade reaction.

<u>IN BATTERY:</u> <u>For low angles of elevation:</u>

$$V_b = \frac{\bar{W}_s L_s - K(h + \cos Ø + s - c \sin Ø) - Pe}{L}$$

$$V_a = \frac{\bar{W}_s (L - L_s) + K[h_t + \cos Ø + (L-c)\sin Ø + s] + Pe}{L} \qquad \text{(lbs)}$$

$$H_a = K \cos Ø$$

For high angles of elevation

$$\left.\begin{aligned} V_b &= \frac{\bar{W}_s L_s + K(c \sin \phi - h_t \cos \phi - s) - Pe}{L} \\ V_a &= \frac{\bar{W}_s (L - L_s) + K[(L-c) \sin \phi + h + \cos \phi + s] + Pe}{L} \\ H_a &= K \cos \phi \end{aligned}\right\} \text{(lbs)}$$

OUT OF BATTERY: For low angles of elevation:

$$\left.\begin{aligned} V_b &= \frac{W_s L_s - \bar{W}_r b \cos \phi - K(h_t \cos \phi + s - c \sin \phi}{L} \\ V_a &= \frac{W_s (L - L_s) + W_r b \cos \phi + K(b + \cos \phi + (L-c) \sin \phi + s}{L} \\ H_a &= K \cos \phi \end{aligned}\right\} \text{(lbs)}$$

For high angles of elevation:

$$\left.\begin{aligned} V_b &= \frac{\bar{W}_s L_s - \bar{W}_r b \cos \phi + K(c \sin \phi - h_t \cos \phi - s)}{L} \\ V_a &= \frac{W_s (L - L_s) + \bar{W}_r b \cos \phi + K(h_t \cos \phi (L-c) \sin \phi + s)}{L} \\ H_a &= K \cos \phi \end{aligned}\right\}$$

With a field carriage where the spade is inserted in the ground, the center of pressure lies a distance "g" inches vertically down. The general equations for the support of a field carriage, therefore become,

For low angles of elevation:

$$V_b = \frac{\bar{w}_s L_s - \bar{w}_r x \cos \emptyset - K(d+g \cos \emptyset) - Fe}{L}$$

$$V_a = \frac{\bar{w}_s (L-L_s)+\bar{w}_r\ x \cos \emptyset + K(L \sin \emptyset - d+g \cos\emptyset)+Fe}{L}$$

$$H_a = K \cos \emptyset$$

$$d = h_t \cos \emptyset + s - c \sin \emptyset$$

For high angles of elevation:

$$V_b = \frac{\bar{w}_s L_s - \bar{w}_r\ x \cos \emptyset + K(d-g \cos \emptyset)-Fe}{L}$$

$$V_a = \bar{w}(L-L_s)+\bar{w}_r\ x \cos\emptyset + K(L \sin\emptyset - d+g \cos \emptyset)+Fe$$

$$H_a = K \cos \emptyset; \quad d = c \sin \emptyset - h_t \cos \emptyset - s$$

In certain types of Barbette mounts, we have the bottom carriage held down by tension bolts to a circular base plate. If we draw a series of parallel chords through the bolts on either side of the axis of the gun, and if we let the distance from those several chords measured from the rear bolt, be L_o, L_1 - - - - L_n. we have, for the maximum tension induced in a tension bolt given by the expression:-

$$T_o = \frac{[Kd-(\bar{w}_s L_s - \bar{w}_r\ x \cos \emptyset)]L_o}{L_o^2 + 2L_1^2 + 2L_2^2 - - - - L_n^2}$$

BENDING IN THE TRAIL AND CARRIAGE.

Considering the section at the attachment of the trail to the carriage, for a constant length of

recoil the maximum bending in the trail occurs at horizontal elevation and is given by the following expression:

$$M_{xy} = \bar{w}_s L_s \left(\frac{h - h_{ys}}{h}\right)$$

where M_{xy} = the max. B.M. at the attachment of trail to carriage.

h = the height of the center of gravity of the recoiling parts (axis of bore practically above the ground when the gun is in its horizontal position.

h_{ys} = the height of the neutral axis of the section above the ground.

$\bar{w}_s$ = weight of entire mount including the gun.

L_s = horizontal distance from the spade to the center of gravity of the weight of the entire mount.

When the recoil varies on elevation, the maximum bending moment in the trail is obtained at the minimum elevation where the short recoil commences, we have,

$$M_{xy} = \bar{w}_s L_x \left(1 - \frac{L_s}{L}\right) + K_s \cos \emptyset_s \left(\frac{L_x}{L} h_t - h_y\right) + Pe \frac{L_x}{L}$$

where

K_s = maximum total resistance to recoil corresponding to short recoil b_s.

$\emptyset_s$ = minimum angle of short recoil.

L_s = distance from trail contact with ground to any distance in the carriage body of trail.

h_y = the height of the neutral axis of the section from the ground.

Pe = maximum powder pressure couple.

STABILITY OF COUNTER RECOIL.

In the design of a field carriage counter recoil stability is a basic limitation. We have for counter recoil stability that,

The equation stability, gives, for variable resistance to recoil, for low angles of elevation consistent with the stability slope,

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = m = C_s \frac{\bar{w}_r \cos \emptyset}{d} \qquad \text{(from } 0^\circ \text{ to } \emptyset_o^\circ \text{ elevation)}$$

$$B = \frac{mKT^2}{m_r} - 2K - 2mE$$

and

$$K = \frac{C_s(\bar{w}_s L_s - \bar{w}_r E \cos \emptyset)}{d - C_s \dfrac{\bar{w}_r T^2}{2m_r} \cos \emptyset} \qquad \text{(lbs)}$$

After an arbitrary elevation $\emptyset_o^\circ$ (approx.5°) the stability of the mount greatly increases with elevations and therefore the stability slope is made to arbitrarily decrease with the elevation arriving at constant resistance to recoil at the elevation corresponding to where the line of action of the resistance to recoil passes through the spade point. To estimate the minimum recoil allowable for the various angles of elevation in this range, we have

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$m = \frac{\bar{w}_r \cos \emptyset}{d_o} \left(\frac{\emptyset_1 - \emptyset}{\emptyset_1 - \emptyset_o} \right) \qquad \text{from } \emptyset_o^\circ \text{ to } \emptyset_1^\circ$$

$$A = m - \frac{2C_s}{d} \bar{w}_r \cos \emptyset$$

$$B = \frac{2C_s}{d} (\bar{w}_s L_s + P.9E\, \bar{w}_r \cos \emptyset) - 1.8\, mE$$

$$C = 0.81\, (mE^2 - m_r V_f^2) - \frac{1.8C_s}{d} \bar{w}_s L_s E$$

$$K_r \leq \frac{\bar{w}_s(L-L_s)}{h}$$ where K_r = the total resistance of counter recoil at horizontal elevation.

$\bar{w}_s$ = weight of entire mount including gun.

L_s = horizontal distance from spade to center of gravity of $\bar{w}_s$.

L = horizontal distance from spade to wheel contact with ground.

Further

$$K_r = -m_r \frac{d^2x}{dt^2}$$

($\frac{d^2x}{dt^2}$ may be obtained from the velocity curve of counter recoil towards the battery position).

and $K_r = H_x' + R + \bar{w}_r \sin \varnothing - F_x$ where H_x' = hydraulic or buffer braking at end of counter recoil. (lbs)

R = total friction resistance

$\bar{w}_r \sin \varnothing = 0$ weight compound equals zero at horizontal elevation.

F_x = recuperator reaction. (lbs)

RECOIL STABILITY

The stability limitation of the resistance to recoil varies in the recoil due to the movement of the recoiling weights. The slope or rate of the variation in the recoil of the equivalent force applied through the center of gravity of the recoiling parts and parallel to the guides that will just overturn the mount, is given by the following expression:

$$m = \frac{\bar{w}_r \cos \varnothing}{d}$$ from $\theta°$ to $\varnothing_0°$

where

m = the stability slope
$\emptyset$ = angle of elevation
d = perpendicular distance from spade to line through center of gravity recoiling parts parallel to the guides.
$\bar{w}_r$ = weight of recoiling parts
$\emptyset_o$ = the initial angle or lower angle of elevation from which the slope is to decrease arbitrarily.

If from $\emptyset_o$ the slope is made to decrease arbitrarily with the elevation, to the elevation $\emptyset$, the angle of elevation corresponding to where the line through the center of gravity of the recoiling parts parallel to the guides passes through the spade point, we have for the stability slope

$$m = \frac{\bar{w}_r \cos \emptyset_o}{d_o}\left(\frac{\emptyset_1 - \emptyset}{\emptyset_1 - \emptyset_o}\right)$$ where the slope is arbitrary.

LENGTH OF RECOIL CONSISTENT WITH STABILITY OF MOUNTS.

The equation of stability, gives, for constant resistance to recoil,

$$\frac{\frac{1}{2} m_r V_f^2}{b - E + V_f T} = \frac{C_s(\bar{w}_s L_s - \bar{w}_r b \cos \emptyset}{d}$$

The solution of this quadratic equation for b, gives:

$$b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where $A = \bar{w}_r \cos \emptyset$
$B = \bar{w}_r \cos \emptyset(V_f T - E) - \bar{w}_s L_s$
$C = \bar{w}_s L_s (V_f T - E) + \frac{m_r V_f^2}{2} \frac{d}{c_s}$

where all units are in feet and pounds.

CHAPTER IV

INTERNAL REACTIONS.

In the design of the various parts of a gun carriage it is of fundamental importance that we have a complete knowledge of the stresses to which each member is subjected, and the variations of such throughout recoil and the position of elevation and traverse.

We have already considered the external reactions on the whole system, and such reactions are useful in computing the stresses in the supporting structure for a gun mount as the strength of concrete emplacements for barbette mounts, or the strength of a railway car or caterpillar frame.

The primary internal reactions within a gun and its mount may be classified as follows:

(a) The mutual reactions between the recoiling parts and the carriage proper or gun mount.

(b) The mutual reaction between the tipping parts or cradle and the top carriage.

(c) The mutual reaction between the top carriage and bottom carriage.

The mutual reaction (a) is between the moveable and stationary part of the total system during the recoil; that of (b) between the moveable and stationary parts during elevation of the gun; and that of (c) between the moveable and stationary parts in traversing the gun.

The mutual reaction (a) may be subdivided into individual or component reactions as follows:

(1) The reactions of the constraints due to the guides or clip reactions at

the two ends of the clips in contact with the guides, which may be subdivided into friction and normal components.

(2) The mutual reaction of the elastic medium connecting the recoiling parts to the carriage proper, that is, the hydraulic brake and recuperator reaction, together with the joint frictions. This will be known as the elastic reaction between the recoiling parts and carriage proper.

The mutual reaction (b) may be subdivided into:

(1) The trunnion reaction between the tipping parts and top carriage.

(2) The elevating arc reaction between the elevating arc of the tipping parts, and the pinion of the top carriage.

The mutual reaction (c) may be subdivided into:

(1) The pintle or pivot reaction between the pintle bearing on the bottom carriage or platform mount and the pintle of the top carriage fitting within this bearing.

(2) The traversing arc reaction, that is, the reaction between the traversing arc of the top and bottom carriage. These are usually roller reactions for platform or pedestal mounts, the rollers being either a part of the top or bottom carriage or else clip reactions field carriage and may be more or less distributed about the arc of contact.

Let X and Y = the coordinates of the center of gravity of the recoiling parts along end perpendicular to the guides with origin at center of gravity of recoiling parts.

x_1 and y_1 = coordinates of front clip reaction measured from the center of gravity of the recoiling parts.

Q_1 = Normal component to guides of front clip reaction.

uQ_1 = Frictional tangentional component of front clip reaction.

x_2 and y_2 = coordinates of rear clip reaction measured from the center of gravity of the recoiling parts.

Q_2 = Rear clip reaction normal component.

uQ_2 = Rear clip reaction frictional component.

$R = nQ_1 + nQ_2$ = total guide friction.

B = elastic reaction (hydraulic breaking and recuperator reaction including friction of joints) assumed parallel to the guides.

F = the total powder pressure on the breech of the gun which necessarily lies along the axis of the bore.

e = the perpendicular distance from center of gravity of recoiling parts to line of action F, that is to axis of bore.

Assuming as in Chapter III, the mount to be hinged at the rear or breech end to its support and resting on a smooth surface at the front end, and if

d = perpendicular distance from hinge to line through center of gravity of recoiling parts, parallel to guides.

d_b= perpendicular distance from hinge to line of action of B.

l_r= horizontal distance from hinge to center of gravitv of recoiling parts in battery.

l_a= horizontal distance from hinge to center of gravity of stationary parts of system (includes stationary parts of tipping parts).

From fig.(1) considering the reactions on the recoiling parts alone, we have from the equations of motion:

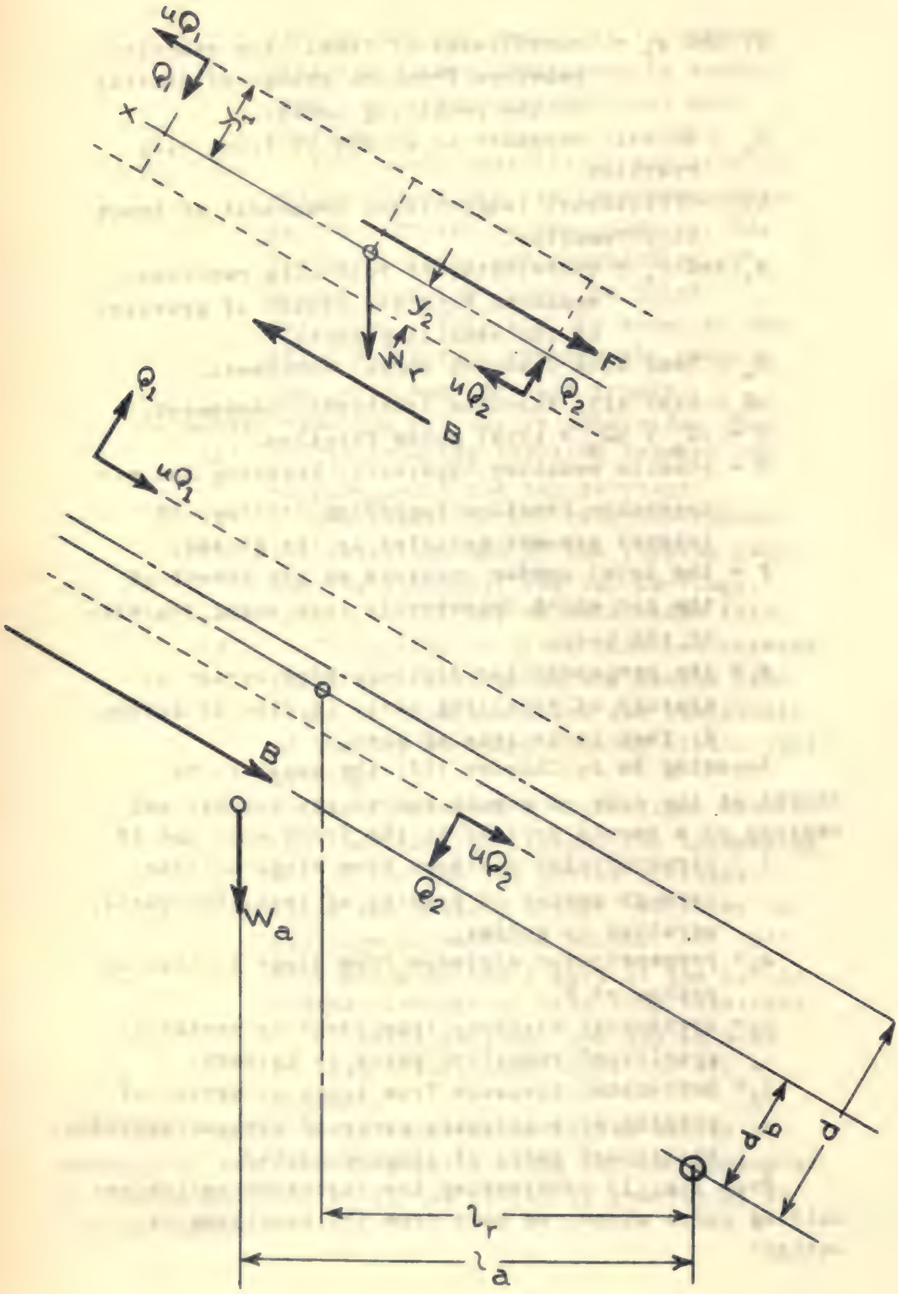

Fig. 1

for motion along the x axis,

$$F - B - uQ_1 - uQ_2 + W_r \sin \emptyset = m_r \frac{d^2x}{dt^2} \quad (1)$$

and since there is no motion along the y axis,

$$Q_2 - Q_1 = W_r \cos \emptyset \quad (2)$$

and taking movements about the center of gravity since there is no angular acceleration

$$B(d-d_b)-u(Q_1y_1 - Q_2y_2)+Fe-Q_1x_1 - Q_2x_2=0 \quad (3)$$

Now in fig.(1) considering the gun carriage or mount including the stationary parts of the tipping parts, we have for the moments of the reaction of the recoiling parts on the gun mount about the hinge A,

$$Q_1(x_1+\frac{l_r-x \cos \emptyset}{\cos \emptyset} + d \tan \emptyset)-Q_2(\frac{l_r- x \cos \emptyset}{\cos \emptyset} +d \tan \emptyset-x_2)$$

$$+uQ_1(d+y_1)+uQ_2(d-y_2)+Bd_b= \Sigma M_{ra} \quad (4)$$

but $uQ_1y_1-uQ_2y_2=B(d-d_b)+Fe-Q_1x_1-Q_2x_2$

and $uQ_1 +uQ_2=R$

Substituting these values in (4), we have,

$$(Q_1-Q_2)(\frac{l_r-x \cos \emptyset}{\cos \emptyset} + d \tan \emptyset)Bd+Rd+Ae=\Sigma M_{ra}$$

that is, $-W_r(l_r-x \cos \emptyset)-W_r \sin \emptyset\ d+Bd+Fe+Rd=\Sigma M_r$

or simplifying and combining, we have,

$$(B+R-W_r \sin \emptyset)d+Fe-W_r(l_r-x \cos \emptyset)=\Sigma M_r \quad (5)$$

at maximum powder pressure, x is usually negligible and the equation reduces to:

$$(B+R-W_r \sin \emptyset)d+Fe-W_rl_r= \Sigma M_{ra} \quad (5')$$

From this we observe that the reaction between the recoiling parts and the mount is equivalent in effect to a force $(B+R-W_r\sin \emptyset)$, the line of action of which is parallel to the axis of the bore or guides and

passes through the center of gravity of the recoiling parts, and a couple of magnitude Fe, due to the powder pressure, together with a component equal to the weight of the recoiling parts and in its line of action assumed concentrated.

Thus the reaction on the gun mount of the recoiling parts, therefore, is equivalent to a single concentrated force, the resultant of $(B+R-W_r \sin \emptyset)$, equal to the total resistance to recoil and a force equal to the weight of the system together with a couple Fe. Since a couple and a single force in the same plane are equivalent in effect to a single force, parallel to the former, and displaced from it equal to the couple divided by the force, the resultant reaction on the mount of the recoiling parts reduces to a single force; the resultant of $B+R-W_r \sin \emptyset$ and W_r, which becomes, since $B+R-W_r \sin \emptyset = K$, equal to

$$J = \sqrt{K^2 + W_r^2 - 2KW_r \sin \emptyset}$$

and the line of action of J makes an angle

$$\theta = \text{Tan}^{-1}\frac{(W_r \cos \emptyset)}{(K+W_r \sin \emptyset)} = \text{Tan}^{-1}\frac{(W_r \cos \emptyset)}{(B+R)}$$

with the axis of the bore and is displaced a distance $\frac{Fe}{J}$, from the center of gravity of the recoiling parts. It is however, more convenient in computation to resolve this resultant into its components, K and W_r together with Fe.

If now we consider the equilibrium of the gun carriage mount, we have for moments about the hinge point, $\Sigma M_{ra} - W_a L_a + V_b\, l = 0$ that is,

$$(B+R-W_r \sin \emptyset)d + Fe - W_r l_r - W_a L_a + V_b\, l = 0 \qquad (6)$$

and since $W_a l_a + W_r l_r = W_s l_s$

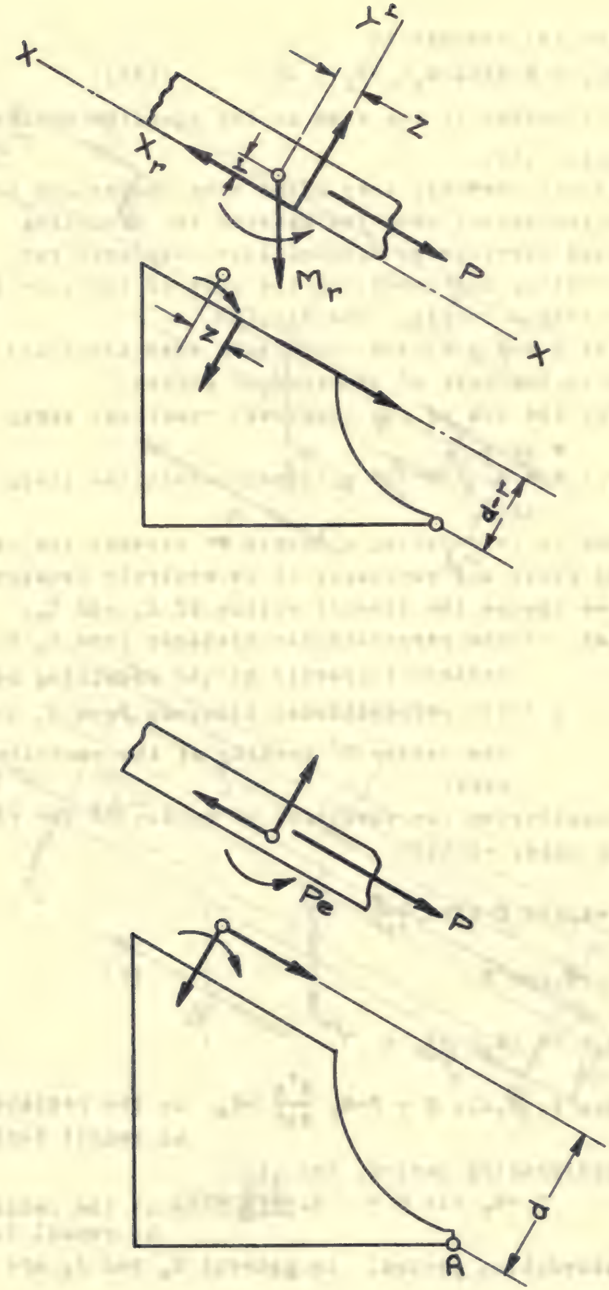

Fig. 2

Equation (6) reduces to

$$(B+R-W_r \sin \emptyset)d+Fe-W_s L_s+V_b\, l = 0 \qquad (6')$$

which of course is the same as the equation obtained in Chapter III.

It is, however, very often more convenient to regard the mutual reaction between the recoiling parts and carriage as divided into component reactions along and normal to the axis of the bore together with a couple. See fig.(3).

Let x and y be the coordinate axes along and normal to the axis of the bore or guides.

X_r = the sum of the component reactions along the x axis.

Y_r = the sum of the component reactions along the y axis.

Now by introducing a couple Mr between the recoiling parts and carriage, it is entirely immaterial where we assume the line of action of X_r and Y_r.

Let r = the perpendicular distance from X_r to the center of gravity of the recoiling mass.

z = the perpendicular distance from Y_r to the center of gravity of the recoiling mass.

Considering the equations of motion of the recoiling mass, we have,

$$\left.\begin{aligned} &F+W_r \sin \emptyset - X_r = M_r \frac{d^2x}{dt^2} \\ &Y_r = W_r \cos \emptyset \\ &M_r = Pe + X_r r - Y_r z \end{aligned}\right\} \qquad (7)$$

But $X_r - W_r \sin \emptyset = F - M_r \dfrac{d^2x}{dt^2} = K_a$ is the resistance to recoil during the accelerating period, and

$X_r - W_r \sin \emptyset = - M_r \dfrac{d^2x}{dt^2} = K_r$ is the resistance to recoil during the retardation period. In general K_a and K_r are

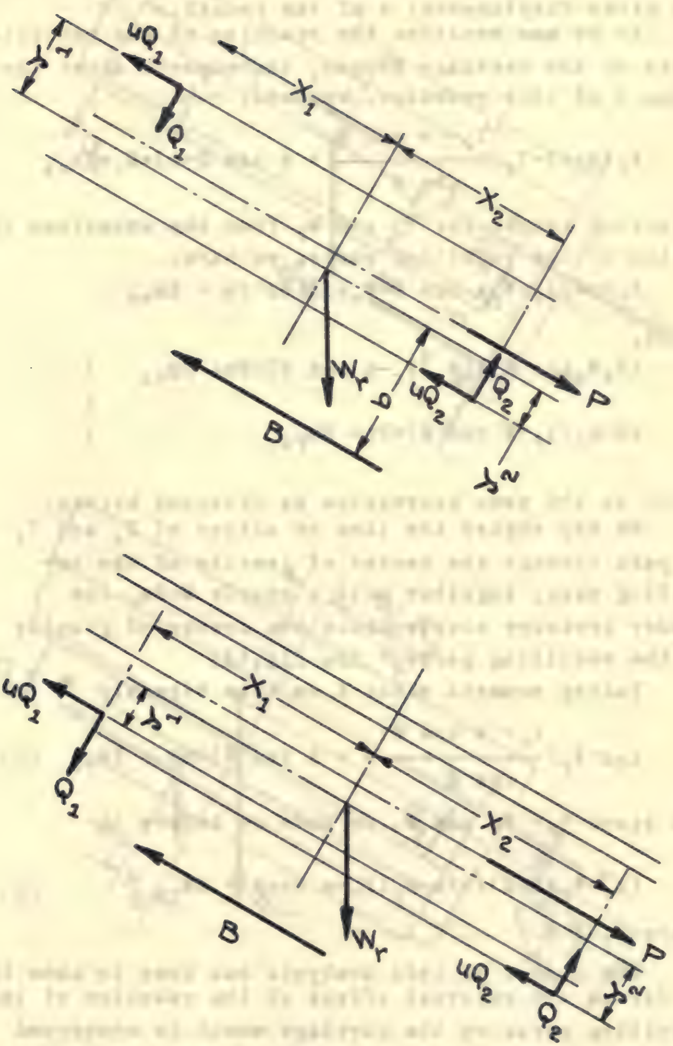

Fig.3

different in value. Hence, let $K = X_r - W_r \sin \phi$ for any given displacement x of the recoil.

If we now consider the reaction of the recoiling parts on the carriage proper, the moments about the hinge A of this reaction, becomes,

$$X_r(d-r)-Y_r\left(\frac{l_r - x\cos\phi}{\cos\phi} + d\tan\phi - z\right)+M_r = \Sigma M_{ra}$$

Inserting values for Y_r and M_r from the equations of motion of the recoiling parts, we have:

$$X_r d - W_r l_r + W_r x\cos\phi - W_r \sin\phi d + Pe = \Sigma M_{ra}$$

hence,

$$(X_r W_r \sin\phi)d - W_r(l_r - x\cos\phi) + Pe = \Sigma M_{ra}$$

or

$$Kd - W_r(l_r - X\cos\phi) + Pe = \Sigma M_{ra}$$

which is the same expression as obtained before.

We may regard the line of action of X_r and Y_r to pass through the center of gravity of the recoiling mass, together with a couple M=Pe, the powder pressure couple about the center of gravity of the recoiling parts. See fig.(4)

Taking moments about A we have directly

$$X_r d - Y_r\left(\frac{l_r - x\cos\phi}{\cos\phi} + d\tan\phi\right) + Pe = \Sigma M_{ra} \quad (8)$$

and since $Y_r = W_r \cos\phi$. we have as before

$$(X_r - W_r \sin\phi)d + Pe - W_r(l_r - x\cos\phi) = \Sigma M_{ra} \quad (9)$$

where $X_r = B + R$

The object of this analysis has been to show that so far as the external effect of the reaction of the recoiling parts on the carriage mount is concerned the exact location of rod pulls or the line of action of the guide frictions, is entirely immaterial, though as we shall see immediately, the value of R, the sum of the guide frictions, does depend upon the line of actions of these pulls together with the friction line

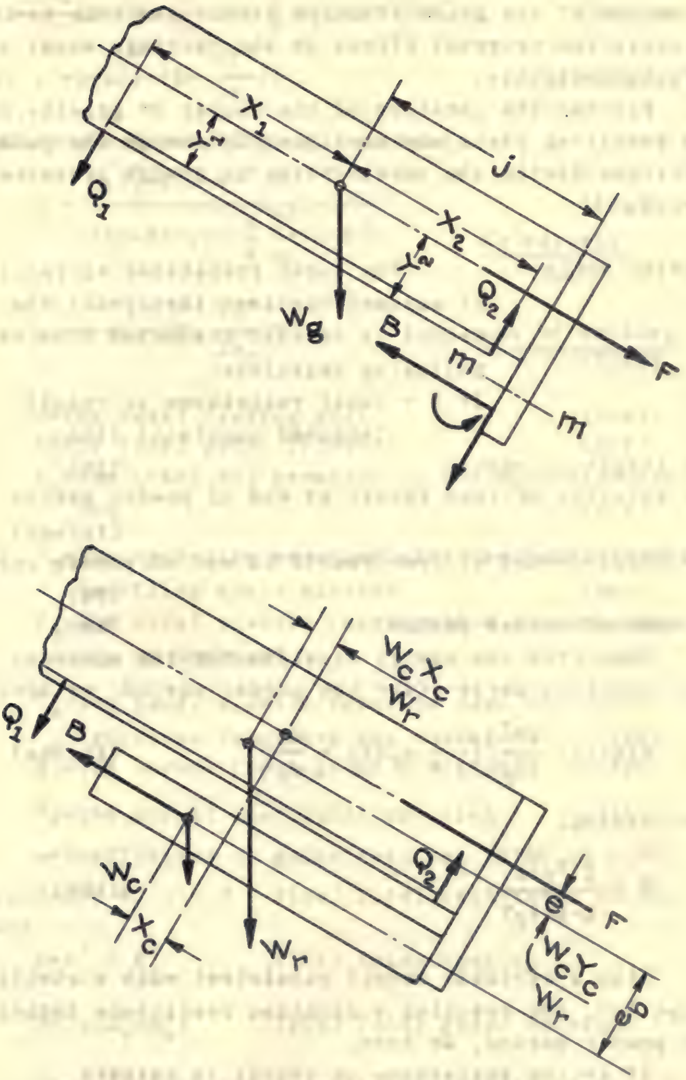

Fig. 4

of action of the guide friction itself and thus indirectly the external effect on the carriage mount is affected slightly.

Further the location of the center of gravity of the recoiling parts may considerably change the guide frictions during the accelerating or powder pressure period.

BRAKING PULLS The total resistance to recoil if assumed constant throughout the recoil is readily evaluated from the following relations:

If K = total resistance to recoil (assumed constant) (lbs)

b = length of recoil (ft)

V_f= velocity of free recoil at end of powder period (ft/sec)

E = displacement of free recoil at end of powder period (ft)

T= time of powder period (sec)

Then from the energy equation for the movement of the recoiling parts after the powder period, we have,

$$K\left[b-\left(E-\frac{KT^2}{2m_r}\right)\right]=\frac{1}{2}m_r\left(V_f-\frac{KT}{m_r}\right)^2 \qquad \text{(ft.lbs)}$$

Simplifying,

$$K=\frac{\frac{1}{2}m_rV_f^2}{b-E+V_fT} \qquad \text{(lbs)}$$

With a variable recoil consistent with a stability slope "m", and assuming a constant resistance during the powder period, we have,

If K= the resistance to recoil in battery

k= the resistance to recoil out of battery

m=stability slope = $\frac{cW_r}{h}$ (h=height of axis of bore above ground)

then $\frac{K+k}{2}[b-(E-\frac{KT^2}{2m_r})]=\frac{1}{2}m_r(V_f-\frac{KT}{m_r})^2$ (ft.lbs)

and $k=K-m[b-(E-\frac{KT^2}{2m_r})]$ (lbs)

Combining and simplifying, we have,

$$K=\frac{m_rV_f^2+m(b-E)^2}{2[b-E+V_fT-\frac{m}{2}\frac{T^2}{m_r}(b-E)]}$$ (lbs) <u>in battery</u>

$$k=K-m[b-(E-\frac{KT^2}{2m_r})]$$ <u>out of battery</u> (lbs)

If

B=the total braking pull (lbs)

R=the total guide friction (lbs)

P_h=the total oil pressure on the hydraulic piston (lbs)

P'_h=the hydraulic reaction plus the joint frictions (stuffing box + piston) (lbs)

P_a=the total elastic reaction (due to compressed air or springs) (lbs)

P'_a=the total elastic reaction plus the joint frictions (stuffing box + piston) (lbs)

Q_1=the normal front guide reaction (lbs)

Q_2=the normal rear guide reaction (lbs)

u=coefficient of guide friction (0.15 to 0.25)

Then $K=B+R-W_r \sin \emptyset$ (lbs) Total resistance to recoil

where

$B=P'_h+P'_a$ (lbs) Total braking

$R=u(Q_1+Q_2)$ (lbs) Total guide friction

The stuffing box friction is usually assumed at from 100 to 150 lbs. per inch of diameter of rod, and if d_u and d_a are the stuffing box diameters of the hydraulic and air cylinders respectively, we have

$P_h' = P_h + 100 d_h$ (lbs)

$P_a' = P_a + 100 d_a$ (lbs)

GUIDE OR CLIP REACTIONS The recoiling mass is constrained to translation parallel to the axis of the bore by the recoiling masses engaging in suitable guides in the cradle of the top carriage. In general the recoiling mass may recoil in a sleeve, a part of the cradle, or along guides considerable below the axis of the bore and the center of gravity of the recoiling parts.

For the former case, considering the external reactions on the recoiling mass, fig.(5)

$$Q_2 - Q_1 = W_r \cos \emptyset$$

$$Q_1 x_1 + Q_2 x_2 + u(Q_1 y_1 - Q_2 y_2) - Fe - B\,b = 0$$ (moments about the center of gravity of recoiling parts)

where $e_b = d - d_b$, then $Q_1 x_1 + (Q_1 + W_r \cos \emptyset) + u(Q_1 y_1 - Q_2 y_2 -$

$$W_r y_2 \cos \emptyset) - Fe - Bb = 0$$

$$Q_1(x_1 + x_2 + uy_1 - uy_2) + W_r \cos \emptyset (x_2 - uy_2) - Fe - Bb = 0$$

Hence

$$Q_1 = \frac{Fe + Be_b - W_r \cos \emptyset (x_2 - uy_2)}{x_1 + x_2 + u(y_1 - y_2)} \qquad (10)$$

Further

$$Q_2 = \frac{Fe + Be_b - W_r \cos \emptyset\, x_2 + W_r \cos \emptyset\, uy_2 + W_r \cos \emptyset\, x_1 + W_r \cos \emptyset\, x_2 +}{x_1 + x_2 + u(y_1 - y_2)}$$

$$\underline{W_r \cos \emptyset\, uy_1 - W_r \cos \emptyset\, uy_2}$$

$$\text{Hence } Q_2 = \frac{Fe+Be_b+W_r\cos\emptyset(x_1+uy_1)}{x_1+x_2+u(y_1-y_2)} \qquad (11)$$

When the guide reactions are below the axis of the bore as in Fig.(4) y_2 remains the same in the above formulae whereas y_1 reverses in sign. Hence for case (2)

$$Q_1 = \frac{Fe+Be_b-W_r\cos\emptyset(x_2-uy_2)}{x_1+x_2-u(y_1+y_2)} \qquad (12)$$

and

$$Q_2 = \frac{Fe+Be_b+W_r\cos\emptyset(x_1-uy_1)}{x_1+x_2-u(y_1+y_2)} \qquad (13)$$

The total guide friction becomes, $R=u(Q_1+Q_2)$ hence

$$R = \frac{2(Fe+Be_b)-W_r\cos\emptyset\, x_2+W_r\cos\emptyset\, uy_2+W_r\cos\emptyset\, x_1+W_r\cos\emptyset y_1}{x_1+x_2+u(y_1-y_2)}\, u$$

$$R = \frac{2(Fe+Be_b)+W_r\cos\emptyset[(x_1-x_2)+u(y_1+y_2)]}{x_1+x_2+u(y_1-y_2)}\, u \qquad (14)$$

Now if $M=x_1+x_2+u(y_1-y_2)$ for case I

or $M=x_1+x_2-u(y_1+y_2)$ for case II

and if $N=(x_1-x_2)+u(y_1+y_2)$ for case I

$N=(x_1-x_2)+u(y_2-y_1)$ for case II

we have therefore, in general that

$$R = \frac{2(Fe+Be_b)+W_r\cos\emptyset\, N}{M}\, u \qquad (15)$$

which gives the total guide friction. The value of the coefficient of friction u ranges from 0.15 to 0.20.

The total braking evidently becomes,

$$B + \frac{2(Pe+Be_b)+W_r \cos \emptyset\ N}{M}\ u = K + W_r \sin \emptyset$$

or $B(M+2uh) = (K+W_r \sin \emptyset)M - (2Pe+W_r \cos \emptyset\ N)u$

hence

$$B = \frac{(K+W_r \sin \emptyset)M - (2Pe+W_r \cos \emptyset N)u}{M+2uh}$$

which gives the total recuperator reaction in terms of the total resistance to recoil.

Denoting as before by

P'_h = the hydraulic reaction plus the joint frictions (stuffing box and piston)

P'_a = the total elastic reaction plus the joint frictions.

e_h = distance from center of gravity of recoiling parts to line of action of hydraulic brake pull P_h

e_a = distance from center of gravity of recoiling parts to line of action of the recuperator reaction P_a

The front and back clip reactions become,

$$Q_1 = \frac{Pe+\Sigma P'_a\ e_a+\Sigma P'_h\ e_h-W_r \cos \emptyset(x_2-uy_2)}{x_1+x_2+u(y_1-y_2)} \qquad (16)$$

and

$$Q_2 = \frac{Pe+\Sigma P'_a\ e_a+\Sigma P'_h\ e_h+W_r \cos \emptyset(x_1+uy_1)}{x_1+x_2+u(y_1-y_2)} \qquad (17)$$

y_1 reversing in sign when the guide reactions are entirely below the axis of the bores. Combining as before and noting that $R=u(Q_1+Q_2)$ we have

$$R = \frac{2Pe+2\Sigma P'_a\ e_a+2\Sigma P'_h\ e_h+W_r \cos \emptyset\ N}{M} \qquad (18)$$

where M and N are the constants referred before.

Now $K = \Sigma P'_a+\Sigma P'_h+R-W_r \sin \emptyset$ (19)

and combining (18) and (19) we get

$$KM=(\Sigma P_a'+\Sigma P_h' - W_r \sin \emptyset)M+n(2Fe+2\Sigma P_a'e_a+2\Sigma P_h'e_h+W_r\cos \emptyset\ N)$$

Simplifying,

$$KM=((\Sigma P_a'-W_r\sin\emptyset)M+\Sigma P_h' \times M+u(2Fe+2\Sigma P_a'e_a+W_r\cos \emptyset\ N)+2u\Sigma P_h'e_h$$

or further simplifying,

$$\Sigma P_h'(M+2e_h)=KM-(\Sigma P_a'-W_r\sin \emptyset)M-u(2Fe+2\Sigma P_a'e_a+W_r\cos \emptyset\ N)$$

Hence,

$$\Sigma P_h'=\frac{M(K-\Sigma P_a'+W_r\sin \emptyset)-u(2Fe+2\Sigma P_a'e_a+N\ W_r\cos \emptyset)}{M+2ue_h}$$

which gives the gross hydraulic pull in terms of the total resistance to recoil, the gross air or spring reaction and the maximum powder force.

APPROXIMATE FORMULAE GUIDE FRICTION — Assuming the reaction between the recoiling parts to be equivalent to a normal force passing through the center of gravity N, a couple M, and the braking and guide friction forces B and R having moment arms about the center of gravity of the recoiling mass equal to d_b and r respectively where r is the mean distance to the guide frictions, we have, for moments about the center of gravity of the recoiling parts,

$Be_h+Rr=M$ neglecting the powder effect which is usually very small and $N=W_r \cos \emptyset$ for the total reaction.

Obviously the actual normal guide reactions, becomes,

$$N_1=\frac{M}{l}-\frac{W_r\cos \emptyset\ x_2}{l}$$

and

$$N_2=\frac{M}{l}+\frac{W_r\ \cos \emptyset\ x_1}{l}$$

where $l = x_1+x_2$ also $R=u(N_1+N_2)$

hence $R = u\left(\frac{2M + W_r \cos \emptyset (x_1 - x_2)}{l}\right)$

Substituting the value of M, we obtain,

$$R = \frac{2uBe_b + u\, W_r \cos \emptyset (x_1 - x_2)}{l - 2ur} \qquad (21)$$

Very often $x_1 - x_2$ is small and in a preliminary design x_1 may be assumed equal to x_2

Hence

$$R = \frac{2uBe_b}{l - 2ur} \qquad (22)$$

which gives an approximate value of the guide friction, useful in a preliminary design - u may be assumed from 0.13 to 0.25.

Very often as in symmetrical barbette mounts, the value of Be_b may be small due to a small value of e_b and a certain limitation arises as to the use of the friction formula previously derived.

When $\frac{W_r \cos \emptyset\, x_2}{l} \geq \frac{M}{l}$

that is, $W_r \cos \emptyset\, x_2 \geqq Be_o + Rr \geqq Be_b$ approx. we have continuous contact along the guides, the distributed guide reaction oalancing the weight comoonent normal to the guides.

For such a condition the guide friction, becomes,

$$R = 0.2 \text{ to } 0.3 W_r \cos \emptyset \qquad (23)$$

INCREASE OF GUIDE FRICTION DURING POWDER PRESSURE PERIOD. If we assume the total oraking B to be constant during the powder pressure period, the guide friction R is augmented by the powder pressure couple together with the increased friction couple.

Let B = the constant braking force

F= the varying powder force

N_1 = the normal reaction of the stationary part on the recoiling mass.

R_1 = the guide friction during the powder pressure period.

M_1 = the reacting couple of the stationary part upon the recoiling mass.

R_2 and M_2 are the corresponding values during the retardation period.

Then, during the powder pressure period, we have

$$F-B-R_1+W_r \sin \emptyset = M_r \frac{d^2x}{dt^2}$$

$$N-W_r \cos \emptyset = 0 \qquad (24)$$

$$Fe+R_1 r+Be-M_1=0$$

and during the retardation period, we have,

$$B+R_2-W_r \sin \emptyset = M_r \frac{d^2x}{dt^2}$$

$$N-W_r \cos \emptyset = 0 \qquad (25)$$

$$R_2 r+Be-M_2=0$$

Further, let $\Delta M = M_1 - M_2$ and $\Delta R = R_1 - R_2$, then subtracting (25) from (24) we have $\Delta M = Fe + \Delta R\, r$ (26)

Now during the accelerating period the normal guide reactions, become,

$$N_1' = \frac{M_1}{l} - \frac{W_r \cos \emptyset\, x_2}{l}$$

$$N' = \frac{M_1}{l} + \frac{W_r \cos \emptyset\, x_1}{l} \qquad (27)$$

and during the subsequent retardation

$$N_1 = \frac{M_2}{l} - \frac{W_r \cos \emptyset\, x_2}{l} \qquad (28$$

$$N_2 = \frac{M_2}{l} + \frac{W_r \cos \emptyset\, x_1}{l}$$

Adding the two equations in (27) and (28) respectively and subtracting (28) from (27) and multiplying by the coefficient of friction n, we obtain obvious expression:

$$\Delta R = \frac{2\Delta M}{l} u \qquad (28)$$

Substituting (29) in (26), we have

$$\Delta M = Fe + \frac{2\Delta M}{l} ur \qquad (29)$$

and $$\Delta M = \frac{Fe}{1 - \frac{2ur}{l}} = \frac{Fel}{l - 2ur} \qquad (30)$$

and substituting in (29), we have for the change of friction during the powder period,

$$\Delta R = \frac{2Feu}{l - 2ur} \qquad (31)$$

Thus the guide friction is continuously augmented always proportional to the total powder pressure, providing the braking is assumed constant. We also note an additional cause of first class importance for the reduction of "e", that is, the importance of locating the center of gravity of the recoiling mass along the axis of the bore.

Another cause for a change in guide friction during the powder period is due to the torque reaction of the rifling, T_r though the total guide friction remains the same.

The normal reaction on the left guide, becomes,

$$N_{1l} = \frac{M}{2_l} - \frac{W_r \cos \emptyset x_2}{2_l} - \frac{T_r}{2dg}$$

$$(32)$$

$$N_{2l} = \frac{M}{2_l} + \frac{W_r \cos \emptyset x_1}{2_l} + \frac{T_r}{2dg}$$

and the same for the right guide, becomes,

$$N_{1r} = \frac{M}{2_l} - \frac{W_r \cos \phi x_2}{2_l} + \frac{T_r}{2dg} \qquad (33)$$

$$N_{2r} = \frac{M}{2_l} - \frac{W_r \cos \phi x_1}{2_l} + \frac{T_r}{2dg}$$

where dg is the distance between guides.

In the gun recoiling in a sleeve this torque must be balanced by the reaction of the key way.

Noting that, $R - u(N_{1l} + N_{1l} + N_{2l} + N_{2r}) = u(N_1 + N_2)$

the total friction remains the same.

It is important to note that the friction on the left guide over that on the right due to this rifling introduces a couple in the plane of the guides which tends to cause rotation about an axis normal to the plane of the guides. Therefore, it is always essential that small side grooves or flanges on the clips be introduced. The additional friction on the flanges is entirely neglegible, but the normal reaction to the flange in extreme cases may be considerable.

During the recoil the guide friction is seldom constant since the distance between clip reactions progressively decreases in the recoil, that is the front clip approaches the rear part of the guide in recoil. When the recoil is long it is desirable on field carriages to have an additional clip near the muzzle which engages in the guide sometime later in the recoil. Due to this cause the guide friction continually increases until the engagement of the outer clip and then we have a sudden drop in the magnitude of the friction.

When the braking pull remains constant and the powder pressure couple is small and no outer clips are introduced during the recoil, the clip reactions should always be designed for the condition of out of battery.

To recapitulate in the limitations in design so far as guide is concerned, we note,

(1) The bearing pressures and consequent friction of the guide are reduced by increasing the distance between the clip reactions nearly directly, consequently for a given guide reaction and friction, we have a minimum distance between clip contacts on the guides.

(2) The guide reactions are reduced by bringing the resultant of the rod pulls through the center of gravity of the recoiling parts.

(3) The moment effect and consequent guide reactions are further reduced by bringing the resultant guide friction line through the center of gravity of the recoiling mass.

(4) It is highly desirable to center the center of gravity of the recoiling mass midway between the guide reactions. This condition is usually impossible to attain especially out of battery, but may be compensated by increasing "l" the distance between the clips, by an additional front clip near the muzzle.

(5) Proper functioning of the recoil may be entirely destroyed by having the center of gravity of the recoiling mass too far below the axis of the bore, thus introducing a powder pressure couple with excessive guide friction during the powder pressure period. This powder pressure couple may cause a "springing" of the guides and considerable heating as well. The center of gravity of the recoiling mass should never exceed 1.5" from the axis of the bore unless a friction disk for rotation during recoil about the trunnions is introduced.

COMPUTATION OF BRAKING PULLS We have seen from the previous discussions that the guide friction is not independent of the braking pulls due to the hydraulic and recuperator reactions. These pulls tend to cause rotation and thus augment the guide friction over that due to the weight component.

The total resistance to recoil is given by:-

(1) <u>when constant during recoil,</u>

$$K=\frac{\frac{1}{2}m_r V_f^2}{b-E+V_f T} \quad \text{(lbs)}$$

where V_f= maximum free recoil velocity (ft/sec)

T = total powder period (sec)

E=free recoil displacement during powder period (ft)

b= length of recoil (ft)

m_r= recoiling mass

(2) <u>when variable consistent with the stability slope "m",</u>

$$K=\frac{m_r V_f^2+m(b-E)^2}{2[b-E+V_f T-\frac{m}{2}\frac{T^2}{m_r}(b-E)]} \quad \text{(lbs)}$$

where

$$K=B+R_g-W_r\sin\emptyset=P_h'+P_a'+u(Q_1+Q_2)-W_r\sin\emptyset \quad \text{(lbs)}$$

and $B=P_h'+P_a'$= total braking

$R_g=u(Q_1+Q_2)$= mean guide friction assumed constant

We have seen $R_g=\dfrac{2uBe_b}{l-2ur}$ lbs.approx.

where u=0.15 to 0.2

e_b= distance from center of gravity of recoiling parts to line of action of B. (in)

B= total hydraulic and recuperator pull (lbs)
l= total distance between clip reactions (in)
r= distance from center of gravity of recoiling parts to mean friction line (in)

then,

$$B+\frac{2uBe_b}{1-2ur}-W_r \sin \emptyset=K \quad \text{(lbs)}$$

$$B\left(1+\frac{2ue_b}{1-2ur}\right)=K+W_r \sin \emptyset$$

hence,

$$B=\frac{(K+W_r \sin \emptyset)(1-2ur)}{1+2u(e_b-r)} \quad \text{(lbs)}$$

Further since,

$$P_a'+P_h'+\frac{2u(P_h'e_h+P_a'e_a)}{1-2ur}=K+W_r \sin \emptyset$$

we have on simplifying

$$P_h'=\frac{(K+W_r \sin \emptyset)(1-2ur)-P_a'[1+2u(e_a-r)]}{1-2u(e_h-r)} \quad \text{(lbs)}$$

Very approximately,

$$R_g=0.3\ W_r \cos \emptyset$$

and

$$B=K+W_r \sin \emptyset-0.3\ W_r \cos \emptyset \quad \text{(lbs)}$$

$$P_h'=K+W_r \sin \emptyset-P_a'-0.3W_r \cos \emptyset \quad \text{(lbs)}$$

INCREASE OF RESISTANCE TO RECOIL DURING POWDER PERIOD

During the powder period, the powder pressure couple may be sufficient to cause a large increase in the guide friction, whereas the braking pulls due to the hydraulic resistance and recuperator reaction are not

affected. From the previous discussion on guide friction, the increment in guide friction equals,

$$\Delta R_g = \frac{2Feu}{1-2ur} \qquad \text{(lbs)}$$

or more exactly

$$\Delta R_g = \frac{2Feu}{M}$$

where

$M = x_1 + x_2 + u(y_1 - y_2)$ Guides above and below axis of bore

$\quad = x_1 + x_2 - u(y_1 + y_2)$ Guides entirely below axis of bore.

Hence the total resistance to recoil becomes during the powder period,

$$K' = K + \Delta R_g \qquad \text{(lbs)}$$

and this value should be used in the computation of the trunnion and elevating gear reactions.

Strictly speaking, the value of K is slightly high, since the augmented friction due to a large powder pressure couple, will diminish the maximum velocity of restrained recoil and thus the resistance to recoil for a given displacement b.

A more exact value of the resistance to recoil can be estimated as follows:

Thus,

$$K\left[b - \left(E - \frac{(K+\Delta R_g)T^2}{2m_r}\right)\right] = \frac{1}{2} m_r \left[V_f - \frac{(K+\Delta R_g)T}{m_r}\right]^2$$

Simplifying, we have,

$$K = \frac{\frac{1}{2} m_r V_f^2 - V_f \Delta R_g T - \frac{\Delta R_g^2 T^2}{2m_r}}{b - E + V_f T - \frac{\Delta R_g T^2}{2m_r}} \qquad \text{(lbs)}$$

where

$$\Delta R_g = \frac{2uF_m e}{x_1 + x_2 + u(y_1 - y_2)} = \frac{2uF_m e}{1 - 2ur} \quad \text{(approx)(lbs)}$$

and

$$F_m = \frac{P_e X_1 + 0.5 P_{ob} X_2}{E} \quad \text{(lbs)}$$

From interior ballistics, we have,

$$P_e = \frac{wv^2}{5.36u} \; ; \; P_{ob} = 1.12 \frac{w}{g} \frac{a^2 b' u_o}{(b' + u_o)^3}$$

w = weight of projectile (lbs)

u_o = total travel up bore (ft)

v = muzzle velocity(ft/sec)

$$b' = u_o \left[\left(\frac{27}{16} \frac{P_m}{P_e} - 1 \right) \pm \sqrt{\left(1 - \frac{27}{16} \frac{P_m}{P_e}\right)^2 - 1} \right] \quad \text{(ft)}$$

$$a = \frac{v(b' + u)}{u} \quad \text{(ft/sec)}$$

where P_m = total maximum powder force (lbs)

Unless the powder pressure couple is excessive, that is the center of gravity of the recoiling parts is considerably below the axis of the bore the above refinement in calculation is unnecessary. When e exceeds 1.5 to 2 inches the above effect becomes of consideration.

INTERNAL STRESS IN THE RECOILING PARTS

It is very important to observe that the braking force B when treating of the external forces on the recoiling masses as in the previous discussions refers always to the reaction of the oil in the hydraulic brake and the spring or com-

pressed air reaction of the recuperator. During the accelerating period the reaction on the gun lug may differ considerably from the braking force B due to the acceleration of the piston and rods where these recoil with the gun or to the acceleration of the recuperator sleigh or slide when the sleigh recoils and the rods are fixed to the carriage.

If now we consider a recoiling mass consisting of a gun together with a single cylinder recoiling with the gun, figure (4) and if we let,

B= the total braking force along the axis of the cylinder.

B'=the normal reaction of cylinder on the gun lug.

j'=the tangential or shear reaction on the gun lug.

M'=the bending moment reaction on the gun lug.

Neglecting the guide friction, let,

Q_1 and Q_2 be the normal guide reactions

x_1' and x_2' the coordinates along the axis of the bore of the clip reactions with origin at the center of gravity of the gun.

x_1 and x_2 the coordinates parallel to the axis of the bore with origin at the center of gravity of the recoiling parts.

x_c and y_c=the coordinates of the center of gravity of the recoiling cylinder with respect to the center of gravity of the gun as origin.

e_b= distance from the center line of the recoil cylinder to a line through the center of gravity of recoiling parts parallel to the axis of the bore.

M_r and W_r= mass and weight of the recoiling parts.

M_c and W_c= mass and weight of the recoiling cylinder.

j = the distance from the shear reaction on the gun lug to the center of gravity of the gun

itself.

F= the maximum powder force along the axis of the bore.

If the mass of the lug is negligible as compared with the mass of the gun, the coordinates of the center of gravity of the recoiling mass with respect to that of the gun becomes

$$x_r = \frac{W_c x_c}{W_r} \text{ and } y_r = e = \frac{W_c y_c}{W_r}$$

Further

$$x_1 = x_1' - \frac{W_c x_c}{W_r} \text{ and } y_1 = y_1' - \frac{W_c y_c}{W_r}$$

$$x_2 = x_2' + \frac{W_c x_c}{W_r} \text{ and } y_2 = y_2' - \frac{W_c y_c}{W_r}$$

Now considering the gun with its lug alone, the reaction of the recoil cylinder on the lug, consists of the pull B' a bending moment $W_c \cos \phi \, (x_c + j)$ and a shear reaction $W_r \cos \phi$.

Taking moments about the center of gravity of the gun, we have $B'(b+e) + W_c \cos \phi \, j - W_c \cos \phi (x_c + j) = Q_2 x_2' + Q_1 x_1'$

$$B' e_b + B' e - W_c \cos \phi \, x_c = Q_1 \left(x_1 + \frac{W_c x_c}{W_r}\right) + (Q_1 W_r \cos \phi)\left(x_2 - \frac{W_c x_c}{W_r}\right)$$

Simplifying, we have,

$$B' e_b + B' e = Q_1 (x_1 + x_2) + W_r \cos \phi \, x_2$$

Considering the recoil cylinder alone we note that during the accelerating period,

$$B' - B + W_c \sin \phi = M_c \frac{d^2x}{dt^2}$$

but from the recoiling mass, we have,

$$\frac{d^2x}{dt^2} = \frac{F-B+W_r \sin \emptyset}{M_r}$$

hence $B' = B - W_c \sin \emptyset + \frac{W_c}{W_r}(F-B+W_r \sin \emptyset)$ and substituting in the previous equation, we have,

$$(B-W_c \sin \emptyset + \frac{W_c}{W_r}(F-B+W_r \sin \emptyset))(e_b+e) = Q_1(x_1+x_2)+W_r \cos \emptyset\ x_2$$

but $y_c = e_b+e$ and $y_r = e$ hence $(e_b+e) = \frac{W_r e}{W_c}$

Therefore, substituting in the above equation, we have,

$$B(e_b+e)-W_r e \sin \emptyset + Fe - Be + W_r c \sin \emptyset = Q_1(x_1+x_2)+W_r \cos \emptyset\ x_2$$

hence $Be_b+Fe = Q_1(x_1+x_2)+W_r \cos \emptyset\ x_2$

Obviously if we consider the recoiling mass, fig. (4) we have, taking moments about the center of gravity

$$Be_b+Fe = Q_1x_1+Q_2x_2 \quad \text{but } Q_2-Q_1 = W_r \cos \emptyset$$

Hence $Be_b+Fe = Q_1(x_1+x_2)+W_r \cos \emptyset\ x_2$ the same equation as obtained above as of course we should expect. The above discussion shows the importance of considering either the mass of the gun with its proper external reactions or the mass of the recoiling parts with its proper external reactions and not confusing the mass, of the gun and recoiling parts, and the coordinates of their center of gravities.

The maximum stress in a section m – m, see fig. (4) of the gun lug obviously occurs when the bending moment due to the weight of the recoil cylinder is a minimum and the braking force B a maximum that is at maximum elevation. In the above discussion the normal reaction between the

piston surface and cylinder was assumed zero. This reaction obviously depends upon the weight and relative deflections of the rods and cylinders. If these weights were equal and at the same distance from the point of support, and with equal elasticity, this reaction becomes zero and we have the bending moment assumed; but since the rods are relatively very elastic as compared with the cylinder in general the moment $W_c \cos \phi \, (x_c + j)$ if anything is augmented.

If I_{mn} is the moment of inertia of the section "m - m", A_{m-m} its area of cross section, and y is the distance to the edge from the neutral axis of the section and "g" the distance from B' to the neutral axis, we have for the maximum fibre stress

$$f_{m-m} = \frac{[B'g - W_c \cos \phi (x_c + J)]y}{I_{m-m}} + \frac{W_c \cos \phi}{A_{m-m}} \qquad (34)$$

where $B' = B - W_c \sin \phi + \frac{W_c}{W_r} (F - B + W_r \sin \phi)$

Since the weight components are small as compared with the powder pressure force and braking for a first approximation, we have,

$$f_{m-m} = \frac{[B + \frac{W_c}{W_r}(F-B)]g}{I_{m-m}} \qquad (35)$$

which is a useful formulae for practical design.

TIPPING PARTS

The tipping parts consist of all the parts that move in elevation with the gun. The two principle parts of the tipping parts are the recoiling parts and cradle, the one moving in recoil and the other remaining stationary. The cradle supports by its guides the

recoiling parts on recoil, it takes the reaction of the braking exerted on the recoiling mass and is supported by trunnions resting in bearings in the top carriage and is further prevented from rotating about these trunnions during the recoil by the reaction between the elevating pinions of the top carriage and the elevating arc of the cradle. When a rocker is introduced between the elevating pinion and cradle for an independent line of sight it should, properly speaking, be included in the tipping parts.

It is of fundamental importance to always balance the center of gravity of the tipping parts about the trunnion axis since with massive parts the elevating process must be done quickly and with the minimum reaction on the elevating pinion of the top carriage.

Let x and y = the coordinates parallel and normal to the axis of the bore.

X and Y = the x and y components of the trunnion reactions.

F = the total powder pressure force.

E = the reaction between the pinion and elevating arc.

j = the radius of the elevating arc.

θ_e = the angle between the "y" axis and the radius to the elevating pinion contact with the elevating arc.

The mutual reaction between the tipping parts and top carriage may be divided into the component reactions X and Y of the trunnions and the elevating arc reaction E.

By D'Alembert's principle, considering the inertia of the recoiling mass as an equilibriating force, we have during the powder pressure period assuming the gun practically in battery, for equilibrium of the tipping parts, that, fig. (5)

$$F - M_r \frac{d^2x}{dt^2} - 2X + W_t \sin \phi + E \cos \theta_e = 0 \qquad (1)$$

for motion along the "x" axis,

$$2Y - W_t \cos \emptyset - E \sin \theta_e = 0 \qquad (2)$$

for motion along the "y" axis, and

$$F(e + s) - M_r \frac{d^2x}{dt^2} \bar{s} - E\,j = 0 \qquad (3)$$

For moments about the trunnions, the weight of the tipping parts having no moment since the center of gravity is at the trunnions <u>in battery.</u>

But $F - M_r \frac{d^2x}{dt^2} = K_a$ the total resistance to recoil during the accelerating period.

Hence equation (3) reduces to: $Fe + K_a s - E\,j = 0$ and the reaction on the elevating arc in battery becomes,

$$E = \frac{Fe + K_a s}{j} \qquad (4)$$

and the trunnion reactions in battery, becomes,

$$\left.\begin{aligned} 2X &= K_a + W_t \sin \emptyset + \frac{(Fe + K_a s)}{j} \cos\theta_e \;{}^{*} \\ 2Y &= W_t \cos \emptyset - \frac{(Fe + K_a s)}{j} \sin \theta_e \end{aligned}\right\} \qquad (5)$$

the resultant trunnion reaction being

$S = \sqrt{X^2 + Y^2}$ making an angle $\tan^{-1} \frac{Y}{X}$ with the "x" axis,= now if, fig. (5)

W_t = total weight of the tipping parts

W_c = weight of the cradle of the tipping parts

W_r = weight of the recoiling parts

l_t = the horizontal distance to the center of gravity of the tipping parts (in battery) always assumed at the trunnions from the hinge point of the top carriage.

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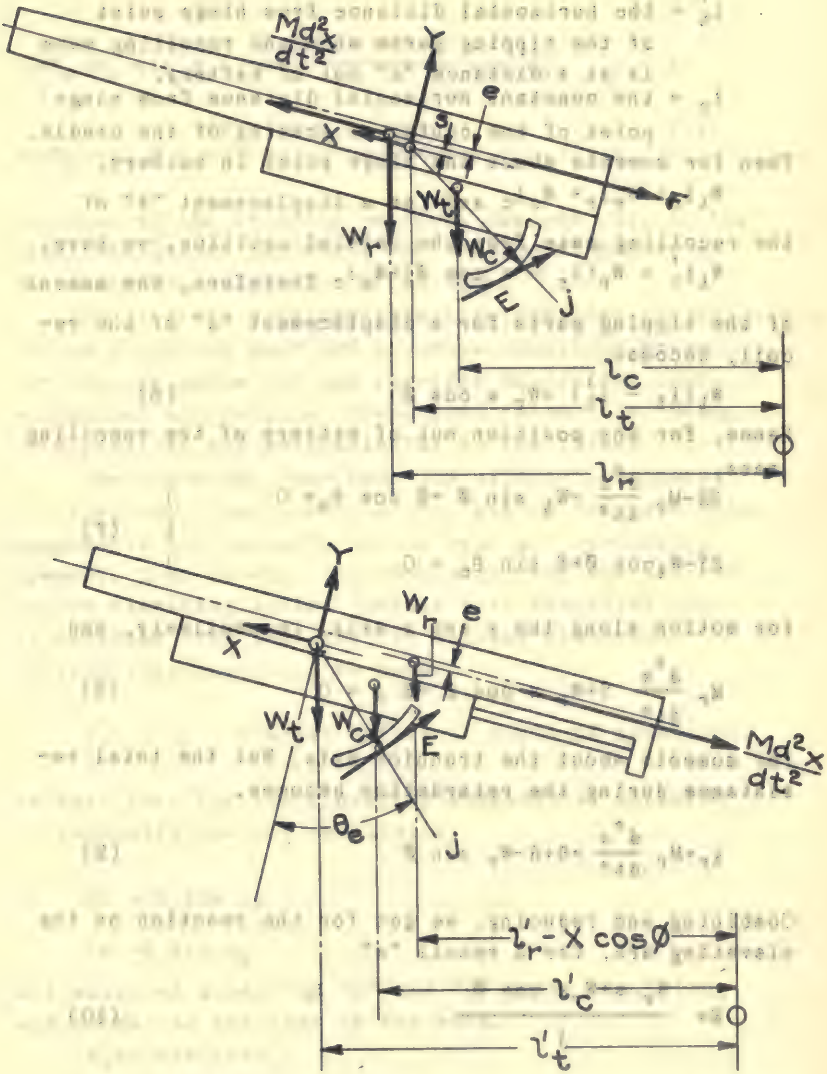
* To account for contact of tooth rack with the pinion or worm of the elevating gear, the reaction E makes an angle approx. 20° with the tangent to pitch lines. Therefore in above equations (eq.5) "j" becomes, "j cos 20" and "cos θ_e" becomes "cos (θ_e+20)" and "sin θ_e" becomes "sin(θ_e+20)".



Md²x/dt²
Y
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Fig. 5

l_t' = the horizontal distance from hinge point of the tipping parts when the recoiling mass is at a distance "x" out of battery.

l_c = the constant horizontal distance from hinge point of the center of gravity of the cradle.

Then for moments about the hinge point in battery,

$W_t l_t = W_r l_r + W_c l_c$ and for a displacement "x" of the recoiling mass from the initial position, we have,

$W_t l_t' = W_r(l_r - x \cos \phi) + W_c l_c$ Therefore, the moment of the tipping parts for a displacement "x" of the recoil, becomes,

$$W_t(l_t - l_t') = W_r x \cos \phi) \qquad (6)$$

Hence, for any position out of battery of the recoiling mass,

$$\left.\begin{aligned} &2X - M_r \frac{d^2x}{dt^2} - W_t \sin \phi - E \cos \theta_e = 0 \\ &2Y - W_t \cos \phi + E \sin \theta_e = 0 \end{aligned}\right\} \qquad (7)$$

for motion along the x and y axis, respectively, and

$$M_r \frac{d^2x}{dt^2} \; S + W_r \; x \cos \phi - E \; j = 0 \qquad (8)$$

for moments about the trunnion axis. But the total resistance during the retardation becomes,

$$K_r = M_r \frac{d^2x}{dt^2} = B + R - W_r \sin \phi \qquad (9)$$

Combining and reducing, we get for the reaction on the elevating arc, for a recoil "x"

$$E = \frac{K_r \; s + W_r x \cos \phi}{j} \qquad (10)$$

and the trunnion reactions for a recoil "x"

$$2X = K_r + W_t \sin \emptyset + \frac{(K_r s + W_r x \cos \emptyset)}{j} \cos \theta_e \quad * \qquad (11)$$

$$2Y = W_t \cos \emptyset - \frac{(K_r s + W_r x \cos \emptyset)}{j} \sin \theta_e$$

which shows the trunnion reaction depends only on the total resistance to recoil and the moment effect of the recoiling weight out of battery.

It is often more convenient to consider the reactions of the elevating gear and trunnion reactions between the tipping parts and top carriage as divided into horizontal and vertical components rather than along axis parallel and perpendicular to the guides.

The elevating gear reaction will be considered positive when the line of action may be resolved into components horizontally to the rear and vertically upwards, that is when the radius joining the trunnion to the elevating pinion contact with elevating rack is measured from the vertical counter clockwise. Calling this angle n_e, we have,

$\theta_e = \emptyset + n_e$ whereas before $\emptyset$= angle of elevation.

We have then for the elevating components, measured horizontally and not vertically.

$$He = E \cos n_e$$

$$Ve = E \sin n_e$$

and measured along the "x" and "y" axis, i.e. along and normal to the axis of the bore,

$$X_e = E \cos (\emptyset + n_e)$$

$$Y_e = E \sin(\emptyset + n_e)$$

---

* More strictly to account for obliquity of tooth contact of elevating mechanism, j becomes j cos 20, cos Θ_e becomes cos $(\Theta_e + 20)$ and sin Θ becomes $\sin(\Theta_e + 20)$.

The horizontal and vertical components of the trunnion reaction become, in battery,

$$\left.\begin{aligned} 2H &= K_a \cos \emptyset + \frac{(Fe+Ks)}{j} \cos n_e \\ 2V &= K_a \sin \emptyset + W_r - \frac{Fe+Ks}{j} \sin n_e \end{aligned}\right)$$

and out of battery,

$$\left.\begin{aligned} 2H &= K_r \cos \emptyset + \frac{(K_r s + W_r x \cos \emptyset)}{j} \cos n_e \\ 2V &= K_r \sin \emptyset + W_t - \frac{(K_r s + W_r x \cos \emptyset)}{j} \sin n_e \end{aligned}\right)$$

and the resultant trunnion reaction becomes,

$$S = \sqrt{H^2 + V^2}$$

INTERNAL REACTIONS OF TIPPING PARTS - ROCKER INTRODUCED.

It is important to observe that X, Y and E are the external reactions exerted by the top carriage on the tipping parts which include a rocker if used. The total reaction on the trunnions include the reaction of the top carriage X and Y and the reaction of the rocker X_r and Y_r. Hence the resultant reaction on the trunnions, become, algebraically,

$$\left.\begin{aligned} X' &= X + X_r \\ \text{and} \quad Y' &= Y + Y_r \end{aligned}\right) \quad (12)$$

= the shear components of the trunnion pins on the cradle.

The reactions on the rocker, alone, therefore, become, the reaction of the top carriage pinion E, the reaction of the trunnion X_r and Y_r reversed and the reaction of the cradle M reversed. In many cases an elevating screw is used between the cradle and rocker and when used, M reversed is the reaction of the elevating screw,

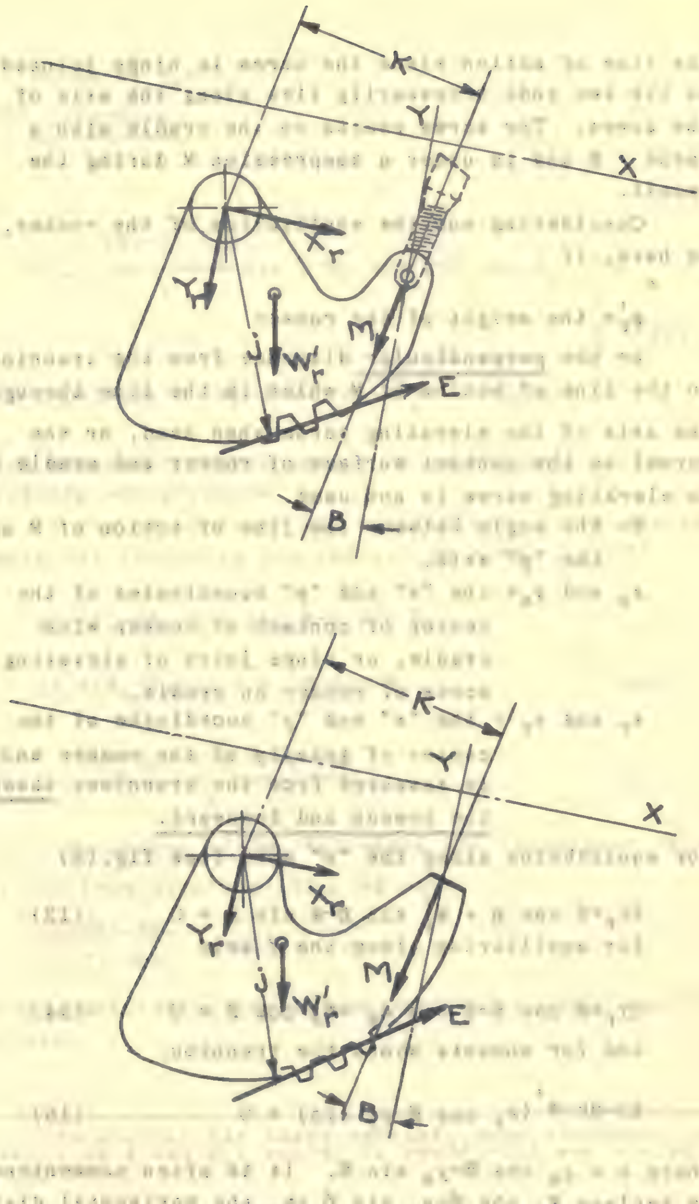

Fig. 6

its line of action since the screw is hinge jointed at its two ends necessarily lies along the axis of the screw. The screw reacts on the cradle with a force + M and is under a compression M during the recoil.

Considering now the equilibrium of the rocker, we have, if

W_r'= the weight of the rocker

k= the perpendicular distance from the trunnions to the line of action of M which is the line through the axis of the elevating screw when used, or the normal to the contact surface of rocker and cradle when an elevating screw is not used.

B= the angle between the line of action of M and the "y" axis.

x_m and y_m= the "x" and "y" coordinates of the center of contact of rocker with cradle, or hinge joint of elevating screw of rocker on cradle.

x_r and y_r = the "x" and "y" coordinate of the center of gravity of the rocker and is measured from the trunnions towards the breech and downward.

For equilibrium along the "x" axis (see fig.(6)

$$2X_r+E \cos \theta + W_r' \sin \emptyset - M \sin B = 0 \qquad (13)$$

for equilibrium along the Y axis

$$2Y_r+M \cos B-E \sin \theta_e +W_r' \cos \emptyset = 0 \qquad (14)$$

and for moments about the trunnion,

$$Ej-Mk-W'(x_r \cos \emptyset - y_r \sin) = 0 \qquad (15)$$

where $k = x_m \cos B+y_m \sin B$. It is often convenient to replace $X_r \cos \emptyset - y_r \sin \emptyset = h_r$ the horizontal distance from the trunnions to the center of gravity of the rocker, then (15) reduces to

$$Ej-Mk-W_r' \, h_r = 0 \qquad (15')$$

hence

$$M = \frac{Ej-W_r' \, h_r}{k} \qquad (16)$$

and

$$\left.\begin{aligned} 2X_r &= \frac{(Ej-W_r' \, h_r)}{k}\sin B - W_r' \sin \emptyset - E \cos \theta_e \\ 2Y_r &= E \sin \theta - W_r' \cos \emptyset - \frac{(Ej-W_r' \, h_r)}{k}\cos B \end{aligned}\right\} \qquad (17)$$

which shows the rocker reactions depend only on the elevating arc pressure.

If now we consider the equilibrium of the tipping parts not including the rocker, we have, fig.(7)

$$\left.\begin{aligned} 2X' &= K_r + (W_t - W_r') \sin \emptyset + M \sin B \\ 2Y' &= (W_t - W_r') \cos \emptyset - M \cos B \end{aligned}\right\} \qquad (18)$$

and

$$M = \frac{K_r s + W_r x \cos \emptyset - W_r' \, h_r}{k} \qquad (19)$$

since the moment of $(W_t - W_r')$ about the trunnions $= -W_r' \, h_r$ but now from equation (12), we have

$$\left.\begin{aligned} 2X &= WX' - 2X_r \\ 2Y &= 2Y' - 2Y_r \end{aligned}\right\}$$

hence substituting the values obtained in (17) and (18), we have,

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* To account for tooth contact, more strictly, replace j to j cos Θ_e; cos Θ_e to cos.(Θ_e+20) and sin. Θ_e to sin(Θ_e+20).

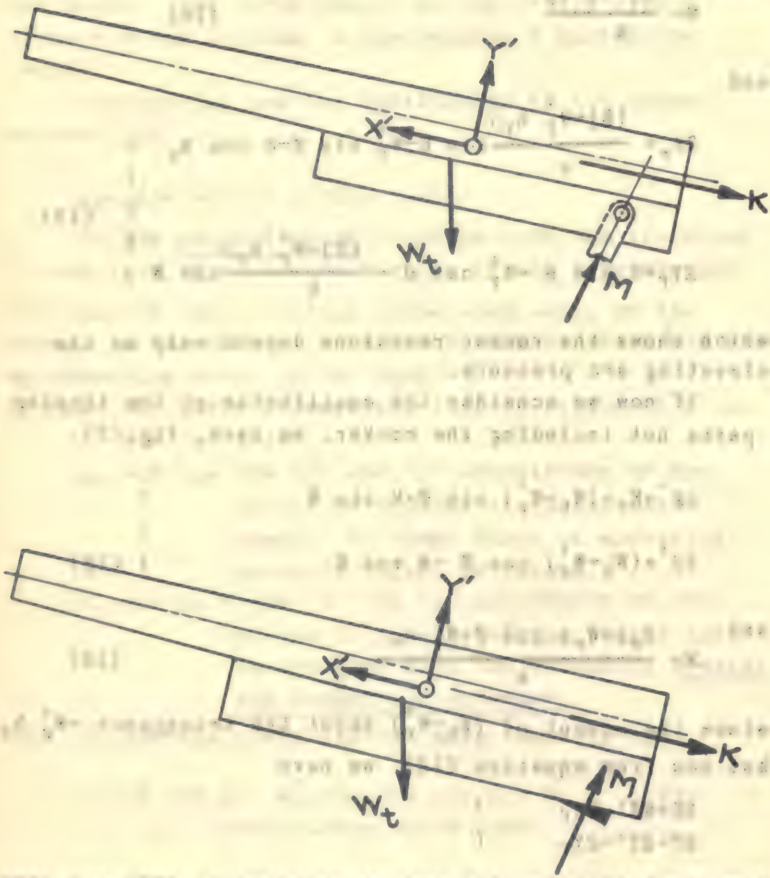

Fig. 7

$$\left\{\begin{array}{l} 2X=K_r+(W_t-W_r') \sin \emptyset+M \sin B-(M \sin B-W_r' \sin \emptyset-E \cos \theta_e) \\ 2Y=(W_t-W_r') \cos \emptyset-M \cos B-(E \sin\theta_e-W_r' \cos \emptyset-M \cos B) \end{array}\right.$$

Simplifying these values reduce to the former values,

$$2X=K_r+W_t \sin \emptyset+E \cos \theta_e$$

$$2Y=W_t \cos \emptyset -E \sin \theta_e$$

and further

$$\frac{K_r s +W_r\ x \cos \emptyset-W_r' h_r}{k} = M = \frac{Ej-W_r' h_r}{k}$$

and therefore as before

$$E=\frac{K_r s+W_r\ x \cos \emptyset}{j}$$

thus checking the formulas derived for the rocker reactions.

TRUNNIONS LOCATED AT THE REAR - BALANCING GEAR OR EQUILIBRATOR. In guns shooting at high elevation such as anti-aircraft guns, mortars, and even howitzers, it is often necessary to locate the trunnions in the rear or near the breech of the gun in order to prevent the breech of the gun from striking the ground during the recoil when the gun is elevated. Obviously it is impracticable to balance the tipping parts about the trunnions without some sort of a balancing gear commonly known however, as an equilibrator.

An equilibrator should balance the tipping parts, since the center of gravity of the tipping parts are now displaced forward of the trunnions, at all angles of elevation when the gun is in battery. It consists sometimes of a cam arc, a chain passing over the contour of the cam arc and connected with a spring cylinder oscillating about the trunnions fixed to the top carriage to take care of the small deflection due

to the change of radius of the cam.

PROCEDURE IN DESIGN OF EQUILIBRATOR:

Let W_t = weight of the tipping parts.

h_t = horizontal distance from the trunnions to the center of gravity of the tipping parts (gun in battery).

r_o = equivalent radius of cam at horizontal elevation.

r_n = final equivalent radius of the cam where the cam arc has turned through the maximum angle of elevation = ϕ.

R = Mean radius of cam.

d_h = deflection of spring at zero elevation.

d_o = deflection of spring at maximum elevation.

C = spring constant.

ϕ = the angle of elevation expresses in radians.

The spring in the spring cylinder is arranged so that it is in general under compression. As the gun elevates, the compression of the spring is decreased in virtue of the motion of the cam. The total motion of the spring becomes,

$R\phi = d_h - d_o$ approx. during the elevation ϕ where

$$R = \frac{r_o + r_n}{2}$$ for a first approximation and

$c\, d_h r_o = W_t h_t$

$C\, d_o r_n = W_t h_t \cos\phi.$

for equilibrium about the trunnions for all elevations.

If now we assume

$$d_h = (\tfrac{2}{3} \text{ to } \tfrac{3}{4})d \text{ solid}$$

and

$$d_o = (\tfrac{1}{4} \text{ to } \tfrac{1}{3})d \text{ solid}$$

Then for a preliminary design, we have,

$$C = \frac{W_t h_t}{(\frac{2}{3} \text{ to } \frac{3}{4})\text{d Solid } r_o}$$

$$C = \frac{W_t h_t \cos \phi}{(\frac{1}{4} \text{ to } \frac{1}{3})\text{ d solid } r_n}$$

$$\text{and } (\tfrac{2}{3} \text{ to } \tfrac{3}{4})\text{d solid } -(\tfrac{1}{4} \text{ to } \tfrac{1}{3})\text{d solid} = (\frac{r_o+r_n}{2})\phi$$

The unknowns in the above equations are C, d solid r_o and r_n; hence if we assume any one of these values the remaining values are determined.

The equivalent radius of the cam may be obtained by a "point by point" method as follows·-

The initial radius, becomes,

$$r_o = \frac{E_t h_t}{Cd_h}$$

Now move the cam an increment angle $\Delta\phi$ where $n\Delta\phi=\phi$ the total angle of elevation, and we have,

$$r_1 = \frac{W_t h_t \cos \phi_1}{C(d_h - r_o\Delta\phi)} \quad \text{and } \Delta d_1 = r_o\Delta\phi$$

then

$$r_2 = \frac{W_t h_t \cos \phi_2}{C[d-(r_o+r_1)\Delta\phi]} \quad \text{and } \Delta d_2 = (r_o+r_1)\Delta\phi$$

$$r_n = \frac{W_t h_t \cos \phi}{C[d_h-(r_o+r_1----r_{n-1})\Delta\phi]} \quad \text{and } d_h - d_o = (r_o+r_1---r_{n-1})$$

$$\Delta\phi$$

Strictly speaking the angle $\Delta\phi$ in the above procedure should be augmented by the angle $\frac{r_n - r_o}{nD}$ radians.

Where D = the perpendicular distance from the trunnions to the extremity of the equivalent cam radius.

From the equivalent radius thus obtained the cam contour may be drawn by drawing in a curve always tangent to the

perpendiculars to these radius, drawn from their extremities.

With a balancing gear or equilibrator, the trunnion reactions are modified and now become if

T = the tension in the chain.

a = the angle T makes with axis "X", (taken along the axis of the bore)

$$2X = K + W_r \sin\emptyset + \left(\frac{Ks + W_r X \cos\emptyset + Fe}{j}\right)\cos\theta_e - T\cos a \quad *$$

$$2Y = W_t \cos\emptyset - \left(\frac{Ks + W_r X \cos\emptyset + Fe}{j}\right)\sin\theta_e + T\sin a$$

and

$$E = \frac{Ks + W_r X \cos\emptyset + Fe}{j}$$

Usually d = 0, and thus the Y component of the trunnion reaction is unaffected.

DIRECT ACTING BALANCING GEAR Another form of balancing gear, for balancing the tipping parts at all angles of elevation about the trunnions which are located to the rear, consists of a spring or pneumatic oscillating cylinder and its rod directly connected between the tipping parts and the top carriage.

In the position of the tipping parts at zero elevation, gives maximum moment and therefore requires the maximum balancing reaction.

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* To account for tooth contact of elevating mechanism, replace j to j cos 20, cos θ_e to cos $(\theta_e + 20)$ and sin θ_e to $\sin(\theta_e + 20)$.

Let

W_t = weight of the tipping parts

h_t = horizontal distance from the trunnions to the center of gravity of the tipping parts (gun in battery).

x_t and y_t = coordinates along and normal to bore from trunnion to center of gravity of tipping parts (gun in battery).

$\emptyset$ = angle of elevation.

$\emptyset_m$ = maximum elevation.

r = radius from the trunnion to the crank pin which connects the tipping parts to the piston rod of the oscillating cylinder.(in.)

R = reaction exerted by the balancing gear along the piston rod of the oscillating cylinder. (lbs)

d_t = moment arm of R about trunnion.(in)

d_h = deflection of spring at horizontal elevation. (in)

d_o = deflection of spring at maximum elevation. (in)

C = spring constant.

R_i = initial balancing reaction (0° elev.) (lbs)

R_t = final balancing reaction ($\emptyset_m^\circ$ elev.)(lbs)

S = stroke of piston in oscillating cylinder.(in)

p_t = final air pressure in pneumatic balancing cylinder (lbs/sq.in.)

p_i = initial air pressure in pneumatic balancing cylinder (lbs/sq.in.)

A = effective area of balancing piston (sq.in.)

V_o = initial air volume (cu.in.)

At any angle of elevation $\emptyset$, we must have,

$R\, d_t = E_t(x_t \cos \emptyset - y_t \sin \emptyset)$

In general the center of gravity of the tipping parts lies approximately along the axis of the trunnions, and therefore, $R\, d_t = W_t\, x_t \cos \emptyset$.

If d_t remains constant, the reaction R should decrease as a cosinefunction in the elevation. Since it is usually impossible to decrease R according to a cosine function, we may so locate the trunnion of the oscillating cylinder so that the product $R\ d_t = W_t x_t \cos \emptyset$. By properly locating the trunnion axis of the oscillating cylinder a very close balancing is possible throughout the elevation either with a spring or pneumatic balancing piston.

For a preliminary layout, we may start by locating the trunnion axis of the oscillating cylinder somewhere depending upon clearance considerations, along a line parallel to the chord joining the assumed initial and final positions of the crank and midway between the chord and middle of arc. The crank turns an angle equal to the total elevation $\emptyset_m$.

Then the initial or horizontal balancing reaction, becomes,

$R_i\ \frac{r}{2}\ (1 + \cos \frac{\emptyset_m}{2}) = W_t\ x_t$ and the final balancing

reaction, becomes for maximum elevation,

$$R_f \frac{r}{2}(1 + \cos \frac{\emptyset_m}{2}) = W_f(x_t \cos \emptyset_m + y_t \sin \emptyset_m)$$

$$= W_t\ x_t \cos \emptyset_m \quad \text{(approx.)}$$

We have, therefore, $\frac{R_f}{R_i} = \cos \emptyset_m$ very roughly, and the total stroke of the piston, becomes, $S = 2r \sin \frac{\emptyset_m}{2}$ (in)

Now with a spring cylinder, if,

$d_h = \frac{2}{3}$ to $\frac{3}{4}$ solid height of spring (in)

$d_o = \frac{1}{4}$ to $\frac{1}{3}$ solid height of spring (in)

then

$d_h - d_o = S$ (in)

and at 0° elev. $R_i = c\, d_h$ at max. elev. $R_f = c\, d_o$.

Hence the required spring, may be approximated by the solution of the following equations,

$$\left.\begin{aligned} &\frac{R_f}{R_i} = \frac{d_o}{d_h} = \cos \phi_m \\ &d_h - d_o = S \\ &C d_h = \frac{2W_t\, x_t}{r\left(1+\cos\frac{\phi_m}{2}\right)} \end{aligned}\right\}$$

With a pneumatic cylinder, we have, since the expansion, may be assumed isothermal,

$$\frac{p_f}{p_i} = \frac{V_o}{V_o + AS} = \cos \phi_m$$ (approx.) from which we

may determine the initial volume V_o, hence,

$$V_o = AS\,\frac{\frac{p_f}{p_i}}{1-\frac{p_f}{p_i}} = AS\,\frac{\cos \phi_m}{1-\cos \phi_m}$$ (approx.) (cu.in.)

Now

$$R_i = p_i A \text{ and } V_o = \frac{R_i S}{p_i}\left[\frac{\frac{p_f}{p_i}}{1-\frac{p_f}{p_i}}\right]$$ (cu.in)

We see, therefore, to decrease the bulk of the cylinder it is important to maintain as high an initial air pressure as possible. It is reasonable to assume the same initial pressure as used in the recuperator brake.

Therefore, in the first approximation of the de-

sign layout of a pneumatic balancing gear, we may start with,-

$$V_o = \frac{2W_t x_t S}{r(1+\cos\frac{\phi}{2})p_i} \left[\frac{\frac{p_f}{p_i}}{1-\frac{p_f}{p_i}}\right] \quad \text{(cu.in)}$$

where $S = 2r\sin\frac{\phi}{2}$, $\frac{p_f}{p_i} = \cos\phi_m$ (approx.)

REACTIONS ON TIPPING PARTS WITH BALANCING GEAR. With the trunnions to the rear of the center of gravity of the tipping parts and a balancing gear introduced, we have a cantilever form of top carriage, and the reactions on the tipping parts are usually approximately in the position shown in fig. (8).

Let

x and y = coordinates along and normal to axis of bore.

R = reaction of balancing gear (lbs)

θ_r= angle between R and y axis.

d_t= moment arm of R about the trunnions at any elevation ϕ.

E = elevating arc reaction (lbs)

θ_e= angle between the y axis and the radius to the elevating pinion contact with the elevating arc.

J= radius of elevating arc from trunnions (in)

X and Y = components along x and y of trunnion reaction. (lbs)

W_t = weight of tipping parts.

x_t and y_t = coordinate of center of gravity of tipping parts from trunnion (in)

W_r = weight of recoiling parts (lbs)

We have, then, two positions to consider, the in and out of battery positions respectively. In

REACTIONS ON TIPPING PARTS WITH DIRECT ACTING BALANCING GEAR

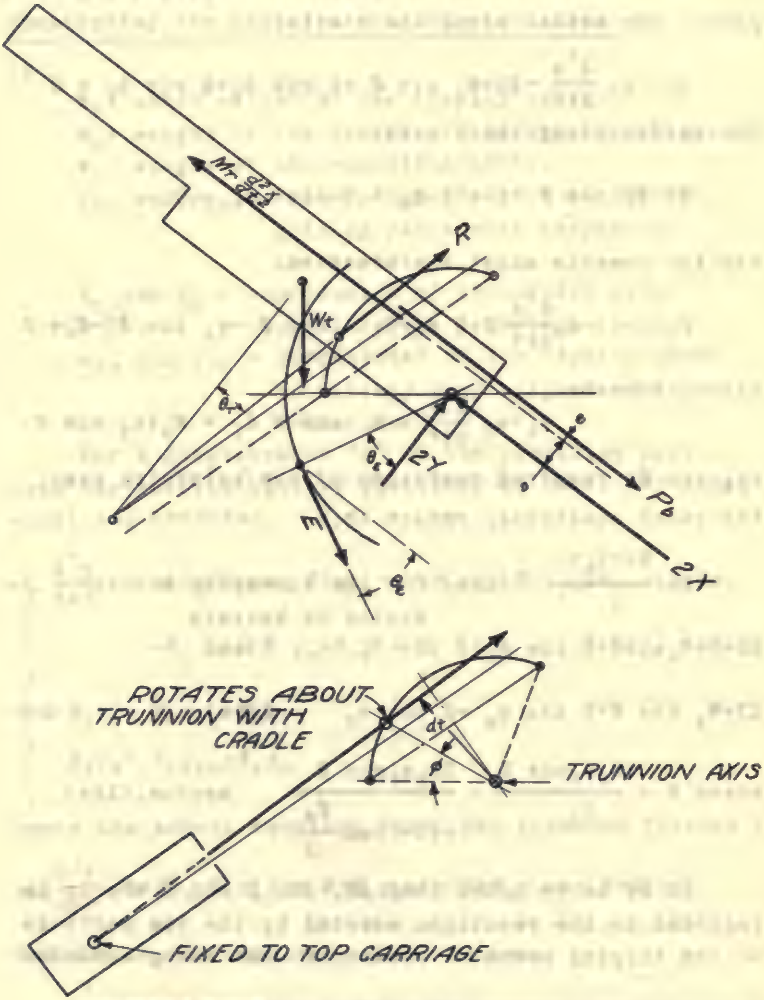

Fig. 8

the battery position we have the effect of the powder pressure couple, while in the out of battery position, we have the moment effect of the recoiling weight.

Considering the Recoiling parts in battery:

We have for the kinetic equilibrium of the tipping parts, for motion along the x axis,

$$P_b - m_r \frac{d^2x}{dt^2} - 2X + W_t \sin \emptyset + E \cos \theta_e + R \sin \theta_r = 0$$

for motion along the y axis,

$$2Y - W_t \cos \emptyset - E \sin \theta_e + R \cos \theta_r = 0$$

and for moments about the trunnion,

$$P_b(e+s) - m_r \frac{d^2x}{dt^2} S + R\, d_t - W_t(x_t \cos \emptyset - y_t \sin \emptyset) - Ej = 0$$

Since, however, $P_b - m_r \dfrac{d^2x}{dt^2} = K$ and $R\, d_t = W_t(x_t \cos \emptyset - y_t \sin \emptyset)$ required condition of the balancing gear, the above equations, reduce to,-

$$E = \frac{Ks + P_b e}{j} \quad \text{(lbs) for the elevating arc reaction in battery}$$

$$2X = K + W_t \sin\emptyset + E \cos \theta_e + R \sin \theta_r \quad \text{(lbs)}$$

$$2Y = W_t \cos \emptyset + E \sin \theta_e - R \cos \theta_r \quad \text{(lbs)}$$

$$\text{where } R = \frac{W_t x_t \cos \emptyset}{d_t} = \frac{2W_t x_t \cos \emptyset}{r(1 + \cos \frac{\emptyset_m}{2})} \quad \text{approx. (lbs)}$$

It is to be noted that 2X, 2Y, E and R are to be regarded as the reactions exerted by the top carriage on the tipping parts, a rocker if used being included

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* To account for tooth contact of elevating mechanism, substitute j cos 20 for j, $E \cos(\theta_e - 20)$ for $E \cos \theta_e$ and $E \sin(\theta_e - 20)$ for $E \sin \theta_e$.

as a part of the tipping parts. The resultant bearing reaction between the top carriage bearing trunnions, becomes,

$$S = \sqrt{X^2+Y^2} \quad \text{(lbs)}$$

Considering the recoil parts out of battery

if

W_t = total weight of the tipping parts

W_c = weight of the cradle of the tipping parts.

W_r = weight of the recoiling parts.

x_{ro} and y_{ro} = battery coordination of the recoiling parts with respect to the trunnions.

x_c and y_c = coordinates of the cradle with respect to the trunnions.

x_{fo} and y_{fo} = coordinates of the tipping parts in battery with respect to the trunnions.

For a displacement "x" of the recoiling parts from the initial battery position, we have, for moments about the trunnion,

$$m_r \frac{d^2x}{dt^2} \cdot S + Rd_t - W_r \cos \emptyset (x_{ro} - x) + W_r \sin \emptyset\, y_{ro} - W_c \cos \emptyset x_c$$

$$+ W_c \sin \emptyset\, y_c - E\, j = 0$$

Now $W_t x_t = W_r x_{ro} + W_c x_c$

$W_t y_t = W_r y_{ro} + W_c y_c$

hence the moment equation about the trunnion reduces to

$$m_r \frac{d^2x}{dt^2} \cdot E + Rd_t - W_t x_t \cos \emptyset + W_t y_t \sin \emptyset + W_r x \cos \emptyset - Ej = 0$$

but due to the balancing gear, we have,

$$Rd_t = W_t (x_t \cos \emptyset - y_t \sin \emptyset)$$

and further $K = m_r \frac{d^2x}{dt^2}$ that is, the inertia resistance, equals the total resistance to

recoil.

Hence

$$E=\frac{K\,s+W_r\,x\cos\phi}{j}\;*$$

For motion along the x axis, we have,

$$2X=K+R\sin\theta_r+E\cos\theta_e+W_t\sin\phi$$

and for motion along the y axis,

$$2Y=W_t\cos\phi+E\sin\theta+E\sin\theta_e-R\cos\theta_r$$

where as before

$$R=\frac{2W_t\,x_t\cos\phi}{r\left(1+\cos\frac{\phi_m}{2}\right)}\text{ roughly.}$$

From the above analysis, we see, therefore, that the elevating arc reaction remains the same with or without a balancing gear, while the trunnion reactions may be increased or decreased according to the location of the line of action of the balancing gear.

INTERNAL REACTIONS OF TIPPING PARTS WITH BALANCING GEAR - ROCKER INTRODUCED.

The rocker reactions depend solely on the elevating gear reaction, and with a balancing gear, the elevating gear reaction is independent of the eccentricity of the center of gravity of the tipping parts from the trunnions. Therefore, the rocker reaction on the trunnion is entirely independent of the reaction exerted by the balancing gear or counterpoise. In brief, the rocker reactions remain the

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* To account for tooth contact of elevating mechanism, substitute j cos 20 for j E cos(θ_e-20) for E cos θ_e and E sin (θ_e - 20) for E sin θ_e.

same with or without a counterpoise or balancing gear. The reactions exerted by the top carriage on the trunnion do however depend on the magnitude and direction of the balancing gear. Therefore, the shear and bending at the section of the trunnion adjoining the cradle must also depend on the balancing gear or counterpoise reaction.

An analytical proof of the reactions is given as follows:-

Let

X and Y = trunnion components of the reaction of top carriage.

X_r and Y_r = trunnion components of the reaction of the rocker.

X' and Y' = shear components of the trunnion pins on the cradle.

W'_r = weight of rocker.

E = elevating gear reaction on rocker

M = cradle reaction on rocker.

j = radius to elevating gear arc.

k = the perpendicular distance from the trunnions to the line of action of M which is the line through the axis of the elevating screw, when used, or the normal to the contact surface of rocker and cradle when an elevating screw is not used.

B = the angle between the line of action of M and the "y" axis.

x_m and y_m = the "x" and "y" coordinates of the cradle hinge joint of rocker elevating screw or the center of contact of rocker on cradle.

x_r and y_r = the "x" and "y" coordinate of the center of gravity of the rocker and is measured from the trunnions <u>towards</u> the breech and downward.

Evidently for the shear at the cradle section of the trunnion,

$$\left.\begin{aligned} X' &= X + X_r \\ Y' &= Y + Y_r \end{aligned}\right)$$

For the angular equilibrium of the rocker,

$$Ej - Mk - W_r'(x_r \cos \phi - y_r \sin \phi) = 0$$

if we let $x_r \cos \phi - y_r \sin \phi = h_r'$, then

$M = \dfrac{Ej - W_r' h_r'}{k}$, where $k = x_m \cos B + y_m \sin B$, that is,

the cradle rocker reaction depends solely on the elevating gear reaction.

For the translatory equilibrium of the rocker,

$$2X_r = M \sin B - E \cos \theta_e - W_r' \sin \phi$$

$$2Y_r = E \sin \theta_e - W_r' \cos \phi - M \cos B$$

which shows the rocker reaction at the trunnion depends only on the elevating arc pressure and therefore is independent of the counterpoise reaction.

Considering the equilibrium of the tipping parts not including the rocker, we have,

$$2X' = K_r + (W_t - W_r') \sin \phi + M \sin B + R \sin \theta_r$$

$$2Y' = (W_t - W_r') \cos \phi - M \cos B - R \cos \theta_r$$

Further if measured from the trunnion axis,

$l_t = x_t \cos \phi - y_t \sin \phi =$ the horizontal distance to center of gravity of tipping parts (recoiling parts in battery)

$l_t' =$ the horizontal distance to center of gravity of tipping parts (recoiling parts out of battery)

$l_r =$ the horizontal distance to center of gravity of recoiling parts in battery.

l_c = the horizontal distance to center of gravity of cradle.

$-h_r'$ = the horizontal distance to center of gravity of rocker measured in a negative direction from the l's.

d_t = moment arm of the counterpoise reaction R about the trunnions.

Then since the moment of the tiooing parts minus rocker about the trunnions is equal to the moment of the weight of the tipping parts minus the moment of the weight of the rocker, we have,

$$W_t l_t' + W_r h_r' = W_r(l_r - x \cos \emptyset) + W_c l_c$$

Now $W_r l_r + W_c l_c = W_t l_t + W_r h_r'$

hence $W_t l_t' + W_r h_r' = W_t l_t + W_r h_r' - W_r\ x \cos \emptyset$

$$= W_t(x_t \cos \emptyset - y_t \sin \emptyset) + W_r h_r' - W_r\ x \cos \emptyset$$

therefore,

$$K_r s + R d_t - W_t(x_t \cos \emptyset - y_t \sin \emptyset) - W_r h_r' + W_r x \cos - Mk = 0$$

but for equilibrium of the tipping parts in battery

$$R d_t = W_t(x_t \cos \emptyset - y_t \sin \emptyset)$$

hence

$$M = \frac{K_r s + W_r x \cos \emptyset - W_r h_r'}{k}$$

Since, however,
$$\left.\begin{array}{l} 2X = 2X' - 2X_r \\ 2Y = 2Y' - 2Y_r \end{array}\right)$$

We have in substituting the previous values for $2X'$ and $2X_r$.

$$2X = K_r + W_t \sin \emptyset + R \sin \theta_r + E \cos \theta_e$$

$$2Y = W_t \cos \emptyset - E \sin \theta_e - R \cos \theta_r$$

$$\text{and } E = \frac{K_r s + W_r x \cos \phi}{j}$$

In the preceeding analysis it is important to note, that the center of gravity of the rocker is assumed to the rear of the trunnions, and the elevating gear reaction is considered positive when the radius to the pinion contact of the elevating rack is measured counter-clockwise with respect to the "y" axis through the trunnions. Evidently when θ_e is negative (i.e. clockwise from "y" axis, $E \cos \theta_e$ remains the same but $E \sin \theta_e$ becomes negative in the above equations.

EFFECT OF RIFLING TORQUE ON TRUNNION REACTION

Due to the rifling, the torque exerted on the gun by the shell must be balanced in considering the equilibrium of the tipping parts by an equal and opposite moment exerted by the top carriage on the trunnions(assuming due to the much greater flexibility of the elevating arc and pinion that the elevating arc reaction is entirely unaffected).

If the rifling is right handed, then in the direction of the muzzle, the Y component of the left trunnion is increased and the Y component of the right trunnion is decreased by the amount equal to the torque of rifling divided by the distance between the trunnion bearings on the top carriage. Usually this affect is quite negligible as compared with the ther forces exerted.

STRENGTH OF THE TRUNNIONS.

The critical section of the trunnions is usually where the trunnion joins the cradle.

Let "mn" represent this section on the trunnion, see fig.(9).

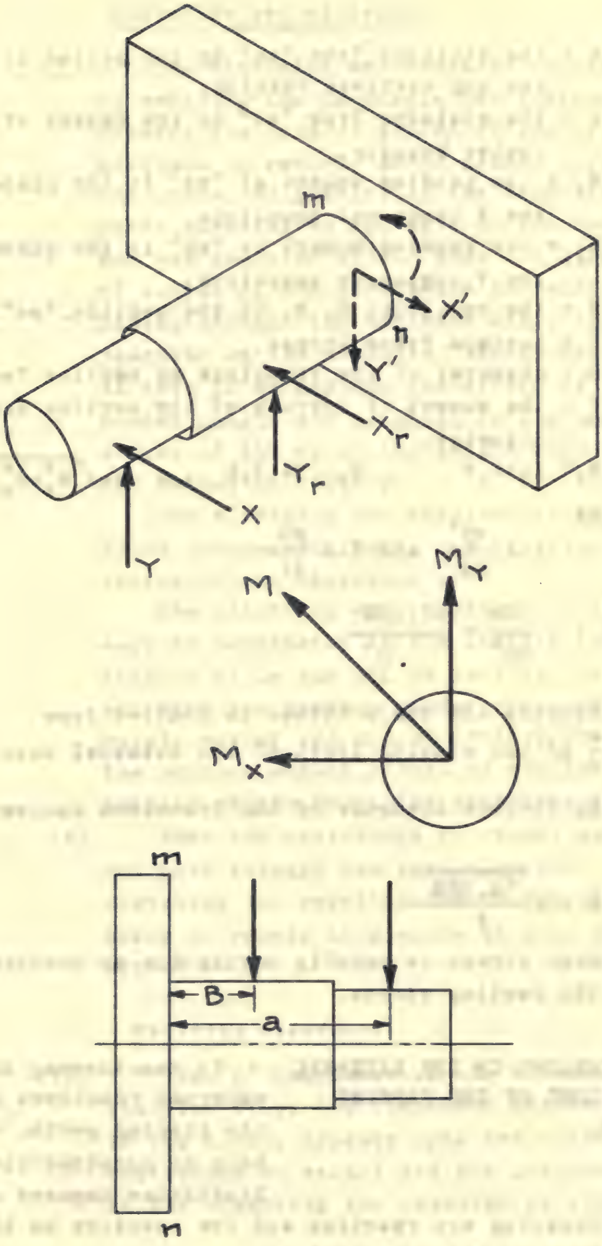


Fig. 9.

a = the distance from "mn" to the center of the top carriage bearing.

b = the distance from "mn" to the center of the rocker bearing.

M_x = the bending moment at "mn" in the plane of the X component reactions.

M_y = the bending moment at "mn" in the plane of the Y component reactions.

M = the resultant B. M. on the section, "mn".

f = maximum fibre stress.

D = diameter of the trunnions at section "mn"

I = the moment of inertia of the section about diameter.

Then $M_x = Xa + X_r b$ $\quad M_y = Ya + Y_r b$ and $M = \sqrt{M_x^2 + M_y^2}$

further

$$I = \frac{\pi D^4}{64} \quad \text{and} \quad f = \frac{MD}{2I}$$

hence $f = \dfrac{32M}{\pi D^3} = \dfrac{10.18M}{D^3}$

Usually the fibre stress is limited from $\frac{1}{2}$ to $\frac{2}{3}$ of the elastic limit of the material used,

and the minimum diameter of the trunnions becomes

$$D = \sqrt[3]{\frac{10.18M}{f}}$$

The shear stress is usually negligible as compared with the bending stress.

LIMITATIONS ON THE EXTERNAL REACTIONS OF THE TIPPING PARTS. In considering the external reactions on the tipping parts, we have to consider the limitation imposed on the elevating arc reaction and the reaction on the trunnions by the top carriage.

ELEVATING ARC REACTION:

(1) The elevating arc reaction is reduced by reducing the perpendicular distance between the line of action of the resistance to recoil, which passes through the center of gravity of the recoiling parts parallel to the axis of the bore, and the trunnion axis. When the line of action of the resistance to recoil passes through the trunnion axis, the reaction on the elevating arc in battery is zero if we neglect the effect of the powder couple and is equal to the moment effect of the recoiling weight when the gun is out of battery.

The elevating arc reaction is reduced proportionally to the increase of the radius of the elevating arc.

The elevating arc reaction should always be considered in the limiting conditions of in and out of battery, that is, with the maximum powder pressure couple acting and out of battery when the maximum moment effect of the recoiling weight about the trunnions exists.

(4) When the resistance to recoil does not pass through the trunnions the elevating arc reaction due to the shortening of recoil is a maxim at max. elevation.

TRUNNION REACTIONS:

(1) The "X" component of the trunnion reaction (i.e. the component parallel to the bore), depends upon the total resistance to recoil and the component of the elevating arc reaction parallel

to the bore as well as the weight component of the tipping parts when the gun elevates.

(2) The "Y" component depends upon the weight of the tipping parts and the component of the elevating arc pressure parallel to the "y" axis.

(3) As the gun elevates the component of the elevating arc pressure parallel to the "x" axis decreases but the weight component increases and due to the shortening of recoil on elevating the resistance to recoil increases.

(4) The component parallel to the "y" axis of the elevating arc reaction increases but in a negative direction, thus tending to decrease the Y component of the trunnion reaction. On high elevation because of the large resistance to recoil for a short recoil, the elevating arc pressure parallel to the "y" axis more than compensates the decreased weight component of the tipping parts thus causing a reversal of direction of the Y component reaction of the trunnions.

(5) Thus in general the X component increases while the Y component decreases, very often reversing on elevating the gun and thus the trunnion bearing contact may shift 90° or over.

STRESSES IN CRADLE OR RECUPERATOR FORGING. The reactions on the cradle or recuperator are: (1) the trunnion reaction of the top carriage on the cradle: (2) the reaction of the elevating arc which is equivalent to a single force in the direction of the elevating pinion re-

action on the elevating arc together with an additional moment: (3) the reaction of the recoiling mass on the guides: (4) a result and reaction parallel to the longitudinal axis of the cradle or the guides due to the various "pulls" exerted on the recoiling mass and (5) a distributed load which is uniform if the cross sections remain the same due to the weight of the cradle.

In an accurate computation of the stresses in a cradle it is necessary from a preliminary layout of the cradle to locate roughly the neutral axis of each section and connect these points for a longitudinal neutral axis line. We may then treat the cradle as a simple beam, taking into account the bending moments caused by eccentric loads such as pull reactions off the neutral axis, guide frictions, etc. The trunnions usually are located considerably above the neutral axis and the X component of the trunnion reaction causes a large eccentric load with a consequent large abrupt change in the bending moment diagram. This is usually a characteristic in the bending moment diagram for all cradles or recuperators using guides.

Let us now consider the various diagrams showing the characteristics for bending moment, direct stress and shear for the "Filloux" cradle as well as for the "240 m/m Schneider Howitzer" cradle representing typical cradles with guides (figures 10 and 11).

Neglecting the weight of cradle as relatively small, and letting

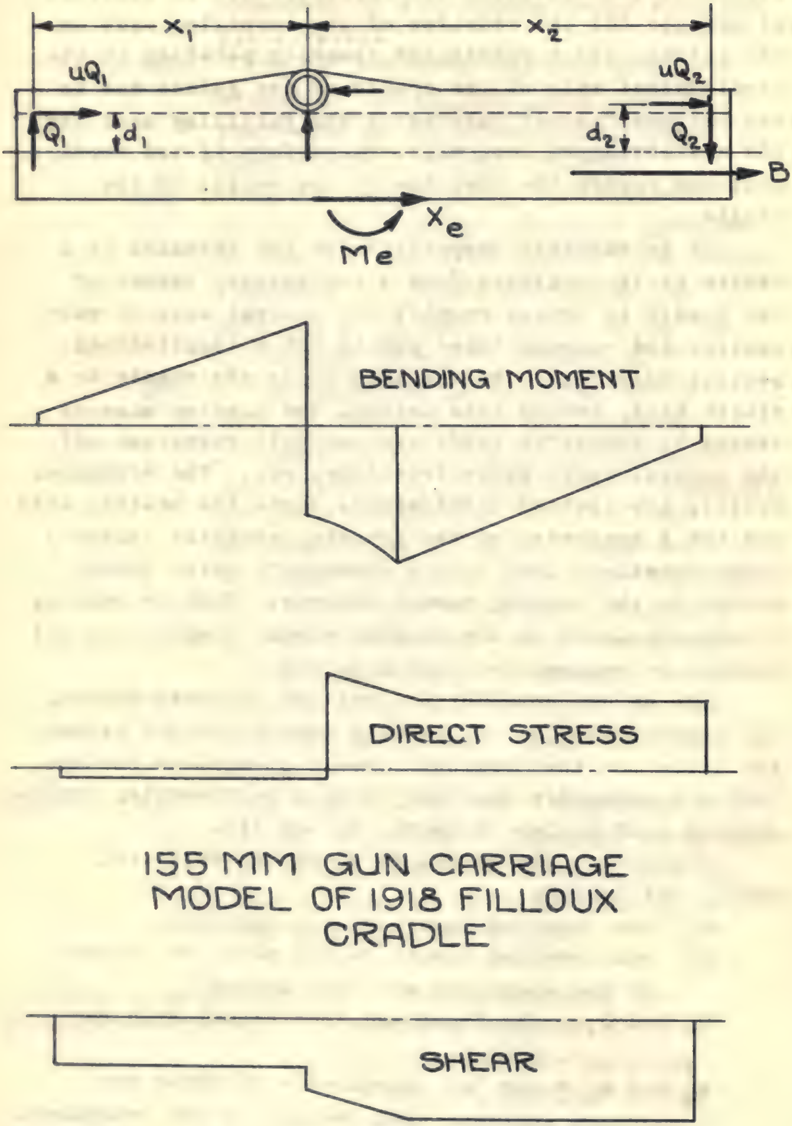
M_t = max. bending moment at the trunnions.

M_c = max. bending moment at the point of contact of the elevating arc with cradle.

Q_1 and Q_2 = the front and rear normal clip reactions.

x_1' and x_2' = the "x" coordinates of these reactions with respect to the trunnions.

d_1 and d_2 = the distance of the friction components of Q_1 and Q_2 from the neutral axis.



155 MM GUN CARRIAGE
MODEL OF 1918 FILLOUX
CRADLE

Fig. 10

B = the resultant of the braking "pulls" reacting on the cradle.

d_b = the distance from the neutral axis to "B"

d_x = the distance from the neutral axis to the trunnions.

X and Y = the trunnion reactions on the cradle.

X_e and Y_e = the elevating arc reaction on the cradle.

Me = the moment exerted by the elevating arc on the cradle.

d_e = the distance from the neutral axis to X_e

x_e = the "x" coordinate of the elevating arc contact with respect to the trunnions.

The bending moment changes at the trunnions by the amount $2X\,d_x$.

Now for guns with "braking pull" reactions on the cradle in the rear as caused by the compression of the oil and air in the recuperator as in the 155 m/m Filloux, we have for the bending moments at the trunnions,

$M_t = Q_1(x_1' + ud_1)$ just to left of trunnion.

$M_t = Q_1(x_1' + ud_1) - 2X\,d_x$ (just to right of trunnion) and at the elevating arc contact,

$M_c = Q_2(x' - x_e + ud_2) - Bd_b$

As a check, we also have,

$$M_c = Q_1(x_1' + x_e + ud_1) + 2Yx_e - 2Xd_x - Me$$

The bending moment M_c is usually distributed causing a parabolic curve as shown in the B. M. diagram of the 155 m/m Filloux.

For guns with "braking pull" reactions on the cradle in the front, due to the tension in the stationary hydraulic piston and recuperator rods, as in the 240 m/m Schneider Howitzer, we have for the bending moment at the trunnions

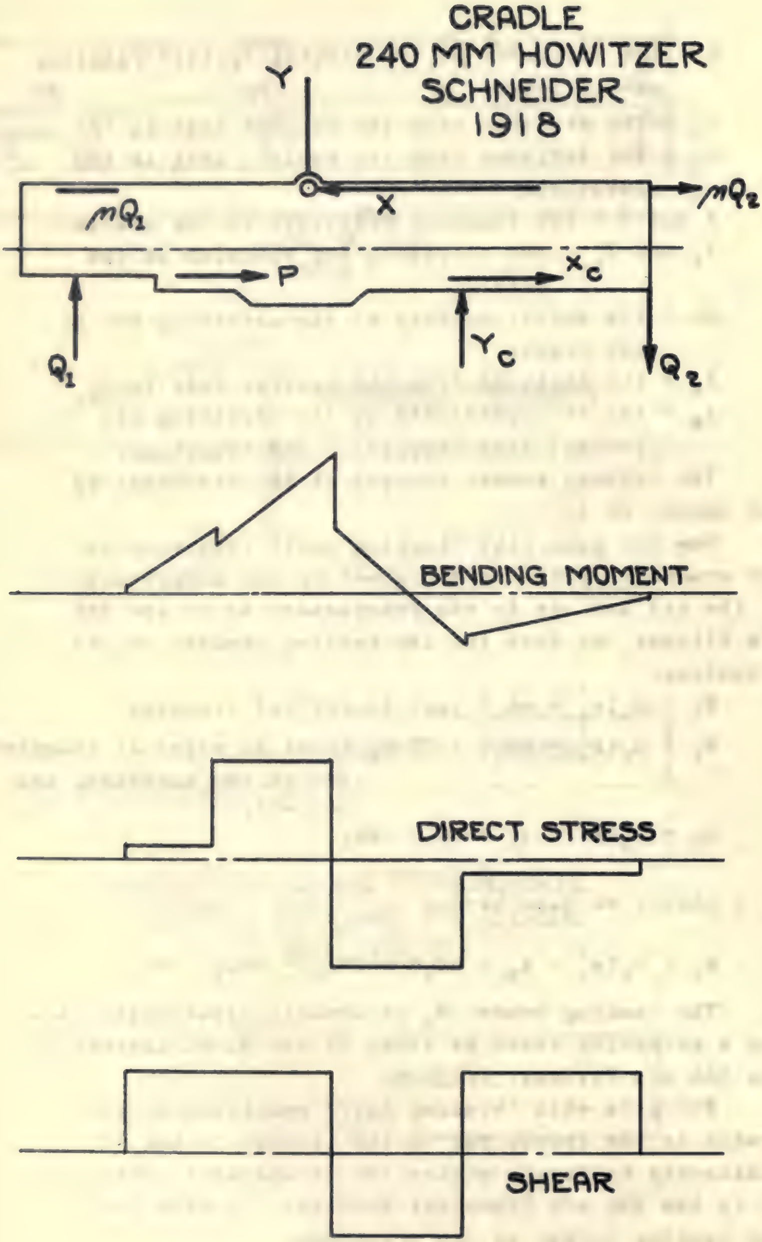

CRADLE
240 MM HOWITZER
SCHNEIDER
1918
Y
X
μQ_1
μQ_2
P
X_c
Q_1
Y_c
Q_2
BENDING MOMENT
DIRECT STRESS
SHEAR

Fig. 11

$M_t = Q_1(x_1' + ud_1) - Bd_b$ (just to left of trunnions) and

$M_t = Q_1(x_1' + ud_1) - Bd_b - 2X\ d_x$ (just to right of trunnion)

Further $M_c = Q_1(x_1' + x_c + ud_1) + 2Yx_e - 2Xd_x - Bd_b$ or as a check

$M_c = Q_2(x_2' - x_c + ud_2)$

In order to compute the maximum fibre stress at a critical section, we must include the direct stress caused by the component of the reactions parallel to the X axis in addition to the fibre stress caused by bending. Therefore a direct stress diagram has been drawn for typical gun carriage cradles.

For the case where the braking reaction is in the rear from the front clip to the trunnion, we have,

A compression = uQ_1 which is small and may be neglected as compared with the bending.

From the trunnion to the rear clip, we have at the trunnions (just to right of trunnions[see diagram]-

a tension $= X - uQ_1$)

at the elevating arc contact section

a tension $= 2X - uQ_1 - X_c$

$= B + uQ_2$

Where the braking reaction is in front, we have from the front clip to the "braking yoke" on the cradle or ac a section through the point of application of the tensions of the rods on the cradle, we have

a compression $= uQ_1$

From the braking yoke section the compression increases to $uQ_1 + B$

at the trunnions

A compression $= uQ_1 + B$(just to right of section)

A tension $= 2X - uQ_1 - B$

$= uQ_1 + X_c$

<u>At the elevating arc section:</u>

A tension $= uQ_2$

$= 2X - uQ_1 - B - X_c$

Shear diagrams for the 155 m/m Filloux and 240 m/m Schneider Howitzer show the variation of the shear in these typical cradles.

To recapitulate if y_t and y_c = the distance to the extreme fibres from the neutral axis at the trunnion and elevating arc section, and if A_t and A_c are the areas of these respective sections, I_t and I_c corresponding moments of inertia, we have for the extreme fibre stress, in the critical sections of the cradle,

$$f_t = \frac{Q_1(x_1' + ud_1)y_t}{I_t} + \frac{uQ_1}{A_t}$$ for the braking reaction in the rear.

$$f_t = \frac{[Q_1(x_1' + ud_1) - Bd_b]y_t}{I_t} + \frac{uQ_1 + B}{A_t}$$ for the braking reaction in the front.

$$f_c = \frac{[Q_2(x_2' + x_c + ud_2) - Bd_b]y_c}{I_c} + \frac{B + uQ_2}{A_c}$$ for the braking reaction in the rear.

$$f_c = \frac{[Q_2(x_2' - x_c + ud_2)]y_c}{I_c} + \frac{uQ_2}{A_c}$$ for the braking reaction in the front.

With Barbette and Naval mounts the gun recoils in a cylindrical sleeve which forms part of the cradle.

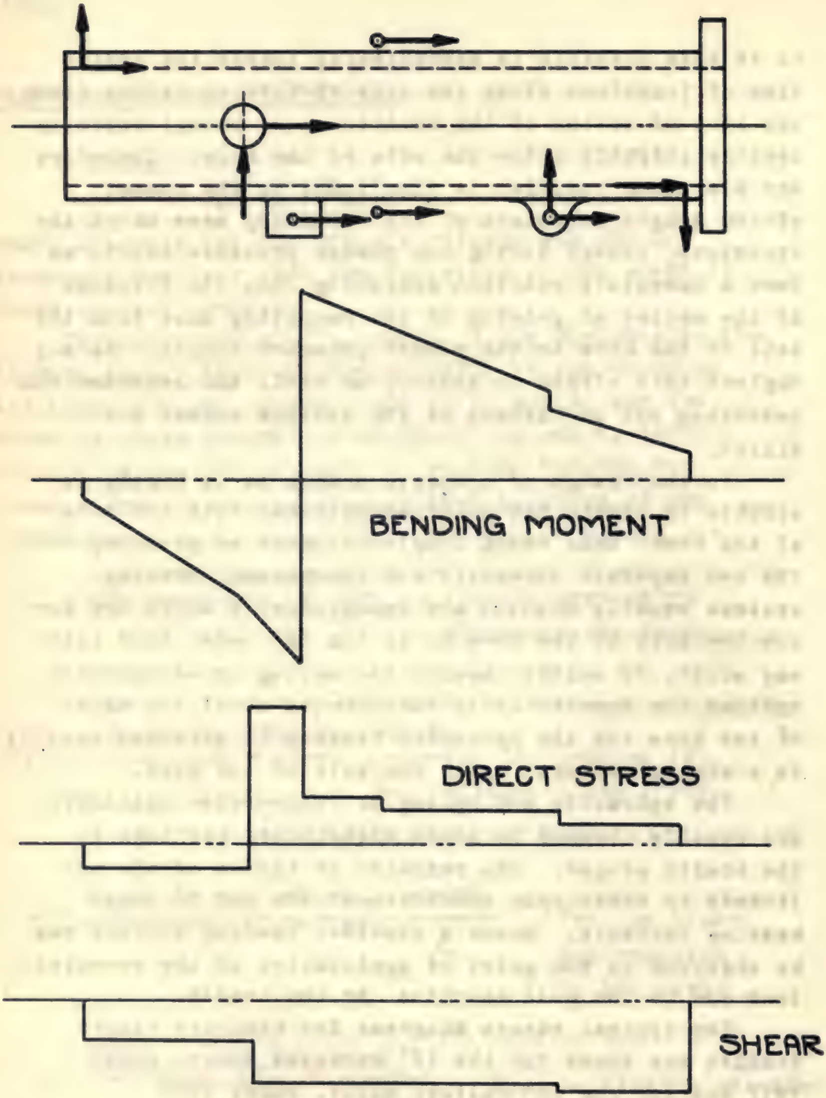
It is thus possible to conveniently locate the center line of trunnions along the axis of bore or rather along the line of action of the resistance to recoil which is usually slightly below the axis of the bore. Therefore the elevating reaction is simply due to the moment of the weight component of the recoiling mass about the trunnions, though during the powder pressure couple we have a momentary reaction depending upon the distance of the center of gravity of the recoiling mass from the axis of the bore to the powder pressure couple. We may neglect this effect in battery as small and consider the reactions out of battery as the maximum stress condition.

In the design of a sleeve cradle it is highly desirable to locate the pulls symmetrical with the axis of the bore, this being completely done by grouping the two separate hydraulic and recuperator braking systems equally distant and symmetrically above and below the axis of the bore as in the 16" model 1918 railway mount. In smaller mounts the spring or recuperator systems are symmetrically distributed about the axis of the bore and the hydraulic braking is affected usually in a single cylinder below the axis of the bore.

The hydraulic and spring or recuperator cylinders are usually clamped in short distributed bearings to the cradle proper. The reaction or thrust of the cylinders is taken upon shoulders at the end of these bearing contacts. Hence a shoulder bearing surface may be regarded as the point of application of the eccentric load due to the pull reaction, on the cradle.

Two typical stress diagrams for barbette sleeve cradles are shown for the 12" barbette mount, model 1917 and for the 16" railway mount, model 1918.

Considering the reactions on the cradle of the 12" barbette carriage, model 1917 in figure (12) we have, for the bending moment diagram, from the front clip to the trunnion section, we have, a uniform increase in the B. M. from uQ_1d_1 to $Q_1(ud+x_1')$



12-INCH BARBETTE CARRIAGE
MODEL OF 1917
CRADLE

Fig. 12

In passing from left to right of the trunnion section there is no abrupt change in B. M. since the X reaction being approximately on the longitudinal neutral axis of the cradle is no longer an eccentric load.

The maximum bending moment occurs at the section through the point of application of the eccentric load due to the reaction of the hydraulic cylinder against the cradle proper.

If X_h' = the distance to this load d_h' = the distance to the neutral axis, where P_h = the hydraulic braking, the bending moment at this section becomes,

$Q_1(ud+x'+X_h')+2Yx_h'$ to left of section, and

$P_hd_h-Q_1)ud+x'+x_h')-2Yx_h'$ to right of section.

In terms of forces to the right of this section the B. M. becomes,

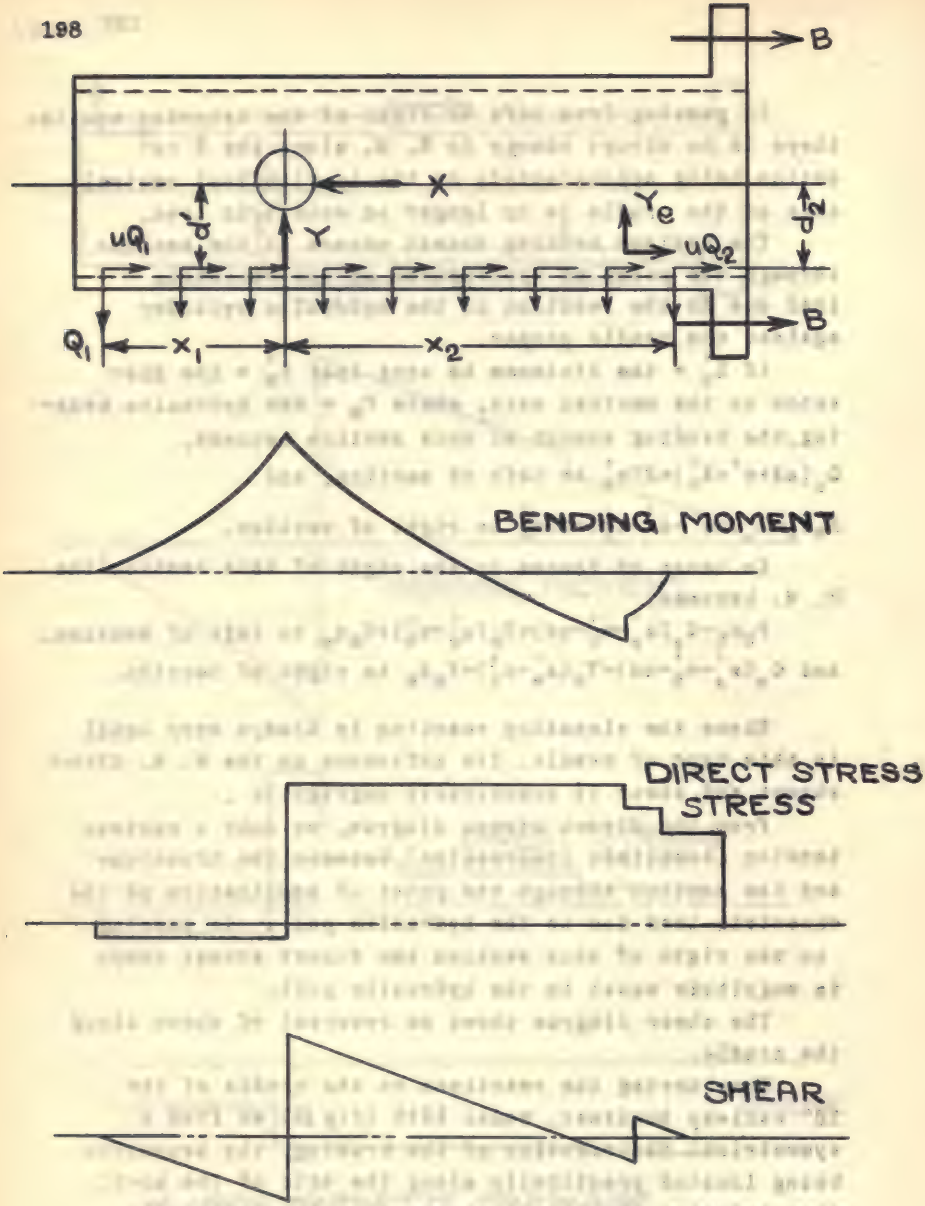
$P_hd_h-Q_2(x_2'-x_h'-ud)+Y_e(x_e'-x_h')+X_ed_e$ to left of section, and $Q_2(x_2'-x_h'-ud)-Y_e(x_e'-x_h')-X_ed_e$ to right of section.

Since the elevating reaction is always very small in this type of cradle, its influence on the B. M. direct stress and shear is practically negligible.

From the direct stress diagram, we note a maximum tension (sometimes compression) between the trunnions and the section through the point of application of the eccentric load due to the hydraulic pull. In passing to the right of this section the direct stress drops in magnitude equal to the hydraulic pull.

The shear diagram shows no reversal of shear along the cradle.

Considering the reactions on the cradle of the 16" railway howitzer, model 1918 (fig 13) we find a symmetrical distribution of the braking, the trunnions being located practically along the axis of the bore, thus reducing the elevating reaction to practically the weight effect of the recoiling mass out of battery and the neutral axis of the cradle through the trunnions and along the axis of the bore. There



16 INCH RAILWAY HOWITZER
MODEL OF 1918
CRADLE

Fig. 13

being no eccentric pull on the recoiling mass we have a distributed load due to the weight of the recoiling mass as shown in the figure

In the B. M. diagram we find the B. M. at the trunnions to be relatively small it being merely equal to that due to a distributed load equal to the weight of the recoiling mass, together with the friction caused by this loading.

Thus, the intensity of loading = $\dfrac{W_r}{x_1+x_2}$

Hence the B. M. at the trunnions, becomes,

$$\frac{x_1^2 W_r}{2(x_1+x_2)} + u\frac{x_1 W_r}{x_1+x_2}d$$

Further between the trunnions and the application of the braking to the cradle by the brake cylinders thrusting against the shoulder bearings for the cylinders on the cradle, we have a tension equal to the total braking, thus causing a direct stress of tension in addition to the bending.

To recapitulate, for barbette sleeve mounts, if y= the distance to the extreme fibres from the neutral axis at the section through the point of application of the maximum eccentric pull due to the hydraulic braking or at the trunnion section when the pulls are symmetrically balanced, A = the area of the cross section and I = the moment of inertia of the section, we have for the extreme fibre stress

For barbette sleeves with eccentric pulls:-
Section at eccentric load -

$$f_t = \frac{(Q_1(ud+x'+x_h')+2Yx_h')y}{I} \pm \frac{X-uQ_1}{A} \text{ at left of section,}$$

and

$$f_t = \frac{(P_h d_h - Q_1(ud+x'+x_h')-2Yx_h')y}{I} \pm \frac{X-P_h-uQ_1}{A} \text{ at right of section.}$$

For barbette sleeves with pulls symmetrically balanced about the axis of the bore:
Section at trunnion:

$$f = \frac{\left[\frac{W_r x_1^2}{2(x_1+x_2)} + \frac{u x_1 W_r d}{x_1+x_2}\right] y}{I} \pm \frac{\left(X - \frac{u x_1 W_r}{x_1+x_2}\right)}{A}$$

STRENGTH OF CYLINDERS AND RECUPERATOR FORGINGS.

The strength of a recuperator forging is a matter of vital importance since in modern artillery the tendency is to use higher and higher pressures consistent with the various cylinder packings used, and to stress the forging higher with smaller factors of safety. Hand in hand with this goes the metallurgical side where improvements in the quality of the steel with higher ultimate strength are constantly being made. High stresses in the recuperator forgings as with high ultimate strength and low factors of safety reduce the weight of the carriage and its cost considerably . Weight of course is of fundamental consideration for mobility. Hence it is of importance to calculate the stresses in the cylinder walls to a considerable degree of accuracy.

The maxim stress in a recuperator forging is a combination of the following:

(1) A bending fibre stress normal to a plane section perpendicular to the longitudinal axis of the forging which is caused by the external reactions exerted on the forging during firing.

(2) A radial compression stress along a radius of the cylinder or normal to a cylindrical surface which is equal to the pressure in the cylinder for the inner surface.

(3) A tangential hoop tension, which is normal to a plane passing through the longitudinal axis of the cylinder.

Obviously these stresses are principle stresses and with the aid of Poisson's ratio we may arrive at the resultant intensity of stress. In a first approximation however it is sufficient to consider the tangential hoop tension alone, the effect and magnitude of the other stresses small.

Consider now a single cylinder 1" long, and subject to an internal pressure p, and external pressure p_1 and of inside radius R_0 and outside radius R_1 respectively.

Further let

r = the inside radius to any differential lamina of the cylinder wall (in inches)

d_r = the radial thickness of the lamina

p_r = the radial compression at radius r (lbs/sq.in)

p_t = the tangential or hoop tension (lbs/sq.in)

E = the modulus of elasticity.

e_l = the longitudiual strain.

e_r = the radial strain

e_t = the tangential or hoop strain.

Then, for the equilibrium of a differential lamina d_r, of length "l" along the axis of the cylinder and a peripheral length equal to the circumference, we have,

$$2p_r lr-2(p_r+dp_r)l(r+dr)=2p_t ldr$$

hence

$$p_t = -p_r-r\frac{dp_r}{d_r} = -\frac{d(rp_r)}{d_r} \qquad (1)$$

Let us further assume plane transverse sections to remain plane under pressure. This assumption is reasonably close to actual conditions for plane transverse sections some distance from closed ends, and in the case of a recuperator forging where the intensity

of longitudinal stress, i. e. the bending stress on transverse sections, is relatively small, except for extreme fibres from the neutral axis of the transverse section.

We are, therefore, not greatly in error in assuming the longitudinal strain to remain constant over the entire cross section, hence,

$$e_l = \frac{1}{E}\left(p_l - \frac{p_t - p_r}{m}\right) = \text{constant}$$

where p_l = intensity of longitudinal stress,

hence $p_t - p_r = k$ (2)

$$p_t - p_r = -2p_r - r\frac{dp_r}{d_r} \text{ or } k + 2p_r = -r\frac{dp_r}{d_r}$$

therefore $$\frac{dp_r}{k+2p_r} = -\frac{d_r}{r}$$

Integrating, $\log(k+2p_r) = -\log r^2 + c$ or $k+2p_r = \frac{c}{r^2}$

hence $p_r = \frac{c}{2r^2} - \frac{k}{2}$ (3)

and

$$p = \frac{c}{2r^2} + \frac{k}{2} \qquad (4)$$

Substituting (3) in (2) where $p_r = p_o$

$r = R_o$ inside conditions

$p_r = p_1$

$r = R_1$ outside conditions

hence $P_o = \frac{c}{2R_o^2} - \frac{k}{2}$

$$P_1 = \frac{c}{2R_1^2} - \frac{k}{2}$$

$$P_o - P_1 = \frac{c}{2R_o^2} - \frac{c}{2R_1^2} \qquad (5)$$

Now eliminating c and k, respectively, we find

$$c = \frac{2(P_0 - P_1)}{R_1^2 - R_0^2} \qquad \text{and } k = \frac{-2(P_0R_0 - P_1R_1^2)}{R_0^2 - R_1^2}$$

Substituting in (5) we have,

$$p_r = \frac{-P_0R_0 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2R_1^2\,(P_0 - P_1)}{R_1^2 - R_0^2}\,\frac{1}{r^2} \qquad (6)$$

$$p = \frac{P_0R_0 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2R_1^2(P_0 - P_1)}{R_1^2 - R_0^2}\,\frac{1}{r^2} \qquad (7)$$

(6), (7): Apparent stress.

The stress corresponding to the actual strain produced in the material, which is the basis of stress limitation imposed, (assuming m = 3 Poisson's ratio), becomes,

$$Ee_t = S_t = E\left(\frac{p_t}{E} + \frac{p_r}{mE} - \frac{p_l}{mE}\right) \qquad (8)$$

$$Ee_r = S_r = -E\left(\frac{p_r}{E} + \frac{p_t}{mE} + \frac{p_l}{mE}\right) \qquad (9)$$

(8), (9): Actual stress corresponding to actual strain

Where (6) and (7) are substituted in (8) and (9) and

$$p_l = p\,\frac{R_0^2}{R_1^2 - R_0^2}$$

the above expression reduces to Clavarine's formula. Birnie's formula is a modification of the above assuming $p_l = 0$, that is no external longitudinal tension. Usually p_l is relatively small and hence (8) and (9) simplifying to,

$$Ee_t = S_t = \frac{2}{3}\,\frac{P_oR_o^2-P_1R_1^2}{R_1^2-R_o^2} + \frac{4}{3}\,\frac{R_o^2R_1^2(P_o-P_1)}{R_1^2-R_o^2}\,\frac{1}{r^2} \qquad (10)$$

$$Ee_r = S_r = \frac{2}{3}\,\frac{P_oR_o^2-P_1R_1^2}{R_1^2-R_o^2} - \frac{4}{3}\,\frac{R_o^2R_1^2(P_o-P_1)}{R_1^2-R_o^2}\,\frac{1}{r^2} \qquad (11)$$

The maximum hoop tension, therefore becomes,

$$S_t = p\,\frac{2R_o^2+4R_1^2}{3(R_1^2-R_o^2)} \qquad (12)$$

which gives slightly higher values than Lame's when simplified, that is,

$$S_t = p\,\frac{R_o^2+R_1^2}{R_1^2-R_o^2} \qquad \text{Lame's formula} \qquad (13)$$

From the above formulae it is evident that the radial compression is always less than the hoop tension.

With large bending fibre stress due to external reactions on the recuperator forging,

p_1 is no longer equal to zero

The maximum stress which is the hoop tension becomes,

$$S_{t_{max}} = p\,\frac{\frac{2}{3}R_o^2+\frac{4}{3}R_1^2}{R_1^2-R_o^2} + \frac{pt_{max}}{3} \qquad (14)$$

THICKNESS OF WALLS BETWEEN ADJACENT CYLINDERS.

Considering two parallel cylinders bored in one forging (fig.) and passing a longitudinal plane section through the center lines or axes of cylinders (1) and (2), we have either half of the forging above or below this plane section in equilibrium under the internal hydrostatic pressures (which now are external with respect either half) and the hoop tensions of the outside and common wall of the

two cylinders. Further if the two cylinders are under pressure p_1 and p_2 respectively and neglecting the small variation of the hoop tension for different radius, we have for a close approximation

$$p_t(T_1 + T_2 + w) = p_1d_1 + p_2d_2$$

where T_1 and T_2 = thickness of cylinder walls (1) and (2) respectively.

w = total width of common wall between the two cylinders.

d_1 and d_2 = the diameters of the respective cylinders.

p_t = the assumed allowable hoop tension fibre stress.

Simplifying,

$$w = \frac{p_1d_1 + p_2d_2 - p(T_t+T_2)}{p_t}$$

For a correction due to the fact that the hoop tension is not constant but varies slightly with the radius, we may augment w by decreasing p_t to $0.9\ p_t$.

Further due to symmetry

$$T_1 = \frac{p_1d_1}{2p_t} \quad \text{and} \quad T_2 = \frac{p_2d_2}{2p_t}$$

and substituting in the previous equation, we have

$$w = \frac{p_1d_1 + p_2d_2}{1.8\ p_t}$$

which gives the minimum thickness of wall between two cylinders under pressures p_1 and p_2 respectively. Evidently the maximum simultaneous pressures in the two cylinders should be considered together.

ALLOWABLE STRESSES IN CYLINDER WALLS. Though this matter will be taken up in detail in practical design applications, certain limitations could profitably be mentioned here.

Cylinders should be tested for strength at pressures considerably higher than would be used in service. It is imperative that even under test pressure the elastic limit is not exceeded. Test pressures should be at least $1\frac{1}{2}$ and preferably twice the maximum service pressures and these test pressures should not exceed $\frac{3}{4}$ the elastic limit of the material or $\frac{7}{8}$ proportional limit.

TOP CARRIAGE The forces exerted on the top carriage are the reactions of the tipping parts and the supporting forces of the platform, or bottom carriage, or ground and axle for a trail carriage. The reaction exerted by the tipping parts on the top carriage may be divided into:-

(1) The trunnion reaction.

(2) The reaction of the elevating arc of the tipping parts on the pinion of the top carriage.

(3) The tension of the equilibrator chain or rod where an equilibrator is used.

These reactions are balanced by the supporting forces exerted at the base of the carriage.

In figure (14) the reactions on the top carriage are shown.

Considering now the reaction of the tipping parts on the top carriage assuming that an equilibrator is not used. Taking moments about the hinge point A(as in previous discussions), we have, when the gun has recoiled a distance X out of battery,

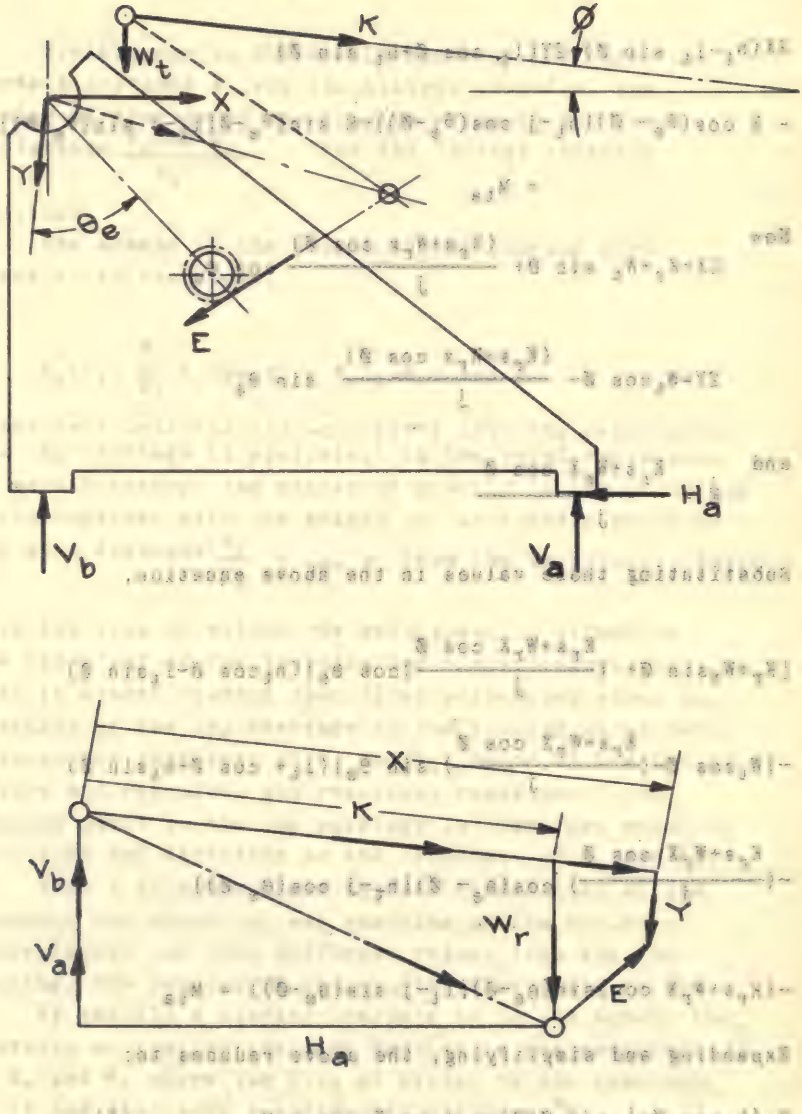


Fig. 14

$$2X(h_t - l_t \sin \emptyset) - 2Y(l_t \cos \emptyset + h_t \sin \emptyset)$$

$$- E \cos(\theta_e - \emptyset)[h_t - j \cos(\theta_e - \emptyset)] - E \sin(\theta_e - \emptyset[l_t - j \sin(\theta_e - \theta))]$$

$$= M_{ta}$$

Now

$$2X = K_r + W_t \sin \emptyset + \frac{(K_r s + W_r x \cos \emptyset)}{j} \cos \theta_e$$

$$2Y = W_t \cos \emptyset - \frac{(K_r s + W_r x \cos \emptyset)}{j} \sin \theta_e$$

and

$$E = \frac{K_r s + W_r X \cos \emptyset}{j}$$

Substituting these values in the above equation,

$$[K_r + W_t \sin \emptyset + (\frac{K_r s + W_r X \cos \emptyset}{j}) \cos \theta_e](h_t \cos \emptyset - l_t \sin \emptyset)$$

$$-[W_t \cos \emptyset - (\frac{K_r s + W_r X \cos \emptyset}{j}) \sin \theta_e](l_t + \cos \emptyset + h_t \sin \emptyset)$$

$$-(\frac{K_r s + W_r X \cos \emptyset}{j}) \cos(\theta_e - \emptyset)[h_t - j \cos(\theta_e - \emptyset)]$$

$$-(K_r s + W_r X \cos) \sin(\theta_e - \emptyset)[l_t - j \sin(\theta_e - \emptyset)] = M_{ta}$$

Expanding and simplifying, the above reduces to:

$$K_r(h_t \cos \emptyset - l_t \sin \emptyset + S) - w_t l_t + w_r X \cos = M_{ta} \qquad (1)$$

Now $h_t \cos \emptyset - l_t \sin \emptyset + S = d$ See Chapter III.

where d is the moment arm of K_r about the hinge point A.

Further due to the displacement of the recoiling parts a distance X from the battery position, the center of gravity of the tipping parts is displaced a distance $\frac{W_r \cos \emptyset_x}{W_t}$ from the initial trunnion position.

The moment of the weight of the tipping parts about A, is therefore,

$$W_t(l_t - \frac{W_r}{W_t} X \cos \emptyset) = W_t l_t - W_r x \cos \emptyset$$

Hence from equation (1) we observe that the reaction on the top carriage is equivalent to the total resistance to recoil through the center of gravity of the recoiling parts together with the weight of the tipping parts acting at a distance $\frac{W_r}{W_t} X \cos \emptyset$ from the trunnions. Therefore the line of action, is equivalent in effect to the resultant of the trunnion and elevating arc reaction. This is almost obvious from first principles since the reaction of the top carriage on the tipping parts must balance the resultant of K_r and W_t: hence by the law of action and reaction, the resultant reaction of the tipping parts on the top carriage is therefore equal in magnitude and direction to the resultant of K_r and W_t.

With a balancing gear we have in addition to the trunnion and elevating arc reaction on the top carriage,(which now have different values from the proceeding) the tension of the equilibrator chain or rod.

By exactly a similar analysis as in the above, the reaction on the top carriage reduces to the resultant of K_r and W_t where the line of action of the component W_t if now displaced a horizontal distance $\frac{W_r}{W_r} X \cos \emptyset$ - the distance which the center of gravity of the tipping parts in battery is placed backwards from the trunnion position, when the balancing gear is used.

In figure (14) is shown the various reactions on the top carriage together with a force polygon. Thus from the space diagrams of reactions obviously the lines of action of the resultant of the trunnion reaction and elevating arc reaction intersect at a common point which necessarily lies along the line of action of the resultant of K_r and W_t where the component of W_t is displaced a horizontal distance

$$\frac{W_r}{W_t} X \cos \emptyset \text{ from the trunnion axis.}$$

In the vector polygon of forces we note that by vector addition, $K+W_t=X+Y+E$

Further for the equilibrium of the top carriage, $X+Y+E+H_a+V_a+V_b = 0$ hence $K+W_t+H_a+V_a+V_b = 0$

The above results are exceedingly valuable in graphical methods as will be outlined later for obtaining the various reactions throughout a gun carriage.

SUPPORTING REACTIONS ON VARIOUS TYPES OF TOP CARRIAGES.

Top carriages have been classified in Chapter I, into (1) the ordinary type with side frames and connected at front or rear by cross beams or transoms, which contain the pivot bearing, (2) pivot yoke type used on small mobile mounts and (3) trail carriage.

The supporting reactions in the ordinary type of top carriage are the H and V components of the pivot bearing which is usually in the front and the V component exerted by the traversing circular guides in the rear. Sufficient horizontal play is allowed so that the reaction of the horizontal traversing guides is only vertical, the H component being taken up entirely at the pivot bearing.

As a typical class (1) top carriage we may illustrate by the Vickers 8",Mark VII, British Howitzer.

Further let

- l = distance between supporting reactions measured horizontally in the direction of the axis of the bore at 0° traverse.
- A = the front pivot point.
- B = the resultant of the distributed vertical reactions of the horizontal traversing arc guide.
- l_t = the horizontal distance to trunnions from B in the direction of the axis of the bore at 0° traverse.
- h_t = height of trunnions above the traversing guides.
- S = the perpendicular distance from the trunnion center to the line of action of the resistance to recoil which necessarily passes through the center of gravity of the recoiling mass.
- h_a = height of horizontal component of pivot reaction above the horizontal traversing guides.
- W_{tc} = weight of top carriage proper.
- l_{tc} = moment arm of W_{tc} about B.

Considering fig.(15) we have for the horizontal component of the pivot reaction, $H_a = K \cos Ø$ and taking moments about B, the center of pressure of the traversing guides, we have,

$$Kd - W_t l_t + W_r x \cos Ø - W_{tc} l_{tc} + V_a l - H_a h_a = 0$$

hence
$$V_a = \frac{W_t l_t + W_{tc} l_{tc} - W_r x \cos Ø - K(d - h_a \cos Ø)}{l}$$

and
$$V_b = \frac{K(d' - h_a \cos Ø) + W_t(l - l_t) + W_{tc}(l - l_{tc}) + W_r x \cos Ø}{l}$$

where for low angles of elevation, $d = h_t \cos Ø + S - l_t \sin Ø$

$$d' = h_t \cos Ø + (l - l_t) \sin Ø + S$$

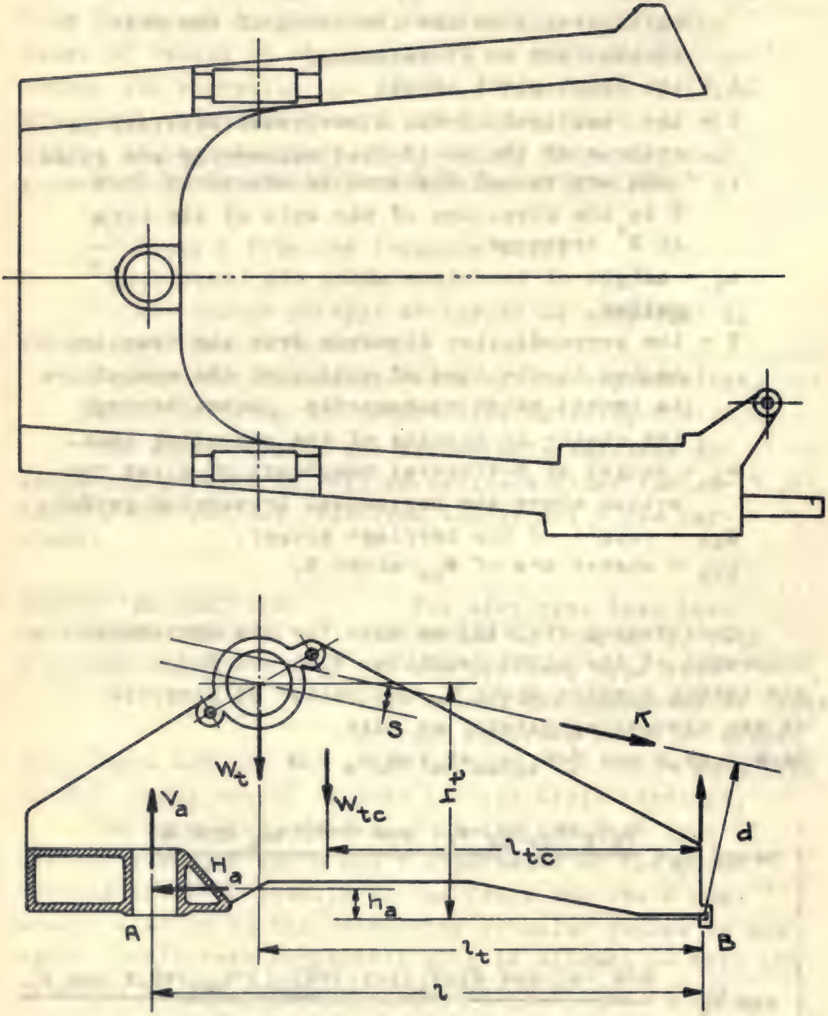


Fig. 15

and for high angles of elevation,

$d=l_t \sin \emptyset - h_t \cos \emptyset - S$

$d'=(l-l_t) \sin \emptyset + h_t \cos \emptyset + S$

y'_{mn} = the horizontal distance along the axle from the center of the wheel bearing pressure.

Considering, max. traverse, right handed, at max. elevation, the reactions on the axle to the left of the section, become,

(1) The components of the trail connecting arm reaction on the axle:- X,Y and Z together with a couple M_{xy} in the horizontal plane.

(2) The vertical reaction exerted by the left wheel, S_a. Therefore at section "mn", we have,

(1) Bending in the vertical plane:
$M_v=Z\ y_{mn}+S_a\ y'_{mn}$ (in lbs.)

(2) Bending in the horizontal plane:
$M_h=X\ y_{mn}-M_{xy}$ (in lbs.)

(3) Shear in the vertical plane:
$X+S_a$ (lbs)

(4) Shear in the horizontal plane:
X (lbs)

(5) Torsion about the y axis, or in the x z plane: $T=X\ Z_{mn}$

(6) A direct thrust:
Y (lbs)

Thus, we have bending in two planes combined with torsion, and a direct thrust as well. Then for a

round section, as would usually be the case, we have,

$$f = \frac{32\sqrt{M_v^2 + M_h^2}}{\pi \ d^3} + \frac{Y}{0.785 \ d^2}$$ Max. normal fibre thrust on outer layer (lbs/sq.in)

$$f_s = \frac{16T}{\pi \ d^3}$$ Extreme value of torsional shear (lbs/sq.in.)

The maximum fibre stress, therefore, becomes,

$$f_m = \frac{1}{2}f + \sqrt{\frac{1}{4}f^2 + f_s^2}$$ (lbs/sq.in)

which should not exceed $\frac{2}{3}$ of the elastic limit of the material to be used.

As a typical class of pivot yoke type, consider the reactions on a Barbette or Pedestal mount, figure (16) and a pivot yoke top carriage used on a trail carriage, figure (16A). In the first type, the lower bearing sustains both horizontal and vertical component reactions, whereas the upper is merely a floating bearing and therefore sustaining only a horizontal component and designed to prevent bending in the lower pivot.

In the second type, the middle bearing has a tapered fit within the axle, and therefore sustains both horizontal and vertical components, but suffers no bending moment since the axle is free to rotate. To prevent the top carriage and mount from rotating about the axle a lower cylindrical vertical pivot fits within an equalizer bar below the axle, the equalizer bar being attached to the trails. (See Theory of Split Trail - next section).

In this type of mount it is more convenient to compute the supporting reactions in terms of the hor-

REACTIONS ON
PIVOT YOKE TOP CARRIAGE

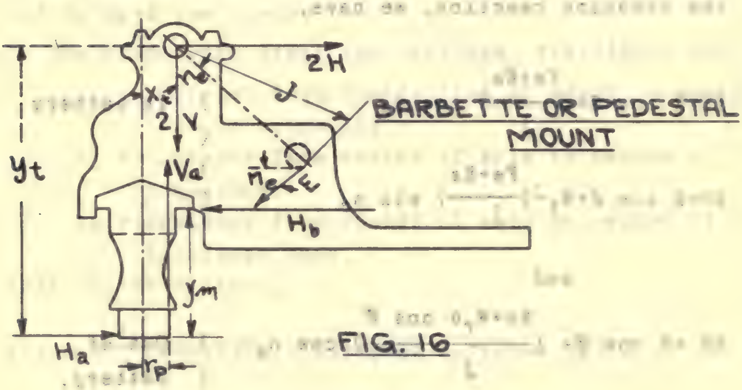


FIG. 16

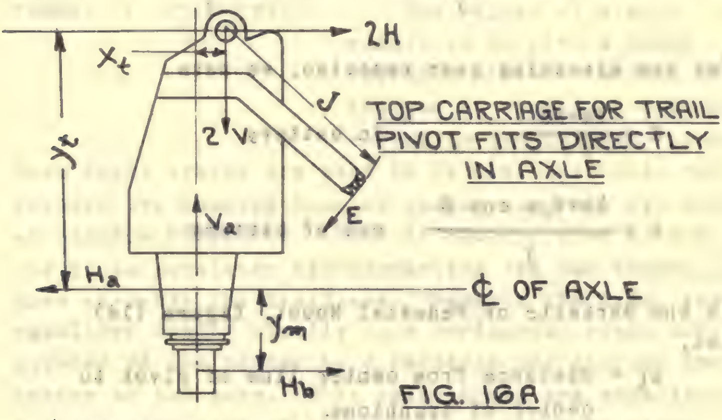


FIG. 16A

izontal and vertical components of the trunnion.

If

n_e = angle made by radius to elevating pinion contact on elevating rack with the vertical.

j = radius of elevating rack

Then for the horizontal and vertical components of the trunnion reaction, we have,

$$\left.\begin{aligned} 2H &= K\cos\emptyset + \left(\frac{Fe+Ks}{j}\right)\cos n_e \\ 2V &= K\sin\emptyset + W_t \mp \left(\frac{Fe+Ks}{j}\right)\sin n_e \end{aligned}\right\} \text{ In battery}$$

and

$$\left.\begin{aligned} 2H &= K\cos\emptyset + \left(\frac{Ks+W_r b\cos\emptyset}{j}\right)\cos n_e \\ 2V &= K\sin\emptyset + W_t - \left(\frac{Ks+W_r b\cos\emptyset}{j}\right)\sin n_e \end{aligned}\right\} \text{ Out of battery.}$$

For the elevating gear reaction, we have

$$E = \frac{Fe+Ks}{j} \quad \text{in battery}$$

$$E = \frac{Ks+W_r b\cos\emptyset}{j} \quad \text{out of battery}$$

In the Barbette or Pedestal Mount, figure (16) let,

x_t = distance from center line of pivot to center of trunnions.

y_t = height of center of trunnion from bottom of pivot.

r_p = radius of pivot bearing,

r_f = radius of floating bearing.

y_m = height between bottom of pivot and top of floating bearing.

Then, $V_a = 2V + E \sin n_e$

and

$$H_b = \frac{1}{y_m}\left\{ 2Hy_t + 2V(x_t - r_p) + E[j + (x_t - r_p)\sin n_e - y_t \cos n_e]\right\}$$

and $H_a = H_b + E \cos n_e - 2H$

In the pivot yoke trail top carriage, fig.(16A), let

x_t = distance from center line of pivot to center line of trunnions

y_t = distance from center of axle to center of trunnions.

y_m = distance from center of axle to center of equalizer beam.

Then, $V_a = 2V + E \sin n_e$

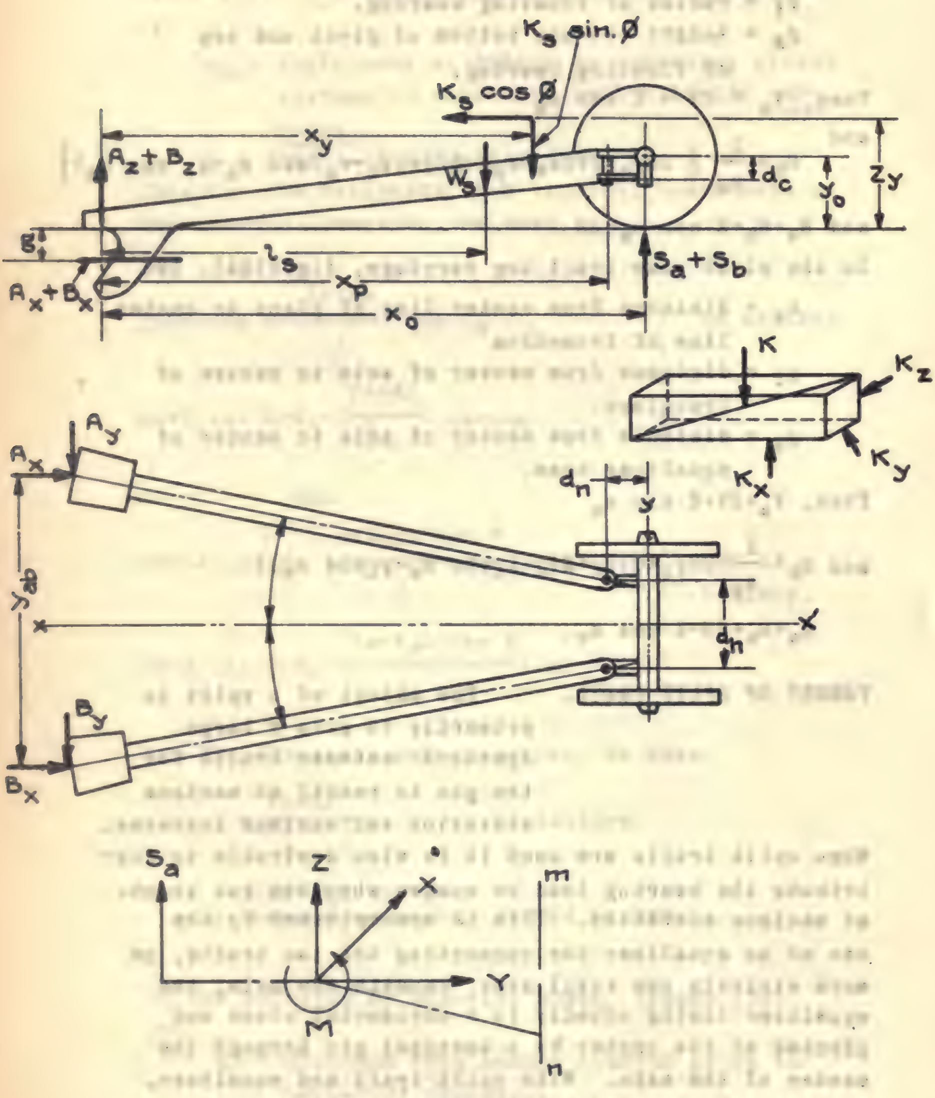
and $H_b = \frac{1}{y_m}[2Hy_t + 2Vx_t + E(j + x_t \sin n_e - y_t \cos n_e)]$

$H_a = H_b + 2H - E \cos n_e$

THEORY OF SPLIT TRAIL. The object of a split is primarily to give a large aperture between trails for the gun to recoil at maximum elevation and maximum traverse. When split trails are used it is also desirable to distribute the bearing load on spades when the gun shoots at maximum elevation. This is accomplished by the use of an equalizer bar connecting the two trails, or more strictly the trail arms, beneath the axle, the equalizer laying usually in a horizontal plane and pivoted at its center by a vertical pin through the center of the axle. With split trail and equalizer, a pivot yoke type of top carriage should be used.

The elements of a slip trail mechanism are :-

(1) The two trails with their spades which are connected by a vertical pin



Ks sin. Ø
Ks cos Ø
xy
Az + Bz
Ws
dc
yo
zy
g
ls
Sa + Sb
Ax + Bx
xp
xo
K
Kz
Ky
Kx
Ay
Ax
dn
y
ya
x
x
dh
By
Bx
Sa
Z
X
m
Y
M
n

Fig. 17

to two trail arms or trail connecting pieces, at either end of the axle.

(2) The trail arm or connecting pieces are free to turn about the axle in a vertical plane and are prevented from sliding along the axle by thrust shoulders. The moment about the axle of the trail reaction is balanced by the moment about the axle of the shear reaction of the equalizer bar.

(3) The equalizer bar is usually designed to rotate in a horizontal plane about a vertical pin through the axle. Thus, with a split trail we have the two trails, their connecting pieces to the axle and equalizer and the equalizer bar, connecting the trail arms and pivoted about a vertical pin which passes through the center of the axle.

We have the following possible motions:-

(1) A free rotation in a horizontal plane of either trail, about the vertical pin in the trail arm.

(2) A constrained rotation in a vertical plane about the axle of either spade, the constraint being due to the equalizer bar.

(3) A constrained rotation in a horizontal plane of the equalizer bar about a vertical pin through the axle.

LOAD REACTIONS ON SPLIT TRAIL AT MAXIMUM TRAVERSE AND MAXIMUM ELEVATION:

Let x and y = horizontal coordinates in longitudinal and transverse directions respectively,

i. e. along and cross wise to the bore at zero traverse.

Z = vertical coordinate.

$A_x A_y A_z$ = the component reactions at the left spade (positive direction towards muzzle (lbs)

$B_x B_y B_z$ = the component reactions at the right spade (lbs)

S_a and S_b = normal vertical reactions for left and right wheel respectively. (lbs)

Q_a and Q_b = shear reactions of equalizer on trails A and B respectively (lbs)

d_h = horizontal distance or span of equalizer between trails which it connects (in)

d_e = vertical distance from center line of equalizer to center line of axle through wheel hub. (in)

ΣM_{av} = moments of the components of A about the axle. (in lbs)

ΣM_{bv} = moments of the components of B about the axle. (in lbs)

x_o = distance from spade to axle. (in)

y_o = distance from ground to center line of axle. (in)

ΣM_{ah} = moments of reactions of A about vertical pin in left trail arm.

ΣM_b = moments of reaction of B about vertical pin in right trail arm.

Taking moments about the center pin of the equalizer, we have,

$Q_a \frac{d_h}{2} = Q_b \frac{d_h}{2}$ hence $Q_a = Q_b = Q$. Therefore, for moments about the axle, we have

$$\Sigma M_a - Q\, d_e = 0$$

$$\Sigma M_b - Q\, d_e = 0$$

hence, $\Sigma M_a = \Sigma M_b$

We have for unknowns,

$$\left.\begin{matrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ S_a & S_b & \end{matrix}\right\} \text{ Eight unknowns.}$$

Equations for solution:

$$\left.\begin{matrix} \Sigma X = 0 & \Sigma Y = 0 & Z = 0 \\ \Sigma M_x = 0 & \Sigma M_y = 0 & M_z = 0 \\ \Sigma M_{ay} = \Sigma M_{bv} & \Sigma M_{ah} = 0 & \\ \Sigma M_{bh} = 0 & & \end{matrix}\right\} \text{ Nine solutions.}$$

We therefore would expect either ΣM_{ah} or ΣM_{bh} not zero This is physically justified since on extreme traverse one of the wheels and trails must be in contact. This is met constructively by usually introducing a show attached to the trail which comes in contact with the wheel upon traversing.

If N = normal reaction between shoe and wheel,

d_n = perpendicular distance from vertical pin on trail arm to vertical plane through wheel, i. e. to line of action of N.

We have for maximum traverse in a right handed rotation,

$\Sigma M_{bh} = N\, d_n$ thus introducing an additional unknown N. The solution is, therefore, statically possible either introducing the above equation or omitting it entirely.

METHOD OF SOLUTION

Assume maximum traverse right handed turn,

Let ϕ_m = maximum angle of elevation.

θ_m = maximum angle of traverse.

K_s = resistance to recoil at maximum elevation (lbs)

x_y = horizontal distance to projection of center of gravity of recoiling parts measured from base line AB. (in)

y_g = 0 assumed approx.

Z_g = height of center of gravity in battery above ground, (in)

g = vertical distance from ground line to center of pressure on spade. (in)

W_s = weight of total system, gun + carriage. (lbs)

l_s = horizontal distance from AB to W_s

y A'B' = distance between vertical trail pins.

Then, the resolved component of K_s through the center of gravity of the recoiling parts, become,

$K_s \cos \phi_m \cos \theta_m$, along the x axis

$K_s \cos \phi_m \sin \theta$, along the y axis

$K_s \sin \phi_m$ along the Z axis

x_p = distance from AB to either vertical trail pin.

y_{ab} = distance between A and B, trails completely spread.

Taking moments about A B, we have,

$$(S_a+S_b)x_o+K_s\cos \phi_m \cos \theta_m(Z_g+g)-K_s\sin \phi_m . x_g-W_s l_s = 0$$

hence,

$$S_a+S_b=\frac{W_s l_s-K_s\cos \phi \cos \theta_m(Z_g+ g)+K_s\sin \phi_m x_g}{x_o} \qquad (1)$$

and $A_z+ B_z= W_s+ K_s \sin \phi_m-(S_a+ S_b)$ (2)

Next take moments in a horizontal plane about the left spade, and we have,

$$\Sigma M_a=B_x\, y_{ab}+K_s\cos \phi_m\sin \theta_m .\, x_g-K_s\cos \phi_m\cos \theta_m \frac{y_{ab}}{2} = 0$$

hence $B_x = \frac{K \cos \phi_m}{y_{ab}} (0.5\, y_{ab} \cos \theta_m - x_g \sin \theta_m)$ (3)

Further $A_x + B_x = K_s \cos \phi_m \cos \theta_m$

hence $A_x = K_s \cos \phi_m \cos \theta_m - B_x$ (lbs) (4)

For moments about the vertical pin in trail arm for left trail, we have, $A_y x_p - A_x\, 0.5\, y_{A'B'} = 0$

hence $A_y = \frac{0.5\, A_x y_{A'B'}}{x_p}$ (lbs) (5)

Now if we take moments about the axle, we have $\Sigma M_{av} = \Sigma M_{bv}$

$$A_z\, x_o - A_x(Z_o+g) = B_z x_o - B_x(y_o+g)$$

therefore
$$A_z - B_z = \frac{(A_x - B_x)(Z_o+g)}{x_o}$$

but $A_z + B_z = W_s + K_s \sin \phi_m (S_a + S_b)$

hence $A_z = \frac{(A_x - B_x)(Z_o+g)}{2x_o} + \frac{W_s + K_s \sin \phi_m - (S_a+S_b)}{2}$ (6)

$$B_z = W_w + K_s \sin \phi_m - (S_a+S_b) - A_z \quad (7)$$

Let

X, Y and Z = components of the reaction of the trail arm on the axle,

M_{xy} = moment reaction of trail arm on axle, in the X Y plane.

Considering moments on the left trail and trail arm together about the axle, we have,

$A_z x_o - A_x(Z_o+g) - Q\, d_e = 0$ hence, the horizontal shear reaction of the equalizer becomes,

$$Q = \frac{A_z x_o - A_a(Z_o+g)}{d_e} \quad (8)$$

Next consider the various reactions on the trail arm, and we have,

$-X+Q+A_x = 0$ along the x axis,
$-Y+A_y = 0$ along the y axis.
$-Z+A_z = 0$ along the z axis.

and further, $-M_{xy}+A_y(x_o-x_p) = 0$ In the x y plane
Therefore, the reactions of trail arm on the axle, becomes,

$$X = A + A_x = \frac{A_z x_o - A_x(Z_o+g)}{d_o} + A_x \quad \text{(lbs)} \qquad (9)$$

$$Y = A_y \quad \text{(lbs)} \qquad (10)$$

$$Z = A_z \quad \text{(lbs)} \qquad (11)$$

$$M_{xy} = A_y(x_o - x_p) \quad \text{(in.lbs)} \qquad (12)$$

STRENGTH OF AXLE MAXIMUM TRAVERSE, MAXIMUM ELEVATION:

This critical section of an axle is at a section near the center where the axle becomes enlarged for holding the vertical pivot of the top carriage. If the axle is made straight, we have no torsion on the section but merely bending in a vertical and horizontal plane. If, however, the axle is underhung for clearance and lowering the top carriage, in addition to the bending we have torsion as well, the magnitude of the torsion depending upon the depth of the underhang.

Let mn be the section under consideration near the center of the axle.

x_{mn} y_{mn} and z_{mn} = the component distances from the center of contact of the trail connecting arm and axle.

REACTION BETWEEN RECOILING PARTS AND MOUNT IN COUNTER RECOIL. During the counter recoil, we may distinguish between the accelerating and retardation period so far as the reactions between

the recoiling parts and mount are concerned. The reactions during the acceleration however are of exactly the same character as during the recoil only of less magnitude. Therefore, from either the point of view of the internal reactions or stability of the mount, we are not concerned with the acceleration period of counter recoil.

Therefore let us consider the various recoiling parts and mount coming into play during the retardation period of counter recoil,-

Let (see figure 18)

x_1 and y_1 = coordinates, along and normal to bore, of front clip reaction with respect to center of gravity of recoiling parts.

x_2 and y_2 = coordinates, along and normal to bore, of rear clip reaction with respect to center of gravity of recoiling parts.

Q_1 = front clip reaction.

Q_2 = rear clip reaction.

W_r = weight of recoiling parts.

$\emptyset$ = unbalanced retarding force exclusive of friction.

$d_\emptyset$ = distance from center of gravity of recoiling parts to line of action of $\emptyset$.

n = coefficient of friction = 0.15 usually.

d^1 = distance from front wheel ground contact to line parallel to bore through center of gravity of recoiling parts.

l_r= horizontal distance from line of action of W_r to front wheel ground contact.

x = displacement along bore or guides from out of battery position.

M_a' = moment of reaction of the recoiling parts on mount about front wheel contact and ground.

Then, for the motion of the recoiling parts, we have,

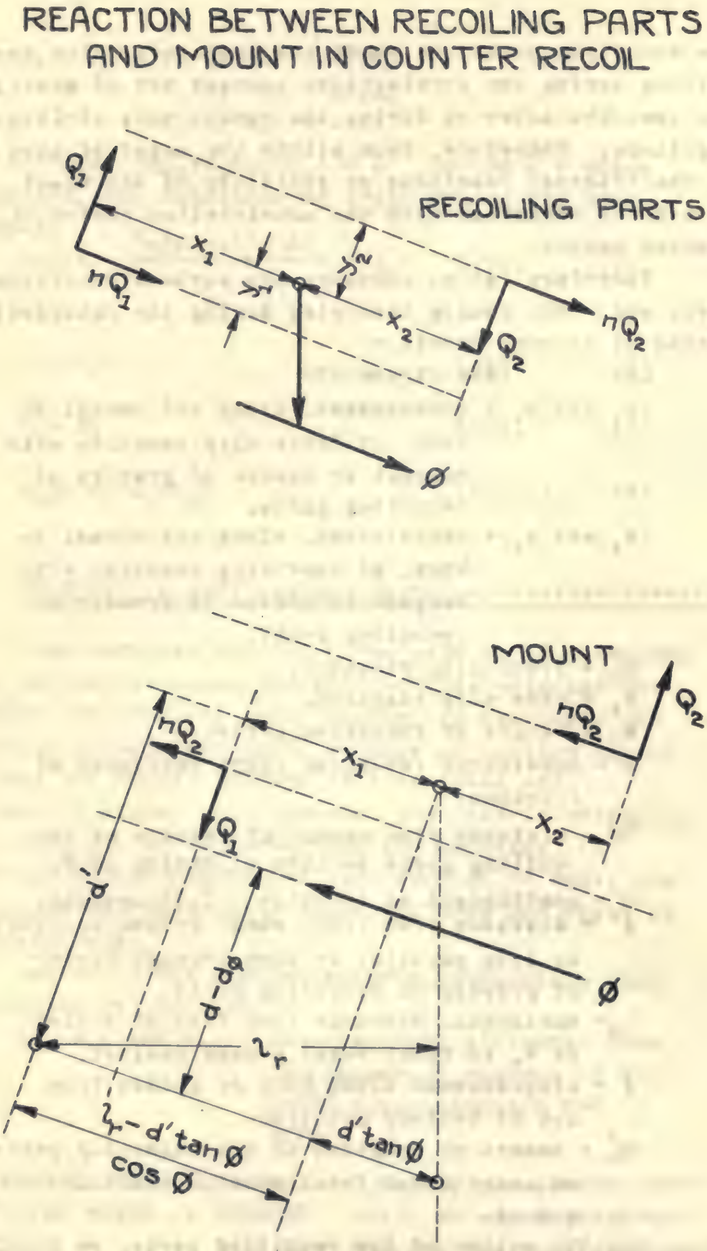

REACTION BETWEEN RECOILING PARTS
AND MOUNT IN COUNTER RECOIL
RECOILING PARTS
Q1
nQ1
x1
x2
y1
y2
nQ2
Q2
Ø
MOUNT
Q2
nQ2
nQ2
Q1
x1
x2
d'
d - dØ
Ø
lr
lr - d' tan Ø
cos Ø
d' tan Ø

Fig. 18

$$\emptyset+n(Q_1+Q_2)+W_r\sin\emptyset=-m_r\frac{d^2x}{dt^2} \qquad (1)$$

$$Q_1-Q_2=W_r\cos\emptyset \qquad (2)$$

$$\text{and } \emptyset\, d_\emptyset-Q_1x_1-Q_2x_2+n\,Q_1y_1-n\,Q_2y_2=0 \qquad (3)$$

Next, considering the reactions on the mount and taking moments, about the point of contact of the front wheels with ground A', we have,

$$\emptyset(d-d_\emptyset)+nQ_1(d'-y_1)+n\,Q_2(d'+v_2)$$
$$-Q_1\left(\frac{l_r}{\cos\emptyset}-d'\tan\emptyset-x_1\right)+Q_2\left(\frac{l_r}{\cos\emptyset}-d'\tan\emptyset+x_2\right)=M_A' \qquad (4)$$

Substituting Eq. (3) and (2) in Eq. 4, we have immediately

$$\emptyset\, d'+n(Q_1+Q_2)d'+W_r\sin\emptyset.d'-W_rl_r=M_A'$$

or

$$(\emptyset+n(Q_1+Q_2)+W_r\sin\emptyset)d'-W_rl_r=M_A'$$

Further from equation (1) $\quad \left(-m_r\frac{d^2x}{dt^2}\right)d'-W_rl_r=M_A'$

Hence, the reaction on the mount during counter recoil is equivalent to the total resistance to recoil acting in a line parallel to the axis of the bore and through the center of gravity of the recoiling parts, together with a component in line and equal to the weight of the recoiling parts.

If further, we let,

F_v = the recuperator reaction.

R_G = total guide friction

R_{s+p} = total packing friction.

B_x' = total counter recoil buffer reaction.

Then $\emptyset = B_x'+R_{s+p}-F_v$

$R_G = n(Q_1+Q_2) = n\,W_r\cos\emptyset$ (approx.)

and the overturning force, passing through the center of gravity of the recoiling parts and parallel to the

bore, becomes,

$$- [F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} - B_x'] = -m_r \, v \frac{dv}{dx} \quad \text{(lbs)}$$

GRAPHICAL CONSTRUCTION AND EVALUATION OF THE REACTIONS IN A GUN CARRIAGE.

Very often it is more convenient to evaluate the various reactions by graphical methods. Graphical constructions are of special value since they give a vivid picture of the relative magnitude of the various reactions. Further the method is comparatively simple and the closing of force and space polygons combined with overall methods gives an admirable check. The accuracy of the method even with rough layouts is sufficient for the computation of the various reactions required.

If we consider the kinetic equilibrium of any piece of the carriage, we have, by introducing the kinetic reactions or inertia forces with the actual reactions exerted on the piece, a dynamic problem reduced to a problem of statics.

For equilibrium of the piece, we have,

$$\left.\begin{array}{l} \Sigma X = 0 \\ \Sigma Y = 0 \\ \Sigma M = 0 \end{array}\right\} \text{ for a coplanor set of forces.}$$

Now the considerations $\Sigma X = 0$, $\Sigma Y = 0$ are met by the vector diagram of reactions having a zero resultant, that is the vector polygon of the piece closing.

The condition $\Sigma M = 0$, requires a consideration of the lines of action of the forces in a space diagram in addition. Since the moments may be taken about any point, there can be no resultant moment existing. The condition $\Sigma X = 0$, $\Sigma Y = 0$ implies the result-

ant force to be zero, but does not imply the existence of a couple. Condition $\Sigma M = 0$.

imolies that a resultant couple cannot exist.

A graphical method, therefore, always consists of two sets of diagrams,

(1) a space diagram of forces and

(2) a vector diagram of forces.

The space diagram requires a layout proportional to the actual piece under consideration and the placing on this diagram the lines of action of the forces. The force diagram requires a layout proportional to the direction and magnitude of the various reactions exerted on the piece. The two diagrams must be carried on simultaneously since the direction of a resultant required in a space diagram, is obtained by the vector addition of the forces which are the components of the resultant. Since Vector addition is commutative, the order of Vector addition is immaterial.

REACTIONS ON THE RECOILING PARTS

The known reactions consist:

(1) The powder force along the axis of the bore P_b. (lbs)

(2) The inertia force along an axis parallel to the bore and through the center of gravity of the recoiling parts - - $m_r \frac{d^2x}{dt^2}$

(3) The weight of the recoiling parts acting vertically through the center of gravity of the recoiling parts - - W_r.

The unknown reactions consist:

(1) The resultant braking force - - - B lbs.

(2) The front and rear clip reactions - - Q and Q_2 lbs.

The lines of actions of these forces however are known or can be readily determined.

Procedure:

Layout a space drawing proportional to the dimensions of the recoiling parts, showing the assumed lines of actions of the various forces. See fig.(19).

Since $P_b - m_r \frac{d^2x}{dt^2} = K$ the total resistance to recoil which is assumed as known.

we have the effect of P_b and $m_r \frac{d^2x}{dt^2}$ equivalent to,

(1) a couple $P_b\, e_b$

(2) a force K through the center of gravity of the recoiling parts parallel to the axis of the bore.

Since a couple and a single force may always be reduced to an equivalent single force, we have (1) and (2) combined into a single force K acting at a distance above the axis through the center of gravity of the recoiling parts equal to

$\frac{P_b e_b}{K}$ (in) (where e_b is in inches)

The reactions Q_1 and Q_2 due to the friction in the cradle sleeve make an angle $u = \tan^{-1} f$ with the normal to the guides, where f = coefficient of friction = 0.15 usually. Hence $u = 8.5^\circ$ approximately. Referring now to the force polygon or diagram, lay off K in the direction and equal to the magnitude of the total resistance to recoil.

Lay off K = a b

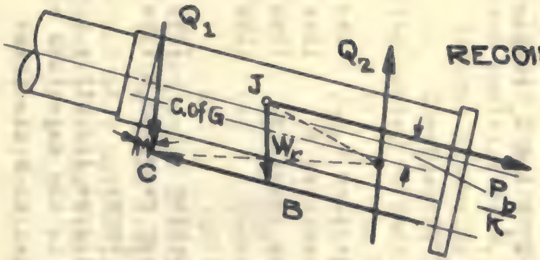
From b lay off b c = W_R, the weight of the recoiling, in magnitude and direction.

Draw $\overline{K + W_R} = a\, c$

SPACE DIAGRAMS

FORCE DIAGRAMS

RECOILING PARTS



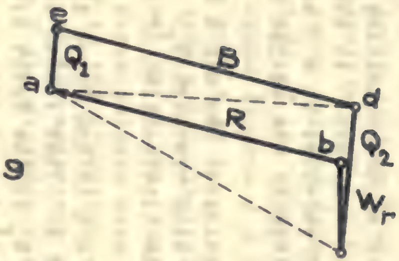
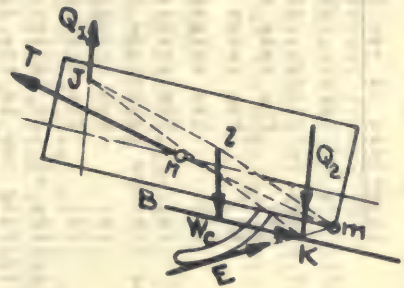


Fig. 19

CRADLE



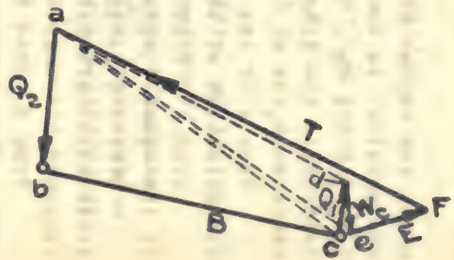


Fig. 20

Referring now to the space diagram lay off K at a perpendicular diatance $\frac{P_b e_b}{K}$ above the center of gravity of the recoiling parts and parallel to the axis of the bore. At the intersection of K and W_R , draw J k parallel to a c until it intersects the line of action of the motion of the reaction Q_2 at k.

From c of the force polygon, lay off c d and fn the direction of the rear clip reaction Q_2.

Draw k c from k to the intersection of Q_1 and B in the space diagram.

Draw a d parallel to k c in the force diagram until it intersects a d at d. This limits and determines the magnitude of Q_2 in the force diagram.

From d, draw d e parallel to B and a e parallel to Q_1. The intersection of a e and d e determines B and Q_1 respectively. Thus from a combination of the space and force diagram we obtain Q_2 B and Q_1 respectively.

REACTIONS ON THE CRADLE.

Referring to figure (20):

The known reactions consist:-

(1) The rear clip reaction Q_2 (lbs)
(2) The front clip reaction Q_1 (lbs)
(3) The weight of the cradle W_C (lbs)
(4) The braking force B (lbs)

The unknown reactions consist:-

(1) The trunnion reaction T (lbs)
(2) The elevating gear reaction E (lbs)

The direction of the latter being known.

Referring now to the force diagram lay off a b = Q_2 in the direction and proportional to the magnitude of Q_2. From b draw c parallel and equal to B the braking force.

Draw a c.

Referring now to the space diagram J k from the intersection of Q_2 and B at k, parallel to a c in the

in the force polygon, to the intersection of Q_1.

In the force diagram, draw c d = Q_1 and parallel to Q_1. draw a d.

In the space diagram draw J L parallel to a d to the intersection of W_c.

In the force polygon draw W_c equal and parallel to W_c the weight of the cradle. Draw a c.

In the space diagram l m parallel to a e to the intersection with E at m.

From m draw m n to the trunnion axis, which gives the line of action of the trunnion reaction T.

In the force polygon draw e f in the direction of E and a F in the direction of T. The intersection at f determines the magnitude of E the elevating gear reaction and T the trunnion reaction.

REACTIONS ON THE TIPPING PARTS.

Locate the trunnions along the resultant of the battery position of W_r and W_c - - - See upper diagram.

Without balancing gear:-

Considering the external forces on tipping parts, we have, the known reactions,

(1) The total resistance to recoil K (lbs)

(2) The weight of the tipping parts W_t (lbs)

The unknown reactions,

(1) The elevating gear reaction E (lbs)

(2) The trunnion reaction T (lbs)

the direction of E being known.

In the space diagram lay off K parallel to the bore and at a perpendicular distance from the center of gravity of the recoiling parts = $\frac{P_b e}{K}$ (in)

In the force diagram, lay off ab = K and bc = W_t.

Draw ac.

In the space diagram, lay off J k from the intersection of K and W_t parallel to ac of the force diagram

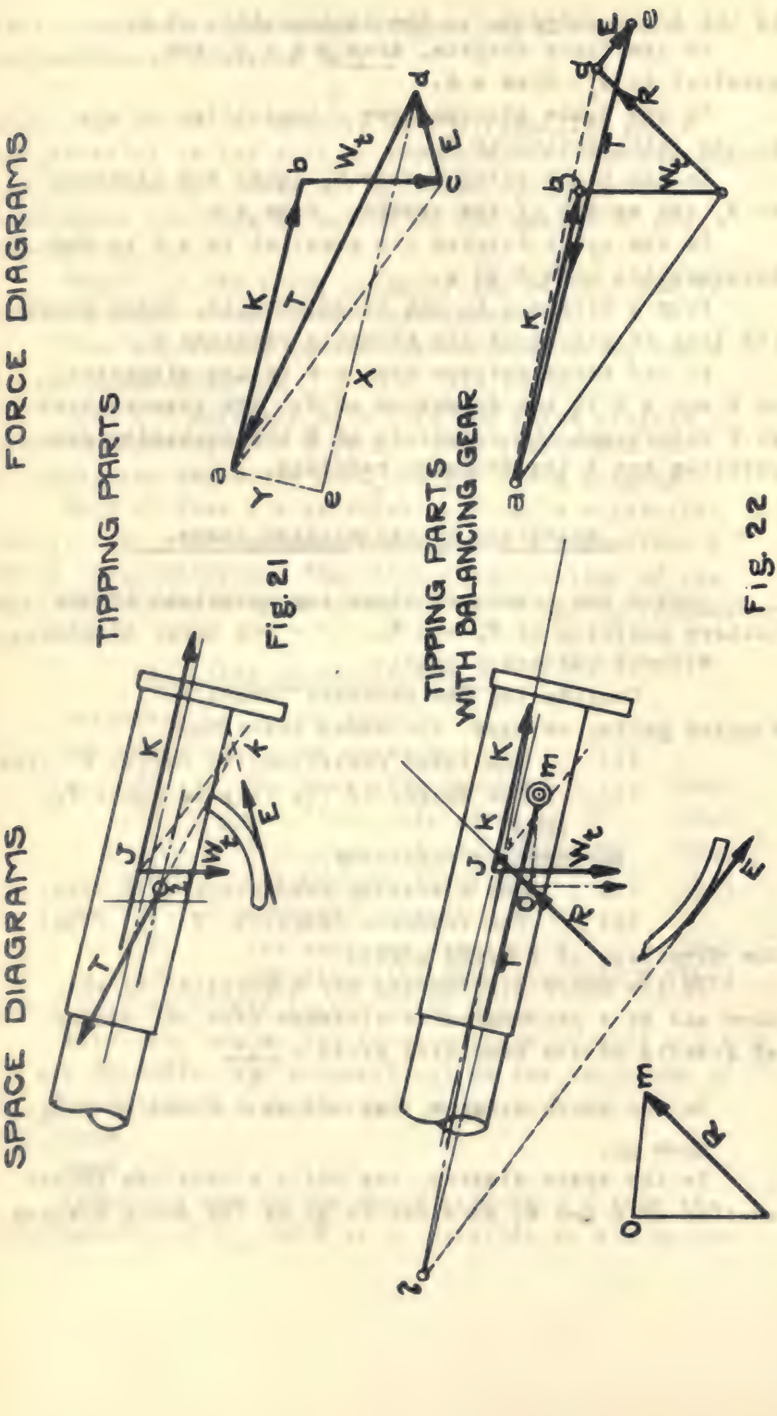

SPACE DIAGRAMS
FORCE DIAGRAMS
TIPPING PARTS
TIPPING PARTS
WITH BALANCING GEAR
K
T
J
E
a
b
c
d
e
X
Y
m
R
o

Fig. 21

Fig. 22

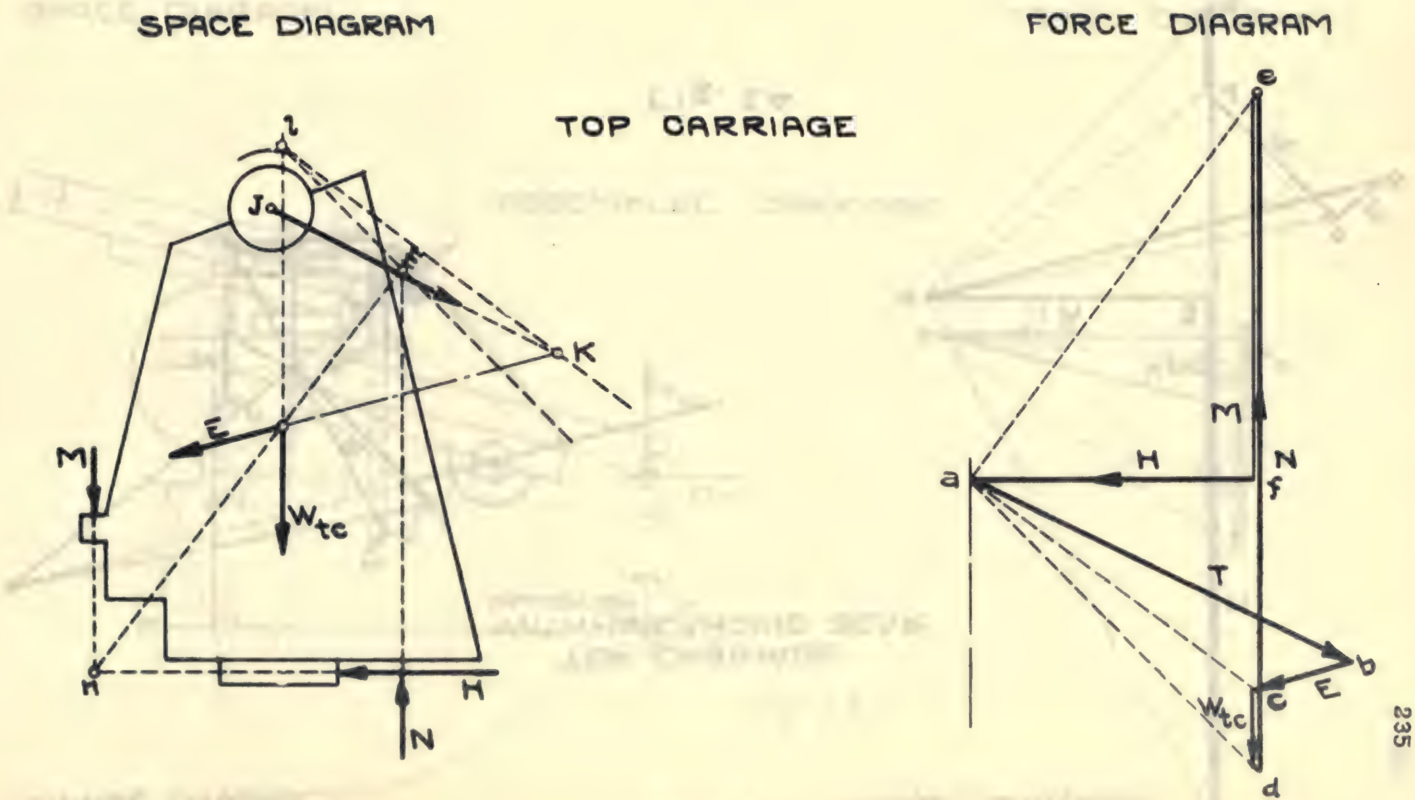

SPACE DIAGRAM
TOP CARRIAGE
FORCE DIAGRAM
l
J
K
E
M
W_{tc}
H
N
n
e
M
N
f
H
a
T
b
E
c
W_{tc}
d

Fig. 23

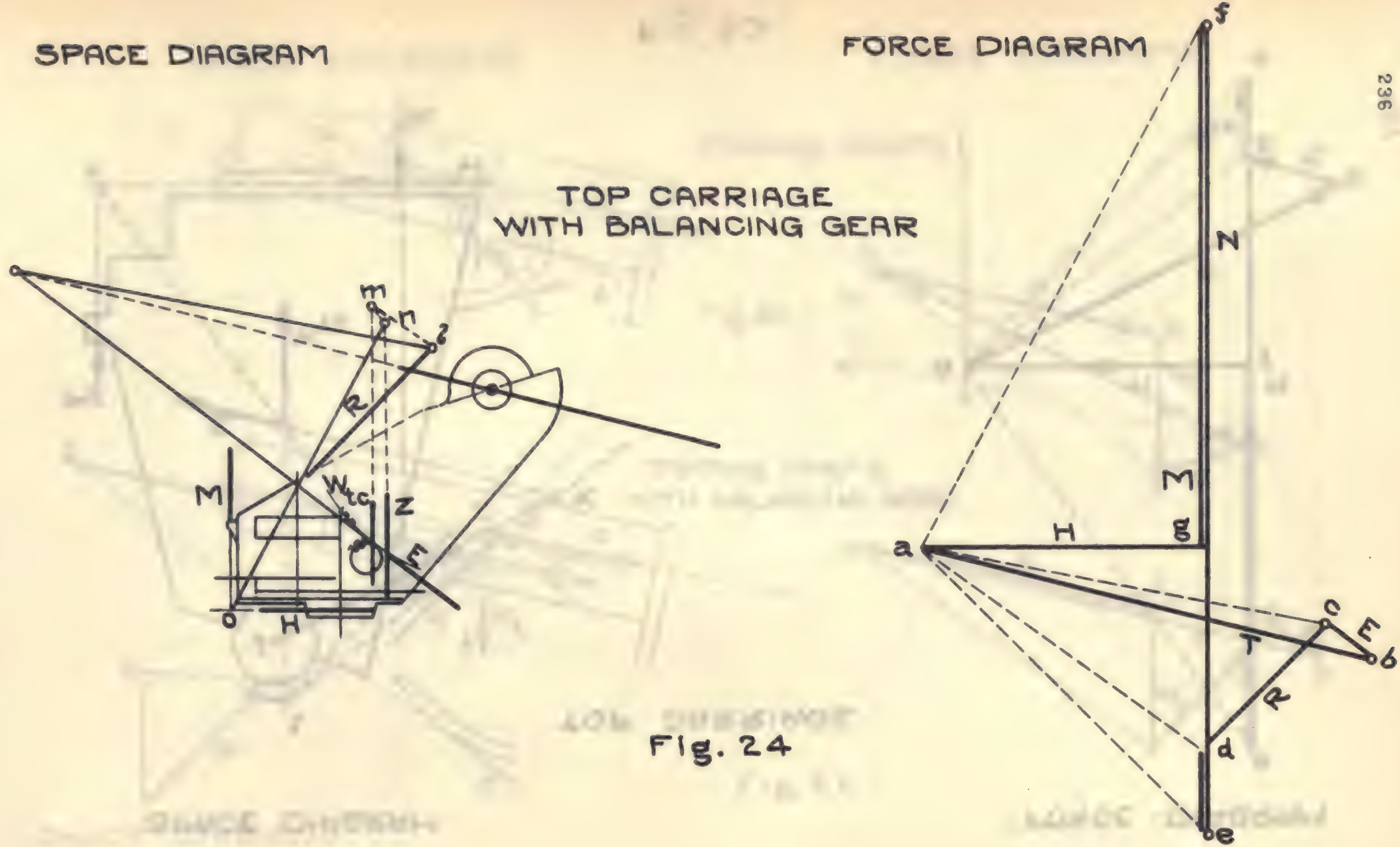

SPACE DIAGRAM
FORCE DIAGRAM
TOP CARRIAGE
WITH BALANCING GEAR
m
n
l
R
M
Wtc
Z
E
o
H
f
N
M
g
H
a
c
E
T
b
R
d
e

Fig. 24

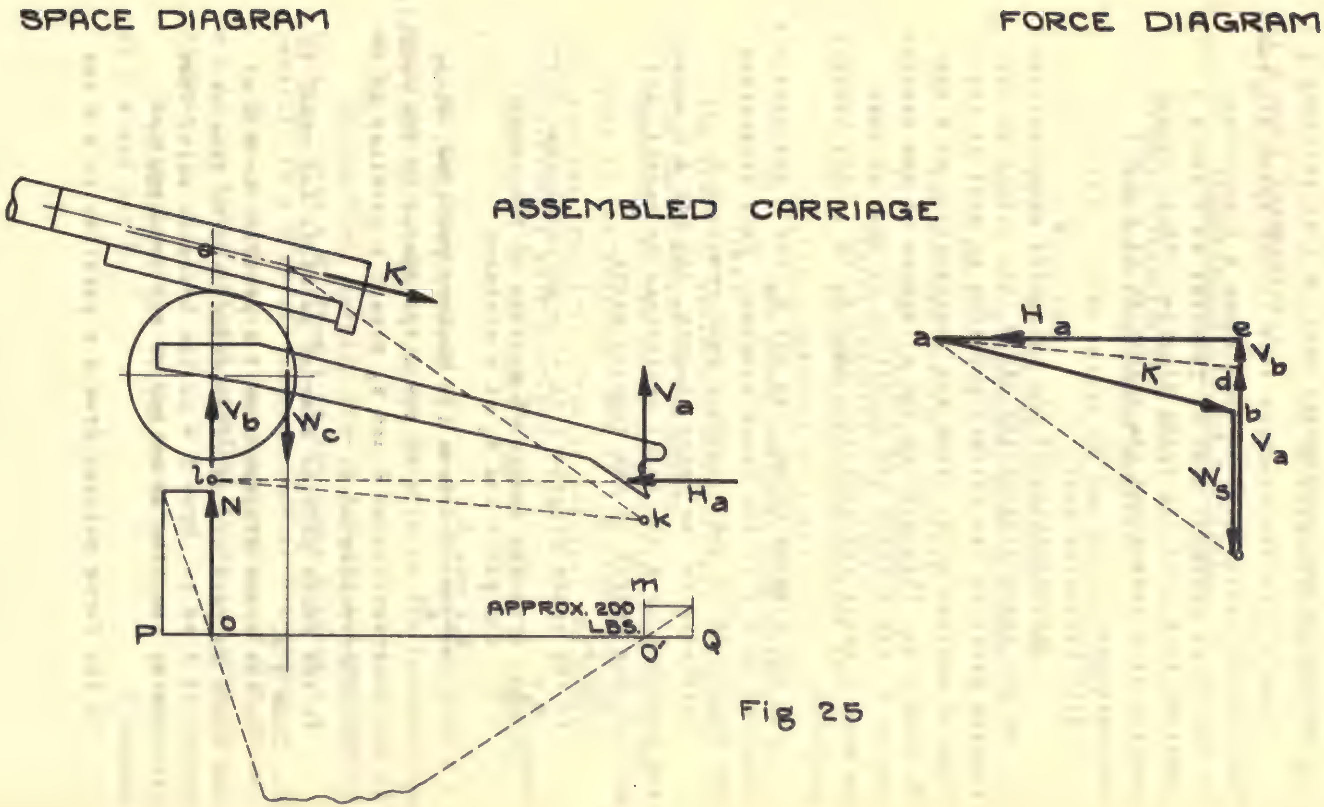

SPACE DIAGRAM
FORCE DIAGRAM
ASSEMBLED CARRIAGE
K
V_a
V_b
W_c
H_a
l
N
k
m
APPROX. 200 LBS.
P
o
O'
Q
a
e
d
b
W_s

Fig 25

and to the intersection of E.

Draw k l to the trunnion axis in the space diagram. The line of action of T is then along h l produced. In the force polygon, draw cd parallel to E and a d parallel to k l of the space diagram. Their intersection at d determines the magnitude of E and T respectivelv.

With balancing gear:-

Determination of the balancing gear reaction R. On the space diagram lay off W_t the weight of the tipping parts in its battery position as well as the line of section of R. From the intersection of R and W_t draw o m. This must be the direction of the resultant of W_t and R since the condition is that we have no moment about the trunnions when W_t is in its battery position.

In the diagram below, lay off W_t and R and draw o.m parallel to o m in the space diagram. This determines the magnitude of the balancing gear reaction R.

Referring to the force diagram, lay off a b equal and parallel to K the total resistance to recoil, and bc = W_t the weight of the tipping parts.

Draw ac.

In the space diagram K is at a perpendicular distance $\frac{B_b e}{K}$ from the center of gravity of the recoiling parts and W_t at a distance $\frac{W_r x \cos \emptyset}{W_t}$ from its battery position, where x is the displacement in the recoil. At the intersection of K and W_t draw j k parallel to ac to the intersection of R.

In the force polygon draw cd parallel and equal to R. Draw a d.

In the space diagram draw l k parallel to a d of the force polygon to the intersection of the line of action of E ac l. Draw l m to the trunnion axis, thus determining the line of action of the trunnion reaction T.

In the force polygon draw d e parallel to m E and

a e parallel to l m, thus determining the magnitude of E and T respectively.

REACTIONS ON THE TOP CARRIAGE

Without balancing gear:-

The known reactions consist:

(1) The weight of the top carriage W_{tc} (lbs)

(2) The trunnion reaction T (lbs)

(3) The elevating gear reaction E (lbs)

The unknown reactions consist:

(1) The horizontal component of the pintle reaction - H (lbs)

(2) The vertical component of the pintle reaction N (lbs)

(3) The front vertical clip reaction M (lbs)

The lines of actions of these forces are given from the construction of the piece.

Referring to the force polygon fig.(25), draw ab =T equal to the magnitude and in the direction of T the trunnion reaction. Draw bc parallel and equal to E the elevating gear reaction.

Draw ac. In the space diagram draw j k parallel to T.

At the intersection of j k and E produced draw k l parallel to a c in the space diagram to the intersection of W_{tc}.

In the force polygon draw c d equal and parallel to W_{tc}. Draw a d From l in the space diagram l m parallel to a d to the intersection of N produced.

From m draw m n to the intersection of H M. Draw a e in the force polygon parallel to mn in the space diagram.

We thus have d e in the force polygon = N and ef = M and ja= H.

Thus the pintle reactions H and N and the clip reaction are determined in magnitude and direction.

With balancing gear:-

The known reactions consist:

(1) The weight of the top carriage W_{tc}

(2) The trunnion reaction T

(3) The elevating gear reaction E

(4) The balancing gear reaction B

The unknown reactions consist:

(1) The horizontal component of the pintle reaction H

(2) The vertical component of the pintle reaction N

(3) The front vertical clip reaction M

The lines of actions of these forces are given from the construction.

Referring now to the force polygon fig.(24) Lay off ab = T and bc = E. Draw ac.

In the space diagram from the intersection of T the trunnion reaction and E elevating reaction produced at K.

Draw k l parallel to ac of the force polygon. Continue in the force polygon c d = R the balancing gear reaction. Draw ad. In the space diagram draw in parallel to ad and the intersection of W_{tc} at m. In the force polygon draw de.

Draw ae.

In the space diagram draw mn parallel to ae to the intersection of N. Fron N draw n o to the intersection of M and H. In the force polygon draw a f parallel to o n. From E in the force polygon draw e f parallel to N to the intersection of e f.

Draw f g and g a as shown.

Thus we determine the reactions M, N, and H respectively.

REACTIONS ON THE ASSEMBLED CARRIAGE GUN AND CARRIAGE TOGETHER.

Location of the weight of the total mount:-

Assuming a static reaction of 200 lbs. under the spade, we lay off o'm = 200 lbs.

Then o N = W_s = 200 under the wheel contact. The resultant of o'm and o n = W_s obtained by the additional construction lines o'q and op. Hence we determine from the triangle of forces the line of action of W_s. The external reactions on the assembled carriage consists of :-

The known reactions -

(1) K = the total resistance to recoil.

(2) W_s = the weight of the total mount.

The unknown reactions -

(1) The horizontal spade reaction H_a.

(2) The vertical spade reaction V_a.

(3) The normal reaction under the wheels V_b.

The direction of these forces are known.

Referring to the force polygon lay off ab = K the total resistance to recoil and bc = weight of total system W_s.

Draw ac.

In the space polygon from the intersection of K and W_s draw j k to the intersection of the reaction V_a.

From k draw k l to the intersection of H_a, V_b at l.

Referring to the force diagram draw ad parallel to l k of the space diagram to the intersection of c e produced.

We thus determine c d = V_a, d e = V_b and e a = H_a.

Thus the reactions H_a, B_a and V_b are determined in magnitude.

PROCEDURE IN THE CALCULATIONS FOR THE PRINCIPLE REACTIONS IN A GUN CARRIAGE MOUNT.

(Illustrated by calculations on 240 m/m Howitzer)

REQUIRED DATA.

Type of Gun		Howitzer
Diameter of bore d (in)		9.45
Total Weight of recoiling parts W_r (lbs)		15790
Weight of Powder Charge W (lbs)		40
Muzzle Velocity v (ft/sec.)		1700
Travel of Shot in Bore u (in)		160
Angle of Elevation ϕ	maximum	60°
	minimum	10°
Length of Recoil b (ft)	short	3.74
	long	3.90
Intensity of Powder Pressure p_m (lbs/sq.in)		32000
Initial Air Volume of Recuperator V_{ai} (cu.in)		2970
Initial Air Pressure of Recuperator P_{ai} (lbs/sq.in)		576

INTERIOR BALLISTICS.

Maximum Powder Pressure on Breech — 2,245,000

$$F = P_b = 0.7854\, d^2 p_m \text{ (lbs)}$$

Maximum Powder Pressure on Base of Projectile p_m (lbs) — 2,005,000

$$P_m = \frac{P_b}{1.12} \text{ (lbs)}$$

Mean Constant Powder Pressure

$$P_e = \frac{wv^2}{5.36u}$$

$$\frac{40 \times \overline{1700}^2}{5.36 \times 160} = 1,350,000$$

l = twice abscissa of Max. Pressure

$$= \frac{27}{16}\frac{P_m}{P_e} - 1 \pm \sqrt{\left(1 - \frac{27}{16}\frac{P_m}{P_e}\right)^2 - 1} \qquad 3.996'$$

P_{ob} = Muzzle Pressure on base of breech

$$= \frac{27}{4} l^2 \frac{u}{(l+u)^3} - P_b \qquad 622,000 \text{ lbs.}$$

Vel. of free recoil: V_f

$$= \frac{wV_m + 4700\, W}{W_r} \qquad 50.25 \text{ ft/sec.}$$

Vel. of free recoil - Shot leaving Muzzle

$$V_o = \frac{wV_m + 0.5W\, V_m}{W_r} \qquad 40.50 \text{ ft/sec.}$$

Time of Shot to Muzzle

$$t_1 = \frac{3}{2}\frac{u''}{12V_v} \qquad 0.01175 \text{ sec.}$$

Time of Expansion of Free Gases:

$t_2 = \frac{2(V_f - V_o)}{P_{ob}} \frac{W_r}{32.2}$ 0.01538 sec.

Free Movement of Gun while shot travels to Muzzle

$X_1' = \frac{u''(w+0.5W)}{12(W_r + w + w)}$ 0.31 ft.

Free Movement of gun during Powder Expansion

$X_2' = \frac{P_{ob}}{W_r} g \frac{t_2^2}{3} + V_o t_2$ 0.7179 ft.

Total free Movement of gun; Powder Pressure Period:

$E = X_1 + X_2$ 1.0279 ft.

Time of Powder Pressure Period.

$r = t_1 + t_2$ 0.02713 sec.

BRAKING PULLS AND STRESSES IN CYLINDERS.

x axis taken along bore: y axis taken normal to bore.

Mass of Recoiling parts

$m_r = \frac{W_r}{32.16}$ $\frac{15790}{32.16} = 491$

Constant of Stability
C = 0.85 to 0.9 — Calculations only for max. elev.

Height of center of gravity of recoiling parts above ground
h (ft) — Calculations only for max. elev.

Stability Slope $m = \frac{cW_r}{h}$ — Calculations only for max. elev.

Total Resistance to Recoil $\frac{\text{Max.Elev.}}{\text{Hor.Elev.}}$ $K = \frac{m_r V_f^2}{2(b-E+V_f T)}$ (lbs)	$\frac{491 \times \overline{50.75}^2}{2(3.74-1.0279+50.75\times.02713)} =$ 152,000
Variable Resistance to recoil in battery (at horizontal elev.) $K = \frac{m_r V_f^2 + m(b-E)^2}{2[b-E+V_f T - \frac{m}{2}\frac{T^2}{m_r}(b-E)]}$ (lbs)	Calculations only for max. elev.
Variable Resistance to Recoil out of battery (at horizontal elev.) $k = K - m(b-E+\frac{KT^2}{2m_r})$	Calculations only for max. elev.
Initial Recuperator Reaction, P_{ai} = approx. $1.3W_r(\sin \text{Ø}_m + 0.15\cos\text{Ø}_m)$ lbs. (unless given)	1.3 × 15790 (sin 60+0.15cos60) = 19300 used 18800 lbs.
Total Initial Recuperator Pull, $P'_{ai} = P'_{ai} + 100\, d_a$ (lbs) d_a = diam. of recuperator rod. (in)	18800+100×2.938 = 19094
Effective Area of Recuperator Piston - A_a (sq.in)	35.766
Initial Air Pressure $p_{ai} = \frac{P_{ai}}{A_a}$ (lbs/sq.in)	$\frac{18800}{32.6} = 576$

Initial Air Volume V_{ai} (cu.in)	2970
Final Air Volume V_{af}(cu.in) $V_{af} = V_{ai} - 12\ A_a b$	$2970 - 12 \times 32.6 \times 3.74 = 1510$
Final Air Pressure $P_{af} = P_{ai}\left(\frac{V_{ai}}{V_{af}}\right)^{1.1}$ (lbs/sq.in)	$576 \left(\frac{2970}{1510}\right)^{1.1} = 1214$
Final Recuperator reaction $P_{af} = P_{af}\ A_a$ (lbs) air; P_{af} = approx. $2P_{ai}$ (lbs) metallic	$1214 \times 32.6 = 39600$
$\perp$ Distance from axis of bore to mean guide contact r(in)	$\frac{3.038 + 3.850}{2} = 3.4444$
Distance between clips l (in)	86.25
$\perp$ Distance from axis of bore to center line of hydraulic piston e_h (in)	16.365
$\perp$ Distance from axis of bore to line of action of recuperator reaction e_a(in)	15.656
Assumed coefficient of guide friction u = 0.15 to 0.25	0.15
Guide friction constant $\frac{2u}{l - 2ur} = A_f$	$\frac{2 \times 0.15}{86.25 - 2 \times 0.15 \times 3.44} = .00352$

Total hydraulic Pull (max. elev.) $P'_h = \dfrac{K+W_r \sin\theta_m - P_a(1+A_f e_a)}{1+A_f e_h}$ (lbs)	$\dfrac{152000+15790\sin 60-18800(1+.0635)}{1+.0663} = 137500$
Total hydraulic Pres- $P_h = P'_h - 100\ d_n$. d_n = diam. of brake rod (in)	2 hydraulic cylinders: $68750-4.72\times100 = 68280$
Effective Area of Hydraulic Piston A_h (sq.in)	31.2
Max. Pressure in Hydraulic Cylinder $p_h = \dfrac{P_h}{A_h}$ (lbs/sq.in)	$\dfrac{68280}{31.2} = 2200$
Inside Diam. of brake cylinder $d_{ih} = 1.13\sqrt{A_h + 0.785 d_h^2}$ (in) (d_n = diam. recuperator rod)	7.874
Outside diam. of brake cylinder d_{oh} (in)	9.450
Hoop tension in brake cylinder wall $p_{th} = p_h\left(\dfrac{d_{oh}^2 + d_{ih}^2}{d_{oh}^2 - d_{ih}^2}\right)$ lbs/sq.in.	$2200\left(\dfrac{9.45^2+7.874^2}{9.45^2-7.875^2}\right) = 12150$

Max. pressure in recuperator cylinder $P_{af}=P_{ai}\left(\frac{V_{ai}}{V_{ax}}\right)^{1.1}$ (lbs/sq.in)	1214
Inside Diam. of recuperator cylinder $d_{ia} = 1.13\sqrt{A_o+0.785d_a^2}$ (in)	7.087
Outside Diam. of recuperator cylinder d_{oa} (in)	8.267
Hoop Tension in Recuperator Cylinder Wall $p_{ta}=p_{af}\left(\frac{d_{oa}^2+d_{ia}^2}{d_{oa}^2-d_{ia}^2}\right)$ (lbs/sq.in)	$1214\left(\frac{\overline{8.267}^2+\overline{7.087}^2}{\overline{8.267}^2-\overline{7.087}^2}\right)=8020$
Inside Diam. of compressed air storage tank d_{ic} (in)	8466
Outside diam. of compressed Air Storage tank d_{oc} (in)	9.45
Hoop Tension in compressed air storage tank $P_{tc}= P_{af}\left(\frac{d_{oa}^2+d_{ia}^2}{d_{oc}^2-d_{ic}^2}\right)$ (lbs/sq.in)	$1214\left(\frac{\overline{9.45}^2+\overline{8.466}^2}{9.45^2-8.466^2}\right) = 11000$
Width of Wall between adjacent cylinders w (in)	- - - - - -
Hoop tension between adjacent cylinders $p_t=\frac{p_h d_{ih}+p_{af}d_{ia}}{1.8\ w}$ (lbs/sq.in)	- - - - -

GUIDE, ELEVATING GEAR AND TRUNNION REACTIONS:

x axis taken along bore:	y axis taken normal to bore.
Coordinates from center of gravity of recoiling parts to front guide reaction x_1 and y_1 (in)	$x_1 = 37.843$ $y_1 = -3.038$
Coordinates from center of gravity of recoiling parts to rear guide reaction x_2 and y_2 (in)	$x_2 = 48,407$ $y_2 = 3.85$
⊥ distance from center of gravity of recoiling parts to brake piston rod axis e_1 (in)	16,365
⊥ distance from center of gravity of recoiling parts to recuperator piston rod axis e_2 (in)	15,656
Max. powder reaction $P_b = F$ (lbs) (See Interior Ballistics)	2,245,000
⊥ distance from center of gravity of recoiling parts to axis of bore e (in)	5.13
Front guide reaction: gun recoiling in sleeve:	- - - - - -

$$Q_1 = \frac{Fe + P'_h e_h + P_a - W_r \cos\emptyset (x_2 - u y_2)}{x_1 + x_2 + u(y_2 - y_1)} \quad u =$$

0.15 to 0.2 (lbs)

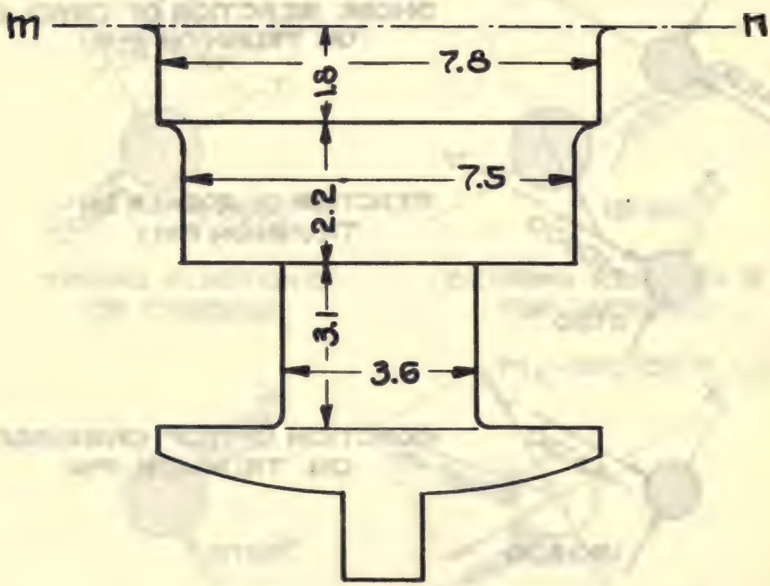
Rear guide reaction: gun recoiling in sleeve $$Q_2=\frac{Fe+P'_h e_h+P'_a e_a+W_r\cos\emptyset(x_1+uy_1)}{x_1+x_2+u(y_1-y_2)}$$ (lbs)	- - - - - - - -
Front guide reaction: gun recoiling in guide below axis of bore $$Q_1=\frac{Fe+P'_h e_h+P'_a e_a-W_r\cos\emptyset(x_2-uy_2)}{x_1+x_2-u(y_1+y_2)}$$	$$\frac{2,245,000\times5.13+137500}{37.84+48.41-0.15\times6.91}$$ $$\underline{\times16.365+19094\times15.66}$$ $$\underline{-7895\times47.83} = 154800$$
Rear guide reaction: gun recoiling in guides below axis of bore $$Q_2=\frac{Fe+P'_h e_h+P'_a e_a+W_r\cos\emptyset(x_1-uy_1)}{x_1+x_2-u(y_1+y_2)}$$	$$\frac{2,245,000\times5.13+137500}{37.84+48.41-0\cdot15\times6.91}$$ $$\underline{\times16.365+19094\times15.66}$$ $$\underline{+7895\times37.38} = 162600$$
Max. guide friction $R'_g = u(Q_1+Q_2)$ (lbs) $u = 0.15$ (approx.)	$R'_g = 0.15(154800+162600)= 47,620$
Weight of Tipping Parts W_t(lbs)	21,021
Max. Resistance to recoil (during powder period)	=137500+19094+47,620-13670=191000
$K' = (P'_h+P'_a+R'_g-W_r\sin\emptyset$) ($)=K+\dfrac{2Feu}{x_1\times x_2-u(y_1+y_2)}$	$$=152000+\frac{2\times2,245,000\times}{85.21}$$ $$\underline{5.13\times0.15} = 192000$$

$\perp$ distance from trunnion axis to line parallel to axis of bore through center of gravity of recoiling parts s (in)	3.73
Radius to pitch circle of elevating arc. j (in)	35.57
Angle between "y" axis and the radius to elevating pinion contact with elevating arc $\theta_e = \emptyset + ne$	60°
Elevating gear reaction (in battery) $E = \frac{Fe+K's}{j}$ (lbs)	$\frac{11,513,884+191,000\times3.73}{35.57}$
Angle of E with horizontal θ_e - - - - -	= 344,000
Top carriage trunnion reaction (in battery with balancing gear) $2X = K+W_r\sin\emptyset+E\cos\theta_e+R\sin\theta_r$ (lbs) $2Y = W_t\cos\emptyset+E\sin\theta_e-R\cos\theta_r$ (lbs) (E is same with or without balancing gear)	Not used.
Top Carriage Trunnion reaction (out of battery with balancing gear) $2X = K+R\sin\theta_r+E'\cos\theta_e+W_t\sin\emptyset$ (lbs) $2Y = W_t\cos\emptyset+E'\sin\theta_e-R\cos\theta_r$ (lbs) (E' is same with or without balancing gear)	Not used.
Estimated Weight of Rocker W_r' (lbs)	Neglected as small

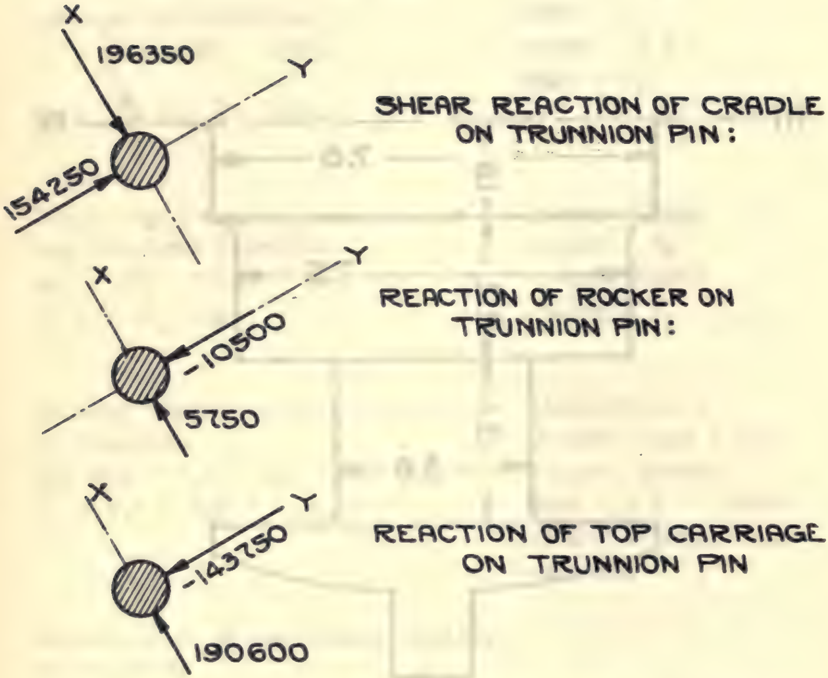
Horizontal Distance from Trunnion to center of gravity of rocker h_r' (in) "measured to rear"	Not used.
Angle between line of action of rocker reaction on cradle and "y" axis. B	+30
$\perp$ distance from trunnion to elevating screw or normal to rocker cradle contact. $k = x_m \cos B + y_m \sin B$ (in) "x_m and y_m coordinates of rocker contact with cradle from trunnion to rear and down".	29.43×.866+15.71 ×0.5 = 33.35
Rocker Reaction on Cradle $M = \frac{Ej - W_r' h_r'}{k}$ (lbs)	$\frac{344000 \times 35.57}{33.35} =$ 367000
Elevating Bear Reaction (out of battery) $E' = \frac{Ks + \bar{W}_r b \cos \emptyset}{j}$ (lbs)	Calculations max. elev. in battery.
Top carriage trunnion reaction (in battery)(X and Y components) $2X = K' + \bar{W}_t \sin \emptyset + E \cos \theta_e$ (lbs) $2Y = \bar{W}_t \cos \emptyset - E \sin \theta_e$ (lbs)	2X=197000+18200+17200=381200 2Y=10510-198,000=-287,500

Top carriage Trunnion Reaction (out of battery) (X and Y components) $2X = K + W_r \sin \text{Ø} + E \cos \theta_e$ (lbs) $2Y = W_t \cos \text{Ø} - E^1 \sin \theta_e$ (lbs)	Calculation at max. elevation in battery.
With balancing Gear: Distance from trunnion to center of gravity of tipping parts (in battery) along x axis: x_t (in)	Not used.
Radius of bell crank (balancing gear) r_c (in)	Not used.
Balancing Gear Reaction: $R = \dfrac{2W_t x_t \cos \text{Ø}}{r_a(1+\cos \frac{\text{Øm}}{2})}$ (lbs) (very approx.) or calculated from layout Øm = max. elevation.	Not used.
Angle made by balancing gear reaction with "y" axis θ_r (See layout)	Not used.
Rocker Trunnion reaction (X and Y components) $2X_r = M \sin B - E \cos \theta_e - W_r^1 \sin \text{Ø}$ (lbs) $2Y_r = E \sin \theta_e - W_r^1 \cos B$ (lbs)	$183500 - 172000 = 11500 = 2X_r$ $297000 - 318000 = -21000 = 2Y_r$

Total shear reaction of trunnion on cradle, $X' = X + X_r$ (lbs)	$190600 + 5750 = 196350 = X'$
$Y' = Y + Y_r$ (lbs)	$-143750 - 10500 = -154250 = Y'$
Total spring reaction of Top Carriage on trunnion $X_s = \frac{W_t}{2} \sin \emptyset$ (lbs)	$10000 \times .866 = 8660 = X_s$
$Y_s = \frac{\overline{W}_t}{2} \cos \emptyset$ (lbs)	$10000 \times 0.5 = 5000 = Y_s$
Total rigid bearing reaction of top carriage trunnion $X_b = X - X_s$ (lbs)	$190600 - 8660 = 181940 = X_b$
$Y_b = Y - Y_s$ (lbs)	$-143750 - 5000 = -149750 = Y_b$
Bending moment at cradle section of trunnion $M_x = X_s a + X_b b + X_r c$ (lbs)	$8660 \times 5.5 + 181940 \times 2.9 + 5750 \times .0.9 = 580780$
$M_y = Y_s a + Y_b b + Y_r c$ (lbs)	$5000 \times 5.5 - 149750 \times 2.9 - 10500 \times 0.9 = -416,950$
Resultant B. M. at cradle section of trunnion $M = \sqrt{M_x^2 + M_y^2}$ (in lbs)	$\sqrt{\overline{580,780}^2 + \overline{416,950}^2} = 716,000$
Max. fibre stress due to bending $f = \frac{10.18\ M}{D^3}$ (lbs/sq.in)	$\frac{10.18 \times 716000}{\overline{7.8}^3}$

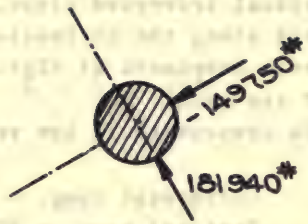


TRUNNION PIN
B. M. AT SECTION m-n:

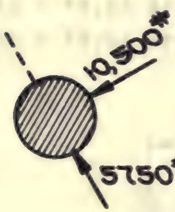


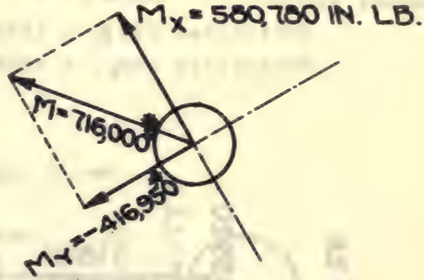
REACTION ON PIN IN X PLANE.

SPRING REACTION OF TOP CARRIAGE



BEARING REACTION OF TOP CARRIAGE





CALCULATIONS FOR STRENGTH OF CARRIAGE AXLE

Proposed 75 m/m St.Chamond

50° Elevation and $22\frac{1}{2}°$ traverse:

Maximum Peak Resistance to Recoil - - - - assumed at 20,000 lbs.

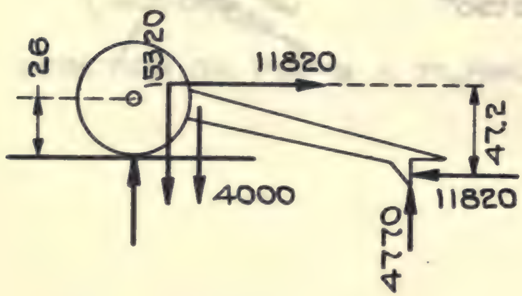
The resistance to recoil may then be divided into a horizontal and vertical component in the vertical plane of traverse. Then, the horizontal component in the vertical traversed plane, may be divided into a component along the horizontal axis of the mount and a transverse component at right angles to the longitudinal axis of the mount.

The components in the vertical traversed plane are:-

Horizontal comp. = 20,000 × cos 50° = 12820 lbs.
Vertical comp. = 20,000 × sin 50° = 15320 lbs.

The longitudinal and transverse horizontal components are:-

Horizontal comp. = 12820 × cos 22.5° = 11800 lbs.
Transverse comp. = 12820 × sin 22.5° = 4900 lbs.



Then, 15320 + 4000 = 19320 (Total Downward Force)

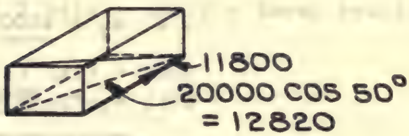
$S \times 130 = 4000 \times 120.25 + 15320 \times 128.2 - 11820 \times 47.2$

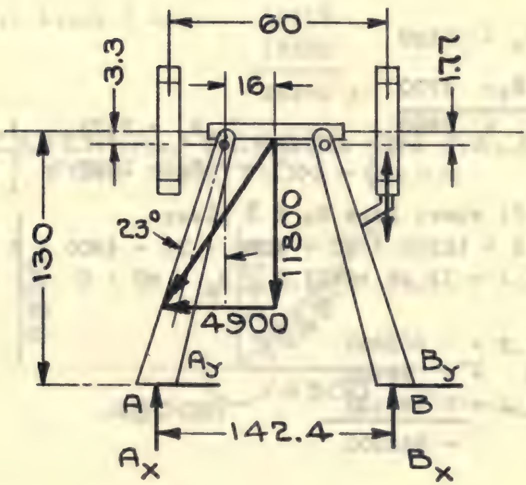
$4000 \times 120.25 = 481000$
$15320 \times 128.2 = \underline{1970000}$
$2451000 \qquad S = 14{,}550 \quad)$

(

$11820 \times 47.2 = \underline{558000} \qquad A_2 + B_2 = \underline{\underline{4{,}770}} \quad)$
1893000

19320
$\underline{14550}$
$4770 \qquad A_2 + B_2 = 4770$





$12800 \times \cos 22\frac{1}{2} = 11800$

$12800 \times \sin 22\frac{1}{2} = 4900$

$\Sigma M_A = B_x \times 142.4 - 11800 \times 71.2 + 4900 \times 128.2 = 0$

$$\begin{array}{r} 11800 \times 71.2 = 840000 \\ 4900 \times 128.2 = 629000 \\ \hline 211000 \end{array}$$

$$\begin{array}{r} 11800 \\ 1481 \\ \hline 10319 \end{array} \qquad \therefore B_x = \frac{211000}{142.4} = 1481$$

$$A_x = 10319$$

ΣM about vertical pin for left trail

$A_y\ 126.38 = 10319 \times 55.2$

$$\begin{array}{r} 4900 \\ 4500 \\ \hline 400 \end{array} \qquad \therefore A_y = 4500$$

$$B_y = 400$$

ΣM axle $= A_z\ 130 - 10319 \times 32 = B_z\ 130 - 1481 \times 3$

$130(A_z - B_z) = 10319 \times 32 - 1481 \times 32$

$= 283000$

$$\begin{array}{rcl} \therefore A_z - B_z & = & 2180 \\ A_z + B_z & = & 4770 \\ \hline 2A_z & = & 6950 \end{array} \qquad \therefore A_z = 3475$$

$$B_z = 1295$$

ΣM about left wheel base in Z Y plane:

$-4900 \times 41.2 + 15300 \times 30 + 4000 \times 30 - 4900 \times 6$
$+ 3475 \times 41.2 - 12.95 \times 101.2 - S_p \times 60 = 0$

$$\begin{array}{rcl} -4900 \times 41.2 & = & -202000 \\ 49000 \times 6 & = & -29400 \\ -1295 \times 101.2 & = & -131100 \\ \hline & & -362500 \end{array}$$

15300 × 30 = 459000	722000
4000 × 30 = 120000	362500
3475 × 41.2 = 143000	359500
722000	

S_B = 5980)
(
S_A = 8570)

Reactions on Trail Axle.

X and Y reaction on vertical pin of left trail:

E_x = 10319 # E_y = 4500

B. M. in XY plane on axle:

$E_y \times 10 = 4500 \times 10 = 45000$ " # XY plane:

Thrust along X axis = 10319 ± Shear reaction of equalizing bar.

Thrust along Y axis = 4500

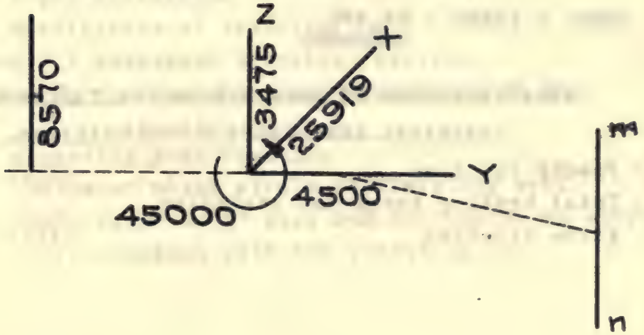
Thrust along Z axis = 3475

Shear reaction of Equalizer bar =

$$\frac{452000 - 331000}{7.75} = 15600 \#$$

Thrust along X axis 10319
15600
25919 #

EXTERNAL FORCES ON AXLE
FOR SECTION -(m-n



Section m-n 5" × 5"

Torsion = 25919 × 2.2 = 57000 (" #)

(B.M.$_{zy}$) = 3475 × 12.2 + 8570 × 26

$$\begin{array}{r} 42300 \\ 223000 \\ \hline 265300 \end{array} \qquad \text{B. M.}_{zy} = 265300 \text{ " \#}$$

(B.M.$_{zy}$) = 25919 × 12.2 − 45000

$$\begin{array}{r} 316000 \\ 45000 \\ \hline 271000 \end{array} \qquad \text{B. M.} = 271000 \text{ (" \#)}$$

$$f_1 = \frac{1}{2} f + \sqrt{\frac{1}{4} f^2 + f_t^2}$$

$$f_x = 2\,\frac{265300 \times 2.56}{5 \times 25} = 12700$$

$$f_y = \frac{271000 \times 6}{125} = f = \frac{13000}{25700}$$

$$J = \frac{\pi\, 5^4}{32} = \frac{625\,\pi}{32} = 61.4$$

$$f_t = \frac{57000 \times 5}{61.4} = 46500$$

$$12850 + \sqrt{\frac{1}{4} \times 25700^2 + 4650^2}$$

$$12850 + 13620 = \underline{\underline{26,470}}$$

RECAPITULATION OF FORMULAE ON THE INTERNAL REACTIONS THROUGHOUT A GUN CARRIAGE.

F = Powder reaction - - - - - - - - - - (lbs)

B = Total braking force not including guide friction (lbs)

b = distance from center of gravity of recoiling parts to line of action of B. (in)

R = total guide friction (lbs)

r = mean distance from center of gravity of recoiling parts to guide friction (in)

e = distance from center of gravity of recoiling parts to line of bore. (in)

P_h = total oil pressure on the hydraulic piston. (lbs)

P'_h = the hydraulic reaction plus the joint frictions (stuffing box at pistons) (lbs)

P_a = the total elastic reaction (due to compressed air or springs) (lbs)

P'_a = the total elastic reaction plus the joint frictions. (lbs)

e_h = distance from center of gravity of recoiling parts to line of action of P_h. (in)

e_a = distance from center of gravity of recoiling parts to line of action of P_a. (in)

d'_h = stuffing box or rod diam. of hydraulic cylinder. (in)

d'_a = stuffing box or rod diam. of air cylinder. (in)

Q_1 = normal front guide reaction (lbs)

Q_2 = normal rear guide reaction. (lbs)

x_1 and y_1 = coordinates from center of gravity of recoiling parts to front guide reaction. (in)

l = distance between line of action of Q_1 and Q_2 (in)

x_2 and y_2 = coordinates from center of gravity of recoiling parts to rear guide reaction. (in)

W_r = weight of recoiling parts. (lbs)

$\emptyset$ = angle elevation.

u = coefficient of friction.

X and Y = component trunnion reactions (lbs)

X_r and Y_r = component rocker reactions at the trunnion (lbs)

E = elevating gear reaction

J = radius to pitch circle of elevating arc. (in)

θ_e = angle between "y" axis and the radius to elevating pinion contact with the elevating arc.

K = total resistance to recoil. (lbs)

s = distance from center of gravity of recoiling parts to trunnion axis measured along the "y" axis. (in)

Total resistance to recoil on recoiling mass, becomes.

$$K = B + R - W_r \sin \emptyset \quad \text{(lbs)}$$

but $B = P_h' + P_a'$

where $P_h' = P_h + 100\, d_h'$) assuming 100 lbs. per
and $P_a' = P_a + 100\, d_a'$ (in diam. for frictions
) in stuffing box.

hence

$$K = P_h' + P_a' + R - W_r \sin \emptyset$$

GUIDE OR CLIP REACTIONS TO GUIDE FRICTION.

Gun recoiling in sleeve, front guide reaction,

$$Q_1 = \frac{Fe + Bb - W_r \cos \emptyset (x_2 - uy_2)}{x_1 + x_2 + u(y_1 - y_2)} \quad \text{(lbs)}$$

and rear guide reaction,

$$Q_2 = \frac{Fe + BbCW_r \cos \emptyset\, (x_1 + uy_1)}{x_1 + x_2 + u(y_1 - y_2)} \quad \text{(lbs)}$$

Gun recoiling in guides below the axis of the bore, front guide reaction,

$$Q_1 = \frac{Fe + Bb - W_r \cos \emptyset\, (x_1 - uy_2)}{x_1 + y_2 - u(y_1 + y_2)} \quad \text{(lbs)}$$

and rear guide reaction,

$$Q_2 = \frac{Fe + Bb + W_r \cos \emptyset (x_1 - uy_1)}{x_1 + x_2 - u(y_1 + y_2)} \quad \text{(lbs)}$$

If $M = x_1 + x_2 + u(y_1 - y_2)$ for sleeve guides

$M = x_1 + x_2 - u(y_1 + y_2)$ for guide below axis of bore

and

$N = x_1 - x_2 + u(y_1 + y_2)$ for sleeve guides

$N = x_1 - x_2 + u(y_2 - y_1)$ for guides below axis of bore.

then the total guide friction equals,

$$R = \frac{2(Fe+Bb)+W_r \cos \emptyset \, . \, N}{M} u \quad \text{(lbs)}$$

and for the total braking force B_1

$$B = \frac{(K+W_r \sin \emptyset)M-(2Fe+W_r \cos \emptyset \; N) \; u}{M + 2 \; u \; h} \quad \text{(lbs)}$$

In terms of the pulls, we have for the clip reactions,

$$Q_1 = \frac{Fe+ \Sigma P'_a e_a + \Sigma P'_h e_h - W_r \cos \emptyset (x_2 - uy_2)}{x_1 + x_2 + u(y_1 - y_2)} \quad \text{(lbs)}$$

$$Q_2 = \frac{Fe+ \Sigma P'_a e_a + \Sigma P'_h e_h + W_r \cos \emptyset (x_1 + uy_1)}{x_1 + x_2 + u(y_1 - y_2)} \quad \text{(lbs)}$$

and the guide friction becomes,

$$R = \frac{2Fe+2\Sigma P'_h e_h + 2\Sigma P'_a e_a + W_r \cos \emptyset \; N}{M} \quad \text{(lbs)}$$

and the hydraulic pull in terms of the total resistance to recoil and recuperator reaction, becomes,

$$\Sigma P'_h = \frac{M(K - \Sigma P'_a - W_r \sin \emptyset) - u(2Fe + 2\Sigma P'_a e_a + N \; W_r \cos \emptyset)}{M + 2ue_h} \quad \text{(lbs)}$$

For approximate calculations, the guide friction equals,

$$R = \frac{2uBd_r}{1-2ur} \quad \text{(lbs)}$$

From the foregoing analysis we observe, that the guide friction and bearing pressures are reduced:

(1) By increasing the distance between the clips.

(2) By balancing the pulls about the center of gravity of recoiling parts or bringing the resultant pull closer to the center of gravity of the recoiling parts.

(3) By bringing the resultant friction line of the guides closer to the center of gravity of the recoiling parts.

(4) By reducing the powder pressure couple Fe, that is by reducing the distance from the center of gravity of the recoiling parts to the center line of bore. The distance from center of gravity of the recoiling mass to the center line of bore should never exceed 1.5 inches unless a friction disk is introduced with angular motion about the trunnion.

Stress on Gun lug:

Let

W_c' = weight of piston and rod or the weight of recoiling cylinder. (lbs)

d_g = distance from center of recoil pull to section "mn" adjacent gun of the gun lug. (in)

I_{mn} = moment of inertia of section. $(in)^3$

y' = distance to extreme fibre from neutral axis. (in)

f_{mn} = max. fibre stress (lbs/sq.in)

then,

$$f_{mn} = \frac{\left[B + \frac{W_c'}{W_r}(F-B)\right] d_g y}{I_{mn}} \quad \text{(lbs/sq.in)}$$

Trunnion and Elevating gear reaction:

When the gun is in battery the tipping parts are balanced about the trunnion axis. This condition

implies that with the gun in battery, the center of gravity of the tipping parts passes through the trunnion axis. When the recoil is limited to a short movement under the breech when the gun is fired at high elevations the center of gravity of the tipping parts is placed forward if the trunnion axis and the balancing gear or counterpoise is introduced, balancing the weight of the tipping parts about the trunnion. The trunnion reactions are modified by the introduction of a balancing gear.

Trunnion and elevating gear reactions when no balancing gear is used:

(a) During the acceleration period,

$$2X = K + W_t \sin \emptyset + \left(\frac{Fe+Ks}{J}\right) \cos \theta_e \quad \text{(lbs)}$$

$$2Y = \overline{W}_t \cos \emptyset - \left(\frac{Fe+Ks}{J}\right) \sin \theta_e \quad \text{(lbs)}$$

$$E = \frac{Fe\ +Ks}{J}$$

(b) During the retardation period,

$$2X = K + W_t \sin \emptyset + \left(\frac{Ks + W_r x \cos \emptyset}{J}\right) \cos \theta_e \quad \text{(lbs)}$$

$$2Y = \overline{W}_t \cos \emptyset - \left(\frac{Ks + W_r x \cos \emptyset}{J}\right) \sin \theta_e \quad \text{(lbs)}$$

$$E = \frac{Ks + W_r X \cos \emptyset}{J} \quad \text{(lbs)}$$

where x = the recoil displacement out of battery.

Rocker Reactions:

The reactions on the rocker are primarily three:

(1) The reaction of the trunnion upon the rocker, X_r and Y_r.

(2) The reaction of the elevating gear, E.

(3) The reaction of the cradle, M, and the weight of the rocker, W_r'.

If k = the perpendicular distance from the trunnions to line of action of M.

B = the angle between the line of action of M and the "y" axis.

h_r' = the horizontal distance to the center of gravity of the rocker from the trunnion.

J = the perpendicular distance from the trunnion axis to the line of action if the elevating gear reaction (i. e. equals the radius of the circular elevating rack on the rocker).

Then, the cradle reaction on rocker, becomes,

$$M = \frac{E_J - W_r' h_r'}{k} = \frac{Fe + Ks - W_r' h_r'}{k} \quad \text{(in battery) (lbs)}$$

$$\text{-------} = \frac{Ks - W_r x \cos \emptyset - W_r' h_r'}{k} \quad \text{(out of battery)(lbs)}$$

approximately $M = \frac{Ks}{k}$

The rocker trunnion reactions become,

$$2X_r = M \sin B - W_r' \sin \emptyset - E \cos \theta_e \qquad \text{(lbs)}$$

$$2Y_r = E \sin \theta_e - W_r' \cos \emptyset - M \cos B \qquad \text{(lbs)}$$

<u>Layout of Balancing Gear:</u>

Two types of balancing gear have been used extensively in gun carriage construction:

(1) A cam with chain type for small field mounts.

(2) A direct acting balancing gear.

For type (1), let

W_t = weight of tipping parts. (lbs)

h_t = horizontal distance from the trunnions to the center of gravity of the tipping parts

(gun in battery) (in)

r_o = equivalent radius of cam at horizontal elevation (in)

r_n = final equivalent radius of the cam where the cam arc has turned through the maximum angle of elevation = $\emptyset$ (in)

R = mean radius of cam. (in)

d_n= deflection of spring at zero elevation (in)

d_o= deflection of spring at maximum elevation (in)

c = spring constant.

$\emptyset$= angle of elevation expressed in radius.

If d_s = deflection of spring at solid height, take

$$\left.\begin{aligned} d_n &= (\tfrac{2}{3} \text{ to } \tfrac{3}{4})d \text{ solid} \\ d_o &= (\tfrac{1}{3} \text{ to } \tfrac{1}{4})\, d \text{ solid} \end{aligned}\right)$$

then

$$c = \frac{W_t h_t}{r_o d_n} = \frac{W_t h_t \cos \emptyset}{r_n d_o}$$

and

$$d_n - d_o = \left(\frac{r_o + r_n}{2}\right) \emptyset$$

To layout the radii of cam, we have $\emptyset$ divided into n parts, then,

$$r_1 = \frac{W_t h_t \cos \emptyset_1}{c(d_n - r_o \Delta\emptyset)}$$

$$r_2 = \frac{W_t h_t \cos \emptyset_2}{x[d_n - (r_o + r_1)\Delta\emptyset]}$$

$$r_n = \frac{W_t h_t \cos \emptyset}{c[d_n - (r_o + r_1 --- r_{n-1})\Delta\emptyset]}$$

With a balancing gear of this type, the trunnion reactions are modified and now become,

if T = the tension in the chain

d = the angle T makes with the axis X(taken along the axis of the bore.)

$$2X = K + W_t \sin \emptyset + \left(\frac{Ks + W_r x \cos \emptyset + Fe}{J}\right) \cos \theta_e - T \cos d \quad \text{(lbs)}$$

$$= 2Y = W_t \cos \emptyset - \left(\frac{Ks + W_r x \cos \emptyset + Fe}{J}\right) \sin \theta_e + T \sin d \quad \text{(lbs)}$$

The elevating gear reaction obviously remains as before that is,

$$E = \frac{Ks + W_r x \cos \emptyset + Fe}{J} \quad \text{(lbs)}$$

for type (2), let

W_t = weight of tipping parts (lbs)

h_t = horizontal distance from the trunnions to the center of gravity of the tipping parts (gun in battery) (in)

x_t and y_t = coordinates along and normal to bore from trunnion to center of gravity of tipping parts (gun in battery)

$\emptyset$ = angle of elevation.

$\emptyset_m$ = max. elevation

r = radius from the trunnion to the crank pin which connects the tipping parts to the piston rod of the oscillating cylinder. (in)

R = reaction exerted by the balancing gear along the piston rod of the oscillating cylinder. (lbs)

d_t = moment arm of R about trunnion (in)

d_h = deflection of spring at horizontal elevation (in)

d_b = deflection of spring at maximum elevation (in)

c = spring constant

R_i = initial balancing gear reaction (0° elev.)

R_t = final balancing gear reaction ($\emptyset_m^o$ elev)

S' = stroke of piston in oscillating cylinder (in)
p_t = final air pressure in pneumatic balancing cylinder (lbs/sq.in)
p_i = initial air pressure in pneumatic balancing cylinder. (lbs/sq.in)
A = effective area of balancing piston (sq.in)
V_o = initial air volume (cu.in)

With a metallic balancing gear, the dimension of the spring may be approximated by the solution of the following equations:

$$\left.\begin{aligned} \frac{d_o}{d_h} &= \cos \phi_m \\ d_h - d_o &= S \\ c d_h &= \frac{2W_t x_t}{r(1+\cos\frac{\phi_m}{2})} \\ S &= 2\, r \sin\frac{\phi_m}{2} \end{aligned}\right\} \text{ from which we may obtain } d_o,\ d_h,\ S_1 \text{ and } c \text{ of the spring.}$$

With a pneumatic balancing gear, we have, for a preliminary approximation,

$$\left.\begin{aligned} V_o &= \frac{2W_t x_t\ S}{r(1+\cos\frac{\phi_m}{2})p_i} \left(\frac{\frac{p_f}{p_i}}{1-\frac{p_f}{p_i}}\right) \text{ (cu.in)} \\ S &= 2r \sin\frac{\phi_m}{2} \text{ (in)} \quad \frac{p_f}{p_i} = \cos\phi_m \text{ (approx)} \end{aligned}\right\}$$

With a direct acting balancing gear, the trunnion reactions are modified and become,

if

R = balancing gear reaction (lbs)
θ_r = angle between R and y axis
d_t = moment arm of R about the trunnion at any elevation $\emptyset$ (in)

when the recoiling parts are in battery:

$$2X = K + W_t \sin\emptyset + E\cos\theta_e + R\sin\theta_r \quad \text{(lbs)}$$

$$2Y = W_t\cos\emptyset + E\sin\theta_e - R\cos\theta_r \quad \text{(lbs)}$$

$$R = \frac{W_t x_t \cos\emptyset}{d_t} = \frac{2W_t x_t \cos\emptyset}{r(1+\cos\frac{\emptyset_m}{2})} \quad \text{(lbs)}$$

$$E = \frac{Ks + P_b e}{J}$$

when the recoiling parts are out of battery :-

$$2X = K + R\sin\theta_r + E\cos\theta_e + W_t\sin\emptyset \quad \text{(lbs)}$$

$$2Y = W_t\cos\emptyset + E\sin\theta_e - R\cos\theta_r \quad \text{(lbs)}$$

$$R = \frac{2W_t x_t \cos\emptyset}{r(1+\cos\frac{\emptyset_m}{2})} \text{ (roughly)} \quad \text{(lbs)}$$

$$E = \frac{Ks + W_r\, x\cos\emptyset}{J} \quad \text{(lbs)}$$

It is evident that the elevating gear reaction remains the same with or without a balancing gear while the trunnion reactions are modified both by the position and magnitude of the balancing reaction.

Strength of the trunnions

The critical section of the trunnions is usually where the trunnion joins the cradle. Let "mn" represent this section. [See fig.(9)].

a = distance from "mn" to center of top carriage bearing.

b = distance from "mn" to center of rocker bearing.

M_x = the bending moment at "mn" in the plane of the X component reactions.

M_y = the bending moment at "mn" in the plane of the Y component reactions.

M = the resultant bending moment on section "mn".

f = max. fibre stress (lbs/sq.in)

D = distance of the trunnion at section "mn"

then

$$M_x = Xa + X_r b \text{ (in lbs) and } M = \sqrt{M_x^2 + M_y^2} \text{ (in. lbs)}$$

$$M_y = Ya + Y_r b \text{ (in. lbs)}$$

hence

$$D = \sqrt[3]{\frac{10.18\ M}{f}} \quad \text{(in)}$$

Stresses in cradle or recuperator forging:

Let

Q_1 and Q_2 = the front and rear normal clip reactions.

x_1' and x_2' = the x" coordinates of these reactions with respect to the trunnions.

d_1 and d_2 = the distance of the friction components of Q_1 and Q_2 from the neutral axis.

B = the resultant of the braking pulls reacting on the cradle.

d_b = the distance from the neutral axis to "B".

I_t = moment of inertia of a cross section at the trunnions.

y_t = distance of extreme fibre from neutral axis at trunnion section.

f_t = fibre stress due to bending and direct pull or thrust at the trunnion section.

I_c = moment of inertia of a cross section at the point of contact of the elevating arc with cradle.

A_c = area of cross section at the point of contact of elevating arc with cradle.

y_c = distance to extreme fibre from neutral axis of elevating arc section.

f_c = fibre stress due to bending and direct pull or thrust at the elevating arc section.

A_t = area of a cross section at the trunnion.

then

$$f_t = \frac{Q_1(x_1' + ud_1)y_t}{I_t} + \frac{uQ_1}{A}$$ for the braking reaction in the rear.

$$f_t = \frac{[Q_1(x_1' + ud_1) - 3d_b]y_t}{I_t} + \frac{uQ_1 + B}{A_t}$$ for the braking reaction in the front.

$$f_c = \frac{[Q_2(x_2' \times x_c + ud_2) - Bd_b]y_c}{I_c} + \frac{B + uQ_2}{A_c}$$ for the braking reaction in the rear.

$$f_c = \frac{[Q_2(x_2' + x_c + ud_2) - Bd_b]y_c}{I_c} + \frac{uQ_2}{A_c}$$ for the braking reaction in the front.

- - - A P P E N D I X - - - - -

APPENDIX CHAPTER IV- INTERNAL REACTIONS.

REACTIONS AND STRESSES INDUCED IN ELEVATING AND TRAVERSING MECHANISMS:

STRESSES DUE TO FIRING. The reaction exerted on the elevating mechanism due to firing equals,

In Battery,

$$E = \frac{Fe + Ks}{J \cos 20}$$

Out of Battery

$$E' = \frac{Ks + W_r x \cos \phi}{J \cos 20}$$

where

- F = max. powder force
- K = Total resistance to recoil
- W_r = weight of recoiling parts.
- x = displacement out of battery.
- J = radius to pitch line of elevating arc from center of trunnions.
- e = $\perp$ distance from axis of bore to center of gravity of recoiling parts.
- S = $\perp$ distance from line parallel to axis of gun through center of gravity of recoiling parts to center of trunnions.

It is highly desirable to reduce the reaction E, since it stresses the teeth of the elevating mechanism. To reduce this, we may,

(1) decrease "e" by so distributing the mass of the recoiling parts as to bring its masses as near coincident with the axis of the bore as possible.

(2) decrease "S" by bringing the trunnion axis along a line through the center of gravity of the recoiling parts and parallel to the axis of the bore.

(3) increase "J" whenever feasible in a construction layout.

In certain types of mounts as those containing a recoiling cylinder, the piston and rods being fixed to cradle, the center of gravity of the recoiling parts is necessarily considerably lowered from the axis of the bore and therefore "e" is inherently large. With large mounts, counterweights or bob weights are sometimes introduced to decrease "e". In this type of mount without a counterweight or bob weight a friction clutch or hand brake are often introduced on the elevating gear shaft or adjacent gear shaft. Then E becomes limited to that required to overcome the friction of the clutch or brake and a large reaction on the elevating mechanism is thus reduced.

With a cone clutch, we have,

$$E = \frac{uPr}{r_e \sin \alpha},$$

where P = total spring load.
r = mean radius of clutch
r_e = pitch radius of gear or pinion.
n = coefficient of friction = 0.15 approx.
2α = cone angle

With a disk clutch, we have,

$$E = \frac{2}{3}\frac{nkP}{r_e}\left(\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\right),$$

where P = total spring load.
r_2 = outer radius: r_1 = inner radius of disk.
k = total no. of friction surfaces.
n = coefficient of friction = 0.15 approx.

FRICTION OF TRUNNIONS AND TRAVERSING PIVOTS. In elevating, or traversing a gun, a large amount of the energy needed is that required to overcome the friction of the pivot about which the gun is traversed.

Trunnion friction:

During the elevating process the load on trunnions equals the weight of the tipping parts, when the trunnion is sufficiently free from binding, the contact is along a narrow strip.

Then

$$nR \sin \varnothing + R \cos \varnothing = \frac{W_t}{2}$$

where $\tan \varnothing = n$

$$\therefore R(\sin \varnothing \tan \varnothing + \cos \varnothing) = \frac{W_t}{2}$$

u = coefficient of friction.
R = normal pressure
r = radius of trunnion.

and the friction moment

$$M_t = R \tan \varnothing \cdot r = \frac{W_t}{2} r \sin \varnothing$$

Since $\varnothing$ is small, $\tan \varnothing = \sin \varnothing$ approx.
hence

$$M_t = n \frac{W_t}{2} r = 0.15 \frac{W_t}{2} r \text{ approx.}$$

In starting n may be as great as 0.25 and proper allowance should be made.

Since the load brought on the trunnions during firing is greatly in excess of that on elevating the gun, the bearing contact may be divided, one part to carry the major of the firing load and the other to carry merely the weight of the tipping parts. This is accomplished constructively by allowing play in the bearing which sustains the firing load, and holding the tipping parts for elevating or transportation merely on a spring cushion, the reaction of the spring, for a deflection just sufficient to lift the tipping parts just clear from the firing bearing, being equal to the weight of the tipping parts. Thus it is possible to reduce the friction by using a

smaller trunnion diameter, in that part of the bearing that is spring borne since the bearing surface for a nominal bearing pressure can be greatly reduced.

Pivot friction in traversing:

This friction will vary considerably according to the type of bearing used. We will consider three types of pivots, 1° flat circular pivot, 2° flat hollow circular pivot, and 3° conical pivot. To estimate the load brought on the pivot, let,

V_a = pivot reaction or load (vertical)

V_b = normal load of traversing guides (vertical)

W_t = weight of tipping parts.

l = horizontal distance between V_a and V_b

l_t = horizontal distance from W_t to V_b

W_c = weight of top carriage.

l_c = horizontal distance from W_t to V_b

then

$$V_a = \frac{W_t l_t + W_c l_c}{l} = \text{load on pivot during traversing.}$$

If M_t = the friction couple exerted at the pivot during the process of traversing we have for the various types of bearings.

1° for flat circular pivot:

$$\text{The friction on an elementary zone} = \frac{uV_a}{\pi r_o^2} 2\pi r\, dr$$

$$\text{The moment of this friction about the center} = \frac{uV_a}{\pi r_o^2} 2\pi r^2 dr$$

$$\text{The total friction} = \frac{2V_a}{r_o^2} n \int_0^{r_o} r^2 dr = \frac{2V_a n r_o}{3}$$

Therefore for a flat circular pivot, letting n = 0.15,

$M_t = 0.1\, V_a r_o$

<u>2° for flat hollow circular pivot:</u>

The total friction evidently becomes,

$$= \frac{2V_a n}{r_2^2 - r_1^2} \int_{r_1}^{r_2} r^2 dr = \frac{2V_a n}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

hence, letting $n = 0.15$

$$M_t = 0.1\ V_a \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

<u>3°for conical pivot:</u>

The intensity of vertical pressure on the projected area of the bearing $= \frac{V_a}{\pi(r_2^2 - r_1^2)} = p_v$

If the cone makes an angle 2α, and p_m equals the intensity of normal pressure, then,

the normal pressure on area $\frac{rd\theta dr}{\sin\alpha} = p_m \frac{rd\theta dr}{\sin\alpha}$

the vertical component of this pressure =

$$p_m \frac{rd\theta dr}{\sin\alpha} \sin\alpha = p_m\, rd\,\theta dr$$

but the pressure on the projected area $rd\theta dr = p_v rd\theta dr$

hence $p_v = p_n = \frac{V_a}{\pi(r_2^2 - r_1^2)}$

the friction on a differential zone $= n \frac{V_a}{\pi(r_2^2 - r_1^2)} \frac{2\pi r dr}{\sin\alpha}$

the total friction moment, therefore becomes,

$$M_t = \frac{nV_a\, 2\,\pi}{\pi(r_2^2 - r_1^2)\sin\alpha} \int_{r_1}^{r_2} r^2 dr = \frac{2}{3} \frac{nV_a}{\sin\alpha} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

If then we let $n = 0.15$

$$M_t = 0.1 \frac{V_a}{\sin\alpha} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)$$

VELOCITY RATIOS OF ELEVATING AND TRAVERSING MECHANISMS Elevating and traversing mechanism consists usually of a train of spur, bevel, helical screw and worm gears.

1°)-Velocity Ratio of spur gear:

Since $w_1 r_1 = w_2 r_2$) w = angular velocity
(r = radius to pitch line.
)
we have $\frac{w_1}{w_2} = \frac{r_2}{r_1} = \frac{n_2}{n_1}$ (n = no. of teeth.
)

2°)-Velocity Ratio of Bevel gears:

Again $w_1 r_1 = w_2 r_2$ where r_1 and r_2 are the outside radii of the gears:

The angle of coning for the first gear, equals,

$$\tan \theta_1 = \frac{r_1}{r_2} \quad (\theta_1 = \tfrac{1}{2} \text{ angle of cone, })$$

and for the second gear

$$\tan \theta_2 = \frac{r_2}{r_1} \quad (\theta_2 = \tfrac{1}{2} \text{ angle of cone})$$

hence
$$\frac{w_1}{w_2} = \tan \theta_2 \text{ and } \frac{w_2}{w_1} = \tan \theta_1$$

Therefore we may take any two common radii in obtaining the velocity ratios, again

$$\frac{w_1}{w_2} = \frac{r_2}{r_1} = \frac{n_2}{n_1}$$

3°)-Helical screw gears:

We have for the velocity of the common normal,

$v_n = w_1 r_1 \cos\theta_1 = w_2 r_2 \cos\theta_2$

$$\therefore \frac{w_1}{w_2} = \frac{r_2 \cos\theta_2}{r_1 \cos\theta_1}$$

but also,

$p_n = p_1 \cos\theta_1 = p_2 \cos\theta_2$

then

$$\frac{w_1}{w_2} = \frac{\dfrac{2\pi r_2}{p_n}\cos\theta_2}{\dfrac{2\pi r_1}{p_n}\cos\theta_1} = \frac{n_2}{n_1}$$

θ_1 = angle between axis of gear #1 and perpendicular to common normal.
θ_2 = angle between axis of gear #2 and perpendicular to common normal.
p_n = common normal pitch.
p_1 = circumferential pitch gear #1.
p_2 = circumferential pitch gear #2.
n_1 = no. of teeth gear #1.
n_2 = no. of teeth gear #2.

$$\text{Hence}\quad \frac{w_1}{w_2} = \frac{n_2}{n_1} = \frac{r_2 \cos\theta_2}{r_1 \cos\theta_1}$$

If $\varnothing$ = the total angle between the axis of the gears in mesh, then

since $p_n = p_1 \cos\theta_1 = p_2 \cos\theta_2$

$$\cos\theta_1 = \frac{p_2 \sin\varnothing}{\sqrt{p_1^2 + p_2^2 - 2p_1 p_2 \cos\varnothing}}$$

$$\cos\theta_2 = \frac{p_1 \sin\varnothing}{\sqrt{p_1^2 + p_2^2 - 2\,p_1 p_2 \cos\varnothing}}$$

therefore $$\frac{w_1}{w_2} = \frac{r_2 p_1}{r_1 p_2}$$

Further the axial pitches, become,

$$m_1 = p_1 \cot \theta_1 \quad \text{and} \quad m_2 = p_2 \cot \theta_2$$

4°)-Velocity Ratio Worm gears:

Though a worm gear is a specified type of helical screw gear when $\emptyset = 90°$. it is convenient to consider this type as a separate classification.

When $\emptyset = 90°$,

$$\cos \theta_2 = \frac{p_1}{\sqrt{p_1^2 + p_2^2}} = \sin \theta_1$$

therefore the axial pitch of one equals the circumferential pitch of the other.

The worm of a worm gear has one to two or three threads while the gear has many threads.

Now, for a single thread worm,

$$\frac{w_g}{w_w} = \frac{r_w \sin \theta}{r_g \cos\theta} = \frac{r_w}{r_g} \tan \theta$$

Directly, we have.

$$w_g r_g = p \frac{w_w}{2\pi}$$

but

$$p \frac{w_w}{2\pi} = \frac{p}{2\pi r_w} w_w r_w = \tan\theta . w_w r_w$$

$$\therefore \frac{w_g}{w_w} = \frac{r_w}{r_g} \tan \theta$$

w_g = ang. velocity of gear wheel.
w_w = ang. vel. of worm wheel.
r_g = pitch radius gear.
r_w = pitch radius of worm
p = axial pitch of worm
θ = angle of helix.

Thus the ratio of angular velocities depends upon the angle of the helix of the worm.

With a "n_w" threaded worm,

$$w_g r_g = n_w p \frac{w_w}{2\pi} \quad \text{and} \quad \frac{w_g}{w_w} = n \frac{r_w}{r_g} \tan \theta$$

In terms of the number of teeth, since $\frac{p}{2\pi r_w} = \tan\theta$

$$\frac{w_g}{w_w} = n_w \frac{\frac{2\pi r_w}{p}}{\frac{2\pi r_g}{p}} \tan\theta = \frac{n_w}{n_g}$$

and for a single threaded worm, since $n_w = 1$

$$\frac{w_g}{w_w} = \frac{1}{n_g} \text{ or } w_g = \frac{w_w}{n_g}$$

<u>Velocity ratio in gear trains:</u>

Combining the previous equations from one pair of elements to the adjacent pair, we finally arrive at the velocity ratios of the first and last wheels of the trains in terms of the number of teeth or radii of pitch circles: In this combination, it is always preferable to set the general equation up in terms of the number of teeth rather than the radii of pitch circles, for then the relations are independent of the type of gearing and velocity ratios between a meshing pair are inversely as the number of teeth or threads.

Thus assume worm #1 to drive worm gear #2, while bevel gear #3 on same shaft as gear #2, drives bevel gear #4, then helical screw gear #5 on same shaft as gear #4, drives helical screw gear #6 and finally gear #7 on gear shaft #6, drives pinion #8.

Since 2 and 3, 4 and 5, and 6 and 7 are on same shafts, we have,

$w_2 = w_3 = w_a$, $w_4 = w_5 = w_b$, and $w_6 = w_7 = w_c$

then,

$$\frac{w_1}{w_2} = \frac{n_2}{n_1}; \quad \frac{w_3}{w_4} = \frac{n_4}{n_3}; \quad \frac{w_5}{w_6} = \frac{n_6}{n_5}; \quad \frac{w_7}{w_8} = \frac{n_8}{n_7}$$

hence $\frac{w_1}{w_2} \times \frac{w_3}{w_4} \times \frac{w_5}{w_6} \times \frac{w_7}{w_8} = \frac{n_2}{n_1} \times \frac{n_4}{n_3} \times \frac{n_6}{n_5} \times \frac{n_8}{n_7}$

therefore $\dfrac{w_1}{w_8} = \dfrac{n_2}{n_1} \times \dfrac{n_4}{n_3} \times \dfrac{n_6}{n_5} \times \dfrac{n_8}{n_7}$

If T_1 = torque on worm shaft #1 and T_8 the torque on pinion shaft and ε the efficiency of the total gearing, then $T_8 w_8 = \varepsilon\, T_1 w_1$

hence

$$T_1 = \frac{T_8}{\varepsilon}\left(\frac{n_1 \times n_3 \times n_5 \times n_7}{n_2 \times n_4 \times n_6 \times n_8}\right)$$

where T_1 = required power torque and T_8 = load torque at end of train.

REACTION BETWEEN GEAR PAIRS:- EFFICIENCY.

The efficiency of spur and bevel gears is high compared with helical screw gearing, especially of the worm gear type. The very large force and velocity ratio attainable by the latter makes this type preferable.

1° Spur Gears:

For approximate calculations, the normal reaction between the teeth will be taken at an angle of 20° with the tangent to the pitch circles. The effect of friction between the teeth is to cause the resultant reaction to make an angle of 25° with the tangent to the pitch circles.

Therefore if T is the torque to be transmitted, the reaction between the teeth R, becomes,

$$R = \frac{T \times 12}{r \cos 25°}$$

where T is measured in (lb.ft)
r is measured in (in.)

When smoother running is required with high velocity ratios helical spur gears have been extensively introduced. If B = the angle between the normal to a tooth surface and the tangent to the circumference (i. e. normal to axis of rotation), then

$$R = \frac{T \times 12}{r \cos 25 \cos B}$$

If b = tooth rim breadth, the mean pressure is distributed along a linear element = b sec B and therefore the pressure on an element becomes per linear inch, proportional to

$$\frac{T \times 12}{r \cos 25.b}$$ the same as in ordinary spur gearing.

2° Bevel Gears:

The reaction between bevel gears takes place at the intersection of the common pitch circles of the cone elements of the gears, and this intersection is in the plane of the axis of the gearing. The neutral reaction between the teeth makes an angle approximately equal to 20° with the normal to this plane due to the contour of the tooth. The tangential component produces no axial thrust. The component parallel to the plane = P tan 20, where P is the tangential component. This component is also perpendicular to the common intersecting line of the two cones. If the cone angle of gear #1 equals 2θ then the cone angle of gear #2 = $2(\frac{\pi}{2} - \theta)$.

The axial thrust for gear #1 becomes, $P \tan 20° \sin\theta$.

The axial thrust for gear #2 becomes, $P \tan 20° \cos\theta$. Further the radial reaction between the teeth and therefore the radial bearing loads for gear #1 and gear #2, becomes,

$$R' = \sqrt{P^2 + (P \tan 20° \cos\theta)^2} = P\sqrt{1 + (\tan 20° \cos\theta)^2}$$

where

$P = \frac{T \times 12}{r}$; and 2θ = the cone angle of gear #1

$2(\frac{\pi}{2} - \theta)$ = the cone angle of gear #2.

3° Helical Screw Gears:

Assuming the axis of the gears to make an oblique angle $\emptyset_1$ the angle θ_1 between the contact line of the teeth and axis of gear #1 is given by the expression

$$\cos\theta_1 = \frac{p_2 \sin\emptyset}{\sqrt{p_1^2+p_2^2-2p_1p_2\cos\emptyset}}$$

while the angle θ_2 between the contact line of the teeth and axis of gear #2, is given by the expression

$$\cos\theta_2 = \frac{p_1 \sin\emptyset}{\sqrt{p_1^2+p_2^2-2p_1p_2\cos\emptyset}}$$

where p_1 and p_2 are the respective circumferential pitches of the two gears.

The reaction between the teeth makes a resultant angle i with the normal to the contact line, where tan i = n the coefficient of friction

Then, if T_1 is the external torque exerted on gear #1, we have $T_1 = R\cos(\theta_1 - i).r_1$

while if T_2 is the torque on gear #2, $T_2 = R\cos(\theta_2+i).r_2$

Work expended = T_1w_1

and $$\frac{w_2}{w_1} = \frac{r_1\cos\theta_1}{r_2\cos\theta_2}$$

Work delivered = T_2w_2

Then the efficiency E becomes,

$$E = \frac{T_2w_2}{T_1w_1} = \frac{\cos(\theta_2+i)\cos\theta_1}{\cos(\theta_1-i)\cos\theta_2}$$

The reaction on the teeth is given by

$$R = \frac{T_1}{r_1\cos(\theta_1-i)}$$

The thrust along gear shaft #1, is

$$R\sin(\theta_1-i) = \frac{T_1}{r_1}\frac{\sin(\theta_1-i)}{\cos(\theta_1-i)}$$

and the thrust along gear shaft #2, is

$$R \sin(\theta_2 + i) = \frac{T_1}{r_1} \frac{\sin(\theta_2+i)}{\cos(\theta_1-i)}$$

The total radial bearing load for shaft of gear #1 balances,

$$R \cos(\theta_1-i) = \frac{T_1}{r_1} \frac{\cos(\theta_1-i)}{\cos(\theta_1-i)} = \frac{T_1}{r_1}$$

and the total bearing load of gear shaft #2 balances,

$$R \cos(\theta_2+i) = \frac{T_2}{r_2}$$

4° Worm Gear:

Though worm gearing is a special case of 3°, a separate analysis will be made due to the greater use of this type of gearing as compared with helical gearing when the shafts are not at right angles.

Let xx' and yy' be the coordinate axis along and perpendicular to the axis of the worm in the plane perpendicular to the radius of the pitch line of the worm through the common pitch point as origin.

Let S = the angle that the contour of the tooth makes with the normal to the xy plane at the pitch point, and θ = the angle of helix.

Let R = normal component between worm and gear tooth,

nR = friction component between worm and gear tooth.

then the axial thrust along worm wheel is

$X = R \cos S' \cos \theta - nR \sin \theta$ and the turning component on the worm is

$Y = R \cos S' \sin \theta + nR \cos \theta$

and the thrust tending to separate the teeth is

$Z = R \sin S'$.

It is to be noted that $\tan S' = \tan S \cos \theta$

If T_w= torque applied to worm gear
T_g= torque on gear wheel
then,

$$T_w = Yr_w \quad \text{and} \quad T_g = Xr_g$$

r_w= radius of worm gear
r_g= radius of gear wheel

To determine the efficiency, we have

$$\varepsilon = \frac{T_g w_g}{T_w w_w} \qquad \text{but } \frac{w_g}{w_w} = \frac{r_w}{r_g} \tan\theta$$

then

$$\varepsilon = \frac{\cos S' \cos\theta - n \sin\theta}{\cos S' \sin\theta + n \cos\theta} \tan\theta$$

$$= \left(\frac{1 - \frac{n \tan\theta}{\cos S'}}{\tan\theta + \frac{n}{\cos S'}} \right) \tan\theta$$

$$\varepsilon = \frac{\tan\theta}{\tan(\theta + k)} \qquad \text{where } k = \tan^{-1}\frac{n}{\cos S'}$$

and $\tan S' = \tan S \cos\theta$

COMBINING THE REACTIONS FROM ONE PAIR TO ANOTHER. In gear transmission we have between two elements, #1 and #2,

$$T_2 w_2 = \varepsilon T_1 w_1$$

w = angular velocity

hence $\frac{T_2}{T_1} = \varepsilon_{12} \frac{w_1}{w_2}$ Likewise between gear elements #3 and #4, $\frac{T_4}{T_3} = \varepsilon_{34} \frac{w_3}{w_4}$

Then if gear #2 is on same shaft as gear #3, we have $T_2 = T_3$ and $w_2 = w_3$ hence

$$\frac{T_4}{T_3} \times \frac{T_2}{T_1} = \varepsilon_{34} \frac{w_3}{w_4} \varepsilon_{12} \frac{w_1}{w_2}$$

$$\frac{T_4}{T_1} = \varepsilon_{12} \varepsilon_{34} \frac{w_1}{w_4}$$

Now the velocity ratio $\frac{w_1}{w_4}$ may be obtained as outlined in previous discussion on velocity ratios.

In the preceeding discussion the inertia effect of the gear elements has been neglected in comparison with the friction developed between the gears.

TORQUE AND POWER REQUIREMENTS FOR ELEVATING AND TRAVERSING MECHANISMS. In elevating ortraversing a gun, we have three important periods:-(a) accelerating period, (b) the period of uniform motion and (c) the retardation period. The maximum torque obviously occurs during the acceleration and power is continued through period (b), while the friction of the mechanism brings the system to rest during period (c).

Let I_t = moment of inertia about the trunnions of the tipping parts.

I_t' = moment of inertia about the vertical traversing pivot of the tipping parts and top carriage.

E = elevating gear (tangential reaction.)

J = radius of elevating arc.

r = radius of traversing arc.

M_t = friction moment of trunnions

M_t' = friction moment of traversing pivot.

Then during the acceleration,

$$E_J - M_t = I_t \frac{d^2\phi}{dt^2} \qquad \text{for elevating the gun}$$

$$E'r - M_t' = I_t' \frac{d^2\theta}{dt} \qquad \text{for traversing the gun}$$

Now M_t and M_t' are constant depending approximately on the weight on the bearing, while on the other hand E and E' depends on the elevating or traversing motor characteristics.

Neglecting the inertia of the gear elements, we have, the torque transmitted varying directly as the number of teeth, that is between any two gear elements,

$$\frac{T_1}{T_2} = \frac{1}{\varepsilon_{12}} \frac{n_1}{n_2} \quad \text{for gear pair } 1 - 2$$

$$\frac{T_3}{T_4} = \frac{1}{\varepsilon_{34}} \frac{n_3}{n_4} \quad \text{for gear pair } 3 - 4$$

- - - - -

$$\frac{T_7}{T_8} = \frac{1}{\varepsilon_{78}} \frac{n_7}{n_8} \quad \text{for gear pair } 7 - 8$$

If gears 2 and 3, 4 and 5, 6 and 7 are assumed on same respective shafts,

$$T_2 = T_3, \; T_4 = T_5, \; T_6 = T_7$$

then

$$\frac{T_1}{T_8} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6} \times \frac{T_7}{T_8} = \left(\frac{n_1}{n_2} \times \frac{n_3}{n_4} \times \frac{n_5}{n_6} \times \frac{n_7}{n_8}\right) \frac{1}{\varepsilon_{12}} \times \frac{1}{\varepsilon_{34}} \times \frac{1}{\varepsilon_{56}} \times \frac{1}{\varepsilon_{78}}$$

Now $T_8 = E\, r_e$ and $\frac{1}{\varepsilon} = \frac{1}{\varepsilon_{12}} \times \frac{1}{\varepsilon_{34}} \times \frac{1}{\varepsilon_{56}} \times \frac{1}{\varepsilon_{78}}$

then

$$T_1 = \frac{E r_e}{\varepsilon} \left(\frac{n_1}{n_2} \times \frac{n_3}{n_4} \times \frac{n_5}{n_6} \times \frac{n_7}{n_8}\right)$$

hence

$$\varepsilon \frac{T_1}{r_e} \left(\frac{n_2}{n_1} \times \frac{n_4}{n_3} \times \frac{n_6}{n_5} \times \frac{n_8}{n_7}\right) J - M_t = I_t \frac{d^2 \phi}{dt^2} \quad \text{for elevating the gun.}$$

$$\varepsilon \frac{T_1'}{r_e}\left(\frac{n_2}{n_1} \times \frac{n_4}{n_3} \times \frac{n_6}{n_5} \times \frac{n_8}{n_7}\right) r - M_t' = I_t' \frac{d^2\theta}{dt^2}$$ for traversing the gun.

and for the sngular velocity ratios, we have,

$$\frac{w_1}{w_8} = \frac{n_2}{n_1} \times \frac{n_4}{n_3} \times \frac{n_6}{n_5} \times \frac{n_8}{n_7}$$

and $w_8 = \frac{J}{r_e} w_t$; - for spur or bevel gears:(elevating)

$w_8 = \frac{r}{r_e'} w_t'$:- for spur or bevel gears:(traversing)

$w_8 = \frac{2\pi J}{np}$ w_t: for worm gear in contact with elevatin are (elevating)

$w_8 = \frac{2\pi r}{n' p}$ w_t': for worm gear in contact with traversing arc (traversing)

CHAPTER V.

RECOIL HYDRODYNAMICS.

OBJECT. The modern recoil system is essentially a hydropneumatic device for dissipating the energy of recoil by so called hydraulic throttling losses, and returning by means of the potential energy stored up in the compression of air, the recoiling mass into battery. The potential energy at the end of recoil required to return the piece into battery is relatively small compared with the energy dissipated by the hydraulic braking. Further the potential energy of counter recoil is in greater part dissipated by the hydraulic counter recoil buffer in the return of the recoiling mass into battery.

In the design of the braking system misunderstanding may result due to imcomplete comprehension of the fundamental principles underlying the hydraulic throttling and the various hydraulic reactions. Hence, in this chapter a resume of the essential principles underlying the hydraulic phase of recoil design will be attempted.

ELEMENTARY HYDRAULIC BRAKE Consider an ordinary tension brake (fig.1) the oil being throttled through apertures in the brake cylinder from the front or rod side of the piston to its rear.

Let a_x = area of the variable apertures or orifice.

A_h = effective area of piston on rod side.

A = total area of cylinder.

a_r = area of rod.

P_h = total hydraulic pull.

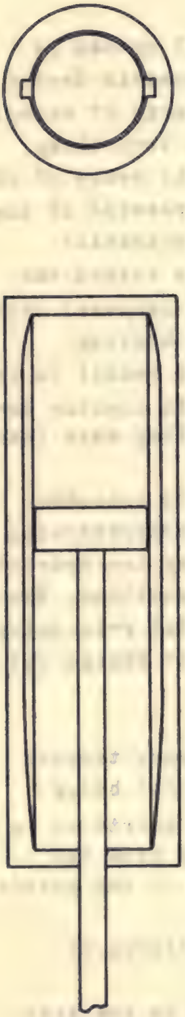


Fig. 1

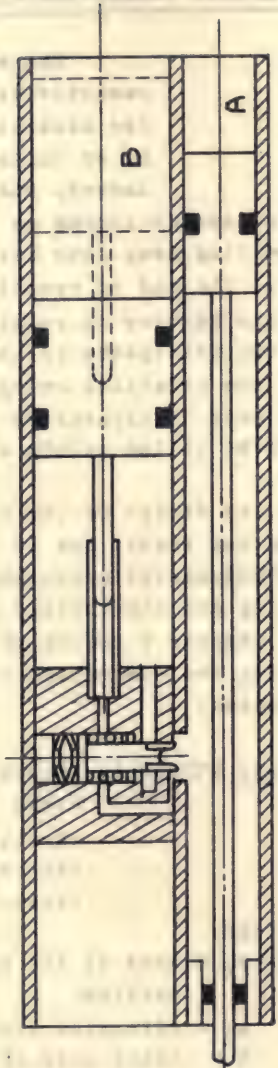


Fig. 2

V_x=velocity of recoil at displacement x.
v_x= velocity of oil through apertures.
D = weight of fluid per unit volume.
p = p_h= intensity of hydraulic pressure.
C = contraction coefficient of orifice.
K = reciprocal of contraction coefficient.

For a displacement dx, the mass of liquid moved by the displacement of the piston, becomes,

$\frac{D\ A_h\ dx}{g}$ and due to the contraction of the liquid in the throttling aperture or orifice, its effective area is reduced to C a_x, therefore, the mass is accelerated to a velocity

$v_x = \frac{A_h\ V_x}{C\ a_x} = \frac{K\ A_h\ V_x}{a_x}$, since $K = \frac{1}{c}$ now the energy

of the jet,
$\frac{\frac{1}{2}\ D\ A_h\ dx}{g}\ v_x^2$ becomes, dissipated by a loss due to sudden expansion in the rear part of the cylinder, where we find a void equal to: $(A-A_h)x= a_r x$. By the principle of virtual work, evidently

$$P_h\ dx= \frac{\frac{1}{2}\ D\ A_h\ dx}{g}\ \left(\frac{A_h\ V_x}{C\ a_x}\right)^2$$

hence

$$P_h= \frac{\frac{1}{2}\ D\ A_h^3\ V_x^2}{g\ c^2 a^2 x} \qquad (1)$$

that is in terms of the liquid pressure

$$p = \frac{\frac{1}{2}\ D\ K^2\ A_h^2\ V_x^2}{g\ a_x^2} \qquad (2)$$

Consider again a brake where the throttling occurs between the hydraulic cylinder A and a recuperator cylinder B containing a floating piston which is contact with the oil on one side and the air on the other. See fig.(2).

Let p = pressure intensity against hydraulic or recoil piston.

A_h = effective area of hydraulic piston.

a_x = throttling area between the two cylinders which we may assume is controlled by a spring.

v_x = velocity through orifice.

V_x = velocity of recoil.

V_a = velocity of floating piston.

A_a = area of floating piston.

p_a = pressure intensity against floating piston.

x' = displacement of floating piston.

Then by the law of continuity, $A_h\, dx = A_a\, dx'$
Due to the contraction and sudden expansion of the liquid from the throttling apertures, the loss due to eddy currents becomes,

$$\frac{\frac{1}{2} D\, A_h\, dx}{g} \left(\frac{A_h V_x}{C\, a_x}\right)^2$$

By the principle of virtual work, we have,

$$P_h\, A_h\, dx - p_a A_a dx' = \frac{\frac{1}{2} D\, A_h\, dx}{g} \left(\frac{A_h V_x}{C\, a_x}\right)^2$$

Neglecting the slight change in the total kinetic energy of the liquid in its virtual displacement. Simplifying, we obtain,

$$p_h - p_a = \frac{\frac{1}{2} D\, K^2\, A_h^2\, V_x^2}{g\, a^2 x} \qquad (3)$$

which gives the drop in pressure through the orifice, or the so called <u>throttling drop</u>, Obviously, $P_h = p_h A_h$, as before,

$$= \frac{\frac{1}{2} D\, K^2 A_h^3 V_x^2}{g\, a_x^2} + p_a\, A_h \qquad (4)$$

PRINCIPLES OF HYDRODYNAMICS. (1) Though in the analysis of recoil brakes, liquid viscosity is an item of importance, the viscosity effect in modifying pressures is, with a few exceptions, small, and therefore, for a first

approximation we will consider an ideal fluid, that is a liquid with no viscosity.

(2) It may be shown by simple analysis in the consideration of a small tetrahedron or triangular prism that the pressure intensity on all planes at a given point within a fluid is the same, the bodily forces such as gravity, inertia resistance etc. in limit being eliminated since they are functions of higher order (three dimensions) than the surface pressures (two dimensions).

(3) By higher analysis it may be shown that fluids flow in so called stream lines and therefore the variation of pressure with velocity at various points along the stream line as well as the change in such due to the acceleration of the fluid as a whole, may be determined by a consideration of the pressures on continuous differential elements. Due to the mutual action between differential elements, we may by simple integration along a stream line determine the pressures at the extremities of a stream line tube, that is the end pressures as well as the terminal velocities.

Consider a differential element A B C D along a stream line, of cross section w of length ds and a circumferential perimeter c.

Let, the intensity of pressure on A D be p, the weight per unit volume be G, then for the pressures on the surface A B - C D and the wall of the tube, we have

$$pw-(p+\frac{dp}{ds}.ds)(w-dw)-pcds \sin \alpha-D\ w\ ds \sin \emptyset = \frac{Dwds}{g}\frac{dv}{dt}$$

but cds sin α = dw. Simplifying and dividing through by w, we have,

$$-dp-D\ ds \sin \emptyset = \frac{Dds}{g}\frac{dv}{dt} \qquad (5)$$

but $dv = \frac{dv}{dt}dt + \frac{dv}{ds}ds$ hence $\frac{dv}{dt} = \frac{dv}{dt} + v\frac{dv}{ds}$ which shows the acceleration is both a time and space

function, inserting in (5) we obtain,

$$-dp - D\,ds\,\sin\phi\,\frac{D\,ds}{g}\left(\frac{dv}{dt} + v\,\frac{dv}{ds}\right) \qquad (6)$$

Integrating from (1) to (2) along a stream line, since the mutual reactions between contiguous particles cancel out, we have,

$$\frac{p_1 - p_2}{D} - \int_{Z_1}^{Z_2} ds\,\sin\phi = \frac{1}{g}\int ds\left(\frac{dv}{dt}\right) + \frac{v_2^2 - v_1^2}{2g}$$

Obviously,

$$\int_{Z_1}^{Z_2} ds\,\sin\phi = Z_1 - Z_2$$

hence

$$\frac{p_1 - p_2}{D} + Z_1 - Z_2 = \frac{1}{g}\int ds\left(\frac{dv}{dt}\right) + \frac{v_2^2 - v_1^2}{2g} \qquad (7)$$

The term $\int ds\,\frac{dv}{dt}$ is of special interest and when it occurs the motion is not steady. This term is theoretical, always existing in a recoil brake, since the fluid in addition to a space variation of velocity due to changes of sections, is on the whole accelerated as well.

To evaluate $\int ds\,\frac{dv}{dt}$ it is necessary to express $\frac{dv}{dt}$ as a function of s. If now we assume the same stream lines to exist whether accelerated or under uniform steady motion, we have, by the equation of continuity, $w_1 v_1 = w_2 v_2 = w_3 v_3$ and

$$w_1\frac{dv_1}{dt} = w_2\frac{dv_2}{dt} = w_3\frac{dv_3}{dt} \qquad (8)$$

hence knowing the acceleration at one section,

$\frac{dv_n}{dt} = \frac{w_1}{w_n}\frac{dv}{dt}$ for any point "n", hence if w is a continuous function of s, we have

$\frac{dv}{dt} = \frac{1}{w}\left(w_1\,\frac{dv_1}{dt}\right) = \frac{1}{f(s)}\,w_1\,\frac{dv_1}{dt}$ hence the line integral of the acceleration along a stream lines, becomes,

$$ds\,\frac{dv}{dt} = w_1\frac{dv}{dt}\,\frac{ds}{f(s)}$$

The line integral of the acceleration may be obtained to a sufficient degree of exactness by dividing stream lines into a linear group of columns of various sections, obtaining the proper acceleration. To form (8) and multiplying by the length of the respective columns and then adding these columns together. The term $\frac{1}{g}\int ds\,(\frac{dv}{dt})$ is found usually to be relatively small compared with the pressure drops due to throttling and the changes of pressure due to changes of section.

Hence (7) reduces to the energy equation for uniform or steady flow, known as Bernoulliis theorem, that is,

$$\frac{p_1}{D}+Z_1+\frac{v_1^2}{2g} = \frac{p_2}{D}+Z_2+\frac{v_2^2}{2g} \qquad (9)$$

The term $\frac{p_1}{D}+Z_1+\frac{v_1^2}{2g}$ is known as the total head at section (1), composed respectively of a pressure head, gravitational head and a velocity head,

(4) When friction, viscosity or turbulent motion occurs Bernoulliis theorem is modified by a friction head h_f.

Considering a tube of a stream, we have for steady motion

$$p_1w_1ds_1-p_2w_2ds_2+Dw_1ds_1(Z_1-Z_2)-dW_{f_{1\,2}} = \frac{Dw_1ds_1}{2g}(v_2^2-v_1^2) \quad (10)$$

Where d W_f corresponds to the differential work due to friction for a differential quantity of flow d Q.

By the equation of continuity, $dQ = w_1ds_1 = w_2ds_2$

hence (9) reduces to

$$p_1-p_2+D(Z_1-Z_2)-\frac{dW_f}{dQ} = \frac{D}{2g}(v_2^2-v_1^2) \qquad (11)$$

but $\frac{dW_{f_{1\,2}}}{dQ} = h_{f_{1\,2}}$, known as the head loss due to friction between 1 and 2, hence

$$\left(\frac{p_1}{D} + \frac{w_1^2}{2g} + Z_1\right) - \left(\frac{p_2}{D} + \frac{v_2^2}{2g} + Z_2\right) = h_{f'_2} \qquad (12)$$

It is to be especially noted that in the flow of a liquid through an orifice as in a recoil brake, the major loss is in the nature of a frictional loss of head due to the contraction and sudden expansion of the liquid through the orifice, thus

$$h_{f'_2} = \frac{\frac{1}{2} K^2 A_h^2 V_x^2}{g a_x^2} = \frac{p_h - p_a}{D}$$ in equation (3), that is

$$\frac{p_1 - p_2}{D} = \frac{\frac{1}{2} K^2 A_h^2 V_x^2}{g a_x^2}$$ since $Z_1 = Z_2$ approximately, also $\frac{v_1^2}{2g}$ and $\frac{v_2^2}{2g}$ are relatively small.

It may be shown by a somewhat similar analysis that in the consideration of friction of or turbulent loss of head due to throttling, that, from (7) we have

$$\left(\frac{p_1}{D} + \frac{v_1^2}{2g} + Z_1\right) - \left(\frac{p_2}{D} + \frac{v_2^2}{2g} + Z_2 = \frac{1}{g} \int \left(\frac{dv}{dt}\right) ds + h_{f'_2} \qquad (13)$$

where if w_o = the area of the jet

v_o = the velocity of the jet.

where $h_{f'_2}$ has the form,

$$h_{f'_2} = \frac{Cv_o^2}{w_o^2}$$

Hence an axact expression for a stream line passing through a jet, and the whole stream line itself under acceleration, becomes,

$$\left(\frac{p_1}{D} + \frac{v_1^2}{2g} + Z_1\right) - \left(\frac{p_2}{D} \frac{v_2^2}{2g} + Z_2\right) = \frac{1}{g} \left(\frac{dv}{dt}\right) ds + \frac{Cv_o^2}{w_o^2} \qquad (14)$$

(5) The pressure variation across a stream line may be obtained by a consideration of a cylinder

the end faces of which are in the outer and inner boundary surface of a stream line tube, and the axle is perpendicular to the stream line axis. We have, if w= cross section of differential cylinder

r = radius of curvature of stream line

d = height of differential cylinder

Ø = angle between r and the vertical

that

$$(p+dp)w-pw+Dwdr\cos Ø = \frac{Dw}{g}dr\frac{v^2}{r} \qquad (15)$$

$$wdp = Dwdr(\frac{v^2}{gr} - \cos Ø)$$

hence $$\frac{dp}{dr} = D(\frac{v^2}{gr} - \cos Ø) \qquad (16)$$

which given the rate of change of the pressure across a stream line with respect to the radius of curvature. Neglecting the weight component, we have,

$$\frac{dp}{dt} - \frac{Dv^2}{gr} \qquad (17)$$

Hence for circular or vortex motion, the change in pressure along the radius, becomes,

$$p_2-p_1 = \frac{D}{g}\int_{r_1}^{r_2}\frac{v^2}{r}\,dr \qquad (18)$$

In particular if the total system acquires the same angular velocity $\frac{d\theta}{dt}$, we have,

$$\frac{v^2}{r} = r(\frac{d\theta}{dt})^2$$

and $$p_2-p_1 = \frac{D}{g}(\frac{r_2^2-r_1^2}{2})(\frac{d\theta}{dt})^2 = D(\frac{v_2^2-v_1^2}{2g}) \qquad (19)$$

since the total head at any point in a fluid equals,

$H = \frac{p}{D} + Z + \frac{v^2}{2g}$ the variation of head across a stream line becomes,

$$\left.\begin{aligned} dH &= \frac{dp}{D}+dZ+\frac{vdv}{g} \\ &= \frac{dp}{D}+dr\cos Ø+\frac{vdv}{g} \end{aligned}\right\} \qquad (20)$$

Substituting Eq.(16) in (29) we have,

$$dH = \frac{v^2}{gr}\,dr + \frac{vdv}{g} \qquad (21)$$

which is the general equation for the change in head across a stream line.

(6) When the flow is radial, evidently the flow outward from circumferences of various radii, becomes, $Q = 2\pi\, rv = 2\pi\, rv$, hence $v_o r_o = vr$

or

$$\frac{v}{v_o} = \frac{r_o}{r}$$ and for steady motion, we have,

$$\frac{p_o}{D} + \frac{v_o^2}{2g} + Z_o = \frac{p}{D} + \frac{v^2}{2g} + Z$$

hence

$$p - p_o = \frac{v_o^2}{2g}\left(1 - \frac{r_o^2}{r^2}\right) \qquad (22)$$

In terms of the total head H, we have,

$$\frac{p}{D} = H - \frac{v_o^2\, r_o^2}{2gr^2} \qquad (23)$$

(7) A free circular vortex occurs when the total head of any annular stream line of the vortex is the same.

That is, for any annular stream line,

$$H = \frac{p}{D} + \frac{v^2}{2g} = \text{const.}$$

To find the distribution of pressure, we have,

$$\frac{dp}{D} + \frac{vdv}{g} = 0$$ and for the flow slowly outward radially, we have,

$$\frac{p - p_o}{D} = \frac{v_o^2 - v^2}{2g} \quad \text{(Neglecting friction)}$$

$$= \frac{v_o^2}{2g}\left(1 - \frac{r_o^2}{r^2}\right)$$

Thus the pressure variation is exactly similar to that of ordinary radial flow.

Now from (21) since $dH = 0$

$$\frac{v^2}{gr}\,dr + \frac{vdv}{g} = 0 \quad \text{hence} \quad \frac{dv}{v} = -\frac{dr}{r}$$ and from the law of continuity

for the flow outward, $v\ r = v\ r$ hence $v = \frac{k}{r}$

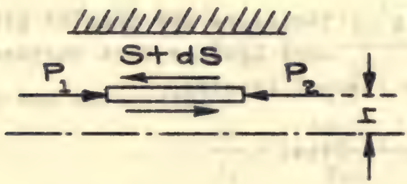
Likewise the flow outward is exactly similar to ordinary radial flow.

THE EFFECT OF THE VISCOSITY OF FLUIDS. The viscosity of a fluid is the shear stress to the distortion of the fluid and this stress is measured by the coefficient of viscosity times the rate of distortion. In other words the viscosity or coefficient of viscosity, becomes,

$$u = \frac{s}{\frac{dv}{dh}} \quad \text{or } s = u\frac{dv}{dh}$$

where v = velocity of a lamina flow (ft/sec)
h = normal to flow lamina (ft)
s = shear

1° Flow between flat surfaces:



$$S = u\frac{dv}{dh}\,.bl$$

$$dS = u\frac{d^2v}{dh^2}\,dh.bl$$

Now considering the forces on a lamina of thickness dh, breadth b and length l, we have, for a constant pressure head $(p_1 - p_2)$

$(p_1 - p_2)\ bdh - dS = 0$ <u>for uniform flow</u>

$$(p_1-p_2)bdh-bln\frac{d^2v}{dh^2}dh = 0$$

$$(p_1-p_2)-lu\frac{d^2v}{dh^2} = 0$$

Integrating, we have $\dfrac{dv}{dh} = \dfrac{(p_1-p_2)h}{ul} + C_1$

when $\dfrac{dv}{dh} = 0$, $C_1=0$

Integrating again, $v = \dfrac{(p_1-p_2)H^2}{2ul} + C_2$ when $v = 0$, $h=\dfrac{H}{2}$

and $C_2 = -\dfrac{(p_1-p_2)H^2}{8ul}$ Hence the distribution of velocity across a section is given by the equation,

$v = \dfrac{(p_1-p_2)}{2ul}(h^2-\dfrac{H^2}{4})$ (ft/sec) as measured from the center. For a differential flow, we have

$dQ = vbdh = \dfrac{p_1-p_2}{2ul}(h^2-\dfrac{H^2}{4})\,bdh$ and for the total flow, summing up on both sides of center line, we have,

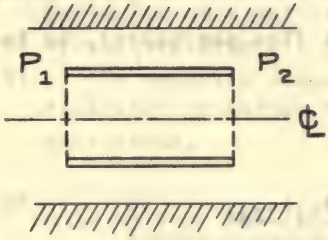
$Q = \dfrac{1}{12}\dfrac{(p_1-p_2)}{ul}bH^3$ Therefore the drop of pressure between flat surfaces in a rectangular channel becomes.

$$p_1-p_2 = \frac{12ul}{bH^3}Q \quad \text{(lbs)}$$

For the particular case of a square section,

$$p_1-p_2 = \frac{12ulQ}{H^4} \quad \text{(lbs)}$$

2° Flow through a circular section:



The viscosity shear becomes, $S = -2\pi r l u \dfrac{dv}{dr}$

$$dS = -2\pi u l d \frac{(r \frac{dv}{dr})}{dr} dr$$

Considering the forces on a cylindrical lamina of thickness dr and length l, we have,

$(p_1-p_2=2\pi r dr - dS' = 0$ for uniform flow

$$(p_1-p_2)r + ul \frac{d(r \frac{dv}{dr})}{dr} = 0$$

Integrating, we have

$$r \frac{dv}{dr} = -\frac{(p_1-p_2)r^2}{2ul} + C_1$$

which may be written

$$\frac{dv}{dr} = -\frac{(p_1-p_2)r}{2ul} + \frac{C_1}{r}$$

when $\dfrac{dv}{dr} = 0$, $r = 0$, hence $\dfrac{C_1}{r} = 0$ i.e. $C_1 = 0$ since

r may have any finite value. Integrating again

$v = -\dfrac{(p_1-p_2)r^2}{4ul} + C_2$ when $v = 0$ for the boundary surface,

$r = R$

hence $C_2 = +\dfrac{(p_1-p_2)R^2}{4\ ul}$ and $v = \dfrac{p_1-p_2}{4\ ul}(R^2-r^2)$

which gives the variation of the velocity over a cross section as a function of the radius from the center.

For the total flow per second, we have

$$Q = \int_o^r 2\pi r dr . v$$

$$= \frac{(p_1-p_2)\pi}{2\ ul}\int_o^r (R^2-r^2)rdr$$

$$= \frac{(p_1-p_2)\pi R^4}{8\ ul}$$

Hence for the drop of pressure through a small orifice where there is no abrupt change in section,

$$p_1-p_2 = \frac{8\ ul}{\pi R^4}\ Q$$ where u = coefficient of viscosity.

$$= \frac{128\ ul}{\pi D^4}\ Q$$

that is the drop of pressure varies as the length and inversely as the 4th power of the diameter of the orifice.

PRINCIPLE OF MOMENTUM AND DYNAMIC REACTIONS. The various formulae previously developed depended upon the application of Bernoulli's theorem, or the energy equation of hydro dynamics. A theorem of equal importance is the principle of momentum and from it with a combination of Bernoulli's theorem, we may compute the various dynamic reactions, that occur in hydro dynamic problems.

If P = the unbalanced reaction on a mass of water and the velocities of the mass is changed in

time t, from v_1 to v_2 ft/sec. then $Pt = m(v_2 - v_1)$. In the application of this principle

(1) the mutual reactions between the particles of water are obviously entirely neglected.

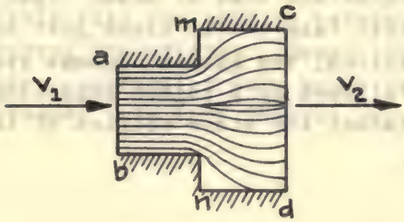
(2) the velocity components in the direction of motion are only to be considered.

DERIVATION OF THROTTLING FORMULAS.

(1) Loss of Head due to sudden expansion.

In the flow of a fluid through an orifice the drop in pressure or loss of head, is primarily due to the sudden expansion or abrupt change from a small to a large section, of the fluid flow. The loss of head is due to the formation of eddies due to the sudden expansion of the flow and the consequent dissipation of energy. If the cross section of the flow is gradually enlarged from that of a small orifice to a large section, no eddies are produced and we have no loss of head. Thus in the Venturi meter the fluid passes from a large section to a very small section and then back again to a large section but since the change in section is gradual, we have no drop in pressure. Hence we have a very important principle that is fundamental in the design of recoil throttling orifices:-

A throttling drop in pressure cannot be produced without a sudden change in section of the flow of the fluid.



Consider a flow of fuild passing through sections, ab, mn and cd respectively. Let the cross sections of the stream be w_1 at ab_1 w_2 at cd and the corresponding pressures be p_1 and p_2 respectively. Let p_0 be the pressure at mn_1 (lbs/sq.ft).

From the energy equation, we have,

$$\frac{p_1}{D} + \frac{v_1^2}{2g} = \frac{p_2}{D} + \frac{v_2^2}{2g} + h_f$$ assuming a continuous uniform flow.

From the principle of momentum, we have

$$p_1w_1+p_0(w_2-w_1)-p_2w_2 = \frac{Q}{g} D (v_2-v_1)$$ where Q = the rate of flow (cu.ft/sec)

D = the density of fluid (lbs/cu.ft). Now from the experiment it is found that $p_0 = p_1$ hence the momentum equation reduces to,

$$(p_1-p_2)w_2 = \frac{Q}{g} D(v_2-v_1) \text{ and } \frac{p_1}{D} - \frac{p_2}{D} = \frac{v_2(v_2-v_1)}{g}$$

hence

$$h_f = \frac{v_2(v_2-v_1)}{g} + \frac{v_1^{2'}-v_2^{2'}}{2g}$$ which simplifies to

$$h_f = \frac{(v_1-v_2)^2}{2g}$$ or in terms of the area of the orifice and the enlarged section, since $w_1v_1 = w_2v_2$

$$h_f = \frac{v_1^{2'}}{2g}(1-\frac{w_1}{w_2})^2 = \frac{v_2^{2'}}{2g}(\frac{w_2}{w_1} - 1)$$

(2) Loss of Head due to sudden contraction.

When the cross section is suddenly diminished beyond the reduced section we have eddying of the flow with a resultant loss of head. This too is really a special case of (1) since just beyond the contracted section, the stream becomes even more contracted, followed by a sudden expansion until the stream reaches the cross section of the contracted area.

Therefore if w_1= the area of the contracted section, then the cross section of the contracted stream becomes, $c_1 w_1$ where c_1= depends upon the preceeding area w and the area of the orifice. In terms of the velocity of the orifice, the loss head equals

$h_f = \frac{v_1^2}{2g}\left(\frac{w_1}{cw_1} - 1\right)^2 = \xi\frac{v_1^2}{2g}$ where if w = the cross section before the sudden contraction of section, the experiments of Weisbach give for

$\frac{w_1}{w}$	c	ξ
0.1	0.624	0.360
0.2	0.632	0.340
0.30	0.643	0.310
0.40	0.659	0.266
0.50	0.681	0.220
0.60	0.712	0.162
0.70	0.755	0.106
0.80	0.813	0.053
0.90	0.892	0.014
1.00	1.00	0.

In the special case when $\frac{w_1}{w} = 0$, that is the area of the orifice w_1 is entirely negligible with the flow from the large cylinder as in a flow from a resevoir, c = 0.6 and ξ = 0.445.

It is to be noted that the above analysis holds only when the length of orifice is sufficiently long to allow the contracted stream in the orifice to expand and completely fill the orifice before expanding in the region beyond the orifice. Hence the loss of head due to sudden contraction only holds for long orifices or entrances into long channel parts.

LOSS OF HEAD AND PRESSURE DROP THROUGH RECOIL ORIFICE. Assuming uniform flow from the recoil cylinder of effective area w, through an orifice of cross section w_1 discharging into a cylinder or channel of cross section w_2. Then from section w to the mid section of the orifice,

$$\frac{p}{D} + \frac{v^2}{2g} = \frac{p_1}{D} + \frac{v_1^2}{2g} + h_{fc} \quad (1)$$

h_{fc} = loss of head due to sudden contraction

and from the mid section of the orifice w_1 to the rear of cylinder or channel cross section w_2,

$$\frac{p_1}{D} + \frac{v_1^2}{2g} = \frac{p_2}{D} + \frac{v_2^2}{2g} + h_{fe} \quad (2)$$

h_{fe} = loss of head due to sudden expansion.

Adding (1) and (2), we have

$$\frac{p}{D} + \frac{v^2}{2g} = \frac{p_2}{D} + \frac{v_2^2}{2g} + h_{fc} + h_{fe}$$

Very often $v = v_2$ approximately and usually the heads corresponding to v and v_2 are small compared with the pressure and throttling heads and therefore the velocity heads may be entirely neglected. We have then,

$$\frac{p - p_2}{D} = h_{fc} + h_{fe}$$

that is the drop in pressure through an orifice is equal to the total head lost due to sudden contraction and expansion.

Now

$$h_{fe} = \frac{v_2^2}{2g}\left(\frac{w_2}{w_1} - 1\right)^2 = \frac{v_1^2}{xg}\left(1 - \frac{w_1}{w_2}\right)^2$$

$h_{fc} = \xi \frac{v_1^{2'}}{2g}$ further $AV = w_1 v_1 = w_2 v_2$, (where V= the velocity of the recoil piston.)

($w=A$ = effective area of recoil piston)

hence

$$h_{fc} = \frac{A^{2'}V^{2'}}{2gw_1^2}(1 - \frac{w_1}{w_2})^{2'}$$

$$h_{fc} = \frac{A^2V^{2'}}{2gw_1^2}\xi$$

When the orifice is in grooves in the cylinder or through orifices in the piston, we have

$v_2=V$ approximately and

$$\frac{w_2}{cw_1} = \frac{A}{cw_1} \text{ approximately}$$

Therefore if $m = \frac{A}{cw_1}$, we have directly,

$$h_{fe} = \frac{V^2}{2g}(m-1)^{2'}$$

)since $w_2=A$ approximately, the effective area of the recoil piston and w_1=effective area of orifice, that is the contracted flow through the orifice. (c = coefficient of contraction of orifice.

Usually the loss of head due to sudden contraction may be entirely neglected as compared with the loss of head due to sudden expansion, hence we have, for a very close approximation of the pressure drop,

(1) With throttling through grooves in cylinder or piston,

$$\frac{p-p_2}{D} = \frac{V^2}{2g}(m-1)^2$$

or

$$\frac{p-p_2}{D} = \frac{A^{2'}V^{2'}}{2gc^2w_1^2}(1-\frac{cw_1}{A})^{2'}$$ where c= coefficient of contraction,

both forms having useful applications.

(2) With throttling through an orifice from the recoil cylinder to the recuperator,

$$\frac{p - p_2}{D} = \frac{A^2 V^2}{2gc^2 W_1^2} \left(1 - \frac{cw_1}{w_2}\right)^2$$

w_1 = area of orifice

w_2 = area of channel leading from orifice.

ANALYSIS OF THE MUTUAL REACTION IN A RECOIL BRAKE. In an ordinary brake cylinder we have a groove in the cylinder or at the circumference of the recoil piston.

As the recoil rod pulls out, a pressure is created on the front side of the piston, due to the forcing of the fluid through the orifice groove. The pressure is by Berboull's theorem, obviously lowered in the vicinity of the orifice due to the increased velocity of the flow.

Hence, with an orifice in the piston, the pressure is not uniformly distributed over the geometrical effective area of the piston.

Therefore, the brake reaction is not equal to the product of the recoil cylinder and the effective area of the recoil piston.

Let X = the total reaction of the fluid on the recoil piston. (lbs)

p = the pressure in the recoil cylinder. (lbs/sq. ft)

A = effective area of recoil piston.
= $0.7854\ (D_r^2 - d_r^2)$ (sq.ft)

where D_r = diam. of recoil cylinder (ft)

d_r = diam. of recoil rod (ft)

A_r = area of recoil brake cylinder = 0.785 (sq.ft)

V = velocity of the recoil piston (ft/sec)

v = velocity of flow through the orifice. (ft/sec)

D = density of fluid (lbs/cu.ft)

w = area of orifice (sq.ft)

c = contraction factor of orifice.

Assume the recoil rod to only extend from one end of the piston. In this case, we have a void in the rear of the piston due to the volume displacement in the front of the piston being less than in the rear of the piston.

Assuming the pressure in the orifice to be small, we have, for the reactions on the fluid from front head of cylinder to a cross section at the center of the orifice:-

$$pA - X = \frac{D\,A\,V}{g}.\,v \qquad (1)$$

and the reactions on the fluid from the orifice to the rear head of cylinder, becomes,

$$Y = \frac{D\,A\,V}{g}\,v \qquad (2)$$

<u>The reaction on the piston</u> = X <u>to the rear</u>

<u>The reaction on the cylinder</u> = pA - Y <u>to the front</u>

Adding (1) and (2) we have, $pA - X - Y = 0$, which is immediately obtained since there is no change in the total momentum of the fluid, as we should expect from first principles, since the fluid acts as a medium for the transmission of the reaction between the recoil cylinder and the recoil piston. Hence $pA - Y = X$.

Now $X = pA - \frac{D\,A\,V}{g}\,v$ which gives the actual reaction exerted on the brake piston. Since $C\,w\,v = A\,V$, by the law of continuity, then

$$v = \frac{AV}{cw} \text{ and } V = \frac{Cwv}{A} \text{ and } X = pA - \frac{D\,A^2\,V^2}{cgw} \quad \text{(lbs)}$$

$$= pA - \frac{D\,c\,w\,v^2}{g} \quad \text{(lbs)}$$

now $p = \frac{Dv^2}{2g}$, from Bernoulli's theorem,

$$\text{Hence } X = \frac{Dv^2}{2g} (A - 2\ cw)$$

$$= \frac{D\ A^2\ V^2}{2gc^2 w^2} (A-2\ cw)\ (\text{lbs})$$

but $p = \frac{D\ v^2}{2g} = \frac{D\ A^2\ V^2}{2gc^2w^2}$ hence $X = p(A-2\ cw)$

That, is the reaction on the piston equals the product of the pressure in the recoil cylinder and the effective area of the recoil piston, where the effective area of the recoil piston equals the annular area between the recoil cylinder and piston rod decreased by twice the contracted area of the orifice.

A physical explanation is that due to the pressure of the orifice, we have the pressure lowered around the orifice. Hence we must not only subtract the area of the orifice, but also an additional equivalent area which is to account for the lowered pressure about the orifice.

Since $c = 0.6$ approx., then $2\ cw = w$ approx., and therefore for practical calculations, the annular area of the recoil piston is merely decreased by the total throttling area through the piston.

When the rod is assumed to extend through both ends of the recoil cylinder, we have a continuous rod in the cylinder and therefore no void is produced during the recoil.

Assuming the same symbols as before, we have, since the total change of momentum of the fluid is nil, $pA - X - Y = 0$. Hence $X = pA - Y$ and the fluid merely transmits the mutual reactions between the recoil cylinder and recoil piston.

Let p_w = the pressure in the orifice. (lbs/sq.ft) then,

$$pA - X_f - p_w w = \frac{D\ A\ V}{g} v$$) X_f = total reaction on front of recoil piston.

for the momentum of the fluid contained in the front

part of the cylinder to the orifice, and

$Y - p_w w - X_r = \dfrac{D\,A\,V}{g}\,v$) X_r = total reaction on
(rear of recoil
) piston

for the momentum of the fluid contained from the orifice to the rear end of the cylinder. Now $X = X_f - X_r$ = total reaction on recoil piston. Due to the sudden expansion of the fluid after leaving the orifice, the pressure on the rear face of the piston, becomes, $p_w(A-w) = X_r$ (assumption from experiment — sudden expansion), hence

$$Y - p_w A = \frac{D\,A\,V}{g}\,v \quad \text{and } X = X_f - p_w(A-w)$$

$$= pA - p_w w - p_w(A-w) - \frac{D\,A\,V}{g}\,v$$

$$= (p - p_w)A - \frac{D\,A\,V}{g}\,v$$

Applying Bernoullis' theorem, we have $p - p_w = \dfrac{Dv^2}{2g}$

hence

$$X = \frac{Dv^2}{2g}A - \frac{D\,A\,V}{g}\,v$$ but by the law of continuity $c\,w\,v = A\,V$ therefore

$$X = \frac{Dv^2}{2g}A - \frac{Dcwv^2}{g}$$

$$= \frac{Dv^2}{2g}(A - 2cw) \quad \text{(lbs)}$$ Since p_w is negligible compared with p, we have

$$p - p_w = p = \frac{Dv^2}{2g}$$

hence, as before $X = p(A - 2cw)$

That is the total reaction on the recoil piston equals the product of the pressure in the recoil cylinder and the effective area of the recoil piston, when the effective area of the recoil piston equals the annular area between the recoil cylinder and piston rod decreased by twice the contracted

area of the orifice.

Since $c = 0.6$, $2cw = w$ approx., and therefore again for practical calculations, the annular area of the recoil piston is merely decreased by the total throttling area through the recoil piston.

DERIVATION OF RECOIL THROTTLING FORMULAS. We may consider the throttling effected in either of the following manners: (1) throttling through grooves in the cylinder wall or through a variable orifice in the piston itself,-(2) throttling through a stationary orifice.

(1) Throttling through a variable orifice in the piston or grooves in the cylinder walls.

Let

A = effective area of the piston, i. e. the cross section of the cylinder minus the cross section of the rod. (sq.ft)

p = the intensity of pressure at the pressure end of the cylinder (lbs/sq.ft)

D = the density of the liquid (lbs/cu.ft)

V = the velocity of the recoil (ft/sec)

w = the area of the orifice (sq.ft)

v = the velocity of flow through the orifice. (ft/sec)

X = the total fluid reaction against the piston (lbs)

Then, we have,(neglecting the small pressure in the orifice)

$$pA - X = \frac{D\ A\ V}{g} v$$ - - - for the momentum generated in the jet,

and $$p = \frac{Dv^2}{2g}$$ - - - - - for the energy of the flow in the jet.

$AV = cwv$ - - - - from the law of continuity of the flow,

then

$$X = pA - \frac{D\ A\ V}{g} v = \frac{DAv^2}{2g} - \frac{D\ A\ V}{g} v$$

$$\therefore X = \frac{DA}{2g}\left(1-\frac{2cw}{A}\right)v^2$$

$$= \frac{DA^3V^2}{2gc^2w^2}\left(1-\frac{2cw}{A}\right) \quad \text{(lbs)}$$

Since the reaction on the cylinder is the difference between the force pA at the pressure end and the reaction of the jet

$\frac{D\ A\ V}{g}v$ flowing from the orifice we have the reaction on the cylinder also equal to

$$pA - \frac{D\ A\ V}{g}v = \frac{DA^3V^2}{2gc^2w^2}\left(1-\frac{2cw}{A}\right) \quad \text{(lbs)}$$

as would be expected from the equality of action and reaction.

With a continuous piston rod through both ends of the cylinder we may neglect the pressure through the orifice and since by experiment the pressure on the rear face of the piston is practically that through the orifice, the reaction on the piston remains the same. Here again the reaction on the cylinder is

$pA-p'A = pA - \frac{D\ A\ V}{g}v$, since $p'A = \frac{D\ A\ V}{g}v$ as would be expected from the equality of action and reaction.

The reaction X on the cylinder may be written

$$X = \frac{DA^2V^2}{2gc^2w^2}A\left(1-\frac{2cw}{A}\right)$$

Further since $p = \frac{Dv^2}{2g} = \frac{DA^2V^2}{2gc^2w^2}$ we have also,

$$X = p(A - 2\ cw)$$
$$= p(A - w) \text{ approximately.}$$

Thus, knowing the pressure in the pressure end of the recoil cylinder to obtain the reaction on the piston, we must multiply this pressure by the effective area of the piston minus the area of the recoil orifice.

(2) Throttling through a stationary orifice.

With a stationary orifice, the throttling

usually takes place between the recoil or brake and recuperator cylinders. The loss of head or pressure drop is mainly due to the sudden expansion of the flow from the orifice, though with a relatively long orifice the loss due to sudden contraction may become appreciable.

If

w = the area of the orifice (sq.ft)

A = the effective area of the recoil piston (sq.ft)

V = the velocity of recoil (ft/sec)

v = the velocity through the orifice (ft/sec)

c = contraction factor of the orifice.

W = the area of the channel leading away from the orifice. (sq.ft)

Then from Bernoulli's theorem, we have

$$\frac{p-p_a}{D} = h_t$$

where p = the pressure in the recoil cylinder.

p_a = the pressure in the recuperator.

h_f = total head lost due to throttling.

h_{fc} = loss of head due to contraction.

h_{fe} = loss of head due to expansion.

Now

$$h_f = h_{fc} + h_{fe}$$

Now $h_{fc} = \xi \dfrac{v^2}{2g}$ where ξ may be taken 0.35 to 0.5 and

and

$$h_{fe} = \frac{v^2}{2g}\left(1 - \frac{cw}{W}\right)^2 \quad \text{hence } h_f = \frac{v^2}{2g}\left[\left(1 - \frac{cw}{W}\right)^2 + \xi\right]$$

In recoil mechanisms W is usually made from 2.3 to 3.0 times w. Then, we have, if c is taken approximately = 0.65

$$\left(1 - \frac{w}{W}\right)^2 = 0.515 \text{ to } 0.614$$

For flow from an orifice into a large reservoir

$\frac{w}{W} \geqq 0$ and $(1 - \frac{w}{W})^2 \leqq 1$

Hence usually

$$[(1 - \frac{cw}{W})^2 + E] = 1 \text{ approximately,}$$

and

$$h_f = \frac{v^2}{2g} \quad \text{hence } p - p_a = \frac{D\,A^2V^2}{2gc^2w^2}$$ for the drop of pressure through

the orifice. The reaction on the recoil piston is,

$$X = pA = \frac{D\,A^2V^2}{2gc^2w^2} + p_aA$$

In recoil design, it is customary to measure areas in sq. inches and pressures in lbs/sq.in. Further the average specific gravity of the recoil oils used in our service may be taken at 0.849 and therefore the density D becomes, D = 62.5 × 0.849 (lbs/cu.ft).

The recoil throttling formulas become, therefore

(1) For throttling through a variable orifice in the piston or grooves in the cylinder walls:-

$$X = \delta\,\frac{K^2A^3V^2}{175\ w^2} \quad \text{(lbs)} \qquad w = \frac{KA^{\frac{3}{2}}V}{13.2}\sqrt{\frac{\delta}{X}} \quad \text{(sq.in)}$$

$$p = \frac{K^2A^2V^2}{175\ w^2} \quad \text{(lbs/sq.in);} \qquad w = \frac{KAV}{13.2\sqrt{p}} \quad \text{(sq.in)}$$

where $K = \frac{1}{c} = 1.6$ to 1.3 approx.

$$\delta = 1 - \frac{2cw}{A}: \qquad c = 0.6 \text{ to } 0.8 \text{ approx.}$$

(2) For throttling through stationary orifices:-

$$X = \delta'\,\frac{K^2A^3V^2}{175\ w^2} + p_aA \quad \text{(lbs)} \qquad w = \frac{KAV}{13.2\sqrt{p-p_a}} \quad \text{(sq.in)}$$

$$p - p_a = \frac{K^2 A^2 V^2}{175 w^2} \qquad \text{(lbs/sq.in)}$$

where K = 1.6 to 1.3 approx. $\delta = (1 - \frac{cw}{W})^2 + \xi$

VARIATION OF THE THROTTLING CONSTANT IN THE RECOIL We have seen the total braking on the recoil piston may be expressed, when throttling through a variable orifice in the piston or through grooves in the cylinder, as

$$X = \delta \frac{K^2 A^3 V^2}{175 w^2} \qquad \text{(lbs)}$$

and when throttling through a stationary orifice, as

$$X = \delta' \frac{K^2 A^3 V^2}{175 w^2} + p_a A \qquad \text{(lbs)}$$

where $\delta = 1 - \frac{2cw}{A}$ and $\delta' = (1 - \frac{cw}{W})^2 + \xi$

Since w varies throughout the recoil, δ and δ' must also necessarily vary in the recoil. Calculations with the omission of the term δ or δ' have been found slightly in error and this error has been ascribed to variations in the contraction factor of the orifice. The contraction factor may also vary but it seems more probable that the error is due to the omission of the term δ or δ'.

With stationary orifices $\frac{cw}{W}$ and ξ can very often be neglected and therefore the variation in the throttling constant can be neglected. With throttling through the piston or by grooves in the cylinders $\frac{2cw}{A}$ is small but not negligible hence with this type of throttling variations in the orifice are more marked.

For a preliminary design δ and δ' may be assumed equal to unity; but on recoil analysis and careful tests δ and its variation in the recoil should be taken into consideration.

CHAPTER VI

DYNAMICS OF RECOIL.

ELEMENTARY PRINCIPLES. The object of the recoil is to reduce greatly the stresses induced in the carriage. Without recoil, the reactions brought on the various parts of the carriage are direct functions of the maximum powder force, which would require a very massive carriage for guns of large caliber.

The mutual reactions created by the powder gases between the gun and the projectile is of very short duration compared with the time of recoil and for a rough approximation may be treated as an impulsive reaction. Neglecting the mass of the powder gases, we have $\int P dt = mv$ and $\int P dt = MV$. Therefore $mv = MV$, where m = mass of the projectile

M = mass of the recoiling parts

v = velocity of projectile

V = velocity of recoil

$\int P dt$ = impulsive reaction of the powder gases.

The momentum generated by the action of the powder gases in the projectile and gun is the same, as is immediately obvious from the principle of conservation of momentum. It is to be further noted that finite forces, as the resistance to recoil, can be neglected in the consideration of impulsive actions, and since the generated velocity of recoil acts for a differential time, the recoil displacement during the impulsive action can also be neglected.

The kinetic energy of the recoiling parts, after the impulsive action, is

$$A = \frac{1}{2} MV^2$$

Since $V = \frac{mv}{M}$, the recoil energy in terms of the

velocity of the projectile becomes, $A = \frac{m}{M}(\frac{1}{2}\, mv^2)$.
Hence the energy of recoil is but

$\frac{m}{M}$ of the energy of the projectile.

The total energy generated by the impulsive action of the powder gases, is, therefore

$$\frac{1}{2}\,(1 + \frac{m}{M})mv^2$$

Obviously the greater M, the smaller the energy of recoil.

The reaction R between the gun and mount for a recoil displacement b, is

$$R = \frac{\frac{1}{2}\, MV^2}{b}$$

or in terms of the velocity of the projectile

$$R = \frac{\frac{1}{2}\, m^2 v^2}{b.M} = \frac{m}{b.M}\,(\frac{1}{2}\, mv^2)$$

The reaction is thereby reduced proportionally to the increase of the recoiling mass M. Hence to reduce the recoil reaction we increase the recoiling mass M and the length of recoil b.

The dynamical relations for an elementary recoil analysis in terms of the relative velocity of the projectile with respect to the gun v_R can be readily obtained as follows:-

$v_R = v + V$ assuming V measured in the direction of recoil from the conservation of momentum

$MV = mv = m(v_R - V)$: hence $V = \frac{m\, v_R}{M + m}$

The energy of recoil is

$\frac{1}{2}\, MV^2 = \frac{1}{2}\, M(\frac{m}{M + m})^2 v_R^2$ and the recoil reaction

becomes

$$R = \frac{\frac{1}{2}\, MV^2}{b} = \frac{1}{2}\,\frac{M}{b}(\frac{m}{M + m})^2\, v_R^2$$

If the recoiling parts are brought to rest by friction alone, $R = u\, Mg$

$$\text{hence } b = \frac{1}{2}\frac{V^2}{ug} = \frac{v_R^2}{2ug}\left(\frac{m}{M + m}\right)^2$$

DOUBLE RECOIL SYSTEM:

When a gun is mounted on a movable mount as a car body or itself rolls along a plane, we have virtually a double recoil system, the upper recoil being between the gun and mount, and the lower between the mount and plane. As a first approximation we will neglect the resistance between the mount and plane as small compared with the upper recoil resistance. Let

M_R = mass of upper recoiling parts
M_c = mass of lower recoiling parts
m = mass of the projectile
v_o = the muzzle velocity of the projectile
V = the initial velocity of the recoiling parts
$\bar{v}$ = the velocity of combined recoil

Then, during the impulsive action, neglecting the mass of the projectile, we have,

for the projectile $\int_0^\tau P dt = mv_o$ (1)

for upper recoiling parts $\int_0^\tau P dt - \int_0^\tau R dt = MV$ (2)

Where R is the vertical reaction between the upper and lower recoiling parts.

Now R is a finite force, $\therefore \int_0^\tau R dt = 0$, if τ is very small. Further the displacement of the upper and lower recoiling parts inappreciable, since

$\int_0^\tau V dt = 0$ and $\int_0^\tau R dt = 0$ respectively

Hence, $mv_o = M_R V$ with no appreciable displacement of either the upper or lower recoiling parts and no momentum imparted to the lower recoiling parts. During the recoil, after the impulsive action, we have

<u>for the upper recoiling parts</u> $\int_0^T Rdt = M_R(V-\bar{v})$

<u>for the lower recoiling parts</u> $\int_0^T Rdt = M_C\bar{v}$

hence, the combined velocity of the system when the relative recoil between the upper and lower recoiling parts ceases, is

$$\bar{v} = \frac{M_R V}{M_R + M_C}$$

If the mutual recoil reaction R between upper and lower recoiling parts is made constant, then

$$R = M_C \frac{\bar{v}}{T} \quad \text{or} \quad T = \frac{M_C M_R}{M_R + M_C} \frac{V}{R}$$

where T is the time of the relative recoil. The relative displacement Z is,

$$Z = \left(\frac{V+\bar{v}}{2}\right) T - \frac{\bar{v}}{2} T = \frac{V}{2} T$$

Substituting for T, we have

$$Z = \frac{M_C M_R}{M_R + M_C} \frac{V^2}{2R}$$ <u>for the relative displacement</u>

The relative displacement can also be obtained from a consideration of the energy relations in the recoil. We have

$$R\left(Z + \frac{\bar{v}}{2} T\right) = \frac{1}{2} M_R (V^2 - \bar{v}^2)$$ <u>for the upper recoil-parts</u>

$$R\left(\frac{\bar{v}}{2} T\right) = \frac{1}{2} M_C v^2$$ <u>for the lower recoil-parts</u>

Subtracting:

$$RZ = \frac{1}{2} M_R (V^2 - v^2) - \frac{1}{2} M_C \bar{v}^2$$

$$= \frac{1}{2} M_R V^2 - \frac{1}{2}(M_R + M_C)\bar{v}^2$$

that is the energy of recoil, $\frac{1}{2} M_R V^2 = RZ + \frac{1}{2}(M_R + M_C)\bar{v}^2$

is dissipated in friction and throttling (RZ) and

remainder is the kinetic energy of the combined masses. Now since,

$$\bar{v} = \frac{M_R V}{M_R + M_c}$$

we have

$$RZ = \frac{1}{2} M_R V^2 - \frac{M_R^2}{(M_R+M_c)} \frac{V^2}{2}$$

$$= \frac{1}{2} \frac{M_R M_c}{M_R + M_c} V^2$$

Therefore as before, the relative displacement becomes

$$Z = \frac{M_R M_c}{M_R + M_c} \frac{V^2}{2R}$$

ELEMENTARY RELATIONS. During the travel of the projectile in the bore of the gun, neglecting for a rough approximation the mass of the powder gases, a mutual reaction is created by the powder gases between the gun and projectile, which generates equal momentum in both projectile and gun provided no extraneous forces are exerted on the gun. The resistance of the recoil brake is very small compared with the powder force, therefore its momentum effect is negligible. After the projectile leaves the bore, further expansion of the gases take place and the reaction due to the momentum generated in these gases causes an additional increment in momentum of the gun. This additional momenta is commonly known as the after effect of the powder gases.

Assuming free recoil of the gun, if

m = mass of projectile

M = mass of the gun or recoiling parts

P = total powder reaction

v = absolute velocity of projectile

V = absolute velocity of gun in the recoil

u = relative velocity of projectile in bore

then during the travel up the bore $\int P dt = mv = MV$ but $u = v + V$ for the relative velocity of the pro-

jectile, hence $m(u-V)=MV$ and the velocity of recoil becomes

$$V = \left(\frac{m}{m + M}\right)u = \frac{mv}{M}$$

Since m is small compared with M, we are not greatly in error in assuming $u = v$ in approximate calculations.

At the end of the travel of the projectile up the bore, we have

$$V_o = \left(\frac{m}{m + M}\right)u_o \text{ and } V_o = \frac{mv_o}{M}$$

After the projectile leaves the bore if P' = the reaction exerted by the gases, then

$$\int_{t_o}^{t_1} P'dt = M(V_f - V_o) = \bar{m}v'$$ where v' = the mean velocity of the gases "$\bar{m}$" after expansion. For a first approximation v' will be assumed a function of the muzzle velocity v_o and we will place $\bar{m}v' = cv_o\bar{m}$

Hence $MV_f = (m+c\bar{m})v_o$. For computations c will be taken equal to 2.3. The energy of free recoil becomes

$$E = \tfrac{1}{2} MV_f^2 = \tfrac{1}{2} M\left(\frac{m + cu}{M}\right)^2 v_o^2$$

hence

$$E = \tfrac{1}{2}\frac{(m+cu)^2 v_o^2}{M}$$

Now the recoil brake exerts a resistance R through a recoil displacement b, hence

$$Rb = \tfrac{1}{2}MV_f^2 \text{ roughly,}$$

and

$$R = \frac{(m+c\bar{m})^2 v_o^2}{2M.b}$$

The recoil reaction R is a measure of the stressed condition of the carriage and very often for a given carriage m, u, v_o and b may one or all be changed. To compare the recoil reactions, we have for the same gun,

$$\frac{R_1}{R_2} = \frac{(m_1+c\bar{m}_1)^2 v_{o1}^2 b_2}{(m_2+c\bar{m}_2)v_{o2}^2 b_1}$$

and for $R_1 = R_2 = R$, then $\dfrac{b_1}{b_2} = \dfrac{(m_1 + c\bar{m}_1)^2 \, v_{O_1}^2}{(m_2 + c\bar{m}_2)^2 \, v_{O_2}^2}$ where c = 2.3 approx., and for $b_1 = b_2 = b$, then

$$\frac{R_1}{R_2} = \frac{(m_1 + c\bar{m}_1)^2 \, v_{O_1}^2}{(m_2 + c\bar{m}_2)^2 \, v_{O_2}^2} \quad \text{where } c = 2.3 \text{ approx.}$$

These equations are important in order to estimate with a given change in the ballistics of a gun, the necessary change in either the recoil or recoil brake reaction.

The energy of recoil may be expressed as

$$E = \frac{m + cu}{M}\left[\frac{1}{2} (m + c\bar{m}) \, v_O^2 \right]$$

$$= \frac{m}{M} \left(\frac{1}{2} m v_O^2 \right) \quad \underline{\text{very roughly}}$$

$$= \frac{m}{M} \text{ (muzzle energy of the projectile) (approx.)}$$

Therefore, to decrease the recoil energy M should be made as large as possible. Since further

$$R = \frac{E}{b}$$

The recoil reaction varies inversely as the recoiling mass, and therefore to decrease R, M should be made large.

EFFECT OF POWDER GASES ON THE RECOIL. The effect of the powder gases on the recoil may be considered during two periods:- (1) while the projectile travels up the bore, (2) after the projectile leaves the bore and the expansion of the gases takes place. In either case an approximate assumption is necessary in order to represent the phenomena with sufficient simplicity.

During the travel of the shot up the bore it will be assumed that the gases expand in parallel lamina, and the motion of any differential lamina to be a linear function of the distance from the base of

the bore to the lamina in question, that is

$$v' = c_1 s + c_2 \quad \text{where } c_2 = -V \text{ and } c_1 = \frac{v+V}{u}$$

v = velocity of projectile
V = velocity of recoil
u = travel of projectile up the bore

hence with free recoil

$$mv + \Sigma \frac{\bar{m}}{u} v' = M_R V \quad \text{during the travel up the bore}$$

but

$$\Sigma \frac{\bar{m}}{u} v' = \frac{\bar{m}}{u} \int_0^u v' ds = \frac{\bar{m}(v-V)}{2}$$

The equation of momentum of the system during the travel up the bore becomes, therefore,

$$mv + \bar{m}\frac{(v-V)}{2} = MV \quad \text{or} \quad V = \frac{(m+0.5\bar{m})v}{M_R+0.5\bar{m}}$$

Further since the relative velocity of the projectile is

$$\frac{du}{dt} = v + V \quad \text{then, } (m+0.5\bar{m})\left(\frac{du}{dt} - V\right) = (M+0.5\bar{m})V$$

therefore

$$V = \frac{(m+0.5\bar{m})\frac{du}{dt}}{M_R+m+\bar{m}}$$

and for the displacement of recoil in terms of the relative displacement of the projectile,

$$X = \frac{(m+0.5\bar{m})}{M+m+\bar{m}}\ .$$

If

P = the reaction of the powder gases on the base of the projectile

P_b = the reaction of the powder gases on the base of the bore of the gun

then, for the powder gases, we have

$$P_b - P = \frac{\bar{m}}{2}\frac{d(v-V)}{dt} = \frac{\bar{m}}{2}\frac{dv}{dt} - \frac{\bar{m}}{2}\frac{dV}{dt} \qquad (1)$$

for the motion of the recoiling parts in free recoil,

$$P_b = M_R \frac{dV}{dt} \quad (2)$$

and for the motion of the projectile

$$P = m \frac{dv}{dt} \quad (3)$$

If the gun moves backwards a displacement X, while the projectile moves forward an absolute displacement x, then

$$X = \int V dt; \quad x = \int v dt \quad (4)$$

From (2) and (3) in (1),

$$M_R \frac{dV}{dt} - m \frac{dv}{dt} = \frac{\bar{m}}{2} \frac{dv}{dt} - \frac{\bar{m}}{2} \frac{dV}{dt}$$

hence

$$(M_R + 0.5\bar{m}) \frac{dV}{dt} = (m + 0.5\,\bar{m}) \frac{dv}{dt} \quad (5)$$

Integrating, we have as before,

$$(M_R + 0.5\bar{m})V = (m + 0.5\bar{m})v \quad (6)$$

and

$$(M_R + 0.5\bar{m})X = (m + 0.5\bar{m})x \quad (7)$$

For the relative displacement

$$u = \int_0^t (V+v)dt \quad \text{or} \quad du = (v+V)dt$$

$$dt = \frac{du}{V+v} = \frac{(M_R + 0.5\bar{m})du}{(M_R + m + \bar{m})\ v}$$

$$\therefore X = \int_0^u V dt = \int_0^u \left(\frac{m + 0.5\bar{m}}{M_R + m + \bar{m}}\right) du$$

hence

$$X = \frac{m + 0.5\bar{m}}{M_R + m + \bar{m}}$$ as was obtained by direct substitution of displacements.

With a <u>constant</u> powder pressure during the travel up the bore, the time of travel becomes,

$$t = \frac{2u_o}{v+V} = \frac{2u_o(M_R + 0.5\bar{m})}{(M_R + m + \bar{m})\ v} = \frac{(2M_R + \bar{m})\ u_o}{(M_R + m + \bar{m})\ v}$$

Actually since the powder reaction varies during the travel up the bore,

$$t = \int_0^{u_o} \frac{du}{v+V} = \frac{M_R+0.5\bar{m}}{M_R+m+\bar{m}} \int_0^{u_o} \frac{du}{v}$$

Since m and $\bar{m}$ are always small compared with M_R, we have

$$t = \int_0^{u_o} \frac{du}{v} \text{ very closely}$$

The relation between P_b and P may be obtained as follows:

$$P_b = \frac{\bar{m}}{2}\frac{d(v-V)}{dt} + P$$

$$= (m+\frac{\bar{m}}{2})\frac{dv}{dt} \quad \text{approximately}$$

hence $$\frac{P_b}{P} = \frac{(m+\frac{\bar{m}}{2})}{m} = (1 + 0.5\frac{\bar{m}}{m})$$

or

$$P_b = P(1 + 0.5\frac{\bar{m}}{m})$$

Since however the linear motion of the powder gases is an assumption, we have more accurately,

$$P_b = (m + B\bar{m})\frac{dv}{dt}$$ where for a first approximation $B = 0.5$

The mean powder pressure lies between P_b and P hence

$$P_m = (1 + B''\frac{\bar{m}}{m}) P$$ where for a first approximation $B'' = 0.3$

ELEMENTARY ENERGY RELATIONS. The Kinetic energy of the powder gases may also be considered a summation of the elementary energies of the differential lamina. Assuming the gases to move up the bore in parallel lamina, with the velocity of any lamina a linear function of the end velocities and neglecting the velocity of the gun as relatively

small compared with that of the projectile, we have, for the kinetic energy of the powder gases,

$$E_p = \frac{1}{2}\frac{\bar{m}}{u}\int_0^u v'^2 ds$$

where
- $\bar{m}$ = total mass of powder gas
- u = travel up bore of projectile
- v' = velocity of any given lamina
- s = distance from base of bore to lamina in question

but $v' = \frac{s}{u} v$

hence

$$E_p = \frac{1}{2}\left(\frac{\bar{m}}{3}\right)v^2$$

The Kinetic energy imparted to the recoiling parts is

$$E_R = \frac{1}{2}\frac{(m+0.5\bar{m})^2 v^2}{M}$$

Further if,

W = the potential energy of the gases at any instant

P_b = the total reaction exerted on the breech of the gun

P = the total reaction exerted on the base of the projectile

X = the displacement of the gun in the direction of its movement

x = the displacement of the gun in the direction of its movement

Q = heat lost in radiation

J = the mechanical equivalent of heat

then, the equation of energy of the powder gases becomes

$$-P_b dX - Pdx = d(E_p+W) + JdQ$$

that is the external work on the powder gas system goes into kinetic, potential or configuration energy and lost heat energy. The above equation may be written $-dW = P_b dX + Pdx + dE_p + JdQ$

Further since $P_b dX = d(\frac{1}{2}\frac{(m+0.5\bar{m})^2 v^2}{M})$

$$Pdx = d(\tfrac{1}{2} mv^2)$$

$$dE_p = d(\tfrac{1}{2}\tfrac{\bar{m}}{3} v^2)$$

We have,

$$-dW = d\left[\frac{1}{2}\left(\frac{(m+0.5\bar{m})^2}{M} + m + \frac{\bar{m}}{3}\right) v^2\right] + JdQ$$

The work done on the system may be represented by an equivalent force P_m acting through a distance corresponding to the travel of the projectile up the bore, then $-dW = P_m\,du + JdQ$ and since $du = dx$, very closely, we have

$$P_m = \left[\frac{(m+0.5\bar{m})^2}{M} + m + \frac{\bar{m}}{3}\right] v\,\frac{dv}{du}$$

Thus the equivalent mass of the system gun, projectile and powder gases, referred to the displacement up the bore is given by the expansion,

$$M_e = \frac{(m+0.5\bar{m})^2}{M} + m + \frac{\bar{m}}{3}$$

RECOIL AND BALLISTIC MEASUREMENTS. The recoil reaction, say, when the gun is mounted on a ballistic pendulum and the reaction of the projectile when fired into a ballistic pendulum, differ by the reaction caused by the expansion and consequent acceleration of the powder. Obviously the smaller the charge the more closely would the swings of these pendulums be alike.

BALLISTIC PENDULUM - GUN MOUNTED ON PENDULUM.

(a) When the powder charge is very small, we have an equal impulsive action on the projectile and gun.

If

d = the distance from the axis of rotation to

the center line of the bore.

M = the mass of the pendulum and gun combined.

k = radius of gyration about the axis of suspension.

θ = angle turned by the pendulum

h = distance from the center of gravity to the axis of suspension.

Then in consequence of the mutual impulse during the fire, $mv.d = Mk^2w$ and the initial angular velocity is, therefore,

$$w = \frac{mv.d}{Mk^2} \quad \text{(rad/sec)}$$

The subsequent motion is given by, $\frac{d^2\theta}{dt^2} = -\frac{gh}{k^2}\sin\theta$

Integrating,

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{2gh}{k^2}\cos\theta + c$$

when $\theta = 0$, $\cos\theta = 1$ and $\frac{d\theta}{dt} = w$

therefore

$$c = w^2 - \frac{2gh}{k^2}$$

and

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{2gh}{k^2}(\cos\theta - 1) + w^2$$

At the maximum swing $\left(\frac{d\theta}{dt}\right) = 0$, and $\theta = \theta_o$, hence

$$w^2 = \frac{2gh}{k^2}(1 - \cos\theta_o)$$

This is immediately evident from the equation of energy, since

$$\frac{Mk^2w^2}{2} = Mgh(1 - \cos\theta_o)$$

The chord of an arc radius "c" is $1 = 2c\sin\frac{\theta_o}{2}$

Further since, $1 - \cos\theta_o = 2\sin\frac{\theta_o}{2}$

$$w = \frac{2\sqrt{gh}}{k}\sin\frac{\theta_o}{2} = \frac{mv.d}{Mk^2}$$

$$\text{hence } v = \frac{M}{m}\frac{k}{d}\, 2 \sin\frac{\theta_o}{2}\sqrt{gh}$$

$$= \frac{M}{m}\frac{k}{d}\frac{l}{c}\sqrt{gh}$$

which means the velocity of the projectile approximately. The radius of gyration may readily be obtained experimentally by noting the time of swing.

(b) When the powder charge is comparable with the weight of the projectile, we have to consider the additional momentum generated by the powder gases.

Assuming the center of gravity of the powder mass to have a mean velocity equal to one-half the velocity of the projectile, we have

(1) during the travel up the bore,

$(m+\frac{\bar{m}}{2})\, v$ as the momentum generated in the gun.

(2) after the projectile leaves the bore we have an additional impulse p due to the expansion of the gases.

Hence the equation for the motion of the ballistic pendulum becomes,

$$d\left[(m+\frac{\bar{m}}{2})v+p\right] = Mk^2 w$$

$$\text{but } w = \frac{2\sqrt{gh}}{k}\sin\frac{\theta_o}{2} = \frac{1}{c}\,\frac{l\sqrt{gh}}{k}$$

$$\text{hence } (m+\frac{\bar{m}}{2})v+p = \frac{Mkl}{cd}\sqrt{gh}$$

If now we repeat the experiment with the powder gases as done in the experiments on the Ballistic Pendulum by Dr. Hutton, we have

$$\frac{\bar{m}}{2}v_o + p = \frac{Mkl_c}{cd}\sqrt{gh}$$

where obviously v_o is greater than v,

Subtracting, we have $mv + \frac{\bar{m}}{2}(v - v_o) = \frac{Mk(l - l_o)}{cd}\sqrt{gh}$

or $v - \frac{\bar{m}}{2m}(v_o - v) = \frac{M}{m}\frac{(l - l_o)k}{cd}\sqrt{gh}$

To account for the powder gases experimentally, Dr. Hutton proposed measuring with and without the projectile as follows:

$$mv + p' = \frac{Mkl}{cd}\sqrt{gh} \qquad \underline{\text{with the projectile}}$$

$$p' = \frac{Mkl_o}{cd}\sqrt{gh} \qquad \underline{\text{without the projectile}}$$

hence
$$v = \frac{M}{m}\frac{(l - l_o)k}{cd}\sqrt{gh}$$

The previous expression indicates this expression in error by the amount
$$\frac{\bar{m}}{2m}(v_o - v)$$

which for small charges is relatively small but for large charges may be appreciable and therefore cannot be neglected. As an approximation, however, in ordinary tests, the method of Dr. Hutton is sufficiently accurate, for the measurement of the velocity of the projectile.

BALLISTIC PENDULUM - IMPULSE OF PROJECTILE

The ballistic pendulum serves as a valuable mechanical means of measuring the velocity of the projectile though this method has been discarded in modern practice. The dynamics involved is worthy however of consideration in the general recoil problem.

The time of penetration is sufficiently short for no appreciable movement of the pendulum.

Let d = the perpendicular distance from the axis to the line of penetration of the pro-

jectile.

J = the distance from the axis to the position of the projectile when the penetration ceases.

B = the angle between "d" and "J"

Then, the impulsive moment of the projectile M_p equals the change in its angular momentum, hence $M_p = mv.d - mJ^2w$ and the corresponding reaction on the pendulum becomes $M_p = Mk^2w$. Therefore $mvd = (mk^2+mJ^2)w$ or $mvJ \cos B = (Mk^2+mJ^2)w$. The initial energy of the system consisting of the pendulum and projectile is, therefore

$$\tfrac{1}{2}(Mk^2 + mJ^2)\,w^2$$

and the work done by the weights in the movement to the maximu swing, becomes, $Mgh(1-\cos\theta)+mgJ[\cos B-\cos(\theta - B)]$ hence, from the principle of energy, we have,

$$\tfrac{1}{2}(Mk^2+mJ^2)w^2=Mgh(1-\cos\theta)+mgJ[\cos B-\cos(\theta-B)]$$

If $B = 0$, the equations reduce to $mvJ = (Mk^2+mJ^2)w$

$$(Mk^2+mJ^2)w^2=2(Mgh+mgJ)(1-\cos\theta_o)$$

Combining these equations and noting that

$1 - \cos\theta = 2\sin^2\frac{\theta_o}{2}$, we have, for the initial velocity of impact for the projectile,

$$v = \frac{2}{mJ}\left[\sqrt{(Mk^2 + mJ^2)(Mh + mJ)g}\,\right]\sin\frac{\theta_o}{2}$$

GENERAL THEORY OF RECOIL. In the preceeding paragraphs the theory of recoil was greatly simplified by assuming the powder period to be of such short duration as to be in the nature of an impulsive action, and therefore the momentum of recoil being generated practically instantaneously. In the theory of impulsive forces, we may neglect finite forces such as the resistance to recoil since the time of action is negligible. Further the displacement in an impulsive action is entirely negligible. This method gives fairly accurate results for long recoil

but when the recoil is shortened the results by this method of computation are only very approximate.

Fortunately due to considerable progress made in interior ballistics of late, the powder reaction can be determined as a function of time and displacement up the bore. It, therefore, becomes a finite force and the recoil problem during the powder period can be treated with a considerable degree of accuracy.

Let P_b = the total powder reaction on the breech in lbs. Its line of action is necessarily along the axis of the bore.

B = the total braking due to the hydraulic and recuperator pulls.

R = the total friction, (guide and packing frictions) in lbs.

K = the total resistance to recoil.

M_r = the mass of the recoiling parts

W_r = the weight of the recoiling mass in lbs.

X = the displacement of the recoiling mass from battery in the direction of the guides.

ϕ = the angle of elevation of the gun

a = the angle of the guides constraining the recoiling mass with respect to the horizontal.

From the theory of energy, we have the fundamental principle:

<u>The work done on the system consisting of the recoiling parts by the powder gases must equal the work done on the system by the total resistance to recoil for the entire recoil, since the energy of the system ar the beginning and end of recoil is zero.</u>

From this theorem we may prove that with a resistance to recoil action throughout the powder period, the energy which the powder imparts to the recoiling mass when free is always greater than the energy which must be developed by the brake in the recoil. The greater the resistance to recoil during the powder period the greater this deviation.

In the following proof the time effect of the powder gases during free and constrained recoil is assumed the same or, in other words, the powder reaction is regarded the same for any given time whether the recoiling mass is contrained or free. Theoretically of course due to the slightly different motions in the two cases, the motion of the powder gases themselves will be slightly different and therefore a slightly different reaction on the breech clock in the two cases. Since, however, the difference in motions is so small and the powder reaction so great, we may entirely neglect this fact and assume the powder force to be entirely a function of time and quite independent of the slightly different motion in constrained and free recoil.

Supposing the gun to recoil along the axis of the bore as is usually the case, the total resistance to recoil evidently may be expressed as: $K = B+R-W_r \sin \emptyset$.

Therefore, the equation of motion for the recoiling mass for constrained recoil, becomes,

$P_b - K = m \frac{dV}{dt}$ and for free recoil, we have $P_b = m_r \frac{dV_f}{dt}$

Integrating for any given time, evidently, $V < V_f$

The work done by the powder for contrained recoil is therefore less than with free recoil, since

$\int_0^{t_1} P_b V \, dt < \int_0^{t_1} P_b V_f \, dt$ where t_1 = the total time of the powder period. Now the work done by the brake must equal the work done by the powder gases in constrained recoil, hence,

$$\int_0^b K dx = \int_0^{t_1} P_b V \, dt$$

and

$$\int_0^b K dx \quad \int_0^{t_1} P_b V_f \, dt$$

but $\int_0^{t_1} P_b V_f dt = \frac{1}{2} m_r V_f^2$ therefore $\int_0^b K dx < \frac{1}{2} m_r V_f^2$

that is, the braking energy or rather the work done by the resistance to recoil provided the braking is effective during the powder period, is always less than the free energy of recoil. When, however, no braking resistance acts during the powder period, the work done by the resistance to recoil or braking energy must equal the free energy of recoil. Therefore, for a given length of recoil, the recoil reaction is reduced by maintaining a resistance during the powder period in a twofold way:

(1) due to the fact that gun recoils over a greater distance, (i. e. the displacement during the retardation and in addition, the displacement during the powder period),

(2) due to the fact that the braking energy is always less than the free energy of recoil.

In the design of a recoil system it is therefore, highly desirable to maintain a large resistance to recoil during the powder period and thus effectively to reduce the required braking and the consequent stresses set up in the carriage, as well as to give better stability to mobile mounts.

GENERAL EQUATIONS OF RECOIL.

(1) When the direction of recoil is not along the axis of the bore. Consider the recoiling parts to be constrained along guides or an inclined plane making an angle "a" with the horizontal, and the axis of the bore to make an angle Ø with the horizontal.

Neglecting the reaction of the projectile normal to the bore, as small compared with the other reactions, we have for the equation of motion for the recoiling mass.

$$P_b \cos(\phi+a) - B - R - W_r \sin a = m_r \frac{d^2x}{dt^2} \qquad (1)$$

hence

$$P_b \cos(\phi+a)-B-R-W_r \sin a)\,dt = m_r\,dv$$

and

$$\int P_b \cos(\phi+a)dt - \int(B+R+W_r \sin a)dt = m_r v$$

but the powder force is measured by the rate of change of momentum imparted to the recoiling mass when free, that is

$$P_b = m_r \frac{dV_f}{dt}$$

hence $P_b \cos(\phi+a)dt = m_r \cos(\phi+a)d\,V_f$

Substituting in the above equation, we have

$$m_r V_f \cos(\phi+a) = \int(B+R+W_r \sin a)dt = m_r V \qquad (2)$$

When the resistance to recoil is constant,

$K = B + R + W_r \sin a$ = a constant, and we have

$$V_f \cos(\phi + a) - \frac{K}{m_r} t = V \qquad (3)$$

Integrating again, we have,

$$\int V_f \cos(\phi + a)\,dt - \frac{Kt^2}{2m_r} = X$$

which gives the displacement from battery of the recoil during the powder period, but

$\int V_f \cos(\phi + a)\,dt = E \cos(\phi + a)$ which is the component displacement for free recoil in the direction of recoil.

The constrained recoil at the end of the powder period, becomes

$$X_1 = E' = E \cos(\phi+a) - \frac{KT^2}{2m_r} \qquad (4)$$

and the corresponding velocity at the end of the powder period, becomes,

$$V_r = V_{f\,max.}\cos(\phi+a) - \frac{KT}{m_r} \qquad (5)$$

where T is the time of the powder period.

From the energy equation in the motion from the end of the powder period to the end of recoil, we

have $\frac{1}{2} m_r v_r = K(b-x_1)$ hence

$$\frac{1}{2} m_r [V_f \cos(\emptyset+a) - \frac{KT}{m_r}]^2 = Kb-K[E \cos(\emptyset+a) - \frac{KT^2}{2m_r}] \quad 6)$$

Expanding and simplifying, we have

$$K[b-E \cos(\emptyset+a)+V_fT \cos(\emptyset+a) = \frac{1}{2}m_rV_f^2 \cos^2(\emptyset+a)]$$

hence

$$K = \frac{\frac{1}{2}m_rV_f^2 \cos^2(\emptyset+a)}{b-(E-V_fT)\cos(\emptyset+a)} \qquad (7)$$

or in terms of the component reactions,

$$B+R+W_r \sin a = \frac{\frac{1}{2}m_rV_f^2 \cos^2(\emptyset+a)}{b-(E-V_fT)\cos(\emptyset+a)} \qquad (7')$$

where

$V_f = \dfrac{wv_o + 4700\,\bar{w}}{w_r}$ from the principle of linear momentum.

E = total free movement of gun during powder period.

T = total time of powder period.

To deduce E and T we proceed as follows: (See Chapter II) Calculate

$$b = u_o[(\frac{27}{16}\frac{p_m}{p_e} - 1) \pm \sqrt{(1 - \frac{27}{16}\frac{p_m}{p_e})^2 - 1}\,]$$

where p_m = max. powder pressure X area of bore

$$p_e = \frac{wv_o^2}{64.4\,u_o}$$

and also,

$$p_{ob} = \frac{27}{4} b^2 \frac{u}{(b+u)^3} p_{bm} \; : \quad (p_{bm} = 1.12\, p_m):$$

then compute

$$V_f = \frac{wv_o + 4700\,\bar{w}}{w_r}; \quad V = \frac{(w + \frac{\bar{w}}{2})v_o}{w_r}$$

where w = weight of projectile

$\bar{w}$ = weight of charge

w_r = weight of recoiling parts

v_o = muzzle velocity

The time of the travel of the projectile up the bore and the time during the expansion of the powder gases are respectively:

$$t_o = \frac{b}{a}\left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right) \quad t_{1o} = \frac{2(V_{f1}-V_{fo})}{p_{ob}} \frac{w_r}{32.2}$$

$$= \frac{3}{2} \frac{u_o}{v_o} \text{ approx.}$$

Therefore the powder period, becomes $T = t_o + t_{1o}$

The free recoil displacement during the travel up the bore, and during the expansion of the gases are respectively:

$$X_{fo} = \frac{u_o(w+0.5\bar{w})}{w_r+w+\bar{w}} \; : \quad X_{f1o} = \frac{p_{ob}}{w_r} g \frac{(t_1-t_o)^2}{3} + V_{fo}(t_1-t_o)$$

Therefore, the total free movement of gun during the powder period, becomes, $E = X_{fo} + X_{f1o}$

NOTE: In the above and further formulae the units employed are :

displacement in feet
velocity in feet per second
force in pounds
mass in pound units

With a void in the recoil cylinder during part of the powder period, equation (7) becomes slightly modified.

Let S = length of void in recoil cylinder

t_s = time of free recoil to end of void

Neglecting,

$R+W_r \sin a$ as small compared with B, we find $K = 0$, until distance S is reached in the recoil.

Therefore we have

$$X_1 = E \cos (\emptyset + a) - \frac{K(T-t_s)^2}{2m_r} \quad (8)$$

$$V_r = V_{f\ max} \cos(\emptyset+d) - \frac{K(T-t_s)}{m_r} \quad (9)$$

where T = time of total powder period. Substituting (8) and (9) in the energy equation,

$$\frac{1}{2} m_r v_r^2 = K(b-x_1)$$

and simplifying, we have

$$K = \frac{\frac{1}{2} m_r V_f^2 \cos^2(\emptyset+a)}{b-[E-V_f(T-t_s)]\cos(\emptyset+d)} \quad (10)$$

To evaluate t_s, the time of recoil with void, we have

$$t_s = \frac{b}{a}(2.3 \log \frac{2u'}{b} + \frac{u'}{b} + 2) \quad (11)$$ See Chapter II.

where

$$u' = \frac{w_r S}{(w + \frac{w}{2})\cos(\emptyset+a)} \quad (12)$$ S being the length of void.

$$b = u_o[(\frac{27}{16}\frac{p_m}{p_e} - 1) \pm \sqrt{(1- \frac{27}{16}\frac{p_m}{p_e})^2 - 1})] \quad (13)$$ See Chapter III

$$a = \frac{(b+u_o)v_o}{u_o} \quad (14)$$

v_o = muzzle velocity in feet

u_o = total displacement up bore in feet

p_m = max. powder pressure, lbs. per sq. in.

$$p_e = \frac{wv_o^2}{64.4\ u_o A_b} \quad (15)$$ mean powder pressure, lbs.per sq. in.

A_b = area of bore of gun.

If, however, the length of void corresponds to a displacement greater than the recoil displacement for the projectile to travel up the bore of the gun, we have,

$$\left.\begin{array}{l} t_s = \frac{b}{a}\left(2.3 \log \frac{2u_o}{b} + \frac{u_o}{b} + 2\right) + t_s' \\ \text{or approx.} \\ t_s = \frac{3}{2}\frac{u_o}{v_o} + t_s' \end{array}\right\} \quad (16)$$

where t_s' is obtained from the solution of the cubic equation,

$$\left[\frac{P_{ob}}{m_r}\left(\frac{t_s'}{2} - \frac{t_s'^3}{6t_{to}}\right) + V_{fo}\right] \cos(\emptyset + a) - X_s' = 0 \quad (17)$$

where

$$V_{fo} = \frac{\left(w + \frac{\bar{w}}{2}\right)v_o}{w_r} \quad : \quad V_{f'} = \frac{wv_o + 4700\,\bar{w}}{w_r}$$

$$t_{to} = \frac{2(V_{f'} - V_{fo})}{P_{ob}} \; \frac{w_r}{32.2}$$

$$\text{and } X_s' = S - \frac{\left(w + \frac{\bar{w}}{2}\right)}{w_r} u_o \cos(\emptyset + a) \quad (18)$$

$$\text{also } P_{ob} = \frac{27}{4} b^2 \frac{u}{(b+u)^3} \quad (1.12\, p_m A_b) \quad (19)$$

Powder reaction on breech when shot leaves muzzle.

<u>CONSTRAINED VELOCITY OF RECOIL:</u>

(1) During powder pressure period. Knowing K from the previous formulae, the constrained velocity of recoil may be computed from the

free velocity curve as follows:

From equation (3) we have, $V = V_f \cos(\emptyset + a) - \frac{Kt}{m_r}$

and the corresponding displacement

$$X = \int_0^t V_f \cos(\emptyset + a)\, dt - \frac{Kt^2}{2m_r}$$

$$= X_f \cos(\emptyset + a) - \frac{Kt^2}{2m_r}$$

Thus we see the free velocity curve of recoil both against time and displacement of free recoil is required in order to compute the constrained velocity curve.

The free velocity curve during the powder period is divided into two periods, (1°) the velocity of free recoil while the shot travels up the bore, and (2°) the velocity of free recoil during the expansion of the powder gases after the shot has left the muzzle.

Leduc's formula gives us a means of computing (1°) while Vallier's hypothesis serves for the computation of (2°).

From Leduc's formula, we have, during (1°) of the powder period,

$$v = \frac{au}{b + u} \qquad (20)$$

$$t = \frac{b}{a}\left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2\right) \qquad (21)$$

where

u = travel up the bore in feet

u_o = travel up the bore to muzzle

v = corresponding velocity of projectile in the bore of the gun (feet per sec)

v_o = muzzle velocity of projectile

t = corresponding time of the travel in seconds.

$$b = u_o\left[\left(\frac{27}{16}\frac{p_m}{p_e} - 1\right) \pm \sqrt{\left(1 - \frac{27}{16}\frac{p_m}{p_e}\right)^2 - 1}\,\right]$$

p_m = max. powder reaction on base of projectile

$$p_e = \frac{wv_o^2}{64.4\ u_o} = \text{mean reaction on base of projectile during travel up bore.}$$

$$a = (b+u_o)\frac{v_o}{u_o}$$

Further from elementary dynamics, (see Chapter II)

$$V_f = \frac{(w+\frac{\bar{w}}{2})v}{w_r} \qquad (22)$$

$$X_f = \frac{(w+\frac{\bar{w}}{2})u}{w_r+w+\bar{w}}$$

or approx.

$$X_f = \frac{(w+\frac{\bar{w}}{2})u}{w_r} \qquad (23)$$

where w = weight of projectile in lbs.
$\bar{w}$ = weight of powder charge in lbs.
w_r = weight of recoiling mass in lbs.

The procedure therefore, to compute the free velocity curve against time and displacement during period (1°) is as follows

(a) Compute b and from it a,
(b) For various displacement up the bore: compute v and t. (Equation 20 and 21).
(c) Then from equations (22) and (23), compute V_f and X_f.

Arrange the data in a table with corresponding values of V_f, X_f and t.

From these values the constrained velocity curve during (1^o) may be computed from equations (3) and (4). From Vallier's hypothesis, we have, during (2^o) of the powder period, for the total pressure on the breech
$P_b = P_{ob} - C(t - t_o)$ (Valliers' hypothesis)

where

$$C = \frac{P_{ob} - P_b}{t - t_o} = \frac{P_{ob}}{t_1 - t_o} = \frac{P^2_{ob}}{2(V_{f'} - V_{fo})m_r}$$

hence

$$P_b = P_{ob} - \frac{P^2_{ob}}{2m_r(V_{f'} - V_{fo})} \quad (t - t_o) \qquad (24)$$

Now, from elementary dynamics, the change of momentum along the axis of the bore, becomes,

$$\int_{t_o}^{t} P_b \, dt = m_r(V_f - V_{fo}) \qquad (25)$$

Substituting (24) in (25) and integrating, we have

$$P_{ob}(t-t_o) - \frac{P^2_{ob}(t-t_o)^2}{4m_r(V_{f'} - V_{fo})} = m_rV_f - m_rV_{fo}$$ and simplifying, we

have, for the free velocity of recoil,

$$V_f = V_{fo} + \frac{P_{ob}}{m_r}(t-t_o)[1 - \frac{P_{ob}(t-t_o)}{4m_r(V_{f'} - V_{fo})}] \qquad (26)$$

The corresponding displacement of free recoil, along the axis of the bore,

$$X_f = X_{fo} + \int_{t_o}^{t} V_f \, dt \qquad (27)$$

where

$$X_{fo} = \frac{w + \frac{\bar{w}}{2}}{w_r} u_o$$ u_o = total travel up the bore in feet.

$$X_f = X_{fo} + \int_{t_o}^{t} V_{fo} dt + \frac{P_{ob}}{m_r}\int_{t_o}^{t}(t-t_o)dt - \frac{P^2_{ob}}{4m^2_r(V_{f'} - V_{fo})}\int_{t_o}^{t}(t-t_o)^2 dt$$

$$= X_{fo} + X_{fo}(t-t_o) + \frac{P_{ob}}{2m_r}(t-t_o)^2 - \frac{P_{ob}^2}{12m_r^2(V_f'-V_{fo})}(t-t_o)^3$$

Simplifying, the displacement of free recoil for time t, becomes,

$$X_f = X_{fo} + \left\{ V_{fo} + \frac{P_{ob}}{2m_r}\left[(t-t_o) - \frac{(t-t_o)^2}{6m_r(V_f'-V_{fo})}\right]\right\}(t-t_o) \quad (28)$$

The following initial values and constants are to be substituted in equations (27) and (28).

$$V_{fo} = \frac{w + \frac{\bar{w}}{2}}{w_r} v_o \qquad X_{fo} = \frac{w + \frac{\bar{w}}{2}}{w_r} u_o$$

$$V_f' = \frac{wv_o + 4700\,\bar{w}}{w_r}$$

$$t_o = \frac{b}{a}\left(2.3 \log \frac{2u_o}{b} + \frac{u_o}{b} + 2\right)$$

$$= \frac{3}{2}\frac{u_o}{v_o} \text{ approximately.}$$

$$b = u_o\left[\left(\frac{27}{16}\frac{P_m}{P_e} - 1\right) \pm \sqrt{\left(1 - \frac{27}{16}\frac{P_m}{P_e}\right)^2 - 1}\,\right]$$

$$a = (b + u_o)\frac{v_o}{u_o}$$

P_m = max. powder reaction on base of projectile.

$P_e = \dfrac{wv_o^2}{64.4\,u_o}$ = mean reaction on base of projectile during travel up bore.

$P_{ob} = \dfrac{27}{4} b^2 \dfrac{u}{(b+u)^3}\, 1.12\, P_m$ = reaction on breech of gun when the shot leaves the muzzle.

The procedure, therefore, to compute the free

velocity curve against time and displacement during period (2°) is as follows:

(a) Compute P_m, P_e, and then b and a as before.

(b) Compute P_{ob}, t_o, $(V_{f'}-V_{fo}$ and X_{fo}.

(c) Then from arbitrary time intervals between $t_1 = T$ and to compute from equations (26) and (28) V_f and X_f

Arrange the data in a continued table as in (1°) with corresponding values of V_f, X_f and t.

From these values the constrained velocity curve during (2°) may be computed from equations (3) and (4).

MAXIMUM VELOCITY OF CONSTRAINED RECOIL:

The condition of maximum velocity of constrained recoil is when the powder reaction exactly balances the resistance to recoil, since before this condition the recoiling mass is accelerated and immediately after it is retarded.

Hence $P_b \cos (\emptyset + a) - K = 0$

$$P_b = P_{ob}(1 - \frac{t - t_o}{T - t_o})$$

$$= P_{ob} [1 - \frac{P_{ob}}{2m_r(V_{f'} - V_{fo})} (t-t_o)] \quad (29)$$

Hence the time at the maximum velocity of constrained recoil, is obtained from either of the following equations:-

$$P_{ob}(1 - \frac{t_m-t_o}{T-t_o}) \cos(\emptyset+a) = P_{ob}[1 - \frac{P_{ob}(t_m-t_o)}{2m_r(V_{f'}-V_{fo})} \cos(\emptyset+a)] = K$$

Solving for t_m, we have

$$t_m = T - \frac{K(T-t_o)}{P_{ob} \cos(\emptyset+a)} \quad (30)$$

$$\text{or } t_m = \frac{2m(V_{f'}-V_{fo})[P_{ob}\cos(\emptyset+a) - K]}{P_{ob}\cos\emptyset + a} + P_{ob}t_o \quad (30')$$

Substituting t_m in (26) and (28), we have,

$$V_{fm} = V_{fo} + \frac{P_{ob}}{m_r}(t_m-t_o)\left[1 - \frac{P_{ob}(t_m-t_o)}{4m_r(V_{f'}-V_{fo})}\right] \quad (31)$$

and

$$X_{fm} = X_{fo} + \left\{V_{fo} + \frac{P_{ob}}{2m_r}[t_m-t_o)\frac{(t_m-t_o)^2}{6m_r(V_{f'}-V_{fo})}](t_m-t_o)\right\} \quad (32)$$

where V_{fm} and X_{fm} are the free velocity and displacement corresponding to the maximum constrained velocity of recoil.

RECAPITULATION OF FORMULAE FOR PRINCIPLE PERIODS DURING POWDER PRESSURE PERIOD.

In the constrained velocity curve during the powder period, we have the following important points:

(a) Velocity and displacement of the recoil when the shot leaves the muzzle.

(b) Maximum velocity and its corresponding displacement of recoil and time.

(c) Velocity time and corresponding displacement at end of the powder period.

Given data:

w_r = wt. of recoiling parts.

v_o = muzzle velocity.

w = weight of projectile.

$\bar{w}$ = weight of powder charge.

u_o = total travel of shot up bore.

P_m = max. powder reaction on base of shot.

P_b = max. intensity of powder pressure assumed X area of bore.

b = length of recoil.

INTERIOR BALLISTIC CONSTANTS REQUIRED FOR VELOCITY CURVE:

P_e = mean average powder reaction on base of shot = $\frac{wv_o^2}{2gv_o}$

B = twice abcissa of max. pressure,

$$= u_o[(\frac{27}{16}\frac{P_m}{P_e} - 1] \pm \sqrt{(1 - \frac{27}{16}\frac{P_m}{P_e})^2 - 1}$$

$$a = \frac{B + u_o}{u_o} v_o$$

$V_{f'}$ = max. velocity of free recoil

$$= \frac{wv_o + 4700\,\bar{w}}{w_r}$$

V_{fo} = velocity of free recoil - shot leaves muzzle

$$= \frac{w + 0.5\,\bar{w}}{w_r}$$

P_{ob} = total pressure on breech when shot leaves muzzle.

$$= \frac{27}{4} B^2 \frac{u_o}{(B+u_o)^3} P_{bm}\ ;\ (P_{bm} = 1.12\ P_m)$$

t_o = time of recoil while shot travels to muzzle

$$= \frac{B}{a}\ (2.3 \log \frac{2u}{B} + \frac{v}{B} + 2) = \frac{3}{2}\frac{u_o}{v_o} \text{ approx.}$$

t_{1o} = time during the expansion of gases after shot leaves muzzle.

$$= \frac{2(V_{f'} - V_{fo})}{P_{ob}} \frac{w_r}{g}$$

$t_1 = T$ = time for total powder period

$= t_0 + t_{10}$

X_{fo} = free movement of gun while shot travels to muzzle

$$= \frac{u_0(w+0.5\,\bar{w})}{w_r + w + \bar{w}} = \frac{w+0.5\,\bar{w}}{w_r} u_0 \quad \text{approx.}$$

$X_{f'o}$ = free movement of gun during expansion of powder gases,

$$= \frac{P_{ob}}{w_r} g \frac{(t_1-t_0)^2}{3} + V_{fo}(t_1-t_0)$$

Total free movement of gun during powder period

$$X_{f'} = E = X_{fo} + X_{f'o}$$

K = resistance to recoil: $\varnothing$ = angle of elevation: a = angle of plane of guides with horizontal,

$$m_r = \frac{w_r}{g}.$$

$$= \frac{\frac{1}{2} m_r V_{f'}^2 \cos^2(\varnothing+a)}{b-(E-V_{f'}T)\cos(\varnothing+a)}$$

CONSTRAINED VELOCITY AND DISPLACEMENTS AT PERIODS (a),(b) and (c).

At Period (a):

V_0 and X_0 = the constrained velocity and displacement in recoil for period (a) when shot leaves the muzzle.

$$V_0 = V_{fo}\cos(\varnothing+a) - \frac{Kt_0}{m_r}$$

$$X_0 = X_{fo}\cos(\varnothing+a) - \frac{Kt_0^2}{2m_r}$$

At Period (b):

t_m = time at max. velocity of constrained recoil.

$$= T - \frac{K(T-t_o)}{P_{ob}\cos(\emptyset+a)}$$

V_{fm} and X_{fm} = velocity and displacement of free recoil at the instant of maximum velocity of constrained recoil.

$$V_{fm} = V_{fo} + \frac{P_{ob}}{m_r}(t_m-t_o)[1 - \frac{P_{ob}(t_m-t_o)}{4m(V_{fo}'-V_{fo})}]$$

$$X_{fm} = X_{fo} + [V_{fo} + \frac{P_{ob}}{2m_r}(t_m-t_o) - \frac{(t_m-t_o)^2}{6m_r(V_{f'}-V_{fo})}](t_m-t_o)$$

V_m and X_m = maximum constrained velocity and corresponding displacement of recoil.

$$V_m = V_{fm}\cos(\emptyset + a) - \frac{Kt_m}{m_r}$$

$$X_m = X_{fm}\cos(\emptyset+a) - \frac{Kt_m^2}{2m_r}$$

At Period (c):

$V_1 = V_r$ = constrained velocity of recoil at end of powder period.

$X_1 = E_r$ = corresponding displacement of constrained recoil at end of powder period.

$$V_1 = V_r = V_{f'}\cos(\emptyset+a) - \frac{Kt_1}{m_r}$$

$$X_1 = E_r = X_{f'}\cos(\emptyset+a) = \frac{Kt_1^2}{2m_r}$$

UNITS TO BE EMPLOYED IN THE ABOVE AND FURTHER FORMULAE:

	BRITISH SYSTEM GRAVITATIONAL UNITS.	METRIC SYSTEM GRAVITATIONAL UNITS.	METRIC SYSTEM GRAVITATIONAL UNITS.
Displacement	in feet = ft.	in meters=m	in centimeters= cm
Velocity	in feet per sec.=ft/sec.	in meters per sec.= m/sec.	in centimeters per sec = cm/sec.
Force	pounds = lbs.	Kilograms = kg.	Kilograms = kg.
Pressure Intensity	lbs.sq.in.	Kg.per sq. cm.	Kg.per sq. cm.
Pressure Area	Sq.inches	Sq.cm.	Sq.cm.
Mass	Lbs/g (g=32.2)	Kgs/g (g=9.81)	Kgs/g g= 981
Time	Seconds =Sec.	Seconds=Sec.	Seconds = Sec.

CONSTRAINED VELOCITY CURVE:

(2) During Retardation Period of Recoil.

After the powder period the recoiling mass is brought to rest by the resistance to recoil. The recoiling mass then reaches the extreme out of battery position.

At the beginning of the 2° period of recoil, the recoiling mass has an initial velocity $V_1 = V_r$, and an initial displacement from battery $X_1 = E_r$.

From the equation of motion, we have $K = - m_r V \frac{dV}{dX}$

Integrating, between the limits X_1 to any given displacement X, and between corresponding velocity V_1 to V_X we have

$$\int_{X_1}^{X} K\, dX = - m_r \int_{V_1}^{V_X} V\, dV \qquad (33)$$

Hence,

$K(X-X_1) = \frac{m_r(V_1^2 - V_X^2)}{2}$ which is the equation of energy during the retardation period of the recoil. Hence

$$V_X = \sqrt{V_1^2 - \frac{2K(X-X_1)}{m_r}} \qquad (34)$$

A simpler and more direct form for computing the constrained velocity during the 2° period of recoil is as follows:

We have, as before $K\, d_X = - m_r\, V\, d\, V$

Integrating between the limits X and b in the displacement and V_X and o in the velocity, we have

$$K(b-X) = \frac{m_r V_X^2}{2} \qquad (35)$$

Hence $V_X = \sqrt{\frac{2K}{m_r}(b-X)}$ (36)

showing that the velocity during the retardation period is a parabolic function of the displacement.

<u>It is to be especially noted that a characteristic of a constant resistance to recoil is a parabolic function of velocity against displacement.</u>

GENERAL EQUATIONS OF RECOIL CONTINUED.- VARIABLE RESISTANCE TO RECOIL.

In the previous formulae the resistance to recoil was assumed constant throughout the recoil. It is however often desirable for stability to decrease the resistance to re-

coil in the out of battery position and thus partially compensate for the decreased stability due to the moment effect caused by the overhang of the recoiling mass in the out of battery position.

With a variable resistance to recoil it is customary to maintain a constant resistance during the powder period and thence decrease the resistance proportional to the displacement to the out of battery position, with a given arbitrary slope "m". See Chapter III.

Let K_o = the constant resistance during the powder period.

V_1 and V_r = the velocity of constrained recoil at the end of the powder period.

b = total length of recoil.

Then the equation of the resistance to recoil against displacement of recoil becomes,

$$\left.\begin{array}{ll} K_o = \text{constant}, & \text{from } 0 \text{ to } X_1 \text{ or } E_r \\ K = K_o - m(X-X_1) & \text{from } X_1 \text{ to } b \end{array}\right\} \quad (37)$$

Further,

$$V_1 = V_r = V_f' \cos(\phi+a) - \frac{K_oT}{m_r} \quad (38)$$

$$X_1 = E_r = E \cos(\phi+a) - \frac{K_oT^2}{2m_r} \quad (39)$$

Now from the equation of motion of the recoiling parts during the retardation, we have, $K\,dX = -\,m_r V\,d_V$

Integrating between limits, X_1 and b: and V_1 and 0,

$$\int_{X_1}^{b} K\,dX = m_r \int_{0}^{V_1} V\,d\,V$$

Hence

$$\int_{X_1}^{b} [K_o - m(X-X_1)]\,dX = \frac{m_rV_1^2}{2}$$

Integrating, we have for the energy equation,

$$K_o(b-X_1) - \frac{m(b-X_1)^2}{2} = \frac{m_r V_1^2}{2} \qquad (40)$$

Substituting (38) and (39) in (40) and neglecting the term $\frac{m}{2}\frac{K_o^2 T^4}{4}$ in the expansion as small, we have

$$K_o = \frac{\left\{\frac{m_r V_f^2{}'}{2}\cos^2(\emptyset+a) + \frac{m}{2}\left[b-E\cos(\emptyset+a)\right]^2\right\}}{b-E\cos(\emptyset+a) + V_f\cos(\emptyset+a)\,T - \frac{m}{2}\frac{T^2}{m_r}\left[b-E\cos(\emptyset+a)\right]} \qquad (41)$$

Thus from the ballistic constants E and T_1 together with the length of recoil, maximum free velocity of recoil and any given arbitrary slope "m", the resistance to recoil maintained constant during the powder period may be computed.

Substituting K_o in place of K in the preceeding formulae during the powder pressure period enables us to compute the retarded velocity curve during the powder pressure period.

During the retardation or second period of recoil we have, $K\,dx = -m_r V\,dV$

Integrating, from the displacement x to the end of recoil, we have

$$\int_x^b K dx = m_r \int_0^{V_x} V\,dV$$

therefore $\int_x^b [K_o - m(X-X_1)]dX = \frac{m_r V_x^2}{2}$

Hence

$$K_o(b-X) - \left|\frac{mX^2}{2}\right|_X^b + mX_1X\Big|_X^b = \frac{m_r V_x^2}{2}$$

and simplifying, we have

$$[K_o - \frac{m}{2}(b+X-2X_1)](b-X) = \frac{m_r V_x^2}{2}$$

Hence

$$V_x = \sqrt{\frac{2[K - \frac{m}{2}(b+X - 2X)](b-X)}{m_r}} \qquad (42)$$

where as before,

m = the arbitrary slope of resistance to recoil.

$$X_1 = E\cos(\emptyset + a) - \frac{K_o T^2}{2m_r}$$

GENERAL EQUATIONS OF RECOIL - Cont.

When the direction of recoil is along the axis of the bore,

(a) Constant resistance to recoil throughout recoil,

let $K = B + R - W_r \sin \emptyset$ = total resistance to recoil

B = total braking R = total friction

E = displacement in free recoil during powder period.

T = corresponding time for free recoil.

then for the motion of the recoiling parts,

$$P_b - K = m_r\frac{dV}{dt} \qquad \text{or} \qquad \int_0^T \frac{P_b}{m_r}\,dt - \frac{KT}{m_r} = V_r$$

but as before $\int \frac{P_b}{m_r}\,dt = V_f$ = max. free velocity of recoil, hence

$$V_r = V_f - \frac{KT}{m_r}$$

and the corresponding displacement, $X_1 = E_r = E - \dfrac{KT^2}{2m_r}$

After the powder period, from the equation of energy,

$$\tfrac{1}{2}m_r V_r = K(b - X_1)$$

or

$$\tfrac{1}{2}m_r\left(V_f - \frac{KT}{m_r}\right)^2 = K\left(b - E + \frac{KT^2}{2m_r}\right)$$

and simplifying, we have $K = \dfrac{\frac{1}{2} m_r V_f^2}{b - E + V_f T}$ (43)

This equation obviously is a special case of equation (7) since when $(\phi + a) = 0$, $\cos(\phi + a) = 1$ and $a = -\phi$.

(b) Variable resistance to recoil.

The resistance to recoil as before is assumed constant during the powder pressure period and thence to decrease uniformly consistent with stability, that is with a stability slope as given in Chapter III on stability.

At the end of the powder period, we have for the constrained velocity of recoil and corresponding displacement,

$$V_r = V_f - \frac{KT}{m_r}$$

and

$$E_r = E - \frac{KT^2}{2m_r}$$

At the end of recoil, the resistance to recoil becomes $K_o - m(b-E_r)$ where m = the stability slope (See Chapter III).

The mean resistance from the end of the powder period to the end of recoil, becomes,

$$\frac{2K - m(b-E_r)}{2} = K_o - \frac{m}{2}(b-E_r) \quad (45)$$

and from the equation of energy of the recoiling mass, we have

$$K_o(b-E_r) - \frac{m}{2}(b-E_r)^2 = \frac{1}{2} m_r V_r^2 \quad (46)$$

Substituting the values of E_r and V_r from (44) in (46) and neglecting the term $\dfrac{m}{2}\dfrac{K^2T^4}{4m^2 r}$

we have,

$$K_o = \frac{m_r V_f^2 + m(b-E)^2}{2\left[b - E + VT - \frac{m}{2}\frac{T^2}{m_r}(b-E)\right]} \quad (47)$$

This equation obviously is a special case of equation (41) since when $(\emptyset + d) = 0$, $\cos(\emptyset + a) = 1$ and $d = -\emptyset$.

(c) Dynamic equation of recoil during powder period.

Since during the powder pressure period, the resistance to recoil is assumed constant even with variable recoil, we have, therefore, the same dynamic equation with either variable or constant resistance to recoil during the powder period.

Dividing the powder period into two intervals t_o and $t_1 - t_o$ while the shot travels up the bore and during the expansion of the powder gases after the shot has left the bore, respectively, we have

(1) During the travel up the bore,

$$V_f = \frac{w + \frac{\bar{w}}{2}}{w_r} \quad \text{and} \quad v = \frac{au}{b + u}$$

$$X_f = \frac{w + \frac{\bar{w}}{2}}{w_r} \quad \text{approx.}$$

Hence

$$V = \frac{(w + \frac{\bar{w}}{2})}{w_r}\left(\frac{au}{b + u}\right) - \frac{Kt}{m_r} \qquad (48)$$

$$X = \frac{(w + \frac{\bar{w}}{2})}{w_r}\, u - \frac{Kt^2}{2m_r} \qquad (49)$$

$$\text{and } t = \frac{B}{a}\left(2.3 \log \frac{2u}{B} + \frac{u}{B} + 2\right) \qquad (50)$$

Thus V, X and t are functions of the parameter u. The ballistic constants a and B have been determined previously in this Chapter as well as in Chapter III in "Interior Ballistics".

When the shot reaches the muzzle,

$$V_o = \frac{(w + \frac{\bar{w}}{2})}{w_r} \frac{(au_o)}{(B + u_o)} - \frac{Kt_o}{m_r} \tag{51}$$

$$X_o = + \frac{(w + \frac{\bar{w}}{2})}{w_o} u_o - \frac{Kt_o^2}{2m_r} \tag{52}$$

$$\left.\begin{aligned} \text{and } t_o &= \frac{B}{a}\left(2.3 \log \frac{2u_o}{B} + \frac{u_o}{B} + 2\right) \\ &= \frac{3}{2}\frac{u_o}{v_o} \text{ approx} \end{aligned}\right\} \tag{53}$$

(2) during the expansion of the powder gases, we have

$$v_o = \frac{au_o}{B + v_o} \; : \; V_{fo} = \frac{w + \frac{\bar{w}}{2}}{w_r} v_o$$

$$X_{fo} = \frac{w + \frac{\bar{w}}{2}}{w_r} u_o$$ where u = the total travel up the bore, the dynamic equation of recoil during this period becomes,

$$P_{ob}\left(1 - \frac{t - t_o}{t_1 - t_o}\right) - K = m_r \frac{dV}{dt} \tag{54}$$

where $P_{ob} = \frac{27}{4} B^2 \frac{u_o}{(B + u_o)^2}$ 1.12 P_m (See Chapter III)

Integrating, we find

$$P_{ob}(t-t_o) \frac{P_{ob}(t-t_o)^2}{2(t_1-t_o)} - K(t-t_o) = m_r(V-V_o) \tag{55}$$

Hence $V = V_o + \frac{P_{ob}(t-t_o)}{m_r}\left[1 - \frac{t-t_o}{2(t_1-t_o)}\right] - \frac{K}{m_r}(t-t_o)$ (56)

The corresponding displacement is obtained by integrating equation (55)

$$\frac{P_{ob}(t-t_o)^2}{2} - \frac{P_{ob}(t-t_o)^2}{6(t-t_o)} - \frac{K(t-t_o)^2}{2} = m_r \int V\, d(t-t_o)$$

$$+ m_r V_o (t-t_o) + \text{Const.} \qquad (57)$$

Now $m_r \int V\, d(t-t_o) = m(X - X_o)$. Hence where $t = t_o$, $X = X_o$ and const. = 0. Simplifying (57) we obtain for the recoil displacement during the second period of the powder period,

$$X = X_o + V_o(t-t_o) + \frac{P_{ob}(t-t_o)^2}{2m_r}\left[1 - \frac{t-t_o}{3(t_1-t_o)}\right] - K\frac{(t-t_o)^2}{2} \qquad (58)$$

To obtain the maximum restrained recoil velocity and corresponding displacement, we must equal the total powder reaction to the total resistance to recoil, that is $P_b - K = 0$

$P_{ob}\left(1 - \frac{t_m-t_o}{t_1-t_o}\right) - K = 0$ where P_{ob} = the pressure on the breech when the shot leaves the muzzle.

t_m = total time to maximum restrained recoil velocity, hence solving for t_m, we have

$t_m = t_1 - \frac{K}{P_{ob}}(t_1 - t_o)$. Substituting in equation (56) and (58) we have

$$V = V_o + \frac{P_{ob}(t_m-t_o)}{m_r}\left[1 - \frac{t_m-t_o}{2(t_1-t_o)}\right] - \frac{K}{m_r}(t_m-t_o) \qquad (59)$$

$$X = X_o + V_o(t_m - t_o) + \frac{P_{ob}(t_m-t_o)^2}{2m_r}\left[1 - \frac{t_m-t_o}{2(t_1-t_o)}\right] - K\frac{(t_m-t_o)^2}{2} \tag{60}$$

At the end of the powder period,
$t = t_1 = T$ and $X = E_r$ and $V = V_r$ and substituting again in eq.(56) and (58), we have

$$V_r = V_o + P_{ob}\frac{(t_1 - t_o)}{2m_r} - \frac{K}{m_r}(t_1 - t_o) \tag{61}$$

$$E_r = X_o + V_o(t_1 - t_o) + \frac{P_{ob}(t_1 - t_o)^2}{3m_r} - K\frac{(t_1 - t_o)^2}{2} \tag{62}$$

(d). Dynamic equation of recoil during the retardation or the pure recoil period.

(1) constant resistance to recoil:

Since the total resistance to recoil is constant, the velocity must be a parabolic function of the displacement of the recoil,

From the principle of energy, we have,

$$K(b-X) = \frac{M_r V^2}{2} \quad \text{hence} \quad V = \sqrt{2\frac{K(b-X)}{m_r}}$$

(2) Variable resistance to recoil

The resistance to recoil out of battery, becomes,

$$k = K - m\left(b - E + \frac{KT^2}{2m_r}\right) \quad \text{where } K = \frac{m_r V_f^2 + m(b-E)^2}{2\left[b - E + V_f T - \frac{m}{2}\frac{T^2}{m_r}(b-E)\right]}$$

The average resistance to recoil in the displacement $b - X$, becomes

$$k + \frac{m}{2}(b-X)$$

From the energy equation, we have,

$$k(b-X) + \frac{m}{2}(b-X)^2 = \frac{1}{2} m_r V^2$$

hence

$$V = \frac{\sqrt{2(b-X)[k + \frac{m}{2}(b-X)]}}{m_r}$$

COMPONENT REACTIONS OF THE RESISTANCE TO RECOIL.

Let K = total resistance to recoil. (lbs. or Kg)

B = total braking. (lbs. or Kg)

R = total friction to recoil. (lbs. or Kg)

P_h = reaction of hydraulic brake. (lbs. or Kg)

P_v = reaction of recuperator. (lbs. or Kg)

p_x = hydraulic brake pressure. (lbs/sq.in) or (Kg/$\overline{m}^2$

A = effective area of hydraulic brake piston. (sq.in. or $\overline{m}^2$)

p_v = recuperator pressure. (lbs/sq.in) or (Kg/$\overline{m}^2$)

A_v = effective area recuperator piston. (sq.in or $\overline{m}^2$)

V_o = initial volume of recuperator. (cu.ft. or $\overline{m}^3$)

X = recoil displacement. (ft. or m)

S_f = final spring reaction. (lbs) or (Kgs)

S_o = initial spring reaction. (lbs) or (Kgs)

The total resistance to recoil then becomes

along the bore	along special guides
$K = B + R - W_r \sin \emptyset$	$K = B + R + W_r \sin \theta$

where $\emptyset$ = angle of elevation, θ = angle of guides. Now in systems where the hydraulic brake is independent of the recuperator system, $B = P_h + P_v$

In systems where the brake and recuperator are connected $B = P_h$

For independent systems

$$P_v = P_{vi}\left[\frac{V_o}{(V_o - Ax)}\right]^k \quad \text{for pneumatic recuperators}$$

$$P_v = S_o + \left(\frac{S_f - S_o}{b}\right) x \quad \text{for metallic recuperators}$$

and $P_{vi} = 1.3\ W_r(\sin \varnothing_m + u \cos \varnothing_m)$ approx.

$$P_h = \frac{cv^2}{w_x^2} \quad \text{where } c = \frac{KA^3}{w_x^2}$$

hence

$$B = P_{vi}\left(\frac{V_o}{V_o - Ax}\right)^k + \frac{cv^2}{w_x^2} \quad \text{for pneumatic recuperators.}$$

$$B = S_o + \left(\frac{S_f - S_o}{b}\right) x + \frac{cv^2}{w_x^2} \quad \text{for metallic recuperators.}$$

For systems where the hydraulic brake and recuperator are directly connected,

$$p - p_v = \frac{c'v^2}{w_x^2} \quad \text{where } c' = \frac{KA^2}{w_x^2}$$

$$p_{vi}A = P_{vi} = 1.3\ W_r(\sin \varnothing + u \cos \varnothing) \text{ approx.}$$

$$p_v = p_{vi}\left(\frac{V_o}{V_o - Ax}\right)^k$$

therefore

$$p = p_{vi}\left(\frac{V_o}{V_o - Ax}\right)^k + \frac{c'v^2}{w_x^2}$$

$$pA = p_{vi}A\left(\frac{V_o}{V_o - Ax}\right)^k + \frac{c'Av^2}{w_x^2}$$

since

$$p_{vi}A = P_{vi} \quad \text{and } c'A = C \quad \text{hence } B = P_h = P_{vi}\left(\frac{V_o}{V_o - Ax}\right)^k + \frac{cv^2}{w_x^2}$$

which is an equation of exactly the same form as for a system where the recuperator is independent of the hydraulic brake.

The general equation for the resistance to recoil becomes,

(a) <u>when the recoil is along axis of the bore:</u>

$$K = P_{vi}\left(\frac{V_o}{V_o - Ax}\right)^k + \frac{cv^2}{w_x^2} + R - W_r \sin \phi, \text{ for pneumatic recuperators.}$$

$$K = S_o + \left(\frac{S_f - S_o}{b}\right)x + \frac{cv^2}{w_x^{2'}} + R - W_r \sin \phi, \text{ for metallic recuperators.}$$

(b) <u>when the recoil is along special guides:</u>

$$K = P_{vi}\left(\frac{V_o}{V_o - Ax}\right)^k + \frac{cv^2}{w_x^2} + R + W_r \sin \theta, \text{ for pneumatic recuperators.}$$

$$K = S_o + \left(\frac{S_f - S_o}{b}\right)x + \frac{cv^2}{w_x^2} + R + W_r \sin \theta, \text{ for metallic recuperators.}$$

$$K = \frac{cv^2}{w_x^2} + R + W_r \sin \theta, \text{ for gravity mounts.}$$

GENERAL EQUATIONS OF COUNTER RECOIL. The function of the recuperator is to return the recoiling mass into battery. The stability of a mount in counter recoil is greatest at the beginning of counter recoil and least at the end of counter recoil or when the gun enters the battery position. To prevent shock and unstableness as the gun arrives in battery it is necessary to introduce some form of counter recoil buffer towards the end of counter recoil. Very often a buffer resistance of varying amount is introduced throughout the counter recoil. In addition we always have the resistance of the guides.

Without a recuperator the recoiling mass must be

returned to battery by the gravity component due to the inclination of the guides with the horizontal. If this inclination is small, the gravity component does not greatly exceed the friction and thence a very elementary buffer may be used, the return velocity being always small.

Let K_v = total unbalanced force in counter recoil.

F_v = recuperator reaction.

B_v' = counter recoil buffer resistance.

w_x' = variable orifice for counter recoil buffer.

A_v = effective area of recuperator piston

p_v = pressure intensity in the recuperator cylinder.

p_a = pressure intensity in the air reservoir.

R = total friction of counter recoil.

During the accelerating period of counter recoil, we have

$K_v = m_R\, v \dfrac{dv}{dx}$ and during the retardation

$$K_v = - m_R\, v \frac{dv}{dx}$$

During the acceleration K_v is necessarily always smaller than the total resistance to recoil, hence during the acceleration counter recoil stability is of no consequence. During the retardation, if

d' = the distance from front hinge or wheel contact with ground in a field mount, to the line of action of the total resistance to recoil.

L = horizontal distance between front and rear supports of mount.

L_s = horizontal distance from rear support to center of gravity of total system with recoil parts in battery.

b = total length of recoil.

W_s = weight of total mount.

Then, for a gun recoiling along the axis of the bore during the retardation, $K_v d' \leq W_s(L-L_s) + W_r(b-X)\cos \emptyset$ and the minimum stability occurs when the gun enters battery, that is $K_v d' \leq W_s(L-L_s)$. The stability slope of counter recoil, becomes

$$m' = \frac{W_r \cos \emptyset}{d'}$$

To consider the components of the total resistance to counter recoil, we have three classifications:

(1) recuperator systems independent of the hydraulic brake and with no throttling between the air and recuperator cylinders.

(2) recuperator systems independent of the hydraulic brake, with throttling between the air and recuperator cylinders.

(3) recuperator cylinders connected directly with the brake cylinder. In all systems an independent buffer may be introduced in either the recuperator or brake cylinder front end. In certain types the buffer acts as a plunger brake within the piston rod of the recoil brake.

Then,

(1) for recuperators independent of the brake cylinder and with no throttling between the air and recuperator cylinders,

$$K_v = F_v - B_x' - W_r \sin \emptyset - R \qquad (1)$$

when

$$F_v = p_a A_v = F_{vi}\left(\frac{V_o}{V_o - A(b-X)}\right)^{1.1}$$

V_o = initial volume

F_{vi} = 1.3 $W_r(\sin \emptyset + 0.3 \cos \emptyset)$ approx.

(2) for recuperators independent of the brake cylinder, with throttling between the air and recuperator cylinders,

$$K_v = p_v A_v - B_x' - W_r \sin \emptyset - R$$

where

$$p_v = p_a - \frac{c'' v^2}{w_o^2} \qquad (w_o = \text{constant orifice usually})$$

$$p_a A_v = F_{vi}\left[\frac{V_o}{V_o - A(b-x)}\right] = F_v$$

$$F_{vi} = 1.3\ W_r(\sin \emptyset + u \cos \emptyset)$$

hence

$$K_v = (p_a - \frac{c'' v^2}{w_o^2})A_v - B_x' - W_r \sin \emptyset - R$$

and since

$$B_x'' = \frac{c'' v^2}{w_o^2} A_v$$

then the equation reduces to same form as (1), that is $K_v = F_v - (B_x' + B_x'') - W_r \sin \emptyset - R$,

(3) for recuperators directly connected with the recoil brake cylinder,

$$K_v = p_v A - B_x' - W_r \sin \emptyset - R$$

where

$$p_v = p_a - \frac{c'' v^2}{w_x^2}$$

(w_x = variable orifice by buffer rod on a floating piston in recuperator or air cylinder.)

$$p_a A = F_{vi}\left[\frac{V_o}{V_o - A(b-x)}\right]^{1.1} = F_v$$

$F_{vi} = 1.3\ W_r(\sin \emptyset + u \cos \emptyset)$ hence,

$$K_v = F_v - (B_x' + B_x'') - W_r \sin \emptyset - R,$$

where $B_x'' = \dfrac{c'' v^2}{w_x^2} A_v$

which is again an equation of same form as (1).

The general equation of counter recoil, therefore, becomes

$$F_v - (B'_x + B''_x) - W_r \sin \emptyset - R = m_R\, v \frac{dv}{dx}$$

where

$$B'_x = \frac{DA_v^3 v^2}{2gc'^2 w_x'^2} = c' \frac{v^2}{w_x'^2}$$

$$B''_x = \frac{DA_v^3 v^2}{2gc''^2\ w_x''^2} = c'' \frac{v^2}{w_x''^2}$$

CALCULATION OF RECOIL CURVES. It is often convenient to calculate the retarded velocity curve against displacement, especially when the resistance to recoil is not made constant. In all cases we have seen the resistance to recoil is in general a function of both the displacement and velocity of recoil, that is the recuperator component of the recoil resistance is a function of the displacement, whereas the brake component is a function of the velocity and the variation of the throttling orifice. Hence $K = f(x_, v)$ and the dynamic equation of recoil is

$$P_b \cos(\theta + \emptyset) - K = m_R \frac{dV}{dt}$$ or when the recoil translates in the direction of the axis of the bore,

$$P_b - K = m_R \frac{dV}{dt}$$

To <u>measure</u> P_b we may consider the momentum imparted by the powder gases in free recoil, then

$$P_b = m_R \frac{dV_f}{dt}$$ or $$\int_{t_1}^{t} P_b dt = m_R(V_f - V_{f_1})$$ Therefore, for the same interval of time $(t - t_1)$ we have

$$m_R(V_f - V_{f_1}) \cos(\theta + \emptyset) - K(t - t_1) = m_R(V - V_1)$$

hence

$$V = V_1 + (V_f - V_{f_1})\cos(\theta+\emptyset) - \frac{K}{m_R}(t - t_1)$$ or when the recoil translates in the direction of the axis of the bore,

$$V = V_1 + (V_f - V_{f_1}) - \frac{K}{m_R}(t - t_1)$$ Further since $X = X_1 + \int_{t_1}^{t} V dt$,

we have

$$X = X_1 + V_1(t - t_1) + \int_{t_1}^{t} V_f dt \cos(\theta+\emptyset) - V_{f_1}(t - t_1)\cos(\theta+\emptyset) -$$

$$\frac{K}{2m_R}(t - t_1)^2$$ now $\int_{t_1}^{t} V_f dt = X_f - X_{f_1}$ hence

$$X = X_1 + [V_1 - V_{f_1}\cos(\theta+\emptyset)](t - t_1) + (X_f - X_{f_1})\cos(\theta+\emptyset) - \frac{K}{2m_R}(t - t_1)^2$$ or when the recoil translates in the direction of the axis of the bore,

$$X = X_1 + (V_1 - V_{f_1})(t - t_1) + (X_f - X_{f_1}) - \frac{K}{2m_R}(t_f - t_1)^2$$

Therefore the velocity and displacement, for any given interval $(t_2 - t_1)$

(a) along guides not parallel to the bore:

$$V_2 = V_1 + (V_{f_2} - V_{f_1})\cos(\theta+\emptyset) - \frac{K}{m_R}(t_2 - t_1)$$

$$X_2 = X_1 + [V_1 - V_{f_1}\cos(\theta+\emptyset)](t_2 - t_1) + (X_{f_2} - X_{f_1})\cos(\theta+\emptyset) - \frac{K}{2m_R}(t_2 - t_1)^2$$

(b) along guides parallel to the axis of the bore:

$$V_2 = V_1 + (V_{f_2} - V_{f_1}) - \frac{K}{m_R}(t_2 - t_1)$$

$$X_2=X_1+(V_1-V_{f_1})(t_2-t_1)+(X_{f_2}-X_{f_1})-\frac{K}{2m_R}(t_2-t_1)^2$$

After the powder period these formulas reduce to=

$$V_2=V_1-\frac{K}{m_R}(t_2-t_1)$$

$$X_2=X_1+V_1(t_2-t_1)-\frac{K}{2m_R}(t_2-t_1)^2$$

and obviously are independent of the direction of the guides with respect to the axis of the bore.

The value of $K=\frac{K_1+K_2}{2}$, which may be closely approximated by a repetition of the substitution in these equations, since from the first substitution we closely approximate V_2 and thereby can determine $K_2=f(X_2V_2)$ for the second substitution.

CALCULATION OF ACCELERATION, TIME AND DISPLACEMENT FROM A GIVEN VELOCITY CURVE:

Recoil and counter recoil velocity curves are usually drawn experimentally as functions of the displacement though they may be drawn as well as functions of the time. The customary method of obtaining a velocity curve, is to set a tuning fork vibrating and allow the vertical oscillations to form a sinuous curve along a narrow soot covered strip recoiling with the gun. Then if f = the frequency of oscillations of the fork, we have for the time of one oscillation, $T=\frac{1}{f}$ If n = the number of oscillations for an interval Δx, the velocity becomes,

$$v=\frac{\Delta x}{\Delta t}\text{, where } \Delta t = nT \quad \text{if } x \text{ is measured in inches,}$$

$$v=\frac{1}{12}\frac{\Delta x}{\Delta t}\quad (\text{ft/sec})$$

To obtain the time as a function of the displacement, since $vdt=dx$

$$t = \int_0^x \frac{1}{v}\,dx$$

and if x is measured in inches, $t = \frac{1}{v}\int_0^x \frac{1}{v}\,dx$

<u>Hence the area under the reciprocal of the velocity curve against displacement is the time of recoil.</u>

We may then draw the velocity curve as a direct function of the time of recoil.

When the recoil velocity is measured as a function of the time, the acceleration is

$\frac{dv}{dt}$ = the slope of the velocity curve

When the recoil velocity is measured as a function of the displacement, the acceleration is,

$v\frac{dv}{dx}$ = the velocity × the slope of the velocity curve.

= the sub-normal of the velocity curve.

If dx is measured in inches, the acceleration is

$12\,v\frac{dv}{dx}$ (ft/sec^2)

From the relations, $v=f(x)$ and $t=\int \frac{1}{v}\,dx = \int \frac{1}{f(x)}\,dx$

we see that the velocity curve may be readily expressed either as a function of the displacement or as a function of the time or both.

CHARACTERISTICS OF RECOIL CURVES. From Proof Firing Tests, recoil curves are obtained for both recoil and counter recoil. From these curves, it is possible to determine the variation of the reactions throughout recoil or counter recoil.

In the analysis of curves during the powder period, since the mutual relation connecting the variation of powder force and the retarded recoil is the common time, it is necessary to express the forces, velocities and displacements as functions of the time.

In the analysis of curves during the retarded recoil and counter recoil it is possible to express the forces and velocities as direct functions of the displacements which considerably simplifies the work.

(1) Powder Pressure Period: Recoil along axis of bore. The equation of recoil is

$$P_b - K = m_R \frac{dV}{dt} \quad \text{where } K = B+R-W_r \sin \phi$$

With a given velocity curve, the velocity and displacement should be tabulated as a function of the time; then for any interval (t_2-t_1) we have

$$(V_{f_2}-V_{f_1}) - (V_2-V_1) - \frac{K}{m_R}(t_2-t_1) = 0$$

$$(X_{f_2}-X_{f_1})-(X_2-X_1) + (V_1-V_{f_1})(t_2-t_1) - \frac{K}{2m_R}(t_2-t_1)^2 = 0$$

If K is assumed constant or found to be constant by brake measurements or if it is determinate as a function of the displacement, we may evaluate V_f the free velocity of recoil. More often however, the free velocity and displacement curves can be evaluated as a function of the time, and knowing the retarded velocity and displacement curve as a function of the time we may calculate the resistance to recoil from the above expressions. Then

$$P_b = m_R \frac{V_{f_2}-V_{f_1}}{t_2 - t_1}$$

and

$$P_b - m_R \frac{dV}{dt} = K \qquad \text{where } m_R \frac{dV}{dt} = m_R V \frac{dV}{dx} = m_R\left(\frac{V_2-V_1}{t_2-t_1}\right)$$

It is to be noted that P_b and $- m_R \frac{dV}{dt}$ are the <u>external</u>

recoil forces during the powder period. Further P_b acts along the axis of the bore and $- m_R \frac{dV}{dt}$ acts through the center of gravity of the recoiling parts parallel to the axis of the bore or guides. If e = the distance from the center of gravity of the recoiling parts to the axis of the bore, we have for the external reactions on the mount a couple $P_b e$ and a force parallel to the axis of the bore, $P_b - m_R \frac{dV}{dt} = K$. The balancing forces are the weights and reactions of the supports. For stability the moment of the weights about the rear support must exceed the moment of $P_b e$ and K about the rear support.

(2) Retardation Period: Recoil along axis of bore. During this period, we have simply

$$\left.\begin{array}{l} m_R \dfrac{dV}{dt} = - K \\[2ex] m_R V \dfrac{dV}{dx} = - K \end{array}\right\}$$

applied through the center of gravity of the recoiling parts, parallel to the axis of the bore,

which together with the weights and balancing support reactions are the external forces on the mount.

It is to be further noted that since

$K = P_h + P_v + R - W_r \sin \emptyset$ we have directly from the velocity curve,

$$P_h = - m_R V \frac{dV}{dx} - P_v - R + W_r \sin \emptyset$$

where $P_v = P_{vi} \left(\frac{V_o}{V_o - A_x}\right)^k$ for pneumatic recuperators

$P_v = S_o + \left(\frac{S_f - S_o}{b}\right) x$ for metallic recuperators

$R = 0.25\ W_r \cos \emptyset + R_p$ approximately where R_p = estimated packing friction.

(3) Counter recoil: C'Recoil along axis of bore.

During the accelerating period of counter recoil, the inertia resistance is directed towards the breech the same as in recoil. Here

$$K_v = m_R\, v \frac{dv}{dx} \quad \text{to the rear}$$

and during the retardation the inertia resistance is directed forward and here,

$$K_v = - m_R\, v \frac{dv}{dx}$$

which together with the weight of the system and balancing supporting reactions are the external forces on the mount.

Since further, during the retardation,

$$- m_R\, v \frac{dv}{dx} = F_v - B_x' - W_r \sin \emptyset - R \quad \text{we have}$$

$$B_x' = F_v + m_R\, v \frac{dv}{dx} - W_r \sin \emptyset - R$$

and

$$F_v = F_{vi} \left[\frac{V_o}{V_o - A(b-x)}\right]^k \quad \text{for pneumatic recuperators}$$

$$= S_o + \left(\frac{S_f - S_o}{b}\right)(b-x) \quad \text{for metallic recuperators}$$

and

$R = 0.15\, W_r \cos \emptyset + R_p$ approximately where R_p = estimated packing friction.

Since critical counter recoil stability is at horizontal elevation, C'recoil curves are usually obtained at horizontal elevation. Then,

$$B_x' = F_v - R + m_R\, v \frac{dv}{dx}$$ for the buffer force where the overturning force is $m_R\, v \dfrac{dv}{dx}$ along the axis of the bore forward.

RECOIL BRAKING WITH A CONSTANT ORIFICE:

As a first approximation we will assume the recuperator reaction not to vary greatly in the recoil.

Then $K = A + Bv^2$ where $A = P_v + R - W_r \sin \emptyset$

B = the hydraulic brake throttling constant.

(1) During the powder period, we have

$$P_b - (A+BV^2) = m_R \frac{dv}{dt}$$

Hence

$$V_2 = V_1 - \frac{A+B\left(\frac{V_1+V_2}{2}\right)^2}{m_R}(t_2-t_1)+(V_{f_2}-V_{f_1})$$

Expanding, we have

$$V_2 = V_1 - \frac{A}{m_R}(t_2-t_1) - \frac{B}{4m_R}(t_2-t_1)(V_1^2+2V_1V_2+V_2^2)+(V_{f_2}-V_{f_1})$$

which is a quadratic equation of the form

$aV_2^2 + bV_2 + c = 0$

and

$$V_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{where } a = \frac{B}{4m_R}(t_2-t_1)$$

$$b = 1 + \frac{V_1 B}{2m_R}(t_2-t_1)$$

$$c = \frac{B}{4m_R}V_1^2(t_2-t_1) - V_1 + \frac{A}{m_R}(t_2-t_1) - (V_{f_2}-V_{f_1})$$

If the intervals are taken very small, then

$$V_2 = V_1 - \frac{A+BV_1^2}{m_R}(t_2-t_1)+(V_{f_2}-V_{f_1})$$

Then solving for V_2 we may repeat with the expression

$$V_2 = V_1 - \frac{A+B\left(\frac{V_1+V_2}{2}\right)^2}{m_R}(t_2-t_1)+(V_{f_2}-V_{f_1})$$

for a closer approximation.

The displacement is obtained from the expression,

$$X_2 = X_1+(V_1-V_{f_1})(t_2-t_1)+(X_{f_2}-X_{f_1})-\frac{A+B\left(\frac{V_1+V_2}{2}\right)^2}{2m_R}(t_2-t_1)^2$$

(2) During the retardation, we have

$$m_R\,V\,\frac{dV}{dx} = -A - BV^2 \quad \text{hence } dx = -\frac{m_R V dV}{A+BV^2}$$

Integrating. we have $X_2-X_1 = \frac{m_R}{2B}\log_e\frac{A+Bv_1^2}{A+Bv_2^2}$

In particular if $X_1 = E_R$ the constrained recoil displacement at the end of the powder period and $v_1 = V_R$ the constrained recoil velocity at the end of the powder period, then, for any displacement $\overline{X}$ and recoil velocity V, we have,

$$X - E_R = \frac{m_R}{2B}\log_e\frac{A+BV_R^2}{A+BV^2}$$

or with common logarithm,

$$X - E_R = 1.15\,\frac{m_R}{B}\log\frac{A+BV_R^2}{A+BV^2}$$

when $V = 0$ the length of recoil becomes,

$$b = E_R + 1.15\,\frac{m_R}{B}\log\left(1+\frac{B}{A}V_R^2\right)$$

As a first approximation, we may take $E_R = E$ the displacement in free recoil during the powder period and $V_R = V_f$ the maximum velocity of recoil, then

$$b = E + 1.15\,\frac{m_R}{B}\log\left(1+\frac{B}{A}V_f^2\right)$$

C H A P T E R   VII

CLASSIFICATION AND CHARACTERISTICS OF RECOIL AND RECUPERATOR SYSTEMS.

Recuperator systems may be divided into:

(1) Hydro pneumatic recuperator systems

(2) Pneumatic recuperator systems

(3) Spring return recuperator systems.

(1) With hydro pneumatic systems, we have two fundamental arrangements:-

(a) The hydraulic brake separate from the hydro pneumatic recuperator. This requires two or more rods, a brake rod and a recuperator rod. Further we have in general two or more cylinders, a brake cylinder, a recuperator cylinder which may have passage way or connection with an air cylinder. The recuperator and part of the air cylinder is filled with oil. The oil may be in direct contact with the air in the air cylinder as in the Schneider and Vickers material or it may be separated from the air by means of a floating piston in the cylinder.

(b) The hydraulic brake cylinder connecting directly with the recuperator cylinder. The oil must be throttled between the recoil and recuperator cylinder, and thus the oil at lower pressure reacts usually on

a floating piston separating the oil and air in the recuperator cylinder. To augment the initial volume for the air in the recuperator an additional air cylinder connecting with the recuperator may be introduced. Thus with this arrangement we have from two to three cylinders.

(2) With pneumatic recoil systems, we have usually,

(a) One or more pneumatic cylinders, according to a satisfactory layout.

The piston compresses the air directly, no oil or other liquid being used for transmitting the pressure.

(3) With a spring return system, we may have various arrangements:

(a) One or more spring cylinders separate from the recoil brake cylinder.

(b) With small guns, the spring concentric and around the recoil brake cylinder.

The potential energy or the energy of compression of the recuperator during the recoil, becomes

FOR PNEUMATIC OR HYDRO PNEUMATIC SYSTEMS:

If

p_{ai} = initial air pressure. (lbs/sq.in)

p_{af} = final air pressure (lbs/sq.in)

$\frac{p_{af}}{p_{ai}} = m$ = ratio of compression

V_o = initial air volume

V_f = final air volume

K_v = recuperator reaction

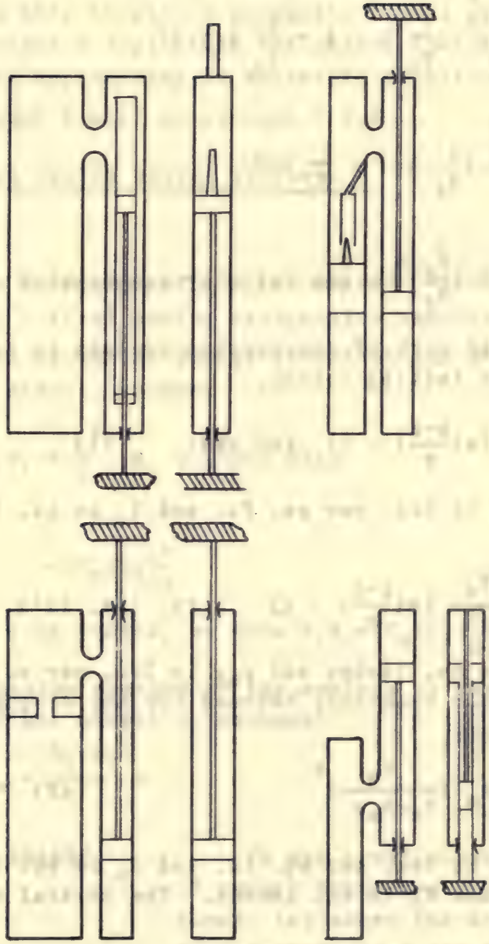


Fig. 1

$$W_c = -\int_{V_o}^{V_f} p_a dV = -p_{ai} V_o^k \int_{V_o}^{V_f} \frac{dV}{V^k}$$

$$= -\frac{p_{ai} V_o^k}{1-k} \left(\frac{1}{V_f^{k-1}} - \frac{1}{V_o^{k-1}}\right)$$

$$= \frac{p_{ai} V_o^k}{k-1} \left(\frac{1}{V_f^{k-1}} - \frac{1}{V_o^{k-1}}\right)$$

Now $m = \frac{p_{af}}{p_{ai}} = \left(\frac{V_o}{V_f}\right)^k$ = the ratio of compression

Therefore, the work of compression becomes in terms of m, and the initial volume,

$$W_c = \frac{p_{ai}V_o}{k-1} \left[m^{\left(\frac{k-1}{k}\right)} - 1\right] \text{ ft. lbs.} \qquad (1)$$

where p_{ai} is in lbs. per sq. ft. and V_o in cu. ft.

$$W_c = \frac{1}{12} \frac{p_{ai}V_o}{k-1} \left[m^{\left(\frac{k-1}{k}\right)} - 1\right] \text{ ft. lbs.} \qquad (1')$$

when V_o is in cu. inches and p_{ai} in lbs. per sq. in. The recuperator reaction, becomes for any displacement X in the recoil,

$$p_a A_a = p_{ai} A_a \left(\frac{V_o}{V_o - A_a x}\right)^k \qquad (2)$$

where p_{ai} is in lbs. per sq. in. and A_a in sq. in. X in inches and V_o in cu. inches. The initial volume becomes,

$$V_o = A_a b \frac{m^{1/k}}{m^{1/k} - 1} \qquad (3)$$

where b = length of recoil. With the oil in direct contact with the air, we will assume that the

temperature remains approximately constant throughout the recoil and k will be taken at 1.1

With a floating piston interposed between the oil and air, or with a pneumatic recoil system, we will assume a negligible radiation, that is the compression approaching an adiabatic condition.

Hence k will be assumed = 1.3

FOR SPRING RETURN SYSTEMS:

If

S_o = initial spring recuperator reaction

S_f = final spring recuperator reaction

Then the potential energy stored in the spring at the end of recoil, becomes

$$P.E. = \int_o^b \left(S_o + \frac{S_f - S_o}{b} x\right) dx$$

$$= (S_o + S_f)\frac{b}{2} \qquad (4)$$

and if b is inches, we have $P.E. = (S_o + S_f)\frac{b''}{24}$

The reaction exerted by the springs at any displacement of the recoil X, becomes

$$K_v = S_o + \frac{S_f - S_o}{b} x \qquad (5)$$

RECOIL BRAKES. In the broad classification of recoil brake systems, we have those: (a) where the brake system is independent of the recuperator system, (b) where the brake system is part of or connects with the recuperator system.

(a) In consideration of independent brake systems, we have a further

classification:-

(1) Brakes with throttling orifice through the recoil piston, the varying aperture during the recoil being produced by the difference in areas of the constant apertures in the piston and the area of the bar or rod of varying depth or diameter fixed to the recoil cylinder and moving through the aperture; or the throttling may be around the piston by varying grooves in the cylinder walls along the cylinder.

(2) Brakes with varying apertures through the recoil piston, the aperture being cut off during the recoil by a rotating disk about the axis of the piston, the disk being rotated during the recoil by a projecting "toe" engaging in a helicoidal groove in the cylinder wall. This form of brake is known as the Krupp valve and is extensively used not only by the Krupp but other types as well.

(3) Brakes with the throttling taking place around the piston,[not through as in (1) and (2)], through a sleeve perforated with holes along the recoil. The piston cuts off the number of holes during the recoil thus decreasing the effective throttling area.

(4) Brakes with the throttling taking place through a spring controlled valve. With independent brake systems the spring valve is contained in the piston. Where the brake is part of the recuperator the throttling takes place through a fixed orifice somewhere between the two cylinders.

(5) Brakes with a constant orifice. Un-

> less the air pressure is fairly large, and the throttling relatively small constant orifice control should be avoided since it gives a large peak in the braking.

In consideration of brake systems as a part of the recuperator, the throttling takes place between an orifice fixed somewhere between the two cylinders, and usually of the spring controlled type though sometimes with high air pressures a constant orifice may be used.

In general it may be stated when the recoil energy is large the throttling may be very satisfactorily controlled, as in brake systems of the type (1),(2) and (3). Where the energy of recoil is small as in small caliber guns, the throttling areas especially at the end of recoil must be small. This can not be satisfactorily met with "bars" or "grooves" due to the inherent tolerance making very often the clearance greater than the required throttling areas towards the end of recoil. This difficulty has been repeatedly met in the design of small recoil systems. On the other hand spring controlled valves are admirably adopted for small recoil systems, since the throttling towards the end of recoil can be finely controlled.

COUNTER RECOIL SYSTEMS OR RUNNING FORWARD BRAKES:

In the classifications of counter recoil systems, we have two general types of systems:

(1) Where the brake comes into action during the latter part of the counter recoil.

(2) Where the brake is effective throughout the counter recoil.

With (1), the buffer action can only take place after a displacement of the void (the displacement

of the recoil piston rod = $A_r \times b$), which with guns of large piston rods may be a considerable part of the counter recoil.

With (2) the buffer must be filled during the recoil, otherwise no buffer or braking action can take place.

The brake with systems where the buffer action takes place towards the end of counter recoil, consists usually of a buffer chamber as an extension of the recoil cylinder in the front and spear buffer projecting from the front side of the piston or with a buffer chamber within the piston rod itself the spear buffer rod being attached to the front head of the cylinder. In the former type we must have a projectory chamber from the cylinder, while in the latter we must have a larger piston rod with consequent larger void to overcome during the counter recoil.

With guns of high elevation in order to meet the demands of counter recoil at maximum elevation, we have a surplus potential recuperator energy in the recuperator and no means of checking or regulating the velocity during the greater part of horizontal recoil: therefore at the initial condition of counter recoil stability, we have unfortunately an inherent condition of a large buffer force, making the mount unstable at the end of counter recoil.

Therefore, this type of counter recoil brake, which is effective only during the latter part of counter recoil should only be used for guns of low elevation.

Counter recoil brakes of type (2) which fill during the recoil end are effective throughout the counter recoil, are really the standard form of counter recoil regulator to meet the varying conditions required in modern artillery. Varying forms of this type of brake are used. Thus in the Filloux and Schneider recoil system the buffer is at the end

of a counter recoil rod which serves also as a throttling bar through the recoil piston. The buffer head is enclosed and slides within a buffer chamber in the piston rod. Apertures near the piston in the piston rod admit the oil during the recoil into the buffer chamber, the oil passing through a valve in the buffer head which can open during the recoil and closes during the counter recoil as in the Schneider material. In the Filloux, though we have a filling in buffer in the recoil piston rod, the buffing action takes place only at the end of counter recoil but a positive filling is ensured. The velocity of counter recoil is maintained low in this system by lowering the recuperator pressure during the greater part of counter recoil by throttling through a constant orifice in the air cylinder.

Various forms of filling in buffers are shown in figs.(1),(2),(4).

APPROXIMATE FORMULA FOR TOTAL RESISTANCE TO RECOIL. If the total resistance to recoil is assumed constant throughout the recoil, we have when the recoil is along the axis of the bore, which usually occurs in practice, that

$$K = \frac{\frac{1}{2} m_r V_f^2}{b - E + V_f T}$$

where

E = free recoil displacement during powder period.

T = time of powder period.

$$V_f = \frac{wv + 4700\,\bar{w}}{w_r} = \text{max. free velocity of recoil. (ft/sec)}$$

b = length of recoil. (ft)

Now

$$E = C_1 V_f T \quad \text{and} \quad T = C_2 \frac{u_0}{v_0}$$

where

u_o = travel up the bore and v_o = muzzle velocity.

Substituting,

$$E = C_1C_2 \frac{V_f}{v_o} u_o = C \frac{V_f}{v_o} u_o$$

This value of E may be obtained in another way,

$$E = C\left(\frac{w+\frac{\bar{w}}{2}}{w_r}\right)u_o = C \frac{V_f}{v_o} u_o$$

hence

$$K = \frac{1}{2} m_r V_f^2\left(\frac{1}{b-C\, C \frac{V_f}{v_o} u_o + C_2 V_f \frac{u_o}{v_o}}\right)$$

$$= \frac{1}{2} m_r V_f^2\left(\frac{1}{b+(C_2-C_1C_2)\frac{u_oV_f}{v_o}}\right)$$

Mr. C. Bethel found from computation on a series of guns of various calibers that the value $(C_2-C_1C_2)$ could be represented very closely to a linear function of the diameter of the bore, that is

$$C_2-C_1C_2 = (V_fT-E)\frac{v_o}{u_oV_f} = k_1 + k_2d$$

where

d = diameter of the bore. (in)

If

u_o= travel up the bore (in)

v_o= muzzle velocity (ft/sec)

V_f = velocity of free recoil (ft/sec)

b = length of recoil (ft)

then

$$C_2 - C_1C_2 = .098 + .0003\, d$$

and we have, $K = \frac{m_r V_f^2}{2} \cdot \frac{1}{\left[b + (.096 + .0003\ d)\frac{u_o V_f}{v_o}\right]}$ (lbs)

(Bethels Formula)

The formula applies to a constant resistance to recoil and is especially useful, since the computation of E and T are not needed.

GENERAL EQUATIONS OF RECOIL AND COUNTER RECOIL.- RECOIL SYSTEMS. The characteristics and functioning of the various recoil systems may be shown in an unique way by a study of the equations of recoil and counter recoil. Let K = the total resistance to recoil assumed constant throughout the recoil. (in lbs)

P_b = powder pressure on breech

p = the pressure in the recoil brake cylinder. (lbs/sq.in)

A = the effective area of the brake piston.(sq.in)

p_v = the recuperator pressure (lbs/sq.in)

A_v = the effective area of the recuperator piston (sq.in)

$B = pA + p_v A_v$ = the total braking. (in lbs)

R_p = the total packing frictions. (in lbs)

R_g = the total guide friction (in lbs)

$R = R_p + R_g$ = the total recoil friction (in lbs)

$\emptyset$ = angle of elevation of the gun.

X = displacement from battery along the recoil (in ft)

b = total length of recoil (ft)

Then during the recoil

$P_b - K = m_r \frac{dv}{dt}$ during the acceleration

$-K = m_r v \frac{dv}{dt}$ during the retardation.

The external force on the mount during the acceleration is

$$P_b - m_r \frac{dv}{dt} = K,$$ as well as the weight of the recoiling parts, and a couple $P_b d_b$, where d_b = distance from the center of gravity of recoiling parts to the axis of the bore.

During the retardation, the external force on the mount in the duration of recoil is,

$$-m_r v \frac{dv}{dx} = K$$, together with the recoiling weight,

(1) when the brake system is independent of the recuperator system, then

$$K = B + R - w_r \sin \emptyset$$

$$= pA + p_v A_v + R - w_r \sin \emptyset$$

Now the hydraulic pull becomes,

$$pA = \frac{C v^2}{W_x^2}$$

where

v = the velocity of recoil at displacement x (ft/sec)

W_x = the throttling are at displacement x.

Further, with pneumatic or hydropneumatic recuperators,

$$p_v A_v = p_{vi} \quad A_v \left(\frac{V_o}{V_o - A_v x}\right)^k \text{ where } k = 1.1 \text{ to } 1.3$$

and with spring return recuperators

$$p_v A_v = S = S_o + \frac{S_f - S_o}{b} x$$

Hence the total resistance to recoil, becomes, with pneumatic recuperators,

$$K = \frac{cv^2}{W_x^2} + p_{vi} \, A_v \left(\frac{V_o}{V_o - A_v x}\right)^k + R - w_r \sin \emptyset$$

and with spring return recuperators

$$K = \frac{cv^2}{w_x^2} + (S_o + \frac{S_f-S_o}{b} x)+R-W_r\sin \emptyset$$

Thus we have four components in the total resistance to recoil,

(a) The hydraulic throttling component which varies as the square of the velocity,

(b) The elastic reaction which increases as a function of the displacement.

(c) The friction component which for convenience may for a first approximation be assumed constant.

(d) The weight component which exists when the gun is elevated and is subtrative since the weight component acts opposite to the braking forces.

(2) With a recoil system where the brake system is part of or connects with the recuperator system, we have $K = pA+R-W_r\sin \emptyset$, where p is the pressure in the recoil cylinder. Now, due to the throttling through the orifice valve between the two cylinders, we have

$$p - p_a' = \frac{c'v^2}{w_x^2}$$ where p_a' = the recuperator pressure on the oil side of the recuperator cylinder.

v = the velocity of recoil. (ft/sec)

w_x = the opening of the orifice at the recoil x.

Further, the pressure in the recuperator at recoil x, in terms of the initial pressure p_{ai}', becomes,

$$p_a' = p_{ai}' \left(\frac{V_o}{V_o-A_x}\right)^k$$ where k = 1.3.

Hence substituting in the recoil equation,

$$K = [\frac{a'v^2}{w_x^2} + p'_{ai} \left(\frac{V_o}{V_o-Ax}\right)^k A]A+R-W_r \sin \emptyset$$

$$= \frac{cv^2}{w_x^2} + p'_{ai} A\left(\frac{V_o}{V_o-A_x}\right)^k + R-W_r \sin \emptyset$$

which is of identical form as the equation for resistance to recoil, where the recuperator system is independent of the braking system.

Thus again, we may consider this recoil system as having the total resistance to recoil divided into, the hydraulic throttling, the elastic, the functional and the weight components.

It is, however, often more convenient to consider the total resistance as divided into "pressure drops". In considering pressure drops we refer the pressure intensities to the effective area of the recoil piston and thus the friction and weight component drop, becomes,

$$\frac{R-W_r \sin \emptyset}{A} \quad \text{(lbs per sq.in)}$$

If R_f = the floating piston friction and A_a the area of the recuperator, the drop across the floating piston becomes,

$$p_a' - p_a = \frac{R_f}{A_a} \quad \text{(lbs.per sq.in)}$$

Therefore the total resistance to recoil <u>in terms of pressure drops</u>, becomes

$$\frac{K}{A} = p + \frac{R-W_r \sin \emptyset}{A}$$

$$= (p-p_a') + (p_a' - p_a) + p_a + \frac{R-W_r \sin \emptyset}{A}$$

$$= \frac{c'v^2}{w_x^2} + \frac{R_f}{A_a} + \frac{R-W_r \sin \emptyset}{A} + p_a$$

STABILITY CONSIDERATION

Now if

K_h = horizontal resistance to recoil

h = height of center of gravity of recoiling parts above the ground.

W_s = weight of the total

l_s = horizontal distance from spade to center of gravity of W_s with recoiling parts in battery.

W_c = weight of carriage proper (not including recoiling parts)

l_c = horizontal distance from spade to center of gravity of carriage proper.

W_r = weight of recoiling parts.

l_r = horizontal distance from spade to center of gravity of recoiling parts in battery.

c = constant of stability

then since $W_s l_s = W_r l_r + W_c l_c$ for any displacement x, the stabilizing moment becomes, $W_r(l_r - x \cos \emptyset) + W_c l_c = W_s l_s - W_r x$. Therefore, with a given margin of stability, we have, $K_h h = c(W_s l_s - W_r x)$ and hence for a constant margin of stability throughout the recoil at horizontal elevation,

$$K_h = \frac{c\, W_s l_s}{h} - \frac{c\, W_r}{h}$$

That is, the resistance tp recoil at horizontal recoil, should decrease with the recoil consistent with this equation.

When a constant resistance is maintained throughout recoil at horizontal elevation, K_h should be limited consistent with stability in the out of battery position.

Advantage of the total resistance to recoil following the stability slope:

(1) More energy is dissipated by the brake during the powder period, by following the stability slope and thus gives a greater decrease of the recoil displacement during the powder period.

(2) The braking forces being higher during the greater part of the recoil, the remaining energy or energy of constrained recoil, is dissipated in a shorter recoil displacement.

Hence the total recoil displacement is decreased over that with a constant resistance to recoil.

Further, since the stability slope causes a smaller resistance to recoil in the out of battery position with a longer recoil, the total resistance to recoil if maintained constant throughout recoil must be smaller, and the recoil displacement greater for a given energy than when the resistance to recoil follows the stability slope.

The relation can be shown analytically as follows: Assuming a constant resistance to recoil maintained during the powder period and varying with the stability throughout the remaining part of the recoil, we have for a variable resistance to recoil throughout recoil,

$$V_r = V_f - \frac{K_o T}{m_r}; \quad E_r = E - \frac{K_o T^2}{m_r}$$

where K_o = the resistance to recoil maintained constant during the powder period.

Since

$$K_h = \frac{c\, W_s l_s}{h} - \frac{c\, W_r}{h} x$$

the stability slope becomes, $m = -\left(\frac{c\, W_r}{h}\right)$

therefore, the resistance to recoil in the out of battery position becomes, $k = K_o - m(b - E_r)$, we have therefore,

$$\left[K_o - \frac{m}{2}(b - E_r)\right](b - E_r) = \frac{1}{2} m_r V_r^2$$

Substituting for V_r and E_r, we have solving for b,

$$b_v = E + \frac{K_o}{m}\left(1 - \frac{m}{2}\frac{T^2}{m_r}\right) \pm \frac{1}{m}\sqrt{K_o^2\left(1 - \frac{m}{2}\frac{T^2}{m_r}\right) - 2m(A - K_o V_f T)}$$

where $A = \frac{1}{2} m_r V_f^2$ = energy of free recoil

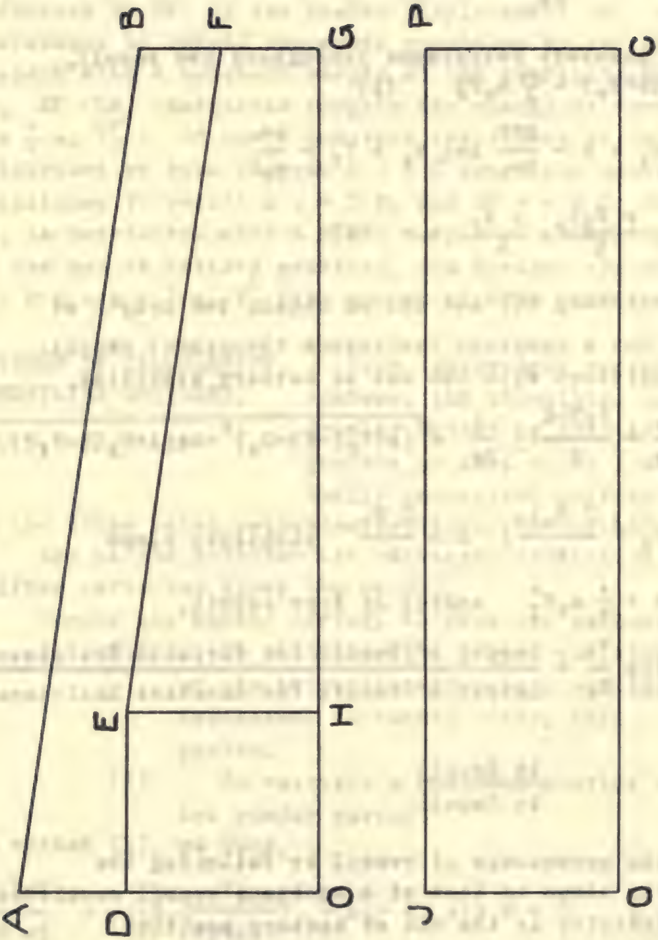


FIG. 2

$$m = \frac{c\ W_r}{h} = \text{stability slope}$$

$$K_o\ (1 - \frac{m\ T^2}{2m_r}) = \frac{c\ W_s l_s}{h}$$

With a constant resistance throughout the recoil,

$$K(b-E_r) = \tfrac{1}{2}\ m_r V_r^2 \qquad (1)$$

where $E_r = E - \dfrac{KT^2}{2m_r}$ and $V_r = V_f - \dfrac{KT}{m_r}$

and $K = \dfrac{c\ W_s l_s}{h} - \dfrac{c\ W_r}{h}\ b \qquad (2)$

Combining (1) and (2) we obtain the length of recoil for a constant resistance throughout recoil, and consistent with the out of battery stability,

$$b_c = \frac{c_s}{2m} - (\frac{V_f T-E}{2}) \pm \frac{1}{2m} \sqrt{[m(V_f T-E)-C_s]^2 - 4m[A+C_s(E-V_f T)]}$$

where

$$C_s = \frac{C\ W_s l_s}{h}; \quad m = \frac{C\ W_r}{h} \quad \text{stability slope}$$

$$A = \tfrac{1}{2}\ m_r V_f^2 \quad \text{energy of free recoil.}$$

The ratio $\dfrac{b_v}{b_c} = \dfrac{\text{Length of Recoil for Variable Resistance to Recoil}}{\text{Length of recoil for constant Resistance to Recoil}}$

gives the percentage of recoil by following the stability slope to that of a constant recoil consistent with stability in the out of battery position.

The relation can be shown graphically, fig.(   ). The ordinates to the line AB represent the maximum possible overturning force consistent with stability. The slope of

$AB = \dfrac{cW_r}{h}$ and the ordinate $oA = \dfrac{C\ W_s l_s}{h}$. Maintaining a constant resistance to recoil during the powder period consistent with stability we have ordinates to DE, in the powder displacement oH. The resistance to recoil decreases according to the EF consistent with a constant margin of the stability. The area OD, EF Go, represents roughly the energy of recoil $A = \frac{1}{2} m_r V_f^2$. If now a constant resistance is to be maintained we have diagram o J P C where the constant resistance to recoil o J = C P, and CP = c B C, that is, is consistent with a given margin of stability in the out of battery position, and further the area $o\ J\ P\ C = A = \frac{1}{2} m_r V_f^2$ (the energy of free recoil).

METHODS OF CALCULATING THROTTLING ORIFICES. With independent recuperator systems, the throttling is usually either by throttling grooves or bars or by a mechanically controlled orifice as in the Krupp valve mechanism previously described.

Let us now consider the necessary throttling orifice variation along the recoil.

During the powder period, we have two methods,

(1) To maintain by a proper variation of the throttling grooves a constant resistance to recoil during this period.

(2) To maintain a constant orifice during the powder period.

In method (1), we have,

$$\frac{C^2 A^3 V^2}{175\ w_x^2} + p_{ai} A_v \left(\frac{V_o}{V_o - AX}\right)^k + R_t - W_r \sin \emptyset = K$$

A constant during the powder period. Therefore

$$w_x = 13.2 \frac{C\ A^{\frac{3}{2}}\ V}{\sqrt{K - p_a - R_t + W_r \sin \emptyset}}$$

where $K = \dfrac{\frac{1}{2} m_r V_f^2}{b-E+V_f T}$ for a constant resistance to recoil.

and

$$K = \frac{m_r V_f^2 + m(b-E)^2}{2\left[b-E+V_f T - \frac{m}{2}\frac{T^2}{m_r}(b-E)\right]}$$ for a variable resistance to recoil.

$p_a = p_{ai} A_v \left(\dfrac{V_o}{V_o - AX}\right)^k = p_{ai} A_v$ approx. during powder period unless the recoil is relatively short.

$R_t = R_g + R_p$ = total friction: guide friction + packing friction.

Further from interior ballistics, av. total powder pressure

$$p_e = \frac{w\, v_o^2}{2g\, u}$$

w = weight of shell
v_o = muzzle vel. (ft/sec)
u = total travel up bore (ft)

Initial pressure on breech in gas expansion $p_{ob} = \dfrac{27}{4} c^2 \dfrac{u}{(b+u)^2}\, 1.12\, p_m$ (lbs) where p_m = total powder pressure. (lbs)

and

$$c = u\left(\frac{27}{16}\frac{p_m}{p_e} - 1\right) \pm \sqrt{\left(1 - \frac{27}{16}\frac{p_m}{p_e}\right)^2 - 1}$$

$t_o = \dfrac{3}{2}\dfrac{u}{v_o}$ approx. $\qquad V_f = \dfrac{w v_o + 4700\, \bar{w}}{W_r}$

$t_{1o} = \dfrac{2(V_f - V_{fo})}{p_{ob}}\dfrac{W_r}{g} \qquad V_{fo} = \dfrac{(W + 0.5\, \bar{w})v_o}{W_r}$

$T = t_o + t_{1o}$ = total powder period (sec)

$$X_{fo} = \frac{W+0.5\bar{W}}{W_r} u \text{ and } X_{f_o'} = \frac{p_{ob}}{W_r} g \frac{(T-t_o)^2}{3} + V_{fo}(T-t_o)$$

and $E = X_{fo} + X_{f_o'}$ + total free recoil during powder period. (ft)

Three points are sufficient for the orifice curve during the powder period and the corresponding constrained velocity and displacement to substitute in the orifice equation with its lay out are:

$$\left.\begin{aligned} V_f &= V_{fo} - \frac{K\ t_o}{m_r} \text{ (ft/sec)} \\ X_o &= X_{fo} - \frac{K\ t_o^2}{2m_r} \text{ (ft)} \end{aligned}\right\} \text{when the shot leaves the muzzle.}$$

$$\left.\begin{aligned} V_m &= V_{fm} - \frac{Kt_m}{m_r} \text{ (ft/sec)} \\ X_m &= X_{fm} - \frac{Kt_m^2}{2m_r} \text{ (ft)} \end{aligned}\right\} \text{the maximum restrained recoil velocity and corresponding orifice.}$$

where

$$V_{fm} = V_{fo} + \frac{p_{ob}}{m_r}(t_m-t_o)[1 - \frac{p_{ob}(t_m-t_o)}{4m_r(V_f-V_{fo})}] \text{ (ft/sec)}$$

$$X_{fm} = X_{fo}+[V_{fo} + \frac{p_{ob}}{m_r}(t_m-t_o) - \frac{(t_m - t_o)^2}{6m_r(V_f-V_o)}](t_m-t_o) \text{ (ft)}$$

and

$$t_m = T - \frac{K(T-t_o)}{p_{ob}} \text{ sec.}$$

$$\left.\begin{aligned} V_r &= V_f - \frac{Kt}{m_r} \text{ (ft/sec)} \\ E_r &= X_f - \frac{Kt^2}{2m_r} \text{ (ft)} \end{aligned}\right\} \text{At the end of the powder period.}$$

After the powder period, that is during the retardation period, we have for a constant resistance to recoil, simply,

$$V_x = \sqrt{\frac{2K}{m_r}(b-x)}$$

and therefore

$$w_x = \frac{CA^{\frac{3}{2}}\sqrt{\frac{2K}{m_r}(b-x)}}{13.2\sqrt{K-p_a-R_t+W_r\sin\phi}} \quad \text{(sq.in)}$$

which gives up the required throttling with a constant resistance to recoil during the retardation period of recoil.

When the resistance to recoil is variable, we have during the retardation period, that

$$\tfrac{1}{2}\, m_r V_x^2 = \left[K - \frac{m}{2}(X+b-2E_r)\right](b-x)$$

$$\text{hence } V_x = \sqrt{\frac{\left[K - \frac{m}{2}(b+X-2E_r)\right](b-x)}{m_r}}$$

and therefore

$$w_x = \frac{CA^{\frac{3}{2}}\sqrt{\frac{2\left[K - \frac{m}{2}(b+X-2E_r)\right](b-x)}{m_r}}}{13.2\sqrt{K-p_a-R_t+W_r\sin\phi}} \quad \text{(sq.in)}$$

which gives the required throttling with a variable resistance to recoil during the retardation period.

With a pneumatic or hydro pneumatic recuperator system,

$$p_a = p_{ai}A_v\left(\frac{V_o}{V_o-A_vX}\right)^k$$

where $k = 1.1$ to 1.3
V_o = initial volume.

With a spring return recuperator, $p_a = S_o = \frac{S_t - S_o}{b} X$

b = length of recoil (ft) where S_o = initial compression of the springs (lbs)

S_t = final compression of the springs (lbs) and

$$\frac{S_t}{S_o} = 2 \text{ approx.}$$

In method (2), with a constant orifice during the powder period, we have

$$p_b - \frac{C^2 A^3 V^2}{175\, w_o^2} - p_a - R_t + W_r \sin \phi = m_r \frac{dV}{dt}$$

Since an integration of this equation is complicated an approximation is made by assuming the recoiling mass to recoil during the powder period "a" given multiple distance of the free recoil displacement when the shot leaves the bore. Let

E_r = length of constrained recoil during powder period, and corresponding length of constant orifice (inches)

u = total travel of shot up bore (inches)

$\bar{a}$ = constant from (2 to 2.5) use 2.24

$\bar{W}$ = weight of powder charge (lbs)

W = weight of projectile (lbs)

P_h = total hydraulic pull (lbs)

w_x = area of orifice (sq.in) at recoil displacement x (in)

A = effective arc of hydraulic recoil piston (sq.in)

c = coefficient of contraction - - - 0.85 to 0.9

d_o = S. G. of fluid = 0.849

Then

$$E_r = \bar{a} \frac{w + \frac{\bar{W}}{2}}{\bar{w}_r} u \qquad (1)$$

Now the total drop in pressure through the recoil orifice, becomes,

$$p = \frac{\frac{1}{2}(d_o\ 62.5)A^2\ V_x^2}{gc^2\ w_x^2}$$ (lbs per sq.ft)(See Hydro dynamics)

or

$$p = \frac{62.5\ d_o}{64.4 \times 144}\ \frac{A^2\ V_x^2}{c^2 w_x^2}$$ (lbs. per sq.in)

During the retardation period of the recoil, we have from the equation of energy,

$$\frac{K(b-x)}{12} = \frac{1}{2}\ \frac{W_r}{g}\ V_x^2 \qquad \text{hence } V_x^2 = \frac{64.4}{12}\ \frac{K(b-x)}{W_r}$$

therefore,

$$p = \frac{62.5\ d_o A^2 K(b-x)}{12 \times 144\ c^2 w_x^2\ W_r}$$

hence

$$w_x^2 = .0361\ \frac{d_o A^3 K(b-x)}{W_r c^2\ P_h} \qquad (2)$$

and

$$w_o^2 = .0361\ \frac{d_o A^3 K(b-E_r)}{W_r\ c^2\ P_h} \qquad (3)$$

which gives the orifice at any displacement x in terms of the total resistance to recoil, K and the total hydraulic pull P_h.

When the resistance to recoil is made to conform with the stability slope, we have

$$V_x^2 = \frac{64.4[K-0.5m(b+X-2E_r)](b-X)}{12\ W_r}$$

$$p = \frac{62.5\ d_o}{12 \times 144}\ \frac{A^3[K-0.5m(b+X-2E_r)](b-X)}{c^2\ w_x^2\ W_r}$$

hence $$w_x^2 = .0361 \frac{d_o A^2 [K-0.5m(b+X-2E_r)](b-X)}{C^2 \; W_r \; P_h}$$

and

$$w_o^2 = .0361 \frac{d_o A^2 [K-0.5m(b+X-2E_r)](b-E_r)}{C^2 \; W_r \; P_h}$$

Further

$$P_h = K + W_r \sin \phi - R_t - p_{ai} \; A_v (\frac{V_o}{V_o - AX})^k$$

for pneumatic or hydro pneumatic recuperators, and

$$P_h = K + W_r \sin \phi - R_t - (S_o + \frac{S_f - S_o}{b} x)$$

for spring return recuperators.

METHODS OF THROTTLING (1) The simplest method of throttling is by varying an orifice through the piston by throttling bars fixed to the recoil cylinder and moving in the apertures through the piston. Let

w_x = the throttling area (sq.in) as previously calculated.

S = width of throttling bar or whole in piston (inches)

h = depth of hole in piston from cylinder surface (in)

d = depth of throttling bar (inches)

d_o = initial of bar (inches)

n = number of bars (usually n = 2

Then the initial or maximum opening

w_o = nX(h-d_o)(sq.in) approx. and for any other point in the recoil,

w_x = nS(h-d) (sq.in) approx,

With grooves in the cylinder wall.

w_x = n S d (sq.in) where d = depth of rectangular groove (in)

(2) When the throttling takes place around a long buffer rod of varying diameter and passing through a circular hole in the piston, as in the Schneider material, we have, if

D = diam. of hole in cylinder (sq.in)

d_x = diam. of buffer rod passing through hole in cylinder (sq.in)

then

$w_x = 0.7854(D^2-d_x^2)$ which gives the variation of the diameter of the buffer rod along the recoil. In the Filloux recoil mechanism, we find grooves of varying depth in the buffer rod, engaging with holes through the piston. The object of this arrangement is to pass from one set of grooves to another by turning the buffer rod on elevating the gun, thus making it possible to shorten the recoil on the elevating the gun.

If n = number of grooves engaged during the recoil.

s = width of groove (in) and d = depth of groove (in)

then $w_x = n\,s\,d$.

(3) When a constant orifice is maintained during the powder period we may use the so called Krupp valve, which has had a wide application for artillery brakes.

The initial orifice is closed uniformly by a disk on the piston rotated by a helicoidal groove in the cylinder wall of constant pitch. Let

$\emptyset_0$ = initial angle moved by valve disk during powder period before engaging the throttling area in the piston.

$\emptyset_1$ = angle moved by valve during the retardation period.

p = pitch of helicoidal groove in cylinder wall (inches)(Linear displacement per complete

KRUPP RECOIL VALVE

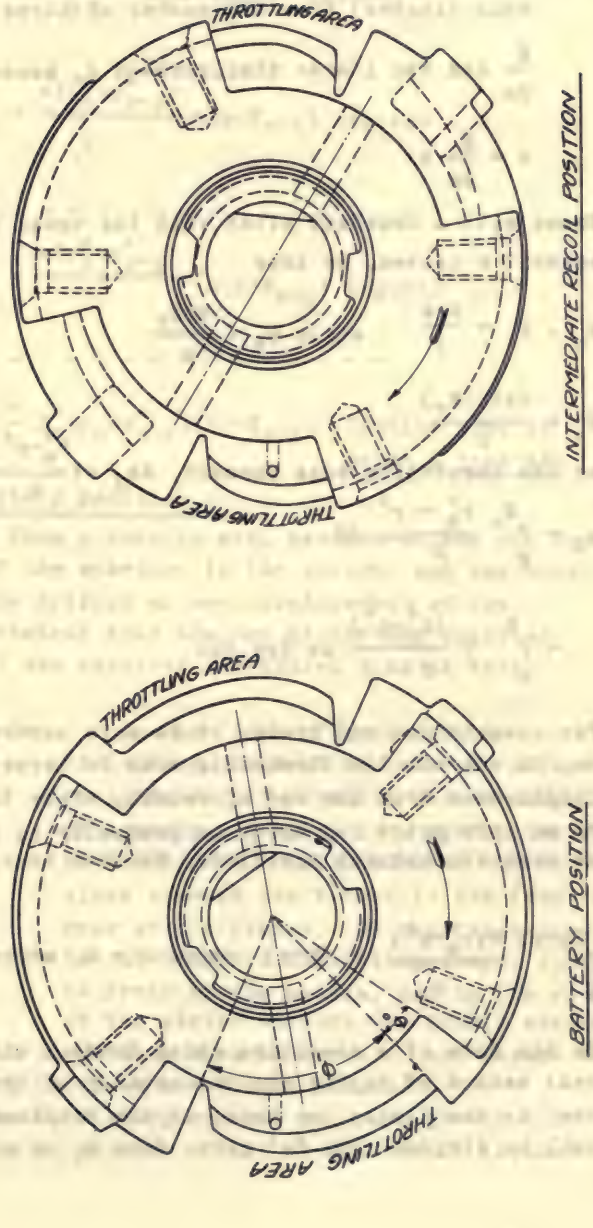


Fig. 3

revolution of disk.)

r_o = radius of cylinder (inches)

r = radius to bottom of throtting opening contour (inches) then the number of turns = $\frac{\phi}{2\pi}$ and the linear displacement x, becomes,

$$x = \frac{\phi}{2\pi} p$$

Hence with a constant pitch with the total recoil displacement b inches, we have

$$\phi_o + \phi_1 = \frac{2\pi b}{p} \quad \text{while } \phi_o = \frac{2\pi E_r}{p}$$

hence

$$\phi_1 = \frac{2\pi(b-E_r)}{p}$$

Further the throttling area becomes, $dw_x = (\frac{r_o^2-r^2}{2})d\phi$

hence

$$w_x = \int_{\phi}^{\phi_1} \frac{r_o^2 - r^2}{2} d\phi$$

$$= \int_x^b \pi \frac{(r_o^2-r^2)}{p} dx \text{ (sq.in)}$$

For computation and design it is more convenient however, to express the throttling area in terms of the displacement from the end of recoil, since the area is zero at this point and opens up gradually to its maximum near the battery position. We have then,

$$w_x = \int_b^{(b-x)} \frac{\pi(r_o^2-r^2)}{p} d\,(b-x) \quad \text{where } r = 0, \text{ where } x = b.$$

In the form of a summation which lends a simple practical method of laying out the contour of the aperture in the piston, we have, if the displacement of recoil is divided into "n" parts from E_r to b,

Starting from the out of battery position,
we have,

$$w_n = \frac{\pi(r_o^2 - r_{n-1}^2)}{p}\Delta(b - X_{n-1}) \quad \text{(sq.in)}$$

$$w_{n-1} = \frac{\pi(r_o^2 - r_{n-2}^2)}{p}\Delta(b - X_{n-2}) \quad \text{(sq.in)}$$

- - - - - - - - - - - - - - - - - - - -

$$w_{n-g} = \frac{\pi(r_o^2 - r_{n-g}^2)}{p}\Delta(b - X_{n-g}) \quad \text{(sq.in)}$$

- - - - - - - - - - - - - - - - - - -

and

$$w_g = \frac{\pi}{p}\Sigma_o^g(r_o^2 - r_{n-g}^2)\Delta(b - X_{n-g})$$ Orifice area at point g <u>from the out of battery position.</u>

Thus from a step by step process we lay out the contour of the aperture in the piston, and the total area of the orifice at any displacement of the recoil, measured from the out of battery position, must equal the required throttling area at this point.

(4) Another form of geometrical throttling, devised in order to effect variable recoil consists essentially of cutting off holes in a perforated sleeve by the piston, the throttling taking place through the sleeve in the front and rear of the piston. We have therefore two distinctive throttling drops, that in front of the piston, and to the rear of the piston through the recoil sleeve.

If w_x = the throttling area in front of the piston at any point in the recoil (sq.in)

w_y = the throttling area to the rear of the piston at any two points in the recoil (sq.in)

p_x = the drop of pressure through the throttling areas w_x, in the sleeve, (lbs/sq.in)

p_y = the drop of pressure through the throttling areas w_y, in the sleeve (lbs/sq.in)

We have for the total drop P

$$P = P_x + P_y$$

$$= \frac{C^2A^2V_x^2}{175\ w_x^2} + \frac{C^2A^2V_y^2}{175\ w_y^2}$$ (assuming the throttling constant C the same)

hence

$$P = \frac{C^2A^2V_x^2}{175}\left(\frac{1}{w_x^2} + \frac{1}{w_y^2}\right)$$

$$= \frac{C^2A^2V_x^2}{175}\ \frac{1}{w_c^2}$$

where w_c is the equivalent throttling area and corresponds to the area obtained in the previous throttling area calculations.

In general
$$\frac{1}{w_c^2} = \frac{1}{w_1^2} + \frac{1}{w_2^2} + - - - \frac{1}{w_n^2}$$

when we have a drop of pressure due to throttling through various orifices in series.

With only two throttling drops, we have

$$w_c = \frac{w_x\ w_y}{\sqrt{w_x^2 + w_y^2}}$$ and $w_x + w_y$ = constant.

From these two equations, we have at the maximum value of w_c,

$$w_x = w_y$$

Hence in laying out the holes in a sleeve valve, we place the piston at its displacement corresponding to maximum throttling, that is at the point of the

maximum retarded or constrained velocity, making the throttling drop on either side equal.

The process of laying out the required orifices and corresponding holes is as follows:

(a) At max. throttling displacement corresponding to max. retarded velocity in the recoil,

$P_x = P_y = \frac{P}{2}$ and $w_x = w_y$ but since we have a void in back of the piston due to the displacement of the piston rod, $P = p$ i.e. the total drop = the pressure in the recoil cylinder.

$$\therefore \quad w_x = w_y = \frac{C\ A\ V_{xm}}{13.2\sqrt{\frac{p}{2}}} \qquad C = \frac{1}{0.8} \text{ approx.}$$

and

$$w_x + w_y = \frac{2\ C\ A\ V_{xm}}{13.2\sqrt{\frac{p}{2}}} = w_c$$

(b) Next move the piston from the position of max. velocity, a unit distance equal to the width of the piston in the direction of recoil.

The area to the rear = w_y =

$\frac{w_c}{2}$, since no openings have been uncovered in the rear.

The area to the front is obtained from the equations, $\frac{1}{w_{c_1}^2} = \frac{1}{w_{x_1}^2} + \frac{1}{w_y^2}$ where w_{co}, w_{xo} etc. are the throttling areas at max. velocity and w_{c_1} w_{x_1} etc are the throttling areas at a distance from the position of max. velocity equal to the first unit displacement, hence

$$w_{x_1} = \frac{w_{c_1} w_y}{\sqrt{w_y^2 - w_{c_1}^2}} \quad \text{(sq.in)}$$

(c) Next move the piston another unit distance in the direction of recoil, the area to the rear,

$$w_{y_2} = \frac{w_c}{2} + \left(\frac{w_c}{2} - w_{x_1}\right) = w_c - w_{x_1} \quad \text{(sq.in)}$$

hence

$$w_{x_2} = \frac{w_{c_2} w_{y_2}}{\sqrt{w_{y_2}^2 - w_{c_2}^2}} \quad \text{(sq.in)}$$

(d) Hence for all succeeding points in the recoil, $w_{yg} = w_c - w_{yg}$ and

$$w_{xg} = \frac{w_{cg} w_{yg}}{\sqrt{w_{yg}^2 - w_{cg}^2}}$$

(e) In the powder pressure period, we move the piston backward towards the battery position from the position of maximum velocity succeeding units to the rear and the process is exactly similar as moving forward in the direction of recoil.

THROTTLING THROUGH A SPRING CONTROLLED VALVE.

With dependent recuperator systems, as in the St. Chamond recoil system, the drop of pressure between the two cylinders (i. e. between the recoil brake and recuperator cylinders= may be obtained by throttling through a spring controlled orifice between the two cylinders. A spring valve, however, may be used with an ordinary recoil brake cylinder, the throttling taking place through a spring controlled orifice in the piston.

Let p = the pressure in the recoil cylinder (lbs/sq.in)

a = the area at base of valve (sq.in)

p_a = pressure in receiving chamber or recuperator (lbs/sq.in)

p_{ai} = initial pressure in recuperator (lbs/sq.in)

p_{af} = final pressure in recuperator (lbs/sq.in)

A_a = effective area at top of valve (sq.in)

a_1 = area of valve stem

S_o = initial spring compression (lbs)

S_f = final spring compression (lbs)

A = effective area recoil piston

h = lift of valve in inches

c = effective circumference at valve opening

Then, at the maximum opening, giving a lift h, we have $p\,a - p_{ai}A_a = S_f$ (lbs) (approx) and when the valve is about closed, $p\,a - p_{af}A_a = S_o$ (lbs)(approx) When $A_a = A$, as with valves in which the spring is entirely enclosed in the recuperator chamber, we have $(p-p_{ai})a=S_f$ when open (approx) and $(p-p_{af})a=S_o$ when closed (approx.)

When the spring is outside of the recuperator chamber, and a valve stem passes through a stuffing box to the outside of the chamber, we have

$p\,a - p_aA_a = pa-p_a(a-a_1)=(p-p_a)a+p_aa_1$ (lbs)

Further at maximum opening of the valve we have for maximum throttling

$$p-p_{ai} = \frac{C_o^2A^2V^2}{175\,C^2h^2} \quad \text{where } C_o = \frac{1}{0.6} \text{ to } \frac{1}{0.8}$$

hence

$$h = \frac{C_oA\,V}{13.2c\sqrt{p-p_a}}$$

which gives the lift of the valve at max. opening and corresponding to a spring reaction = S_f lbs.

Therefore knowing p, p_{ai} and p_{af} and solving for the total lift h, we have, for the spring required:

Initial load ....S_o (lbs)

Final load ....S_f (lbs)

Total lift ....h (in)

Spring constant ....$\dfrac{S_f-S_o}{h}$ (lbs per in)

which completely specifies the spring required to properly function the valve during the recoil.

Now the pressure in the recoil cylinder, is

$$p = \frac{K+W_r \sin \phi\, R_t}{A} \qquad \text{(lbs/sq.in)}$$

and in the recuperator cylinder, $p_a = p_{ai}\left(\dfrac{V_o}{V_o-AX}\right)^k$

(lbs/sq.in)

The maximum throttling opening occurs, at displacement X_m in the recoil, that is at the point during the powder pressure period, where the powder reaction just balances the recoil reaction. This is slightly before the end of the powder period and for an approximation we have,

$$X_m = \frac{\bar{a}_1 (w+\frac{\bar{W}}{2})u}{w_r} \qquad \text{where } \bar{a}_1 = 2 \text{ approx.}$$

Further the maximum constrained velocity may be taken at, $V_r = g\, V_f$ where $g = 0.88$ at short recoil
$= 0.92$ at long recoil.

Therefore at maximum opening of the valve (lift h") we have,

$$\left(\frac{K+W_r \sin \phi - R_t}{A}\right)a - p_{ai}\left(\frac{V_o}{V-AX_m}\right)^k A_a = S_f$$

and at the end of recoil,

$$\left(\frac{K+W_r \sin \phi - R_t}{A}\right)a - p_{ai}\left(\frac{V_o}{V_o - Ab}\right)^k A_a = S_o$$

Now due to the hydraulic throttling,

$$h = \frac{C_o\ A\ V_r}{13.2\sqrt{\frac{K+W_r\ \sin\phi - R_t}{A} - p_{ai}\frac{V_o}{V_o - AX_m}}}$$

Thus we have a complete specification for the design of the spring. If now, $p_s = K_s + W_r \sin\phi - R_t$ = pull at short recoil, max. elev. (lbs), $P_h = K_h - R_t$ = pull at long recoil, zero elev. (lbs) F_{vi} = initial recuperator reaction, required to hold gun at max. elev. in battery (lbs), $F_{vf} = m\ F_{vi}$ = final recuperator reaction at the end of recoil (lbs)

We have, at short recoil, max. elevation, at the beginning of recoil,

$$S_f = \frac{p_s}{A}a - \frac{F_{vi}}{A}a \text{ (lbs)}$$

$$= \frac{p_s - F_{vi}}{A}a + \frac{F_{vi}}{A}a_1 \text{ (lbs) with springs}$$

functioning outside recuperator chamber, and at the end of recoil,

$$S_o = \frac{p_s}{A}a - \frac{mF_{vi}}{A}A_a \quad \text{(lbs)}$$

$$= \frac{p_s - m\ F_{vi}}{A}a\ \frac{mF_{vi}}{A}a_1 \text{ (lbs) with springs functioning out-}$$

side recuperator chamber.

The corresponding max. lift at short recoil becomes,

$$h = \frac{C_o A^{\frac{3}{2}}\ V}{13.2\ c\ \sqrt{p_s - F_{vi}}} \qquad \text{now } S_f - S_o = \frac{F_{vi}}{A}a_a(m-1)$$

and the spring constant, lbs. per linear inch, becomes,

$$S = \frac{S_f - S_o}{h} = \frac{13.2\ c\ A_a(m-1)F_{vi}\sqrt{p_s - F_{vi}}}{C_o A^{\frac{3}{2}}\ V} \qquad \text{(lbs.per in)}$$

From the above equations, we see, therefore, that the load on the spring is large at short recoil and proportional to the difference of the pull at max. elevation and the initial recuperator reaction and this load is increased proportionally to the valve stem area and load on the air. Therefore to decrease the load on the springs, the valve stem should be made as small as possible, only sufficient to carry the spring load. The lift varies inversely as the square root of the difference of the pull at max. elevation and the recuperator reaction, and when this difference is large as in short recoil, the lift is proportionally small. Finally the spring constant (that is the slope of the load - deflection chart) increases with the load on the air and with the square root of the difference of the max. pull and the initial air recuperator reaction. On the other hand, if the compression ratio is low, approaching 1, or if the annular area or the effective area on top of the valve is small, that is, using a large valve stem, we must have a spring of considerable deflection for a given change in load. When $A_a = 0$, or $F_{vi} = 0$, we have no change in load in the spring and the valve would open a given lift h, with a corresponding spring reaction, As the gun recoils, if the lift and corresponding throttling area remained constant, the pressure would drop proportionally to the square of the velocity. This, therefore, causes a gradual closing of the valve since the spring reaction must decrease, and we have a throttling in between an ideal spring controlled orifice and that with a constant orifice. Even with this arrangement we have a vast improvement over that of a constant orifice and the peak in the throttling is greatly reduced.

Now, at long recoil, horizontal elevation, at the beginning of recoil,

$$S_f = \frac{P_h}{A} a - \frac{F_{vi}}{A} A_a \text{ (lbs)}$$

$$= \frac{P_h - F_{vi}}{A} a \text{ (lbs)}$$ with spring functioning inside recuperator chamber as is usually the case at long recoil (See St. Chamond Chapter), at the end of recoil,

$$S_o = \frac{P_h}{A} a - \frac{mF_{vi}}{A} A_a \text{ (lbs)}$$

$$= \frac{P_h - mF_{vi}}{A} a \text{ (lbs)}$$ with spring functioning inside recuperator chamber.

The corresponding max. lift at long recoil, becomes,

$$h = \frac{C_o a^{\frac{3}{2}} V}{13.2 c \sqrt{P_h - F_{vi}}} \text{ (inches)}$$

Further

$$S_f - S_o = \frac{F_{vi}}{A} a (m-1) \text{ (lbs)}$$ and the spring constant, lbs. per linear inch, becomes,

$$S = \frac{S_f - S_o}{h} = \frac{13.2 c\, a(m-1) F_{vi} \sqrt{P_h - F_{vi}}}{C_o a^{\frac{3}{2}} V}$$

From these equations we see the load on the springs is relatively small as compared with short recoil, the deflection h large and the spring constant small.

Thus, in comparing the requirements of spring characteristics at short and long recoil respectively, we have,

(1) Short recoil and max. elev. = A large spring reaction and small deflection with a spring constant having a steep load deflection slope.

(2) Long recoil and horizontal elev.= A small spring reaction and large deflection with a spring constant having a small load deflection slope.

To meet the requirements of (1) in the St. Chamond recoil system we find Belleville spring used; and in (2) the use of a weak spiral spring.

When a spring valve is used without a recuperator, the spring valve is usually located in the piston of the hydraulic cylinder. In the design and working of this valve the following points are important:

Let

P_{hi} = the initial hydraulic pull (lbs)

P_{hf} = the final hydraulic pull (lbs)

A = the effective area of the recoil piston (sq.in)

a = the area at the base of valve (sq.in)

P_{ai} = initial recuperator reaction

P_{af} = final recuperator reaction

R_t = total recoil friction

Then $P_{hi} + P_{ai} + R_t - W_r \sin \emptyset = K$ at the beginning of recoil, and

$P_{hf} + P_{af} + R_t - W_r \sin \emptyset = K$ at the end of recoil,

hence $P_{hi} = K + W_r \sin \emptyset - R_t - P_{ai}$: $P_{hf} = K + W_r \sin \emptyset - R_t - R_{af}$.

At the beginning of recoil,

$S_f = \frac{P_{hi}}{A} a$ (lbs) the pressure in the back of the valve being negligible.

At the end of recoil,

$S_o = \frac{P_{hf}}{A} a$ (lbs)

The throttling at the beginning of recoil, becomes

$$b = \frac{C_o A^{\frac{3}{2}} V}{13.2\, c \sqrt{P_{hi}}}$$

Further

$S_f - S_o = \frac{P_{af} - P_{ai}}{A} a$ (lbs) and the spring constant, becomes,

$$S = \frac{S_f - S_o}{h} = \frac{13.2\, a\, c\, \sqrt{P_{hi}(P_{af} - P_{ai})}}{C_o\, A^{\frac{3}{2}}\, V}$$

The above equations show that the maximum load on the spring depends upon the maximum hydraulic load, the assembled load on the minimum hydraulic load at the end of recoil, the lift varying inversely as the square root of the maximum hydraulic load and the spring constant or the compression deflection slope of the spring being proportional to the difference between the final air and initial recuperator reaction and the square root of the maximum hydraulic reaction.

The spring throttling valve has been used successfully with an ordinary hydraulic recoil brake cylinder, designed for approximately constant pull throughout the total recoil as in the lower brake cylinders of a double recoil system or in the brake cylinders of a gun or sliding carriage mount. Of course it is impossible to maintain an absolute constant braking resistance throughout the recoil as previously discussed but a sufficient approximation can be obtained to justify its use.

In the design of constant braking with a spring control, we have a spring valve seated in the piston.

If the throttling takes place mainly through the valve seat, we have $p\, a = S_o + S\, h$ where p = pressure in the recoil cylinder, (lbs/sq.in)

S_o = initial spring load (assembled load)(lbs)

S = spring constant (lbs/in)

a = the effective area at the base of the valve.

h = lift of valve (inches)

Now

$$h = \frac{C_o A\, V}{13.2 c \sqrt{p}}, \quad C_o = \frac{1}{0.6} \text{ to } \frac{1}{0.8}$$

If the valve is bevelled the throttling area becomes in place of $c\, h$,

$$w = \pi\, D\, h \sin \phi$$

where D = mean diam. of the bevel portion of the valve (in)

$\varnothing$ = angle of bevel measured with respect to the central axis of the valve,

hence

$$h = \frac{C_o\ A\ V}{13.2\ \pi\ D \sin\varnothing \sqrt{p}},\quad C_o = \frac{1}{0.6} \text{ to } \frac{1}{0.8}$$

To design the spring we may adjust S_o to give a suitable value of the spring constant S, by the formula,-

$$S = \frac{pa - S_o}{h}$$

RECOIL THROTTLING WITH A "FILLING IN" COUNTER RECOIL BUFFER. When a buffer or regulator is desired to act throughout the counter recoil, the counter recoil buffer chamber must be filled during the recoil.

The filling of the counter recoil buffer chamber during the recoil, affects the recoil throttling in two ways:

(1) The total oil displaced by the recoil piston does not pass through the recoil throttling grooves: a part passing into the buffer chamber in the process of filling it in the recoil.

(2) In the buffer chamber, we have more or less pressure during a part of the recoil, since if the throttling into the buffer chamber is just sufficient to fill during the max. vel. of recoil, we will have if the pressure in the recoil cylinder remains constant an over filling during the latter part of recoil and therefore pressure in the buffer chamber, since the throttling drop is decreased due to the decreased velocity of recoil.

Therefore the total hydraulic reaction on the piston rod is somewhat modified.

Let p = intensity of pressure in recoil cylinder (lbs/sq.in)

A = effective area in recoil piston (sq.in)

A_b = effective area of buffer (sq.in)

V_x = recoil velocity (ft.sec)

w_x = recoil throttling area (sq.in)

a_o = entrance throttling area for filling buffer chamber in the recoil (sq.in)

P_b = intensity of pressure in buffer chamber (lbs/sq.in)

Then, during the recoil, we have, for the tension in the rod "P_h"

$$P_h = p\,A - p_b\,A_b \quad \text{(lbs)} \quad (1)$$

The drop of pressure due to throttling through the filling in hole to the buffer chamber, becomes for continuous filling,

$$P_b = p - p_b = \frac{C_o'^2\,A_b^2\,V_x^2}{175\,a_o^2} \quad \text{(lbs/sq.in)} \quad (2)$$

hence

$$P_h = p(A-A_b) + \frac{C_o'^2\,A_b^3\,V_x^2}{175\,a_o^2} \quad \text{(lbs)} \quad (3)$$

$$p = \frac{P_h - \dfrac{C_o'^2\,A_b^3 V_x^2}{175\,a_o^2}}{A - A_b} \quad \text{(lbs/sq.in)} \quad (4)$$

Further, with continuous filling, we have, for the velocity through the recoil throttling orifice,

$$v_x = \frac{(A-A_b)\,V_x}{w_x} \quad \text{(ft/sec)} \quad (5)$$

and therefore,

$$p = \frac{C^2(A-A_b)^2\,V_x^2}{175\,w_x^2} \quad \text{(lbs/sq.in)} \quad (6)$$

Combining (4) and (6) we have,

$$w_x = \frac{C_o(A-A_b)^{\frac{3}{2}}\,V_x}{13.2\sqrt{P_h - \dfrac{C_o'^2 A_o^2 V_x^2}{175\,a_o^2}}}$$

which gives the required recoil throttling area (assuming a density of the liquid = 53 lbs. cu.ft.) in terms of the total pull P_h, the recoil constrained velocity V_x and the constant filling in entrance area to the buffer chamber a_o.

If the density of the liquid is different from that of hydroline oil = 53 lbs/cu.ft.we have,

$$w_x = \sqrt{\frac{C_o^2 D(A-A_b)^3 V_x^2}{288g\left(P_h - \dfrac{C_o'^2 D A_b^2 V_x^2}{288\,g\,a_o^2}\right)}} \quad \text{(sq.in)}$$

$$= \frac{C_o V_x}{12}\sqrt{\frac{D(A-A_b)^3}{2g\left(P_h - \dfrac{C_o'^2 D A_b^2 V_x^2}{288\,g\,a_o^2}\right)}} \quad \text{(sq.in)}$$

where D = weight of liquid per cu. ft.

If we have several contractions in the filling in passage to the buffer chamber, we have approximately assuming the same contraction factor for the flow C_o',

$$\frac{1}{a_o^2} = \frac{1}{a_1^2} + \frac{1}{a_2^2} + - - - - \frac{1}{a_n^2}$$

Determination of a_o:

If we desire a continuous filling of the counter

recoil buffer chamber during the recoil, with a constant entrance throttling area for filling the buffer chamber, we must design a_b for throttling at maximum velocity of recoil, since the throttling drop varies with the square of the velocity and is a maximum at maximum velocity, and the pressure in the recoil cylinder remains approximately constant during the recoil.

If now, the throttling drop is just equal to the pressure in the recoil cylinder at maximum velocity, since the throttling drop is less at all other velocities and the pressure head the same, we have a pressure in the buffer chamber continuously rising during the latter part of the recoil.

If the throttling drop at maximum velocity is less than the pressure head in the recoil cylinder, we have a void in the buffer chamber during the first part of recoil when the velocity of recoil is large, and therefore, not <u>continuous filling</u>.

For continuous filling, therefore $p_{max} \geq p_b$ at max. vel. of recoil and therefore max. recoil pressure, that is

$$p_{max} \geq \frac{C_o'^2 A_b^2 V_{max}^2}{175\, a_o^2} \quad \text{hence } a_o \geq \frac{C_o' A_b V_{max}}{13.2\sqrt{p_{max}}}, \text{(sq.in)}$$

which gives the proper entrance throttling area required for filling the buffer continuously during the recoil.

Since, however, the buffer over fills during the greater part of the remainder of recoil, a_o can be made smaller than required for a <u>continuous filling throughout the recoil</u> and yet have a complete filling of the buffer chamber. In order that the buffer chamber may completely fill, (though not continuously throughout the recoil) we have, for the time of recoil, roughly

$$t = \frac{m_r V}{pA} \qquad \text{approx.}$$

and assuming the pressure in the buffer chamber at any time of the recoil small,

$$p = \frac{v^2}{2g} D \quad \text{(lbs.per sq,ft)}$$

For the filling of the buffer chamber,

$$\frac{a_o v t}{C_o} = A_b b, \text{ where } b = \text{length of recoil (ft)}$$

hence $\dfrac{a_o}{C_o}\sqrt{\dfrac{2gb}{D}}\, t = A_b\, b$ and $A_o = C_o A_b A\, b \sqrt{\dfrac{pD}{2g}}$ (sq.in)

where b = length of recoil (in)

A_b = area of buffer (sq.in)

C_o = contraction constant of orifice $= \dfrac{1}{0.6}$ to $\dfrac{1}{0.8}$

p = pressure in recoil cylinder (lbs/sq.in)

A = effective area of recoil piston (sq.in)

D = density of liquid (lbs/cu.ft)

Since the pressure in the buffer is probably small by this method of filling, we may neglect the total buffer reaction in modifying the tension or pull in the rod. Further the throttling in the "filling in" buffer, becomes,

$$p = \frac{C_o'^2 A_b^2 V_x^2}{175\, a_o^2} \quad \text{approx.}$$

hence $A_b V_x = Q_b$ constant - that is, the flow into the buffer may be assumed constant throughout the recoil, hence for the main recoil throttling, we have,

$$p = \frac{C_o^2 (AV - Q_b)^2}{175\, w_x^2}$$

and

$$w_x = \frac{C_o (AV_x - Q_b)}{13.2\sqrt{p}}$$

Since, however, p_b (the pressure in the buffer) actually rises even in this method somewhat towards the end of recoil, Q_b <u>decreases with</u> AV_x and there-

fore by slightly modifying the true contraction constant C_o, we have,

$$w_x = \frac{C_o A V_x}{13.2\sqrt{p}}$$

which is sufficiently exact for ordinary design.

For correct filling of the buffer chamber, the filling throttling area to the buffer should be made variable. We may plot this variable area against recoil and take its mean value as an approximation for the proper throttling area for filling the buffer chamber.

The condition for ideal filling of the buffer chamber are, that $C\,U_x a_x = A_b V_x$ and $P_b = 0$ throughout the recoil, where u_x = the throttling velocity into the filling in buffer,

p_b = the pressure in the buffer chamber

c = the contraction constant of the orifice.

a_x = the variable buffer filling throttling area (sq.in)

By Bernoulli's theorem, we have,

$$p = \frac{DU_x^2}{288g} \quad \text{and} \quad p = \frac{Dv_x^2}{288g} \qquad \text{(lbs/sq.in)}$$

where v_x = the velocity through the recoil throttling orifice.

D = the weight of the fluid per cu. ft.

hence

$$A_a = \frac{A_b V_x}{C\,U_x} = \frac{A_b V_x \sqrt{D}}{12c\sqrt{2gp}} \qquad \text{(sq.in)}$$

and since $P_h = p\,A$

$$A_a \sqrt{\frac{A_b^2\ A\ D\ V_x^2}{288\ c\ g\ P_h}} \qquad \text{(sq.in)}$$

$N\,w\,\dfrac{V_x^2}{P_h}$ is a variable in the recoil, and therefore the recoil throttling areas become modified at any instant, such that,

$$C_a A_x U_x + C_w w_x v_x = A\,V_x \text{ hence } w_x = \frac{AV_x - C_a a_x u_x}{C_x\ v_x}$$

$$= \frac{AV_x - C_a A_x \sqrt{\dfrac{288\ gb}{D}}}{C_w \sqrt{\dfrac{288\ gb}{D}}}$$

$$w_x = \sqrt{\frac{D\ A^2 V_x^2}{288\ g\ P_n}} - A_x \frac{C_a}{C_w}$$

Constructive difficulties make it impractical to vary A_x according to the above theory in an ordinary design but by making $a_x = a_o$ a constant, and assuming p_b small, we have from the above formula, that the recoil throttling area equals the throttling area computed as if no buffer existed in the recoil, and lessened by a constant area $A_o \frac{C_a}{C_w}$

VARIABLE RECOIL:- VARYING THE RECOIL AS THE GUN ELEVATES.

Stability consideration: As the gun elevates the overturning moment decreases, since the perpendicular distance from the spade point or the point where the mount tends to overturn on recoil, to the line of action of the total resistance to recoil decreases on elevation. Therefore, since the initial recoil energy is practically constant, it is possible to decrease the length of recoil considerably as the gun elevates and yet maintain stability. When the line of action of the resistance to recoil passes through the spade point, the overturning moment is independent of the magnitude of the recoil reaction, and therefore theoretically the recoil can be made as small as the strength of the carriage can stand.

Therefore, the recoil limitations on elevating the gun are clearance at maximum elevation, as well as clearance considerations at intermediate elevations, and the limitation imposed by stability for various elevations of the gun.

The recoil may be cut down in any arbitrary manner provided, that consideration be given to strength, clearance and stability at all angles of elevation. The maximum length of short recoil depends upon clearance considerations at maximum elevation, while the minimum length of long recoil depends upon stability at horizontal elevation.

To investigate the stability limitations on the length of recoil at low angles of elevation, let

C = constant of stability = $\dfrac{\text{Overturning moment}}{\text{Stabilizing moment}}$ = 0.85

A_r = initial recoil constrained energy = $\frac{1}{2} m_r V_r^2$ (ft/lbs)

V_r = 0.9 V_f restrained recoil velocity (ft/sec)

$V_f = \dfrac{w\ v + \bar{w}\ 4700}{\bar{w}_r}$ = free velocity of recoil (ft/sec)

u = travel up bore (in ft)

E_r = displacement of gun during powder period = $2.25\ \dfrac{(w + 0.5\ \bar{w})u}{w_r}$ (in ft)

d = moment arm to line of action of total resistance to recoil (ft)

b = length of recoil (ft)

Then, $\dfrac{A_r}{b - E_r} = \dfrac{C(W_s l_s - W_r\ b\cos\emptyset)}{d}$ and solving for b, we have

$$b = \frac{(W_s l_s + W_r E_r \cos\emptyset°) \pm \sqrt{(W_s l_s + W_r E_r \cos\emptyset)^2 - 4W_r\cos(W_s l_s E_r + \frac{dA_r}{C})}}{2\ W_r \cos\emptyset}\ \text{(ft)}$$

which gives us the limiting recoil consistent with stability for low angles of elevation, <u>with a constant resistance throughout the recoil.</u>

When the resistance to recoil is made to conform with the stability slope, we have,

$$\frac{1}{d}\int_{E_r}^{b} (W_s l_s - W_r\, x \cos \emptyset)\, dx = \frac{A_r}{A}$$

Solving, we have $\frac{1}{d}[W_s l_s (b-E_r) - \frac{W_r \cos\emptyset}{2}(b^2 - E_r^2)] = \frac{A_r}{c}$

Hence, we have, the quadratic equation in terms of b

$$b^2 - \frac{2W_s l_s}{W_r \cos \emptyset}\, b + 2\left(\frac{\frac{dA_r}{c} + W_s l_s E_r - \frac{W_r \cos \emptyset}{2} E_r^2}{W_r \cos \emptyset}\right) = 0$$

Solving for b: we have,

$$b = \frac{1}{W_r \cos\emptyset}[W_s l_s - \sqrt{(W_s l_s)^2 - 2W_r \cos \emptyset(\frac{A_r}{c} d + W_s l_s E_r - \frac{W_r \cos\emptyset}{2} E_r^2)}]$$

which gives us the limiting recoil consistent with stability for low angles of elevation, <u>with a variable resistance throughout the recoil conforming with the stability slope.</u>

<u>METHOD OF DECREASING THE LENGTH OF RECOIL:</u>

In the layout design of varying the recoil on elevation, it is highly desirable to maintain a constant recoil equal to that at horizontal recoil for the first few degrees of elevation and then begin cutting down the length of recoil, to the minimum recoil at max. elevation, since by this method the margin of stability increases as the gun elevates and therefore exact stability at horizontal recoil is

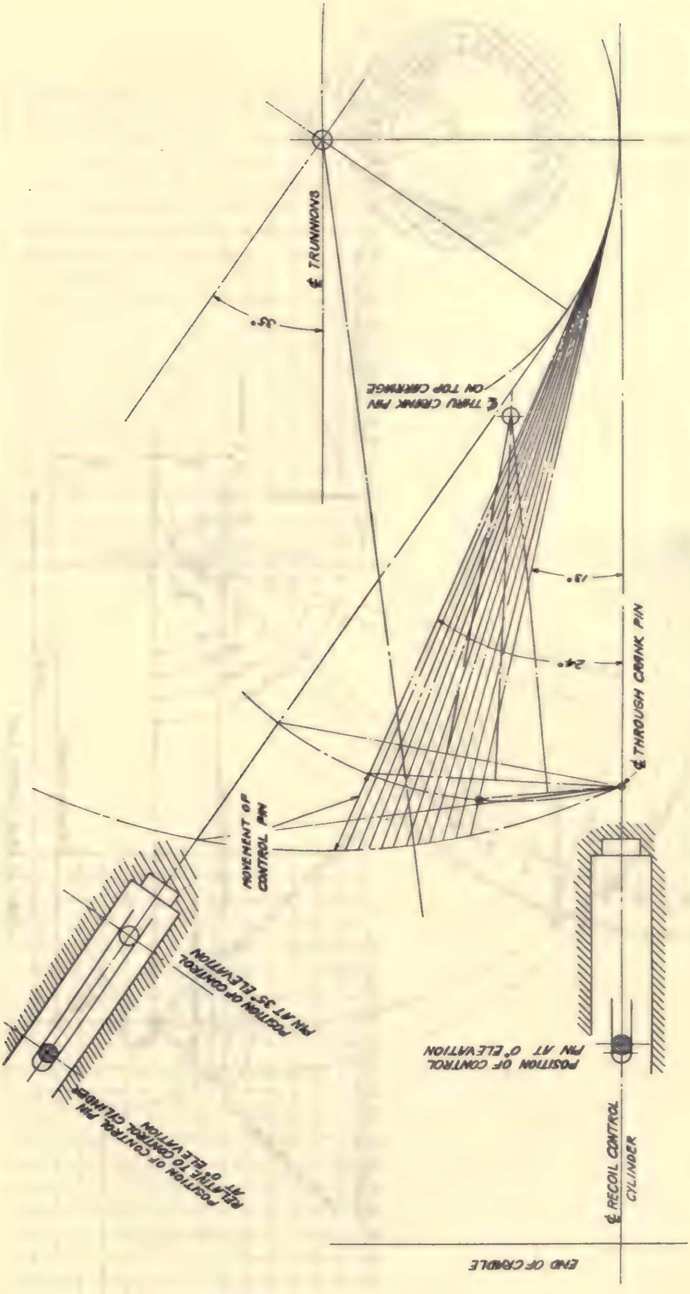
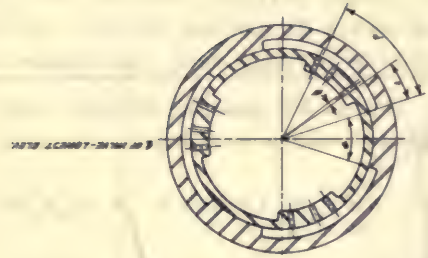
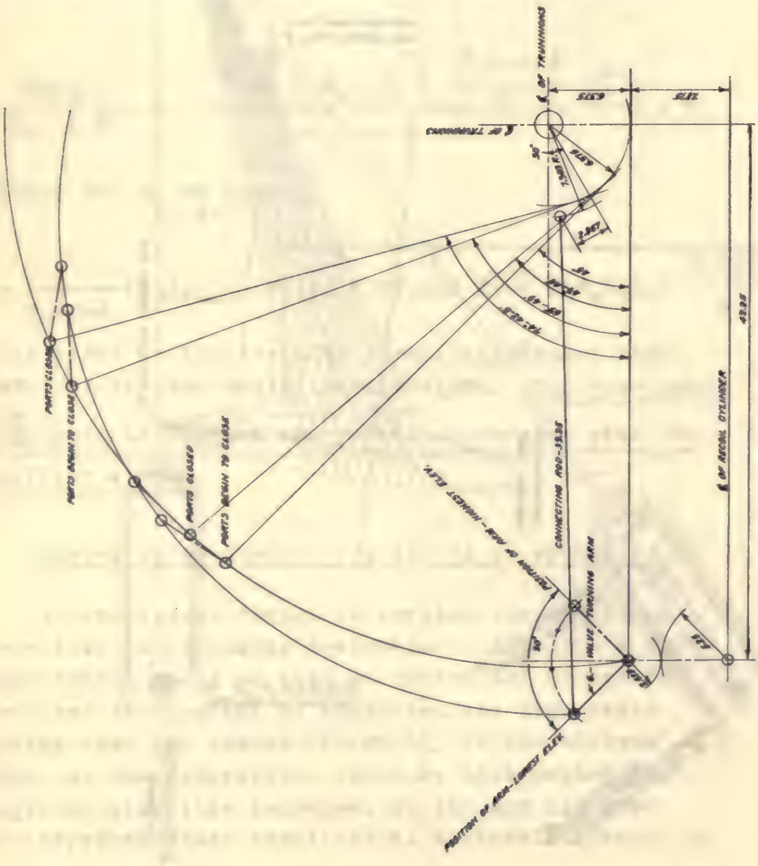

℄ TRUNNIONS
35°
℄ THRU CRANK PIN ON TOP CARRIAGE
13°
24°
℄ THROUGH CRANK PIN
MOVEMENT OF CONTROL PIN
POSITION OF CONTROL PIN AT 35° ELEVATION
POSITION OF CONTROL PIN RELATIVE TO CONTROL CYLINDER AT 0° ELEVATION
POSITION OF CONTROL PIN AT 0° ELEVATION
℄ RECOIL CONTROL CYLINDER
END OF CRADLE

Fig. 4



VALVE



LAYOUT OF VALVE TURNING GEAR

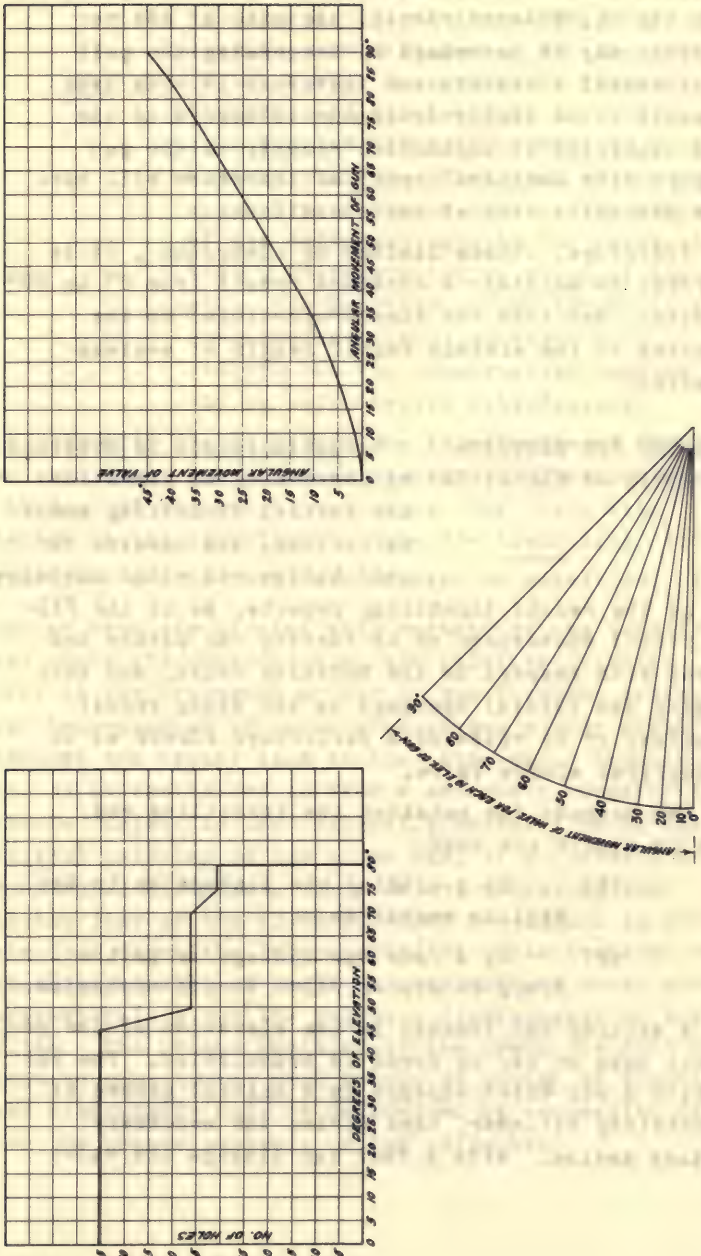

ANGULAR MOVEMENT OF VALVE
ANGULAR MOVEMENT OF GUN
DEGREES OF ELEVATION
NO. OF HOLES
ANGULAR MOVEMENT OF VALVE FOR EACH 10° ELEV. OF GUN

Fig. 6

no longer of vital consideration as horizontal fire is seldom used. In certain types of recoil systems as in the St. Chamond recoil, the size of the recuperator may be decreased by increasing the pull at horizontal elevation and therefore in this type of recoil it is highly desirable to design to the exact stability at horizontal recoil, as the gun elevates with constant recoil we therefore will have ample stability even at low elevations.

Therefore, unless limited by clearance , it is desirable to maintain a constant recoil from 0° to 20° elevation, and then cut down proportional to the elevation to the minimum recoil length at maximum elevation.

MECHANISM FOR REDUCING THE RECOIL ON ELEVATION. Variable recoil is obtained by decreasing on elevation the initial throttling areas by turning, the counter recoil buffer rod which contains sets of the recoil throttling grooves, as in the Filloux recoil mechanism; or by turning the piston and its rod with respect to the rotating valve, and thus changing the initial openings in the Krupp recoil mechanism; or by rotating a perforated sleeve as in the American sleeve valve.

Two methods for rotating the throttling rod, valve or sleeve are used,

(1) by a sliding bar linkage as in the Filloux mechanism or

(2) by a four bar linkage as in the Krupp or sleeve valve recoil mechanism.

With a sliding bar linkage in the elevation of the gun, a cross head or bar is moved in translation. The bar contains a pin which engages in a helical groove of the rotating cylinder, thus giving the necessary rotatory motion. With a four bar linkage the valve

is turned directly in the movement of the linkage during the elevation of the gun.

(1) In a layout of the sliding bar linkage, the distance of the translation of the bar or cross head is fixed by the pitch of the helix on the rotating cylinder and the angle turned to be turned by the cylinder. The pitch of the helix may not be constant that is the slope of the helix may vary in the revolution. With a uniform pitch or slope of the helix, the decrease in the length of recoil against elevation may not be uniform but for constructive reasons it may be sufficiently satisfactory.

Knowing the length of the translation of the slide we may layout the valve mechanism. In the sliding bar linkage of the recoil mechanism, the crank with center at the trunnions is made the fixed link, while the frame of the mechanism rotates on elevation. If now we draw two circles with centers at the trunnions and crank pin respectively, the relative displacement of the crosshead or bar is the distance between the intersection of these circles and a line drawn through the center line of the slide bar. Constructively, it is convenient to draw a secondary constructive circle tangent to the projectile center line of the initial position of the slide bar, i. e. usually at horizontal elevation. Then at any elevation the center line of the slide bar must be tangent to this circle. Hence the intersection of these tangents with the base circles of radii at trunmion and crank pin respectively gives the relative displacement of the slide. The proper position of the crank pin with respect to coordinates with origin at center of trunnions can practically only be determined by successive trials for the proper movement of the slide bar.

(2) In a layout of a four bar linkage the angle of rotation of the valve during the elevation of the gun is ascertained from the design of the recoil throttling. The gear turning the valve may mesh with another gear and from the gear ratio and the maximum turning of the valve the angle turned by the valve crank can be determined.

Knowing the angle turned by the valve crank or valve arm we may then layout the valve mechanism. The four bar linkage consists of the frame connecting the trunnion and valve center; the fixed trunnion crank connecting the trunnion and connecting rod; the connecting rod connecting the fixed trunnion crank and the valve crank or arm; and finally the valve or arm connecting the connecting rod with the valve center. The fixed member of the four bar linkage is the"fixed trunnion crank" joining the trunnion to the connecting rod. If we draw two circles from the fixed centers of the trunnion and trunnion crank pin respectively, the center of the valve travels along the circular path with center at the trunnion, while the crank pin of the valve arm moves in a circular path with center at the fixed trunnion crank pin. It is important to note that the relative position of the valve crank arm should be measured from the tangent to the circle with center at the trunnions. The relative angle turned by the valve crank is therefore the difference between the final angle with respect to the tangent of the trunnion circle when at maximum elevation and the initial angle with respect to the tangent of the trunnion circle when at minimum, usually horizontal elevation.

Constructively, it is convenient to draw a secondary constructive circle tangent to a horizontal line through the center of the valve arm. Then the position of the valve center at any elevation is the intersection of the tangent to this secondary

circle at the given elevation with the base trunnion circle of the valve.

If we lay off from this intersection the length of valve arm to the intersection of the trunnion crank pin base circle, we have the position of the valve arm for this elevation. For the angle turned we note the angle made by the valve arm with the tangent to the trunnion base circle at the valve center, and the initial angle of the valve arm with the tangent at horizontal elevation. The difference between these angles is the angle turned by the valve arm, which multiplied by the gear ratio gives the actual angle turned by the valve.

ON THE LENGTH OF RECOIL WITH A STATIONARY SPRING CONTROLLED ORIFICE.

As before for a grooved orifice we have from the equation of energy:

$$K(b-x) = \frac{1}{2} m_R v_x^2 \quad (1)$$

where b = length of recoil (ft)

x = recoil displacement (ft)

v_x = recoil velocity at displacement x (ft/sec)

m_R = mass of recoiling parts

and for the total resistance to recoil, for a dependent recoil system $K = pA + R - W_r \sin \emptyset$

where p = pressure in the recoil cylinder (lbs/sq.ft)

R = total friction (lbs)

A = effective area of recoil piston (sq.ft)

Now

$$p - p_a = \frac{D A^2 V_x^2}{2gc^2 w_x^2} \quad \text{and} \quad (p-p_a)A = \frac{D A^3 V_x^2}{2gc^2 w_x^2} = \frac{C V_x^2}{w_x^2}$$

then since

$$K = \frac{CV_x^2}{w_x^2} + p_a A + R - W_r \sin \emptyset \quad (2)$$

Combining (1) and (2), $w_x^2 = \dfrac{2KC(b-x)}{m_R(K - p_a A - R + W_r \sin \emptyset)}$

the ratio $C' = \frac{K}{K - p_a A - R + W_r \sin \emptyset}$ is approximately constant, since the variation of the weight component $W_r \sin \emptyset$ and the recuperator reaction $p_a A$ is small compared with K. Then

$$w_x^2 = \frac{2C_o}{m_R} (b-x) \text{ where } C_o = C'C.$$

Therefore the orifice variation is a parabolic function of the recoil displacement and is independent of the initial velocity and therefore variation in the ballistics, and is practically independent of the weight component and therefore of the elevation of the gun.

In general, independent of the method of throttling the length of recoil is practically independent of variation in the ballistics of the gun or in the variation of the elevation of the gun.

ON THE LENGTH OF RECOIL WITH A GROOVED ORIFICE. During the retardation period of the recoil, we have, from the equation of energy,-

$$K(b-x) = \tfrac{1}{2} m_r V_x^2$$

where b = length of recoil (ft)
x = recoil displacement (ft)
V_x = recoil velocity at displacement x (ft/sec)
m_r = mass of the recoiling parts
K = total resistance to recoil (lbs)

hence

$$V_x^2 = \frac{2K(b-x)}{m_r} \qquad (1)$$

Now the pressure in the recoil brake, becomes,

NOTE: Not confirmed by observed data. Editor.

$$p = \frac{D\,A^2 V_x^2}{2gC^2 W_x^2} \quad \text{(lbs/sq.ft)}$$

$$P_h = p\,A = \frac{D\,A^3 V_x^2}{2gC^2 W_x^2} = \frac{CV_x^2}{W_x^2} \qquad (2)$$

P_h = total hydraulic pull (lbs)
A = effective area of recoil piston (sq.ft)
D = weight per cu.ft. of fluid (lbs/cu.ft)
C = contraction constant of orifice
where

$$C = \frac{DA^3}{2gC^2}$$

Combining (2) with (1), we have $W_x^2 = \frac{K}{P_h}\,\frac{2C}{m_r}(b-x)$ (3)

If now we assume $\frac{K}{P_h}$ to always remain a constant C'

and placing c C' = C_o, we have $W_x^2 = \frac{2C_o}{m_r}(b-x)$ (4)

which is an equation of remarkable physical significance. We find the orifice variation to be a parabolic function of the displacement and is quite independent of the initial recoil velocity. Therefore with the same weight of recoiling parts, the recoil displacement is practically the same for all values of the initial recoil velocity. Since the initial velocity depends upon the ballistics of the gun, we may completely change the ballistics of the gun and yet with grooved orifices the length of recoil remains practically unchanged.

In the following discussion the ratio $\frac{K}{P_h}$ was assumed to remain constant; the change in the length of recoil depends therefore on the change in the ratio $\frac{K}{P_h}$.

Let us examine this ratio for the change under two conditions,

(1) As the gun elevates where the weight component is brought into effect.

(2) For different ballistics of the gun, where the initial velocity is changed.

Now for case (1),

$$K = 0.45 \frac{m_r V_f^2}{b}$$

and assuming the same length of recoil, K is a constant and independent of the elevation.

If $\frac{K}{P_h}$ is to remain constant, its reciprocal must remain constant for all elevations.

Since $K = P_h + F_v + R_t - W_r \sin \phi$ where P_h = total hydraulic pull (lbs)

F_v = recuperator reaction (lbs)

R_t = total friction (lbs)

Hence

$$\frac{P_h}{K} = \frac{K - F_v - R_t + W_r \sin \phi}{K} = 1 - \frac{F_v + R_t - W_r \sin \phi}{K}$$

Since F_v and K remains a constant for all elevations, in order that

$\frac{K}{P_h}$ or its reciprocal $\frac{P_h}{K}$ remain a constant, we must have $R_{t_1} - W_r \sin \phi_1 = R_{t_2} - W_r \sin \phi_2$

To consider extreme conditions, let us consider, horizontal and max. elevation, then $R_{t(\phi_0)} = R_{t\phi_m} - W_r \sin \phi_m$

where ϕ_m = the angle of elevation at max. elevation.

Now $R_t = R_g + R_p$ where R_g = the total guide friction

R_p = the total packing friction

Now R_g is proportional to the total braking $= K + W_r \sin \phi$ due to the pinching action of the guides, and the packing friction remains practically constant since P_r does not change greatly. Hence on elevation,

$R_{t\phi_m} > R_{t\phi_0}$ usually except for large guns with balanced pulls.

From actual numerical calculations on a series of guns, the term R_{to} was found to be slightly greater than

$R_{t\phi} - W_r \sin \phi_m$. Therefore, $\frac{K}{P_h}$ remains practically constant.

(1) <u>The length of recoil with the same grooved orifices is practically independent of the elevation of the gun.</u>

In case (2) with different ballistics, we have roughly, $K_1 = 0.45\, m_r V_1^2$

$K_2 = 0.45\, m_r V_2^2$

and as before the reciprocal of the ratio $\frac{K}{P_h}$, becomes,

$\frac{P_h}{K} = 1 - \frac{F_v + R_t - W_r \sin \phi}{K}$ therefore for a constant ratio, we should have,

$$\frac{F_v + R_t - W_r \sin \phi}{K_1} = \frac{P_v + R_t - W_r \sin \phi}{K_2}$$ which obviously is impossible.

But $F_v + R_t - W_r \sin \phi$ is always small compared with K, hence the difference of the above terms must be correspondingly smaller.

Hence though the ratio $\frac{K}{P_h}$ changes with different ballistics, the change is very small.

(2) <u>The length of recoil with the same grooved orifices is practically independent of the ballistics of the gun.</u>

NOTE: The above discussion on length of recoil is retained as a point for discussion. The author's conclusions are not however well confirmed by observed data. Editor.

COUNTER RECOIL: ELEMENTARY DISCUSSION. In the design of a counter recoil system, we are concerned with either counter recoil stability when the gun enters the battery position or with the buffer pressure in the counter recoil regulator. In the former, we are concerned with the overall force, that is the total force towards the end of counter recoil, while in the latter, with the c'recoil buffer or regulator reaction. Let

K_v = total resistance to counter recoil (lbs)
F_v = total recuperator reaction (lbs)
B_x' = counter recoil regulator or buffer force (lbs)
R_t = total friction (lbs)
w_x = throttling area of c'recoil regulator(sq.in)
C' = throttling constant
A_b = area of buffer (sq.in)
v = velocity of c'recoil (ft/sec)

The critical angle of elevation for counter recoil functioning is at horizontal elevation. Then $K_v = B_x' + R_t - F_v$ and for horizontal c'recoil stability in a field carriage, we have

$$K_v \leq \frac{W_s l_s + W_r(b-x)}{h}$$

where l_s' = distance from total weight of system to forward overturning point, usually the front wheel base (ft)

x = displacement in c'recoil from out of battery position (ft)
b = length of recoil (ft)
h = height of center of gravity of recoiling parts from ground (ft)

We may express $W_s l_s'$ in terms of the static load on the spade then, $T\,l = W_s l_s'$

where l = distance between spade and wheel contact with ground. Then

$$K_v \leq \frac{T\,l + W_r(b-x)}{h}$$

where T = 150 to 200 (lbs)

If the ground is assumed to exert a downward pressure on the spade comparable with the load T, we have

$$K_v = 0.85 \frac{2T\ l + W_r(b-x)}{h}$$

which gives the limitation of the magnitude of the total unbalanced force towards the end of counter recoil.

For simplicity in the following discussion a constant regulator reaction will be assumed acting throughout the counter recoil. This method of control was used by the Krupp and the earlier material of the Schneider in France.

SPRING RETURN.

Let S_o = initial or battery load on spring column (lbs)

S_f = final or out of battery load on spring column (lbs)

C_t = spring constant

Then $F_{vi} = S_o, \quad F_{vf} = S_f$

and the recuperator reaction, in terms of the c'recoil displacement x, becomes,

$$F_v = S_o + \frac{S_f - S_o}{b}(b-x) = S_o + S(b-x) \text{ where } S = \frac{S_f - S_o}{b} = \text{the spring constant,}$$

then $m_r v \dfrac{dv}{dx} = -K_v$

$$= -(B_x' + R_t - F_v)$$

therefore

$$\frac{m_r v^2}{2} = -B_x' x - R_t x + \left(S_o + S b - \frac{Sx}{2}\right) x$$

which is the general equation of c'recoil, with a constant regulator reaction and spring return. When $x = b$, $v = 0$, hence

$$-B_x' b - R_t b + \left(S_o + Sb - \frac{Sb}{2}\right) b = 0$$

hence

$$B_x' = S_o + \frac{Sb}{2} - R_t \text{(lbs)}$$

This same value may be obtained by a consideration of

the potential energy stored in the recuperator.
The potential energy of the recuperator, becomes,

$$W_o = \int_o^b S_o\, dx + \int_o^b \frac{S_f - S_o}{b} x\, dx$$

$$= S_o b + \frac{S_f - S_o}{b}\frac{b^2}{2}$$

$$= \frac{S_o + S_f}{2} b \qquad \text{(ft.lbs)}$$

We have, then, from the principle of energy,

$W_o = R_t b + B_x' b = \dfrac{S_o + S_f}{2} b$ since $S_f = S\,b + S_o$

hence $B_x' = S_o + \dfrac{Sb}{2} - R_t$

Substituting this value in the energy equation and simplifying, we have $m_r v^2 = Sx(b-x)$ hence $S = \dfrac{m_r v^2}{(b-x)x}$

and $B_x' = (S_o - R_t) + \dfrac{m_r v^2\, b}{2(b-x)x}$ (lbs)

which gives the value of the constant regular reaction.

Now

$B_x' = \dfrac{C^2 A_b^3 v^2}{175 W^2}$ (lbs) where C = the reciprocal of the contraction factor of the regulator orifice.

A_b = effective area of buffer

w_x = variable regulator orifice, and since,

$$v^2 = \frac{S(b-x)x}{m_r}$$

$$B_x' = \frac{C^2 A_b^3\, s(b-x)x}{175 m_r w_x^2}$$

and therefore $w_x^2 = \dfrac{C^2 A_b^3 s}{175 m_r B_x'} (bx - x^2)$

hence $w_x = C_o \sqrt{x(b-x)}$ Value of throttling orifice of regulator (sq.in)

where $C_o = \dfrac{C\ A_o^{\frac{3}{2}}\ \sqrt{s}}{13.2\ \sqrt{m_r B_x'}}$

$B_x' = S_o + \dfrac{Sb}{2} - R_t$

The unbalanced force of c'recoil, becomes,

$$m_r\ v\ \frac{dv}{dx} = -\ (B_x' + R_t - F_v)$$

$$= -\ (S_o + \frac{Sb}{2} - S_o - Sb + Sx)$$

$$= \frac{Sb}{2} - Sx = S(\frac{b}{2} - x)$$

Hence the unbalanced force decreases with the displacement of c'recoil, reverses to a negative value at mid stroke.

The initial unbalanced force at the beginning of c'recoil, equals

$$\frac{Sb}{2} = (\frac{S_f - S_o}{2b})b = \frac{S_f - S_o}{2}$$

The overturning force at the end of c'recoil, becomes

$$-\frac{Sb}{2} = \frac{S_f - S_o}{2}$$

GENERAL EQUATIONS OF COUNTER RECOIL. The functioning of counter recoil may best be studied by a consideration of the physical aspects of the dynamic equation for counter recoil. Let

- p_a' = intensity of pressure of the oil in the air cylinder (lbs/sq.in)
- w_{ax} = counter recoil throttling area between air and recuperator cylinders (sq.in)
- A_v = effective area of recuperator piston (sq.in)
- K_v = total resistance to counter recoil (lbs)
- F_v = actual or equivalent recuperator reaction at any displacement "x" from the out of battery position (lbs)
- w_x = variable buffer orifice at c'recoil dis-

placement x for buffer counter recoil throttling (sq.in)

Then during the counter recoil for a spring, pneumatic or similar recuperator system, we have,

(1) the recuperator reaction acting to displace the gun forward into battery F_v (lbs)

(2) the weight component resisting F_v - - $W_r \sin \emptyset$ (lbs)

(3) the guide friction $R_g = n W_r \cos \emptyset$ approx. since the pinching action of the guides is small on counter recoil and we therefore have an approximation of pure sliding friction throughout the greater part of counter recoil. This reaction also resists F_v.

(4) the packing friction R_{s+p} resisting F_v (lbs)

(5) the throttling through the return of the recoil apertures together with the counter recoil buffer throttling. The throttling through the recoil is small as compared with the buffer throttling and may be neglected or else included with the buffer throttling. The throttling is proportional to the square of the velocity of counter recoil and inversely as the square of the throttling orifice, that is, the buffer braking becomes,

$$H'_x = \frac{c'_o v^2}{w_x^2}$$ (lbs) <u>and resists</u> F_v

Therefore, we have

$$F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} - \frac{C'_o v^2}{w_x^2} = m_r v \frac{dv}{dx} \quad (1)$$

which is the differential equation of counter recoil.

With a hydro pneumatic recuperator system it is possible to regulate counter recoil by lowering the pressure in the recuperator cylinder for the greater part or the entire recoil, by throttling the oil through an orifice between the air and recuperator cylinders. Introducing a buffer chamber in the air cylinder with a buffer attached to a floating piston, gives a simple means for varying the orifice and thus reducing the pressure in the recuperator cylinder or in the recoil cylinder to a value consistant for the proper movement of the recoiling parts in counter recoil.

The pressure in the recuperator cylinder due to throttling through the orifice between the air and recuperator cylinders, becomes,

$$p_v = p_a' - \frac{C_o''' v^2}{w_{ax}^2}$$

Hence, for the motion of the recoiling parts in counter recoil, we have,

$$p_v A_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} - \frac{C_o' v^2}{w_x^2} = m_r v \frac{dv}{dx}$$

or substituting for p_v, we have

$$p_a' A_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} - \left(\frac{C_o''}{w_{ax}^2} + \frac{C_o'}{w_x^2}\right) v^2 = m_r v \frac{dv}{dx} \quad (2)$$

where $C_o'' = A_v C_o'''$

Now $p_a' A_v$ may be regarded as the equivalent recuperator reaction, that is $F_v = p_a' A_v$ and further assuming the regulation to be entirely effected through the throttling in the recuperator, we have, for eq.(2)

$$F_v - W_r(\sin\emptyset + n \cos \emptyset) - R_{s+p} - \frac{C_o''}{w_{ax}^2} v^2 = m_r v \frac{dv}{dx} \quad (3)$$

which is exactly similar to the previous equation of counter recoil for a simple spring recuperator system.

The external force on the total mount, is

$K_v = m_r v' \frac{dv'}{dx}$, together with the weight of the recoiling parts W_r.

During the acceleration,

$$K_v = m_r v \frac{dv}{dx}$$ acts **towards** the breech, and

during the subsequent retardation,

$$K_v = m_r v \frac{dv}{dx}$$ acts towards

the muzzle. During the acceleration K_v is necessarily always less than K the total resistance to recoil since,

$$K = F_v + R + \frac{Cv^2}{w^2} - W_r \sin \emptyset, \quad \text{for the recoil and}$$

$$K_v = F_v - R - \frac{C'v^2}{w_x^2} - W_r \sin \emptyset, \quad \text{for the counter recoil,}$$

therefore

$$K - K_v = 2R + \frac{Cv^2}{w^2} + \frac{C'v^2}{w_x^2}$$, roughly assuming total friction the same on

recoil and counter recoil. Hence, in the design of a counter recoil system we are only concerned with counter recoil stability, and not at all with the reaction during the acceleration. If we let, further,

W_s = weight of total system (lbs)

l_s' = horizontal distance from front hinge or contact of wheel and ground to the center of gravity of the total system in battery (ft)

C' = constant of counter recoil stability

$$= \frac{\text{Overturning counter recoil moment.}}{\text{Stabilizing counter recoil moment.}}$$

d' = perpendicular distance from front hinge or contact of wheel and ground to line of action of K_v through center of gravity of recoiling parts (ft)

then, for stability at angle of elevation $\emptyset$, we have

$$\frac{C_o'v^2}{w_x^2} + R_{s+p} + W_r(\sin\emptyset + n\cos\emptyset) - F_v = C'\left(\frac{W_s l_s' + W_r(b-x)\cos\emptyset}{d'}\right) \qquad (2)$$

$$\text{and } -m_r v \frac{dv}{dx} = C'\left[\frac{W_s l_s' + W_r(b-x)\cos \emptyset}{d'}\right] \tag{3}$$

which gives us the velocity curve against displacement consistent with counter recoil stability. Substituting v in (2) enables us to determine the variable orifice w_x consistent with counter recoil stability, since F_v is a known function of x.

During the acceleration, we have

$$F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} - \frac{C_o' v^2}{w_x^2} = m_r v \frac{dv}{dx}$$

and since we are not concerned with stability, for <u>minimum time</u> during the acceleration K_v should be made a maximum, that is the hydraulic braking term $\frac{C' v^2}{w_x^2}$ should be made zero, hence

$$F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} = m_r v \frac{dv}{dx} \tag{4}$$

Let further v_m = maximum velocity of counter recoil (ft/sec)

x_m = corresponding displacement to maximum velocity from out of battery position (ft)

Then, for ideal counter recoil, that is the counter recoil functioning in minimum time and consistent with stability, we have,

$$-\int_{v_m}^{0} m_r v\, dv = \frac{C'}{d'} \int_{x_m}^{b} [W_s l_s' + W_r(b-x) \cos \emptyset] dx \tag{5}$$

from which we obtain,

$$\frac{m_r v_m^2}{2} = \frac{C'}{d'} \left[W_s l_s' b + W_r \frac{(b-x_m)^2}{2} \cos \emptyset\right] \tag{5'}$$

To determine x_m, we have

$$\int_0^{x_m} [F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p}]dx = \int_0^{v_m} m_r\, v\, dv$$

hence

$$\int_0^{x_m} F_v\, dx - [W_r(\sin\emptyset + n \cos \emptyset)+R_{s+p}] = \frac{C'}{d'}[W_s l_s' b + W_r \frac{(b-x_m)^2}{2} \cos \emptyset\ ] \qquad (6)$$

and knowing F_v as a function of x, we may solve for x_m. Substituting in (5') we easily obtain v_m which gives the maximum velocity of counter recoil.

Thus we see during the acceleration it is desirable to make, K_v a maximum, that is

$$K_{v_{max}} = F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p}$$

and during the retardation K_v should be consistent with counter recoil stability, that is

$$K_v = - m_r\, v \frac{dv}{dx} = C_s' \frac{[W_s l_s + W_r(b-x) \cos \emptyset]}{d'}$$

which can be obtained by increasing the buffer or counter recoil regulator, such that,

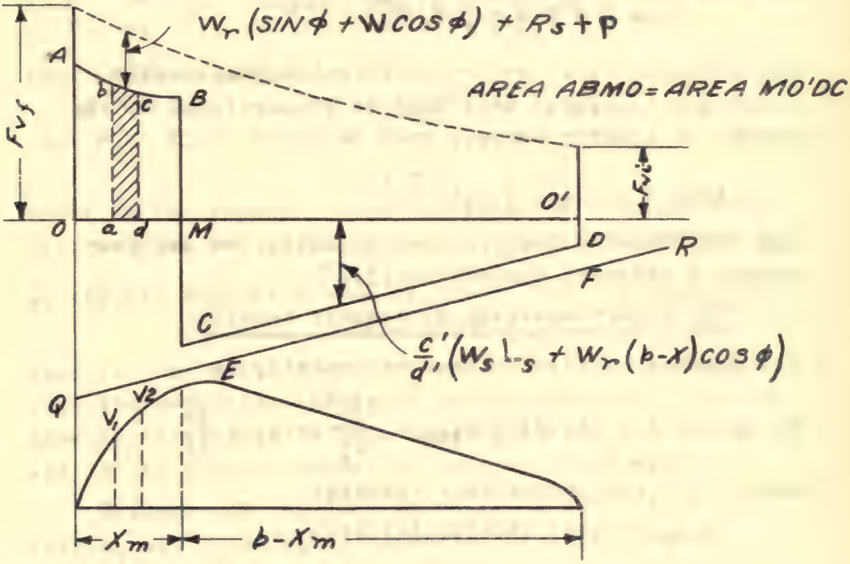
$$\frac{C_o' v^2}{w_x^2}, + R_{s+p} + W_r(\sin \emptyset + W \cos \emptyset) - F_v = C_s'[\frac{W_s l_s' + W_r(b-x) \cos \emptyset}{d'}]$$

A simple graphical solution of the above analysis may be made as follows:

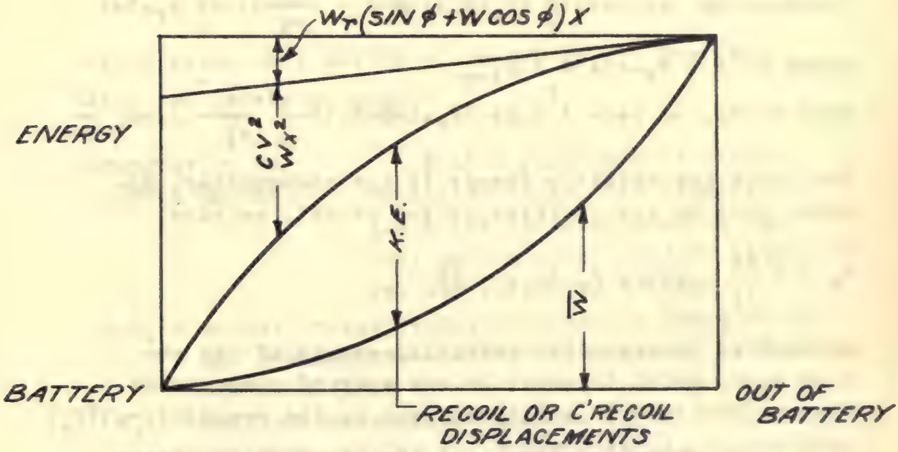
Lay off the recuperator reaction $F_{vf}-F_{vi}$ and from the ordinates of this curve subtract $W_r(\sin \emptyset + W \cos \emptyset)+R_{s+p}$ which gives the unbalanced reaction proportional to the ordinates to AB, during the acceleration period. Draw in below OO', CD parallel to the counter recoil stability slope Q R, such that

$\frac{O'D}{O'F} = \frac{MC}{ME} = C'$, the constant of counter recoil stability assumed. Then we locate M such that the area OABM = area M O' D C. Since OABM is pro-

IDEAL COUNTER RECOIL: MIN. TIME CONSISTENT WITH COUNTER RECOIL STABILITY



C'RECOIL ENERGY PLOTS



$\overline{W}$ = POTENTIAL ENERGY OF RECUPERATOR

K.E. = KINETIC ENERGY OF RECOILING PARTS IN C'RECOIL

$\frac{cV^2}{W_x^2}$ = TOTAL HYDRAULIC BRAKING IN THE C'RECOIL

Fig. 7

portional to the work done during the acceleration, we have

$$\text{Area } O\ A\ P\ M = \frac{1}{2} M_r V_m^2$$

The velocity curve may be constructed graphically since any increment area abcd is proportional to the change of kinetic energy, that is

$$\text{Area } a\ b\ c\ d = \frac{1}{2} m_r (v_1'^2 - v_1^2)$$

and thus knowing the previous velocity, we may construct a velocity curve directly.

The energy equation of counter recoil:

The dynamic equation of counter recoil, is

$$F_v - (n \cos \emptyset + \sin \emptyset) W_r - R_{s+p} - \frac{C_o' v^2}{w_x^2} = m_r\ v\ \frac{dv}{dx}$$

where F_v = the recuperator reaction

R_{s+p} = total packing friction.

$\frac{C_o' v^2}{w_x^2}$ = hydraulic buffer resistance

Integrating, we have $\int_0^x (F_v - W_r \sin\emptyset - R_t - \frac{C_o' v^2}{w_x^2}) dx = \int_0^v m_r v\ dv$

where $R_t = m\ W_r \cos \emptyset + R_{s+p}$

Separating, we have $\int_0^x F_v dx - (W_r \sin\emptyset + R_t) X - \int_0^x \frac{C_o' v^2}{w_x^2} dx = m_r \frac{v^2}{2}$

Now since the relative energy in the recuperator, depends only on the position in the recoil, we have,

$$F_v = - \frac{dW}{de} \quad \text{since } F_v\ dx = - dW$$

where W is the relative potential energy of the recuperator, which is equal to the work of compression (approximately) for a displacement in the recoil $(b-x)(F_v)$

If W = the potential energy of the recuperator in the out of battery position,

$$- \int_{W}^{W_x} \frac{dW}{dx} \cdot dx - (W_r \sin \emptyset + R_t)x - \frac{C_o' v^2}{w_x^2} dx = \frac{m_r v^2}{2}$$

from which we obtain

$$(W_o - W_x) - (W_r \sin \emptyset + R_t)x - \int \frac{C_o' v^2}{w_x^2} dx = \frac{m_r v^2}{2}$$

which is the general energy equation of counter recoil. Obviously at any displacement in the counter recoil x,

$$W_x + (W_r \sin \emptyset + R_t)x + \int \frac{C_o' v^2}{w_x^2} dx + \frac{m_r v^2}{2} = W_o \quad \underline{\text{a constant}}$$

That is, the total energy at any recoil x, is divided into the potential energy of the recuperator, the work done by friction, the work done by buffer throttling and in the kinetic energy of the recoiling mass.

Between any two displacements in the counter recoil x_1 and x_2 we have, approximately, provided the points are sufficiently close:

$$(W_{x_1} - W_{x_2}) - (W_r \sin \emptyset + R_t)(x_2 - x_1) - \frac{C_o' v^2}{W_{x_1}}(x_2 - x_1) = m_r \frac{(v_{x_2}^2 - v_{x_1}^2)}{2}$$

which gives us a method of computing v_{x_2} knowing v_{x_1} from the previous point.

COMPUTATION OF COUNTER RECOIL. With a given set of counter recoil orifices, the velocity and force curve of counter recoil may be calculated by either of the two following methods:

If F_v = actual or equivalent recuperator reaction at any displacement "x" from the out of battery position (lbs)

F_{vi} = initial recuperator reaction (lbs)

w_x = variable orifice for counter recoil throttling at displacement "x" from the out of battery position (sq.in)

C_o' = counter recoil throttling constant

n = coefficient of guide friction

R_{s+p} = total c'recoil packing friction (lbs)

A_v = effective area of recuperator piston (sq.in)

V_o = initial volume of recuperator (cu.in)

x = counter recoil displacement (ft)

METHOD I - LOGARITHMIC METHOD.

The dynamic equation of c'recoil, becomes

$$F_v - W_r(\sin \emptyset + n \cos \emptyset) - R_{s+p} - \frac{C_o' v^2}{w_x^2} = m_r v \frac{dv}{dx}$$

If we let, $R = W_r(\sin \emptyset + n \cos \emptyset) + R_{s+p}$

then $F_v - R - \dfrac{C_o' v^2}{w_x^2} = m_r v \dfrac{dv}{dx}$

Now F_v and w_x are both functions of x and therefore the equation cannot be readily integrated. If, however, we take a small interval F_v and w_x may both be assumed constant during this interval. Considering any two points x_1 and x_2 in the counter recoil, we have

$$\int_{x_1}^{x_2} dx = \int_{v_1}^{v_2} \frac{m_r v \, dv}{A - \dfrac{C_o' v^2}{w_{x_2}'^2}} \quad \text{where } A = F_v - R$$

Rearranging, we have

$$\int_{x_1}^{x_2} dx = -\frac{m_r w_x^2}{2C_o'} \int \frac{d\left(A - \dfrac{C_o' v^2}{w_{x_2}'^2}\right)}{A - \dfrac{C_o' v^2}{w_{x_2}'^2}}$$

hence integrating, we find

$$x_2-x_1 = -\frac{m_r w_{x_2'}^2}{2\,C_0'}\left[\log_e\left(A-\frac{C_0'v^2}{w_{x_2'}^2}\right)-\log_e\left(A-\frac{C_0'v_1^2}{w_{x_2'}^2}\right)\right]$$

and

$$-\frac{2C_0'(x_2-x_1)}{m_r\, w_{x_2'}^2} = \log_e\left(A-\frac{C_0'v_2^2}{w_{x_2'}^2}\right) - \log_e\left(A-\frac{C_0'v_1^2}{w_{x_2'}^2}\right)$$

therefore

$$\log\left(A-\frac{C_0'v_2^2}{w_{x_2'}^2}\right) = \log\left(A-\frac{C_0'v_1^2}{w_{x_2'}^2}\right) - \frac{2C_0'(x_2-x_1)}{2.3 m_r w_{x_2'}^2}$$

where $A = F_v - W_r(\sin\emptyset + n\cos\emptyset) - R_{s+p}$ (lbs) from which we may construct the velocity curve.

The advantage of this method is that a small variation of F_v and w_x has a negligible effect on the equation of motion and therefore fairly intervals may be taken provided the throttling orifice of counter recoil is not changing rapidly. During the buffer period where the throttling changes rapidly small intervals must be taken.

The total unbalanced force acting on the recoiling parts during counter recoil, is

$$m_r\, v\,\frac{dv}{dx} = m_r\, v\,\frac{\Delta v}{\Delta x} \qquad \text{(approx.)}$$

From this the unbalanced force (total accelerating or retarding force)

$$F_v - W_r(\sin\emptyset + n\cos\emptyset) - R_{s+p} - \frac{C_0'v^2}{w_x}$$

may be calculated and plotted.

To compute the recuperator reaction at any point, we have for spring recuperators,

$$F_v = S_o + \frac{S_f - S_o}{b}(b-x)$$

where S_o = initial or battery spring reaction (lbs)

S_f = final or out of battery spring reaction (lbs)

and for, pneumatic or hydro pneumatic,

$$F_v = p_{ai}\, A_v \left[\frac{V_o}{V_o-12A_v\ (b-x)}\right]^k$$

$$= F_{vi}\left[\frac{V_o}{V_o-12A_v(b-x)}\right]^k$$

where b= length of recoil (ft)

x = c'recoil displacement from out of battery position (ft)

V_o= initial volume (cu.in)

To compute R_{s+p}, we have, R_{s+p}=100 to 150 Σd for ordinary packing

where d = diam. of any one of the various recoil rods (in)

$$R_{s+p}=\Sigma(C_1+C_2p)=\Sigma[0.15(.05\ \pi w_p d_p p_{max})+0.75(.05\pi w_p d_p p)]\ \text{(lbs)}$$

where w_p = width of the various packings (in)

d_p = diam. of the annular contacts of the various packings (in)

p_{max} = the design pressure, usually the max. pressure in the cylinder to which the packing is subjected to (lbs/sq.in)

p = actual pressure during the various points in the counter recoil to which certain parts of the packing are subjected to (lbs/sq.in)

Obviously since p is variable, R_{s+p} must be variable during the counter recoil but an average value of p may be assumed and the corresponding R_{s+p} can be used with sufficient accuracy.

Computation of the throttling resistance $C_o' \frac{V^2}{w^2}$

(1) with a filling in buffer, the counter recoil regulation being effective throughout the counter recoil:

we may neglect the small throttling through the apertures of the recoil orifice, and then,

$$C_o' \frac{v^2}{w^2} = \frac{C'^2 A_b^3 v^2}{175 w_x^2} \qquad \text{(lbs)}$$

where C' = the reciprocal of the throttling constant

A_b = area of the buffer (sq.in)

w_x = buffer throttling area (sq.in)

(2) with some form of spear buffer, the buffer action being effective only during the latter part of counter recoil,

we have three stages:

(a) the void displacement with no regulation.

(b) throttling through the recoil apertures which cannot be neglected due to the much higher velocity of c'recoil than in case (1).

(c) throttling through the buffer orifice, the throttling resistance being large as compared with the resistance due to throttling through the recoil orifice, the latter being neglected.

In (b), we have,

$$C_o' \frac{v^2}{w^2} = \frac{c^2 (A+a_r)^3 v^2}{175 \; w_{xr}^2}$$

where A = effective arc of recoil piston (sq.in)

a_r = area of recoil rod (sq.in)

w_{xr} = area of recoil throttling grooves (sq.in)

In (c), we have, as in (1)

$$C_o' \frac{v^2}{w^2} = \frac{C^2 A_b^3 v^2}{175\ w_x^2}$$

where A_b = area of buffer (sq.in)

w_x = buffer throttling area (sq.in)

With a hydro pneumatic recoil system,
In this type it is possible to lower the pressure in the recuperator by throttling through a constant orifice.

Now it has been shown, that

$$p_v A_v = (p_a' - \frac{C_o''' v^2}{w_o^2}) A_v = F_v - \frac{C_o'' v^2}{w_o^2}$$

Hence $\dfrac{C_o' v^2}{w_x^2} = \dfrac{C^2 A_v^3 v^2}{175\ w_o^2}$

At the end of recoil if a spear buffer in the recoil brake cylinder also functions,

$$\frac{C_o' v^2}{w_x^2} = \frac{C^2 A_v^3 v^2}{w_o^2} + \frac{C^2 A_b^3 v^2}{w_x^2}$$

where A_v = the effective area of the recuperator piston (sq.in)

w_o = the c' recoil throttling area between the air and recuperator cylinders (sq.in)

METHOD II - THE ENERGY METHOD.

From the energy equation, we have, for any arbitrary interval,

$$(W_{x_1} - W_{x_2}) - (W_r \sin \emptyset + R_p)(w_2 - x_1) - \frac{C_o' v^2}{w_{x_1}}(x_2 - x_1) = m_r\left(\frac{v_{x_2}^2 - v_{x_1}^2}{2}\right)$$

where W_{xn} = the recuperator potential energy at the point "n" in the counter recoil (ft.lbs)

To compute W_{xn} we proceed as follows,
With a spring recuperator,

$$W_{xn} = \int [S_o + \frac{S_f - S_o}{b}(b-x)]\ d(b-x)\ \text{(ft.lbs)}$$

$$= S_o(b-x) + \frac{S_f - S_o}{2b}(b-x)^2 \qquad \text{(ft.lbs)}$$

where S_o = initial spring recuperator reaction (lbs)

S_f = final spring recuperator reaction (lbs)

b = length of recoil (ft)

x = displacement in counter recoil (ft)

With a pneumatic or hydro pneumatic recuperator,

$$W_{xn} = \int_o^{b-x} F_v d(b-x) = F_{vi} \int_o^{b-x} \left(\frac{V_o}{V_o - A_v(b-x)}\right)^k d(b-x) \quad \text{(ft.lbs)}$$

where k = 1.1(oil in contact with air)

= 1.3 oil and air separated by floating piston or pure pneumatic)

A_o = effective area of recuperator (sq.ft)

V_o = initial volume (cu.ft.)

F_{vi} = initial recuperator reaction (lbs)

Integrating, we have

$$W_{xn} = \frac{F_{vi} V_o^k}{A_v(k-1)} \left(\frac{1}{V^{k-1}} - \frac{1}{V^{k-1}}\right) \quad \text{(ft.lbs)}$$

where $V = V_o - A_v(b-x)$. Further since,

$$p_{ai} V_o^k = p_a V^k \quad \text{or} \quad \frac{p_a}{p_{ai}} = \left(\frac{V_o}{V}\right)^k$$

then,
$$W_{xn} = \frac{p_{ai}}{k-1}(V_o^k \;\; V^{1-k} \;\; V_o)$$

$$= \frac{p_a V - p_{ai} V_o}{k-1} \qquad \text{(ft.lbs)}$$

Hence, when V is in cu. ft., A_v in sq. ft. and $b-x$ in ft, we have

$$V = V_o - A_v(b-x) \qquad \text{(cu.ft)}$$

$$F_v = F_{vi}\left(\frac{V_o}{V}\right)^k \qquad \text{(lbs)}$$

$$W_x = \frac{F_v V - F_{vi} V_o}{A_v(k-1)} \qquad \text{(ft.lbs)}$$

Usually it is more convenient to express V is in cu. in., A_v in sq. in and b-x in ft.

$$V = V_o - 12A_v(b-x) \qquad \text{(cu.in)}$$

$$F_v = F_{vi}\left(\frac{V_o}{V}\right)^k \qquad \text{(lbs)}$$

$$W_x = \frac{F_vV - F_{vi}V_o}{12A_v(k-1)} \qquad \text{(ft.lbs)}$$

To compute F_v, we have $\log \frac{F_v}{F_{vi}} = k \log \frac{V_o}{V}$, a linear logarithmic equation and therefore may be readily plotted. Therefore, we may make a table for computation of the potential energy of the recuperator as follows:

		V			W_x
x	$12A_v(b-x)$	V	$k \log \frac{V_o}{V}$	F_v	$\frac{F_vV - F_{vi}V_o}{12A_v(k-1)}$

We have, from the energy equation

$$W_o - W_{x'} - (W_r \sin\emptyset + R_p)x_1 \text{ - - - - - - - - - - - - - - - - } = m_r \frac{v_1^2}{2}$$

$$W_{x'} - W_{x^2} - (W_r \sin\emptyset + R_p)(x_2 - x_1) - \frac{C_o' v_1^2}{w_{x_2'}^2} + m_r \frac{v_1^2}{2} = m_r \frac{v_2^2}{2}$$

$$W_{x^2} - W_{x^3} - W_r \sin\emptyset + R_p)(x_3 - x_2) - \frac{C_o' v_2^2}{w_{x_3^2}^2} + m_r \frac{v_2^2}{2} = m_r \frac{v_3^2}{2}$$

- -

- -

$$W_{x(n-1)} - (W_r \sin\emptyset + R_p)(x_n - x_{n-1}) - \frac{C_o' v_{n-1}^2}{w_{xn}^2} + m_r \frac{v_{n-1}^2}{2} = 0$$

The solution of these equations, may be put in a table form:

x	W_x	$W_{x'} - W_{x^2}$	$(W_r \sin\emptyset + R_p)\Delta x$	w_x	$\frac{C_o' v^2}{w_x^2}$	$m_r \frac{v_{n-1}^2}{2}$	$m_r \frac{v_n^2}{2}$	v
o	W_o							
x_1	$W_{x'}$	$W_o - W_{x'}$	$(W_r \sin\emptyset + R_p)x_1$	$w_{x'}$			$m_r \frac{v_1^2}{2}$	v_1
x_2	W_{x^2}	$W_{x^1} - W_{x^2}$	$(W_r \sin\emptyset + R_p)$ $(x_2 - x_1)$	$w_{x_2'}$	$\frac{C_o' v_1^2}{w_{x_2'}^2}$	$m_r \frac{v_1^2}{2}$	$m_r \frac{v_2^2}{2}$	v_2
x_3	W_{x^3}	$W_{x^2} - W_{x^3}$	$(W_r \sin\emptyset + R_p)$ $(x_3 - x_2)$	$w_{x_3^2}$	$\frac{C' v_2^2}{w_{x_3^2}^2}$	$m_r \frac{v_2^2}{2}$	$m_r \frac{v_3^2}{2}$	v_3
x_n	o	$W_{x(n-1)}$	$(W_r \sin\emptyset + R_p)$ $(x_n - x_{n-1})$	w_{xn}	$\frac{C_o v_{n-1}^2}{w_{xn}^2}$	o	o	

From the above table we may plot the velocity curve. To obtain the unbalanced force (accelerating or re-

tardation force of c'recoil) we have,

$$F_v - W_r(\sin\phi + n\cos\phi) - R_p - \frac{C_o' v^2}{w_x^2} = m_r\left(\frac{v_2^2 - v_1^2}{\Delta x_2'}\right) \quad \text{(lbs)}$$

RELATIVE ADVANTAGES OF THE LOGARITHMIC AND ENERGY METHOD FOR COMPUTATION OF COUNTER RECOIL:

In the design and computation of a c'recoil system, we are either concerned with counter recoil stability which is the primary limitation on c'recoil for small caliber mobile carriages, or with the maintaining of a low and constant buffer pressure, where c'recoil is no longer a consideration and the potential energy of the recuperator is large, as in large caliber artillery.

In the former case, it is important that the total unbalanced resistance to c'recoil or the total retardation towards the end of counter recoil, either remain constant or follow the c'recoil stability slope. In the latter case, however, it is important to maintain as low buffer pressure as possible and thus a constant buffer resistance is used in spite of the total resistance to c'recoil rising towards the end of c'recoil. In the calculation of the total accelerating or retarding force in c'recoil, the logarithmic method and the simple dynamic equation of c'recoil are preferable since we are only concerned with the total unbalanced force on the recoiling mass. During the first period of c'recoil a constant throttling orifice is usually used for regulation and large intervals may be taken by the logarithmic method. During the retardation the total resistance to c'recoil is usually constant and therefore we have the simple dynamic relation of a mass being brought to rest by a constant force. With a constant buffer force, the energy method is preferable since the work done by the buffer and corresponding kinetic energy and therefore the velocity of c'recoil

can be quickly estimated.

Estimation of the buffer resistance of c'recoil, with constant buffer force and corresponding velocity of c'recoil:

(1) If the buffer force acts only during the latter part of c'recoil, we have, three periods:

(a) the accelerating period, corresponding to the void displacement.

$$(W_o-W_a)-(W_r\sin\emptyset+R_p)X_a=m_r\frac{v_a^2}{2}\quad\text{(ft.lbs)}$$

(b) the retardation period where throttling takes place in a reverse direction through the recoil apertures only.

$$(W_a-W_b)-(W_r\sin\emptyset+R_p)(X_b-X_a)-\int_{x_a}^{x_b}\frac{C_o'v^2}{w_x^2}\,dx=\frac{m_r}{2}(v_b^2-v_a^2)$$

If we neglect the term, $\int_{x_a}^{x_b}\frac{C_o'v^2}{w_x^2}\,dx$ as small, we have immediately

$$(W_o-W_b)-(W_r\sin\emptyset+R_p)X_b=m_r\frac{v_b^2}{2}\quad\text{(ft.lbs)}$$

(c) the retardation period where the running forward brake or c'recoil buffer comes into action: assuming a constant buffer force, we have $\frac{C_o'v^2}{w_x^2}=B_x'$

and

$$W_b-(W_r\sin\emptyset+R_p)(b-x_b)-B_x'(b-x_b)=-m_r\frac{v_b^2}{2}$$

Substituting for $(-m_r \frac{v_b^2}{2})$ from the previous equation, we have

$$B_x'(b-x_b) = W_o - (W_r \sin \emptyset + R_p)b$$

hence

$$B_x' = \frac{W_r - (W_r \sin \emptyset + R_p)b}{b - x_b} \quad \text{(lbs)}$$

If A_b = the area of the buffer (sq.in)

$d_b = b - x_b$ = length of the buffer (ft)

b = length of recoil (ft)

p_b = the average buffer pressure (lbs/sq.in)

then we have for the average buffer pressure,

$$p_b = \frac{W_o - (W_r \sin \emptyset + R_p)b}{d_b A_b}$$

where

$$W_o = \frac{F_{vf} V_f - V_{vi} V_o}{A_v(k-1)} \qquad F_{vf} = m\, F_{vi}$$

$$V_f = V_o - A_v b \qquad m = \text{ratio of compression}$$

To compute the velocity curve during the buffer action, we have

$$W_b = W_x - (W_r \sin \emptyset + R_p)(x - x_b) - \int_{x_b}^{x} \frac{C_o' v^2}{w_x^2}\, dx = \frac{m_r}{2}(v_x^2 - v_b^2)$$

Since W_x and v vary at each point, the above equation may be divided into a step by step process, i. e.

$$W_b - W_{x_1} - (W_r \sin \emptyset + R_p)(x_1 - x_b) - \frac{C_o' v_b^2}{w_{b1}^2}(x_1 - x_b) = \frac{m_r}{2}(v_{x_1}^2 - v_b^2)$$

$$W_{x_1} - W_{x_2} - (W_r \sin \emptyset + R_p)(x_2 - x_1) - \frac{C_o' v_1^2}{w_{1_2}^2}(x_2 - x_1) = \frac{m_r}{2}(v_{x_2}^2 - v_{x_1}^2)$$

From the velocity curve and buffer pressure p_b

(2) Where the buffer force acts throughout the c'recoil.

At the beginning of c'recoil, the recoil apertures are small and the throttling through them during the c'recoil cannot be neglected. Since, however, this additional throttling is effective only for a short distance at the very beginning of counter recoil we have as a close approximation, for the average buffer force of c'recoil (assumed constant)

$$p_b = \frac{W_o-(W_v \sin \emptyset + R_p)}{A_b}$$

where as before, $W_o = \dfrac{F_{vf}V_f - F_{vi}V_o}{A_v(k-1)}$: $F_{vf} = mF_{vi}$: $V_f = V_o - A_v b$

COUNTER RECOIL SYSTEMS. Counter recoil systems may be broadly classified into:
(1) Those in which the brake comes into action during the latter part of counter recoil.
(2) Those in which the brake is effective throughout the counter recoil.

With (1) we have, usually some form of spear buffer which comes into action towards the end of recoil.

With (2) we have, usually a "filling in" type of buffer, the buffer being filled during the recoil and acting throughout the counter recoil.

Type (2) gives obviously far better counter recoil regulation than with type (1) where in the latter, we have considerable free counter recoil and corresponding high velocities before the buffer action takes place. This is especially true for long recoil guns.

(1) <u>Counter recoil systems, where the brake is only effective during the latter part of counter recoil.</u> The counter recoil

functioning may be divided into three periods:

(a) The acceleration period during the void displacement.

(b) The retardation period where throttling takes place in a reverse direction through the recoil apertures only.

(c) The retardation period where the running forward brake comes into action.

During period (a), we have

$$F_v - R_{s+p} - W_r(\sin \phi + n \cos \phi) = m_r v \frac{dv}{dx}$$ and the void displacement

$$X_{oa} = \frac{a_r b}{A} \quad \text{(ft)}$$

where a_r = area of recoil rod (sq.in)

A = effective area of recoil piston (sq.in)

b = length of recoil (ft)

hence

$$\int_0^{X_{oa}} [F_v - R_{s+p} - W_r(\sin\phi + n \cos \phi)dx = \frac{m_r v_a^2}{2}$$

As an approximation, we have

$$\left(\frac{F_{vf}+F_{va}}{2}\right) x_a - [R_{s+p} + W_r(\sin \phi + n \cos \phi)] x_a = \frac{m_r v_a^2}{2}$$

where F_{vf} = the max. recuperator reaction

F_{va} = the recuperator reaction at the end of the void displacement.

$$v_a = \sqrt{\frac{[(F_{vf}+F_{va}) - 2(R_{s+p}+W_r(\sin\phi + n \cos\phi)]X_a}{m_r}} \quad \text{(ft/sec)}$$

where v_a is usually the max. velocity of counter recoil. During period (b), we have

$$F_v - R_{s+p} - W_r(\sin \phi + n \cos \phi) - \frac{C_o v^2}{w_h^2} = m_r v \frac{dv}{dx}$$

where $\frac{C_o v^2}{w_h^2}$ = hydraulic braking reaction due to throttling through the recoil apertures.

Now the constant C_o is different from that of recoil since the area of displaced fluid and contraction of orifice on the return motion are different from these factors in the recoil. However, for a first approximation, we may assume C_o the same both in recoil and counter recoil. If V_a is the velocity of recoil, with total hydraulic pull P_h at displacement $b - X_{oa}$ in the recoil, we have

$$\frac{C_o v^2}{w_h^2} = \frac{v_a^2}{V_a^2} P_h \quad \text{approximately}$$

and therefore, approximately,

$$\left(\frac{F_{va}+F_{vb}}{2}\right) - R_{s+p} - W_r(\sin \emptyset + n \cos \emptyset) - \frac{v_a^2}{V_a^2} P_h(x_b - x_a) = \frac{m_r}{2}(v_b^2 - v_a^2)$$

where F_{va} = recuperator reaction at end of the void

F_{vb} = recuperator reaction at entrance to buffer.

From the above equation, knowing v_a we may readily compute v_b. During period (c), we have

$$F_v - R_{s+p} - W_r(\sin\emptyset + n \cos \emptyset) - \frac{C_o v^2}{w_h^2} - \frac{C_o' v^2}{w_x^2} = m_r v \frac{dv}{dx}$$

where $\frac{C_o' v^2}{w_x^2}$ = the hydraulic braking due to the counter recoil buffer.

Now the term $\frac{C_o v^2}{w_h^2}$ is small compared with $\frac{C_o' v^2}{w_x^2}$ and may be neglected, especially during the latter part of period (c).

Period (c) is the critical period of the counter recoil since the reaction,

$$\frac{C_o'v^2}{w_x^2} + W_r(\sin \emptyset + n \cos \emptyset) + R_{s+p} - F_v \leq \frac{W_s l_s'}{d'}$$

where $\frac{W_s l_s'}{d'}$ = the max. stabilizing force in battery.

The buffer throttling should be designed, either with

$$-K_v = \frac{C_o'v^2}{w_x^2} + W_r(\sin \emptyset + n \cos \emptyset) + R_{s+p} - F_v = C_s' \frac{W_s l_s'}{d'}$$

<u>a constant</u> or

$$-K_v = \frac{C_o'v^2}{w_x^2} + W_r(\sin \emptyset + n \cos \emptyset) + R_{s+p} - F_v = C_s'\left(\frac{W_s l_s'}{d'}\right) + W_r(b-x)\cos \emptyset$$

that is consistent with the stability slope of counter recoil. By the latter method the buffer action may have a somewhat shorter displacement in the recoil and yet maintain the same factor of stability as in the former.

At this point it is well to emphasize, that a <u>constant</u> buffer resistance is entirely inconsistent with counter recoil stability since the total counter recoil resistance becomes greater in the battery position than at the entrance to the buffer. Therefore, a longer buffer is required, for the same mean resistance to counter recoil.

(2) <u>Counter recoil systems, where the brake is effective throughout the recoil.</u>

In this type of counter recoil, it is customary to regulate the maximum velocity attained during the acceleration period to a low value, by the use of a constant orifice throughout the accelerating period. A constant orifice during the first period of c'recoil has distinctive advantage since it gives a satisfactory control together with simplicity from a fabrication point of view.

During the latter part of the counter recoil, it is obviously necessary to introduce a variable orifice in order that the recoiling mass may be brought to rest

gradually. We have, therefore,

(a) The accelerating period with a constant orifice.

(b) The retardation period with a variable orifice.

We have the two systems of regulation:

(1) By a buffer brake control in the recoil hydraulic brake cylinder throughout the counter recoil.

(2) By lowering the pressure in the recuperator cylinder by throttling through an orifice between the air and recuperator cylinders.

With simple spring or pneumatic recuperator systems we must use a regulation system similar to type (1). With hydro-pneumatic recoil systems we may use type (2) alone, as in the St.Chamond or Puteaux brakes, or a combination of type (1) and type (2) regulation, as in the Filloux and Vickers recoil mechanisms.

In either type (1) or type (2) regulation, for the running forward brake effective throughout counter recoil, we have, exactly the same characteristic dynamic equation.

With a simple recuperator of a spring or pure pneumatic type, we have for the equation of motion,

$$F_v - W_r(\sin \emptyset + u \cos \emptyset) - R_{s+p} - \frac{C_o' v^2}{w_x^2} = m_r \, v \frac{dv}{dx}$$

whereas with a hydropneumatic, assuming the pressure lowered in the recuperator cylinder by throttling between the air and recuperator cylinders, we have,

$$\left(p_a' - \frac{C_o''' v^2}{w_x^2}\right)A_v - W_r(\sin \emptyset + u \cos \emptyset) - R_{s+p} = m_r \, v \frac{dv}{dx}$$

and if we let $p_a' A_v = F_v'$ the equivalent recuperator reaction

$\frac{C_o'''v^2}{w_v^2} A_v = \frac{C_o''v^2}{w_v^2}$ we have, as before

$$F_v - W_r(\sin \emptyset + u \cos \emptyset) - R_{s+p} - \frac{C_o''v^2}{w_v^2} = m_r\, v\, \frac{dv}{dx}$$

Further, for the critical condition of counter recoil stability, that is counter recoil at horizontal elevation, in type (1), the reaction on the carriage, consists of:

(1) F_v acting to the rear

(2) $m\, W_r + R_{s+p}$ acting forward

(3) $\frac{C_o'v^2}{w_x^2}$ acting forward

It may be easily shown that the resultant of these reactions acts in a line, through the center of gravity of the recoiling parts, the effect of the reaction of the guides being to transfer the various resistances and pulls to the center of gravity of the recoiling parts.

In recoil systems of type (2), the reaction on the carriage consists of:

(1) $(p_a' - \frac{C_o'''v^2}{w_v^2})A_v$ acting to the rear

(2) $n\, W_r + R_{s+p}$ acting forward.

The line of action of the resultant as before passing through the center of gravity of the recoiling parts. But the effect of

$(p_a' - \frac{C_o'''v^2}{w_v^2})A_v$ is exactly the same as $F_v - \frac{C_o''v^2}{w_v^2}$

where $F_v = p_a'A_v$ and $\frac{C_o''v^2}{w_v^2} = \frac{C_o'''A_v v^2}{w_v^2}$

Hence so far as the reactions on the mount and motion

are concerned, the hydro pneumatic and spring return types of recuperators are exactly similar.

<u>For the first period of counter recoil:</u> Assuming a constant orifice during the first part of recoil, we have

$$F_v - W_r(\sin\emptyset + u\cos\emptyset) - R_{s+p} - \frac{C'v^2}{w_o^2} = m_r v \frac{dv}{dx}$$

If we let, $R = (W_r \sin\emptyset + u\cos) + R_{s+p}$ then

$$F_v - R - \frac{C v^2}{w_o^2} = m_r v \frac{dv}{dx}$$

Now F_v is a function of x, and therefore the equation cannot be readily integrated. But since F_v does not vary greatly over a short interval, we may assume mean values of F_v for a few given intervals. The advantage of the integration of the equation, is that we may greatly reduce the number of intervals as compared with that of a step by step process and obtain sufficiently exact results.

Considering any two points x_1 and x_2 in the counter recoil, we have

$$\int_{x_1}^{x_2} dx = \int_{v_1}^{v_2} \frac{m_r v\, dv}{A - \frac{C_o v^2}{w_o^2}} \qquad \text{where } A = F_v - R$$

Rearranging, we find

$$\int_{x_1}^{x_2} dx = -\frac{m_r w_o^2}{2C_o} \int \frac{d\left(A - \frac{cv^2}{w_o^2}\right)}{A - \frac{cv^2}{w_o^2}}$$

hence

$$x_2 - x_1 = -\frac{m_r w_o^2}{2C'_o}\left[\log_e\left(A - \frac{cv_2^2}{w_o^2}\right) - \log_e\left(A - \frac{cv_1^2}{w_o^2}\right)\right]$$

and

$$-\frac{2C(x_2-x_1)}{m_r w_o^2} = \log_e\left(A - \frac{C_o v_2^2}{w_o^2}\right) - \log_e\left(A - \frac{C_o v_1^2}{w_o^2}\right)$$

therefore

$$\log\left(A - \frac{C_o' v_2^2}{w_o^2}\right) = \log\left(A - \frac{C_o' v_1^2}{w_o^2}\right) - \frac{2C(x_2-x_1)}{2.3 m_r w_o^2}$$

where $A = F_v - W_r(\sin \phi + u \cos \phi) - R_{s+p}$ (lbs)

From this equation knowing the velocity at the beginning of any arbitrary interval and with the mean recuperator reaction for the interval we may compute the velocity at the end of the interval. Further fairly large intervals may be assumed provided the recuperator reaction does not vary greatly at the limits of the interval.

The velocity curve is computed by this method to $x = b_h - d$ from the out of battery position, where d = length of the retardation or variable orifice period, in ft. and b = length of recoil in ft.

The velocity v_b at the end of the acceleration period is usually taken at approximately 3.5 ft/sec. at horizontal elevation, though a more rational assumption of the velocity should be based on the following:

Let h = height of bore from ground in ft. (horizontal c'recoil)

d = length of buffer or variable throttling interval (ft)

W_s = weight of total system gun + carriage (lbs)

l_s' = horizontal length from W_s to contact of wheeled ground (ft)

C_s = factor of stability (= 0.85 usually)

R_h' = counter recoil reaction at horizontal elevation

k = proportional distance of d that the c'recoil energy is to be dissipated along. k = 0.7 to 0.9

Then, the counter recoil reaction at horizontal elevation, becomes,

$$R_h' = \frac{\frac{1}{2} m_r v_b^2}{kd} = C_s \frac{W_s l_s'}{h}$$

With variable recoil, assuming the length of recoil to be at short recoil one half of that at long recoil, in order to have sufficient displacement for acceleration at maximum elevation the buffer or variable throttling should not take place at horizontal recoil for over 1/3 to 1/4 the recoil.

Hence $d = 0.33$ to $0.25\ b_h = 0.3\ b_h$ approx.

$$v_b \sqrt{\frac{0.6\ C_s k\ W_s l_s' b_h}{m_r\ h}} = 3.62 \sqrt{\frac{W_s C_s' b_h}{w_r\ h}} \quad \text{(ft/sec)}$$

Knowing v_b we may estimate the proper size of the counter recoil constant orifice. Actually the maximum velocity of counter recoil is attained shortly after the out of battery position and at this position the acceleration is zero. But since the retardation is very slight until the variable orifice is encountered, we may assume the recoiling mass to move with uniform velocity at the entrance to the buffer or variable throttling. Therefore at horizontal recoil,

$$F_v - n\ W_r - R_{s+p} - \frac{C_o' v^2}{w_o^2} = 0$$

Hence, the constant orifice becomes, $w_o = \sqrt{\dfrac{C_o' v_b^2}{F_v - nW_r - R_{s+p}}}$

where $v_b = 3.62 \sqrt{\dfrac{W_s l_s' b_h}{W_r\ h}}$ where for a spring or pneumatic return recuperator system

$$C_o' = \frac{C'^2 A_b^3}{175}$$

C' = reciprocal of orifice contraction factor and

A_b = area of buffer (sq.in)

F_v = recuperator reaction at displacement $X = b_h - d$ (ft) and for hydro pneumatic recuperator system,

$$C_o' = \frac{C'^2 A_v^3}{175}$$

C' = reciprocal of orifice contraction factor.
A_v = effective area of recuperator piston (sq.in)
$F_v = p_a' A_v$ lbs.
p_a' = pressure of oil in air cylinder.

<u>For second period of counter recoil:</u> During this period it is customary to maintain a constant total retarding force which at horizontal elevation becomes,

$$F_v - n\,W_r - R_{s+p} - \frac{C_o' v^2}{w_x^2} = R_h' \qquad \text{(lbs)}$$

where $R_h' = C_s \dfrac{W_s l_s'}{h}$ (lbs)

Since the counter recoil reaction is constant during the retardation, the velocity is a parabolic function of the displacement, that is

$$v = 8.03\sqrt{\frac{R_h'(b-x)}{W_r}} \qquad \text{(ft/sec)}$$

Substituting this value of v in the following equation, we have

$$w_x = \sqrt{\frac{C_o' v^2}{F_v - n\,W_r - R_{s+p}}} \qquad \text{(sq.in)}$$

where for a spring or pneumatic recuperator, $C_o' = \dfrac{C'^2 A_b^3}{175}$

C' = reciprocal of orifice contraction factor
A_b = area of buffer (sq.in)
F_v = recuperator reaction at displacement x, for a hydro pneumatic recuperator system, $C_o' = \dfrac{C'^2 A_v^3}{175}$

C' = reciprocal of orifice contraction factor
A_v = effective area of recuperator piston
$F_v = p_a' A_v$
p_a' = pressure in oil in air cylinder (lbs/sq.in)

COUNTER RECOIL FUNCTIONING WITH VARIABLE RECOIL. With a variable recoil, the requirements of proper counter recoil functioning for all elevations are more difficult to obtain. At horizontal recoil we must meet the condition of counter recoil stability, whereas at maximum elevation, the time period of the counter recoil, for rapid fire, must not be too long. Since the recoil at maximum elevation is a fraction of that at horizontal recoil, the recuperator reaction at the beginning of counter recoil at maximum elevation is necessarily smaller than that at horizontal elevation. Further at maximum elevation we have the weight component resisting motion. Therefore, the accelerating force is necessarily considerably smaller than at horizontal elevation and the velocity attained at maximum elevation becomes a function of that at horizontal recoil. In the design of a counter recoil system in order to obtain sufficient velocity in the counter recoil at maximum elevation, it is important that a proper compression ratio be used. This in turn effects the initial volume of the recuperator and therefore the entire layout of the recuperator forging. It is here important to emphasize that proper functioning of counter recoil can not be attained by increasing pressure where an improper ratio of compression is used.

The following analysis gives a rough approximation as to the requirementts to be met for proper counter recoil functioning at all elevations with a variable recoil.

It will be assumed that the recoil at maximum elevation is reduced to one half that at horizontal recoil and that a constant orifice is maintained until the latter third or fourth of the counter recoil. We have therefore a constant orifice which is the same for the accelerating period of counter recoil at maximum elevation or horizontal recoil.

If now

F_{vi} = initial recuperator reaction (lbs)

F_{vf} = final recuperator reaction (lbs)

F_{vm} = recuperator reaction at middle of horizontal or long recoil (lbs)

v_s = maximum velocity of counter recoil at maximum elevation (ft/sec)

v_h = maximum velocity of counter recoil at horizontal elevation (ft/sec)

w_o = area of constant orifice (sq.in)

C_o' = throttling constant

R_{s+p} = stuffing + packing friction (lbs)

$\emptyset_m$ = maximum elevation

As a first approximation, we will assume, the maximum horizontal counter recoil velocity to be attained after a displacement equal to one half the recoil. Hence

$$F_{vm} - n\,W_r - R_{s+p} - \frac{C_o' v_h^2}{w_o^2} = 0 \qquad (1)$$

At maximum elevation, the maximum velocity of counter recoil will be attained somewhat after a displacement equal to half the recoil, but we are not greatly in error in assuming the same recuperator reaction F_{vm}. Hence

$$F_{vm} - W_r(\sin\emptyset + n\cos\emptyset) - R_{s+p} - \frac{C_o' v_s^2}{w_o^2} = 0 \qquad (2)$$

Subtracting (2) from (1), we have

$$W_r[\sin\emptyset - (1 + \cos\emptyset)n] = \frac{C_o'(v_h^2 - v_s^2)}{w_o^2}$$

hence

$$\frac{C_o'}{w_o^2} = \frac{W_r[\sin\emptyset - n(1 + \cos\emptyset)]}{v_h^2 - v_s^2}$$

We have therefore for required recuperator reaction at the middle of the recoil

$$F_{vm} = n\,W_r + R_{s+p} + \frac{v_h^2}{v_h^2 - v_s^2}\,W_r[\sin\emptyset - n(1+\cos\emptyset)]$$

If we assume values for v_h and v_s for design approximations, we may take, $v_h = 3.5$ ft per sec.

$v_s = 2.5$ ft per sec.

then, $F_{vm} = n\,W_r + R_{s+p} + 2W_r[\sin\emptyset - n(1+\cos\emptyset)]$

If we take a large coefficient of guide friction we neglect R_{s+p}; hence if $n = 0.3$,

$F_{vm} = 0.3\,W_r + 2W_r[\sin\emptyset_m - 0.3(1+\cos\emptyset_m)]$

To obtain the minimum allowable ratio of compression, for spring recuperators, we have $2(F_{vm} - F_{vi}) = F_{fv} - F_{vi}$ and $F_{vf} = F_{vi} + 2(F_{vm} - F_{vi})$ hence

$$m = \frac{F_{vf}}{F_{vi}} = \frac{2(F_{vm} - 0.5F_{vi})}{F_{vi}}$$

With a pneumatic or hydropneumatic recuperator, we have $2.5(F_{vm} - F_{vi}) = F_{vf} - F_{vi}$ (approx.) and

$F_{vf} = F_{vi} + 2.5(F_{vm} - F_{vi})$ hence

$$\frac{F_{vf}}{F_{vi}} = n = \frac{2.5F_{vm} - 1.5F_{vi}}{F_{vi}} = \frac{1.5(1.66F_{vm} - F_{vi})}{F_{vi}}$$

RECUPERATORS.

GENERAL CONSIDERATIONS. After the recoil the recoiling mass must be brought into battery and this must take place at any elevation of the gun and held there until the next cycle of the firing. Obviously sufficient potential energy must be stored during the recoil to overcome the counter recoil friction and the weight component at maximum elevation throughout the count-

er recoil. Further in order that the counter recoil may be made in mimimum time, an excess potential energy is required over that required for friction and gravity, in order that a rapid acceleration at the beginning of counter recoil may be attained. Finally in the battery position an excess recuperator reaction is necessary over that for balancing the weight component and overcoming the friction in case of a slight slipping back of the piece in the battery position.

Therefore a satisfactory recuperator must satisfy the following requisites:

(1) The initial recuperator reaction should have a marginal excess over that required to balance the friction in battery and the weight component at maximum elevation.

(2) The potential energy of the recuperator at the end of recoil must be sufficient to overcome the work of friction and gravity at maximum elevation during the recoil and rapidly accelerate the gun at the beginning of counter recoil.

INITIAL RECUPERATOR REACTION. In general the size or bulk of the recuperator whether spring or hydro pneumatic depends upon the magnitude of the initial recuperator reaction. It becomes, therefore, important to estimate the required initial recuperator reaction to a considerable degree of accuracy. This is especially true in certain types of recoil systems where the size of the forging, especially for guns of high elevation, depends directly upon the magnitude of the initial recuperator reaction and it becomes very important to make this a minimum.

Let R_g = guide friction (lbs)

R_{pv} = packing friction of recuperator (lbs)

F_{vi} = initial recuperator reaction (lbs)

e_v = distance down from center of gravity of recoiling parts to line of action of F_v (in)

Q_1 = front normal clip reaction (lbs)

Q_2 = rear normal clip reaction (lbs)

x_1 and y_1 = coordinates of front clip reaction (in)

x_2 and y_2 = coordinates of rear clip reaction (in)

n = coefficient of guide friction = 0.15 approx.

$\emptyset_m$ = angle of maximum elevation.

l = distance between clip reactions (in)

Considering the recoiling mass at maximum elevation in battery, case of slight slipping back from the battery position, we must have (see fig)

$$F_{vi}-R_{pv}=n(Q_1+Q_2)+W_r\sin\emptyset_m$$

or

$$F'_{vi}=n(Q_1+Q_2)+W_r\sin\emptyset_m \quad (1)$$

and normal to the guides, $Q_2-Q_1=W_r\cos\emptyset$ (2)

and taking moments about the center of gravity of the recoiling parts,

$$F'_{vi}\ e_v-Q_1x_1-Q_2x_2+nQ_1y_1-n\ Q_2v_2 = 0 \quad (3)$$

Substituting (2) in (3), we have,

$$F'_{vi}\ e_v-Q_1x_1-Q_2x_2-W_r\cos\emptyset x_2+n\ Q_1y_1-n\ Q_1y_2-n\ W_r\cos\emptyset\ y_2 = 0$$

$$\text{hence}\quad Q_1 = \frac{F'_{vi}\ e_v-W_r\cos\emptyset(x_2+n\ y_2)}{x_1+x_2+n(y_2-y_1)} \quad \text{(lbs)} \quad (4)$$

and solving for Q_2,

$$F'_{vi}\ e_v-Q_2-W_r\cos\emptyset)x_1-Q_2x_2+n(Q_2-W_r\cos\emptyset)y_1-nQ_2y_2 = 0$$

$$\text{hence}\ Q_2 = \frac{F'_{vi}\ e_v+W_r\cos\emptyset(x_1-ny_2)}{x_1+x_2+n(y_2-y_1)} \quad (5)$$

Hence, with sleeveguides,

$$R_g = n(Q_1 + Q_2) = \frac{2F'_{vi}\ e_v + W_r \cos\emptyset_m [x_1 - x_2 - n(y_1 + y_2)]}{x_1 + x_2 + n(y_2 - y_1)} n \quad (6)$$

With grooved guides y_1 becomes negative and

$$R_g = n(Q_1 + Q_2) = \frac{2F'_{vi}\ e_v + W_r \cos\emptyset_m [x_1 - x_2 - n(y_2 - y_1)]}{x_1 + x_2 + n(y_2 + y_1)} n \quad (7)$$

Since with grooved guides, $y_2 = y_1$ approx., also $x_1 + x_2 = l$ = distance between clip reactions, and $y_1 = y_2 = e_r$ = mean distance to guide friction, we have,

$$R_g = \frac{2F'_{vi}\ e_v + W_r \cos\emptyset_m (x_1 - x_2)}{l + 2\ n\ e_r} n \text{ (with grooved guides)} \quad (8)$$

$$R_g = \frac{2F'_{vi}\ e_v + W_r \cos\emptyset_m (x_1 - x_2)}{l} n \text{ (with sleeve guides)} \quad (9)$$

Substituting in eq.(1) we have

$$F'_{vi} = \frac{2F'_{vi}\ e_v + W_r \cos\emptyset (x_1 - x_2)}{l + 2\ n\ e_r} n + W_r \sin\emptyset_m$$

hence

$$F'_{vi} = \frac{W_r \left[\sin\emptyset_m + \dfrac{n \cos\emptyset_m (x_1 - x_2)}{l + 2\ n\ e_r}\right]}{1 - \dfrac{2\ e_v n}{l + 2n\ e_r}} \text{ (lbs)} \quad (10)$$

and for the initial recuperator reaction,

$$F_{vi} \geqq R_p + \frac{W_r \left[\sin\emptyset_m + \dfrac{n \cos\ \emptyset_m (x_1 - x_2)}{l + 2n\ e_r}\right]}{1 - \dfrac{2\ e_v\ n}{l + 2\ n\ e_r}}$$

$$=k\left\{R_p = \frac{W_r[\sin\emptyset_m + \dfrac{n\cos\emptyset_m(x_1-x_2)}{1+2\,n\,e_r}]}{1 - \dfrac{2\,e_v\,n}{1+2\,n\,e_r}}\right\} \text{ where } k = 1.1 \text{ to } 1.2 \quad (11)$$

l = distance between clip reactions (in) with 3 clips —— $l = \frac{b}{2}$

with 4 clips: —— $l = b$

b = length of recoil (in)

Estimation of Recuperator Packing Friction R_p:

With hydro pneumatic recuperator systems, the packing friction is usually a linear function of the recuperator pressure. Assuming a given initial intensity of pressure $p_{v\ max}$ lbs/sq.in. in the recuperator, we have, $R_p=C_p p_{v\ max}$.

The packing friction in the recuperator is divided into the suffing box friction plus the recuperator piston friction. To estimate these frictions we must know the diameter of the recuperator piston rod and recuperator piston.

To roughly estimate these diameters, we have for the effective area of the recuperator piston,

$$A_v = \frac{1.3W_r(\sin\emptyset_m+0.3\cos\emptyset_m)}{p_{v\ max}} \quad \text{(sq.in)}$$

for the required area of the recuperator rod,

$$a_v = \frac{2.6W_r(\sin\emptyset_m+0.3\cos\emptyset_m)}{f_m} \quad \text{(sq.in)}$$

where f_m = allowable fibre stress in rod material.

Then the diameter of the piston, becomes,

$$D_v = \sqrt{\frac{A_v+a_v}{0.7854}} \quad \text{(in)}$$

and the diameter of the rod becomes, $d_v = \sqrt{\dfrac{a_v}{0.7854}}$ (in)

If w_{sv} = width of stuffing box packing of recuperator(assumed)(in)

w_{pv} = width of piston packing of recuperator (assumed)(in)

then assuming the pressure normal to the cylinder or surface of the rod to be made equal to the hydrostatic pressure in the cylinder, we have

$R_p = (.05\pi\ w_{pv}\ D_v + .05\ \pi\ w_{sv}\ d_v)p_{v\ max.}$

$= .05\pi(w_{pv}\ D_v + w_{sv}\ d_v)p_{v\ max.}$ (lbs)

where .05 = approx. coefficient of friction of the packing.

Approximate Initial Recuperator Reaction:

For preliminary calculations, especially when the type of packing and arrangement of cylinders has not been considered we may neglect the recuperator packing friction by increasing the coefficient of guide friction.

Without pinching action of the guides in battery the guide friction, $R_g = 0.15\ W_r \cos \phi$ (approx) (lbs). To account, for a possible pinching action, as well as the packing friction, for elevations up to 65°, approx. $R_g = 0.30\ W_r \cos \phi$ (lbs) and the required initial recuperator reaction, to allow for possible variations, should be increased from 20% to 30% over that required to hold the gun in battery. Hence $F_{vi} = 1.3\ W_r(\sin\phi_m + 0.3 \cos\phi)$ (lbs). With guns of very high elevation, $R_g = 0.3 \cos \phi$ becomes negligible. However, the packing friction remains the same whereas the guide friction is comparable with that at horizontal recoil due to the pinching action of the guides at maximum elevation. Therefore, it is desirable to use an approximate formula taking these factors

into consideration. We have, approx. $R_g = \frac{2n\, e_b F_{vi}}{l}$

where n = 0.1 to 0.2. If we take n = 0.3 to account for the recuperator packing friction, we have at high elevations,

$$F_{vi} = 1.3(W_r \sin \emptyset_m + \frac{0.6\, F_{vi}\, e_b}{l}) \quad \text{(lbs)}$$

where e_b = distance from bore to line of action of F_{vi} (assumed)(in)

l = distance between clip reactions (in)

with 3 clips ----- $l = \frac{b}{2}$

with 4 clips ----- $l = b$

b = length of recoil (in)

ENERGY REQUIREMENTS FOR PROPER RECUPERATION. The initial recuperator reaction is designed to be somewhat greater than that required to hold the gun in battery at maximum elevation, against the guide and packing frictions. Further, the recuperator reaction, being necessarily derived from a potential function, must therefore increase with the displacement out of battery. The work done by the recuperator, therefore, is in excess of that required and we have, always, an excess potential energy over that required to bring the gun into battery. This excess energy is dissipated by the counter recoil regulator. We have, therefore, merely a transfer of part of the recoil energy, dissipated by means of the recuperator, ultimately in the counter recoil. The total heating or rather the average in a recoil cycle is quite independent of the magnitude of the compression. However, with high compression ratios, we have extreme local heating where the radiation is small and therefore injurious effects are likely to result with the air packings in hydro pneu-

matic recoil systems. Further excessive potential energy stored in the recuperator, requires careful counter recoil regulation, and as stability on counter recoil is far more sensitive than on recoil, we have more difficulty in meeting the rigid requirements of counter recoil stability. Finally with excessive recuperator energy to maintain low counter recoil regulator or buffer pressures requires a cumbersome and large counter recoil regulator whereas it is far simpler constructively to dissipate the recoil energy during the recoil.

Therefore excessive recuperator energy is undesirable for the following reasons:

(1) Localized heating resulting with hydro pneumatic recuperators, is injurious to the packing.

(2) Difficulty in counter recoil regulation and meeting counter recoil stability requirements.

(3) Constructive difficulties due to a bulky counter recoil buffer or regulator required to maintain moderate pressures in the buffer chamber.

On the other hand, the mean recuperator reaction must be sufficient not only to balance the weight component of the recoiling parts and frictions, but enough to accelerate the recoiling parts to a given minimum velocity for counter recoil at all angles of elevation. Since it is constructively complicated and more or less impractical to introduce varying counter recoil regulation as the gun elevates in the majority of the types of recoil systems are designed on the bases of given maximum velocity at horizontal elevation consistent with counter recoil stability and a given minimum velocity at maximum elevation, consistent with reasonable time of counter recoil at maximum

elevation. Usually the recoil is shortened at maximum elevation. We are not greatly in error in assuming the respective velocities to be attained at a displacement corresponding to the mean recuperator reaction, which is roughly from one half to two thirds away from the battery position.

We have, then, with a variable recoil, if

F_{vm} = mean recuperator reaction (lbs)

R_{s+p} = total packing friction in counter recoil (lbs)

C_o' = throttling constant of regulator

w_o = throttling orifice of regulator (sq.in)

v_h = velocity of horizontal c'recoil (ft/sec)

v_s = velocity of c'recoil at maximum elevation (ft/sec)

n = coefficient of guide friction,

for the motion of the recoiling parts at horizontal recoil,

$$F_{vm} - n\,W_r - R_{s+p} - \frac{C_o' v_h^2}{w_o^2} = 0$$

for the motion of the recoiling parts at maximum elevation,

$$F_{vm} - W_r(\sin \phi_m + n \cos \phi_m) + R_{s+p} - \frac{C_o' v_s^2}{w_o^2} = 0$$

Subtracting, we obtain

$$\frac{C_o'}{w_o^2} = \frac{W_r[\sin \phi_m - n(1+\cos \phi_m)]}{v_h^2 - v_s^2}$$

and

$$F_{vm} = n\,W_r + R_{s+p} + \frac{v_h^2}{v_h^2 - v_s^2} W_r[\sin \phi - n(1+\cos \phi)]$$

We see, therefore, that the mean recuperator reaction required depends greatly on the square of the horizontal c'recoil velocity and inversely as the difference between the squares of the horizont-

al and maximum elevation, c'recoil velocities. Since v_h is more or less fixed by c'recoil stability limitations, whereas v_s depends upon the time allowed for counter recoil functioning at maximum elevation, F_{vm} becomes more or less fixed and therefore the required excess potential energy of the recuperator.

Assuming design values of v_h = 3.5 ft/sec. and v_s = 2.5 ft/sec. with an increased coefficient of guide friction to compensate for the packing friction, n = 0.3, we have

$F_{vm} = 0.3W_r+2W_r[\sin\emptyset_m-0.3(1+\cos\emptyset_m)]$ which gives a rough approximation as to the value of the mean recuperator reactions required.

CALCULATION OF THE MEAN RECUPERATOR REACTION AND THE ENERGY STORED IN THE RECUPERATOR.

SPRING RECUPERATORS. With spring return recuperators, we have the recuperator reaction increasing proportionally with the recoil. If $F_{vi} = S_o$ = the initial spring recuperator reaction (lbs)

$F_{vf} = S_f$ = the final spring recuperator reaction (lbs)

b = length of recoil (ft)

Then

$$F_{vm} = \frac{S_f+S_o}{2} = \frac{F_{vi}+F_{vf}}{2} \quad \text{(lbs)}$$

hence $F_{vf} = 2F_{vm}-F_{vi}$ (lbs)

The potential energy stored in the recuperator for s displacement x, becomes

$$W = \int_o^x \left(S_o+\frac{S_f-S_o}{b}x\right)dx$$

$$= S_o x + \frac{S_f - S_o}{2b} x^2 \text{ (ft.lbs)}$$

and the total potential energy required at the end of recoil, becomes

$$W = (S_o + S_f)\frac{b}{2} = (F_{vi} + F_{vf})\frac{b}{2} \text{ (ft.lbs)}$$

With hydro pneumatic or pneumatic recuperators, we have the recuperator reaction increasing as an exponential function of the recoil displacement. If

p_a = intensity of air pressure in recuperator at any displacement in the recoil X (lbs/sq.ft)

p_{ai} = initial pressure in the recuperator (lbs/sq.ft)

p_{af} = final or maximum pressure in the recuperator (lbs/sq.ft)

$m = \frac{p_{af}}{p_{ai}}$ = ratio of compression

A_v = effective area of recuperator piston (sq. in)

V = volume of recuperator at displacement x (cu.ft)

V_o = initial volume of recuperator (cu.ft)

V_f = final volume of recuperator (cu.ft)

x = recoil displacement (ft)

b = total length of recoil (ft)

Then,

$$p_a V^k = p_{ai} V_o^k$$

where k = 1.1 for oil in contact with air

= 1.2 for oil separated from air by a floating piston.

Since $V = V_o - A_v x$, for a recoil displacement x, we have

$$p_a = p_{ai} \left(\frac{V_o}{V_o - A_v x}\right)^k$$

or in terms of the total recuperator reaction

$$F_v = F_{vi}\left(\frac{V_o}{V_o - A_v x}\right)^k \quad \text{since } F_v = p_a A_v$$

The work of compression, becomes

$$W_x = -\int_{V_o}^{V} p_a \, dV = -p_{ai} V_o^k \int_{V_o}^{V} \frac{dV}{V^k} \quad \text{(ft.lbs)}$$

$$= \frac{-p_{ai} V_o^k}{1-k}\left(\frac{1}{V^{k-1}} - \frac{1}{V_o^{k-1}}\right) \quad \text{(ft.lbs)}$$

$$= \frac{p_{ai} V_o^k}{k-1}\left(\frac{1}{V^{k-1}} - \frac{1}{V_o^{k-1}}\right) \quad \text{(ft.lbs)}$$

Since $F_{vi} = p_{ai} A_v$, we have for the work of compression in terms of the total initial recuperator reaction

$$W_x = \frac{F_{vi} \; V_o^k}{A_v (k-1)}\left(\frac{1}{V^{k-1}} - \frac{1}{V_o^{k-1}}\right)$$

where as before $V = V_o - A_v x$. At the end of recoil, we have substituting, for V, $V_f = F_o - A_v b$

$$W_x = \frac{p_{ai} V_o^k}{k-1}\left(\frac{1}{V_f^{k-1}} - \frac{1}{V_o^{k-1}}\right)$$

now $m = \frac{p_{af}}{p} = \left(\frac{V_o}{V_f}\right)^k$ = the ratio of compression.

The total work of compression in terms of "m" becomes

$$W_b = \frac{p_{ai} \; V_o}{k-1}\left(m^{\frac{k-1}{k}} - 1\right) \text{ (ft.lbs)}$$

It is customary to measure the pressure in lbs. per sq.in. rather than lbs. per sq. ft. and the volume in cu. in. The above formulas, become

$$W_x = \frac{p_{ai} \; V_o^k}{12(k-1)}\left(\frac{1}{V^{k-1}} - \frac{1}{V_o^{k-1}}\right) \text{ (ft.lbs)}$$

$$W_b = \frac{p_{ai}V_o}{12(k-1)}(m^{\frac{k-1}{k}} - 1) \text{ (ft.lbs)}$$ or in terms of the initial recuperator reaction F_{vi} and the effective area of the recuperator piston A_v(sq.in) we have

$$W_x = \frac{V_{vi}\ V_o^k}{12A_v(k-1)}\left(\frac{1}{V^{k-1}} - \frac{1}{V_o^{k-1}}\right) \text{ (ft.lbs)}$$

$$W_b = \frac{F_{vi}\ V_o^k}{12A_v(k-1)}(m^{\frac{k-1}{k}} - 1) \text{ (ft. lbs)}$$

and $p_a A_v = F_v = p_{ai}\ A_v\left(\frac{V_o}{V_o - A_v x}\right)$ (lbs)

where x = recoil displacement (inches)

A_v= effective area of recuperator piston(sq.in)

V_o= initial volume (cu.in)

p_{ai}=initial recuperator pressure (lbs/sq.in)

$V = V_o - A_v x$ (cu.in)

The mean recuperator reaction, becomes,

$$F_{vm} = \frac{F_{vi}V_o}{A_v b(k-1)}(m^{\frac{k-1}{k}} - 1) \text{ (lbs)}$$ where A_v is in sq. ft., b in ft., and V_o in cu.ft.

Since $\frac{p_{af}}{p_{ai}} = m = \left(\frac{V_o}{V_f}\right)^k$

$\frac{V_o}{V_f} = m^{\frac{1}{k}}$ and $V_f = \frac{V_o}{m^{\frac{1}{k}}}$ hence

$V_o\left(1 - \frac{1}{m^{\frac{1}{k}}}\right)A_v b$ and $A_v b = V_o\left(\frac{m^{\frac{1}{k}} - 1}{m^{\frac{1}{k}}}\right)$ therefore

$$F_{vm} = F_{vi}\left(\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}\right)\left(\frac{m^{\frac{k-1}{k}} - 1}{k-1}\right) \text{ (lbs)}$$ which gives the mean recuperator reaction in terms of the initial recuperator reaction and the ratio of compression

$$m = \frac{F_{vf}}{F_{vi}}$$

Since $F_{vi} = 1.3W_r(\sin\emptyset_m + 0.3 \cos \emptyset_m)$(approx.) (lbs) we will have

$$\left(\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}\right)\left(\frac{m^{\frac{k-1}{k}} - 1}{k-1}\right) = \frac{0.3W_r\left(\frac{v_h^2}{v_h^2 - v_s^2}\right)W_r[\sin\emptyset_m - 0.3(1-\cos\emptyset_m)]}{1.3W_r(\sin\emptyset_m + 0.3 \cos\emptyset_m)}$$

$$= \frac{0.3 + \left(\frac{v_h^2}{v_h^2 - v_s^2}\right)[\sin\emptyset_m - 0.3(1-\cos\emptyset_m)]}{1.3(\sin\emptyset_m + 0.3\cos\emptyset_m)}$$

If we assume $v_h = 3.5$ ft/sec. and $v_s = 2.5$ ft/sec., then

$$\frac{v_h^2}{v_h^2 - v_s^2} = 2 \text{ approx.}$$

hence

$$\left(\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}\right)\left(\frac{m^{\frac{k-1}{k}} - 1}{k-1}\right) = \frac{0.3 + 2[\sin\emptyset_m - 0.3(1-\cos\emptyset_m)]}{1.3(\sin\emptyset_m + 0.3\cos\emptyset_m)}$$

From the above equations, we note that the proper ratio of compression depends on the angle of elevation and is entirely independent of the weight of the recoiling parts. The compression ratio does depend upon the value assumed for the initial recuperator reaction, the higher the initial recuperator reaction the lower ratio of compression. The compression ratio increases with the elevation for proper functioning of counter recoil at max. elevation.

If now we construct a table with values of m, and the corresponding values .

$\left(\dfrac{m^{\frac{1}{k}}}{m^{\frac{1}{k}}-1}\right)$ and $\dfrac{m^{\frac{k-1}{k}}-1}{k-1}$ and their product for $k = 1.1$ and 1.3 respectively, we may determine m by inspection and interpolation, provided we know the max. angle of elevation. If we let,

$$A = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}}-1}; \quad B = \frac{m^{\frac{k-1}{k}}-1}{k-1};$$

$$C = \frac{0.3+\left(\frac{v_h^2}{v_h^2-v_s^2}\right)[\sin\emptyset_m-0.3(1-\cos\emptyset_m)]}{1.3(\sin\emptyset_m+0.3\cos\emptyset_m)}$$

then, where k =1.1

m	A	B	C
1.3	4.717	0.23	1.084
1.5	3.247	0.37	1.201
1.75	2.508	0.51	1.279
2.00	2.138	0.64	1.368
2.30	1.883	0.78	1.468

and where k = 1.3

m	A	B	C
1.3	5.464	.296	1.129
1.5	3.732	.326	1.219
1.75	2.840	.456	1.306
2.00	2.420	.577	1.395
2.30	2.113	.703	1.486

From the above tables, curves were plotted with values of C against m for k = 1.1 and 1.3 respectively.

In order to compare the probable velocities obtained in the counter recoil at maximum and horizontal recoil for a given ratio of compression m, or on the other hand if given values of velocity at horizontal and maximum elevation are wanted the following method enables us to determine the proper value of the ratio of compression m.

If we plot for various values of m, the corresponding value of $\frac{v^2}{v_h^2-v^2}$

for a mean max. elevation rg at 63°, against v_h as horizontal abscissa and v_s as ordinates, we obtain, a series of curves for the various values of m, which having decided upon the ratio of compression to be used enables us to determine immediately the velocity of c'recoil at max. elevation for any given velocity at horizontal recoil.

Now,

$$(1) \qquad C = \frac{0.3 + \frac{V_h^2}{V_h^2-V_s^2}[\sin\emptyset - 0.3(1-\cos\emptyset)]}{1.3(\sin\emptyset + 0.3\cos\emptyset)}$$

Ø = angle of elevation. (In this series of calculations, the angle of elevation will be considered only at 65°).

.·. Ø = 65°.

Sin Ø = .906308

Cos Ø = .422618

Sin Ø − 0.3(1−cosØ)= .906308 − .3(1 − .422618)= .7330

1.3(sinØ+.3cosØ)= 1.3(.906308+ .3 x .422618)=1.343

Various values of C (equation #1) are given in table on preceding page.

The only unknown in the equation #1 is the expression

$\frac{V_h^2}{V_h^2-V_s^2}$. Let $\frac{V^2}{V_h^2-V_s^2}$ = K. Taking the various values of C as given in the preceding table and substituting in formula #1, we get the following values of "K", for the given values of "C":

1.1

C	K
1.084	1.576
1.201	1.790
1.279	1.933
1.368	2.096
1.468	2.279

1.3

C	K
1.129	1.659
1.219	1.824
1.306	1.983
1.395	2.145
1.486	2.312

$$K = \frac{V_h^2}{V_h^2-V_s^2}$$

Now to show the relation V_h and V_s, a curve will be plotted for each value of "K" as calculated and recorded in the table :

$$\frac{V_h^2}{V_h^2-V_s^2} = K \;; \quad V_h^2 = KV_h^2 - KV_s^2 \;; \; KV_s^2 = KV_h^2 - V_h^2 \;;$$

$$V^2 = \frac{KV_h^2 - V_h^2}{K}; \quad V_s = \sqrt{\frac{KV_h^2 - V_h^2}{K}} \text{ Formula \#2.}$$

Now for each value of K, assume values of V_h, from 0 to 10 and substitute in Formula #2, and obtain various corresponding values of V_s. These values of V_s plotted against values of V_h enables us to plot the curve, the corresponding values of V_s and V_h for each value of "K".

1.1 SET OF CURVES

M

When K = 1.576 — 1.3

V_h	1	2	3	4	5	6	7	8	10
V_s	.604	1.21	1.81	2.42	2.79	3.62	4.08	4.84	6.05

When K = 1.790 — 1.5

V_h	1	2	3	4	5	6	7	8	10
V_s	.663	1.33	1.98	2.66	3.32	3.96	4.64	5.31	6.64

When K = 1.933 — 1.75

V_h	1	2	3	4	5	6	7	8	10
V_s	.693	1.39	2.08	2.78	3.47	4.16	4.85	5.55	6.93

When K = 2.096

V_h	1	2	3	4	5	6	7	8	10
V_s	.722	1.44	2.17	2.89	3.62	4.34	5.06	5.78	7.22

When K = 2.279

V_h	1	2	3	4	5	6	7	8	10
V_s	.748	1.50	2.24	2.99	3.74	4.48	5.24	5.99	7.49

1.3 SET OF CURVES.

										M
When K = 1.659										1.3
V_h	1	2	3	4	5	6	7	8	10	
V_s	.63	1.28	1.89	2.52	3.15	3.78	4.41	5.04	6.3	
K = 1.824										1.5
V_h	1	2	3	4	5	6	7	8	10	
V_s	.67	1.34	2.01	2.69	3.36	4.03	4.70	5.37	7.12	
K = 1.983										1.75
V_h	1	2	3	4	5	6	7	8	10	
V_s	.704	1.40	2.06	2.82	3.52	4.22	4.92	5.63	7.04	
K = 2.145										2.00
V_h	1	2	3	4	5	6	7	8	10	
V_s	.734	1.46	2.19	2.92	3.65	4.38	5.11	5.85	7.30	
K = 2.312										2.3
V_h	1	2	3	4	5	6	7	8	10	
V_s	.753	1.50	2.26	3.01	3.76	4.50	5.27	6.02	7.53	

SPRING RECUPERATORS. Spiral spring columns, enclosed in cylinders for protection, are extensively used to bring the recoiling parts back into battery from the out of battery position. For small guns, spring recuperators are more useful, since they are simple in construction compact and readily adaptable to a gun mount. With large guns, however, the energy required for recuperation is large and therefore the spring columns become excessively heavy, since the weight of the springs is proportional to the potential energy stored within the springs.

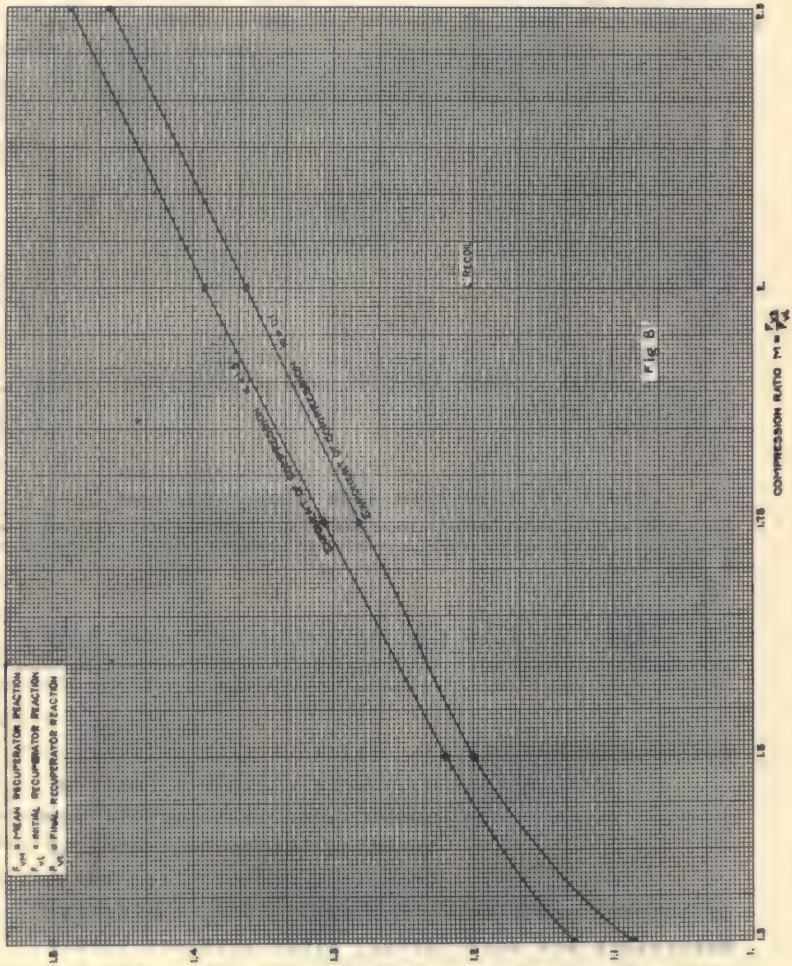
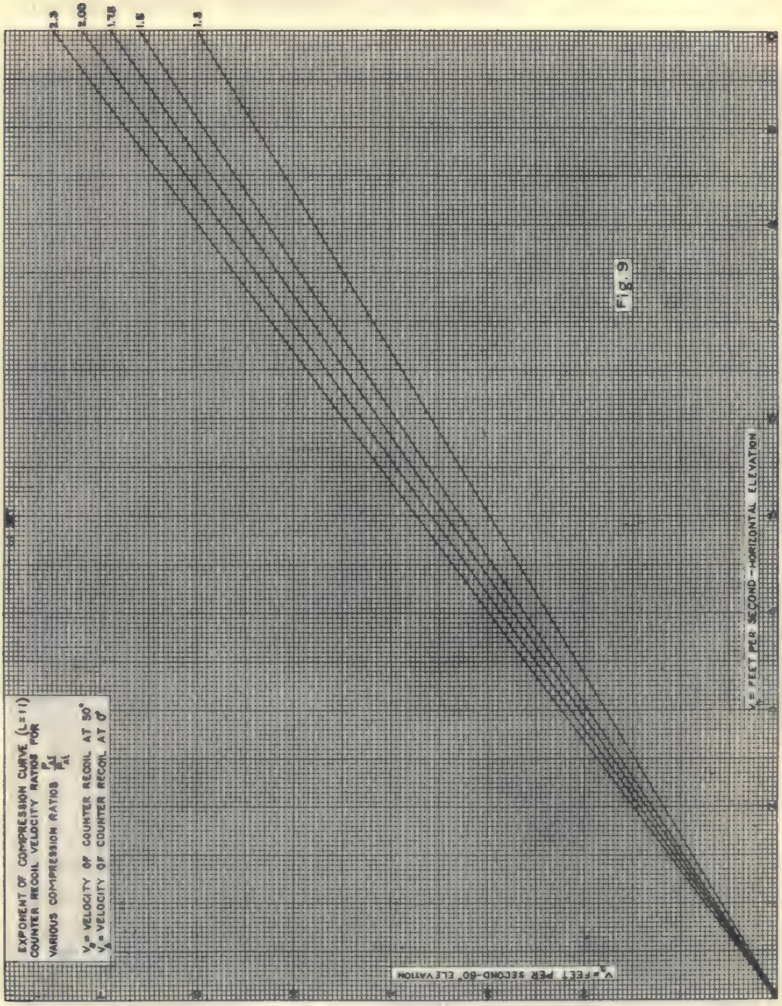

P_{vm} = MEAN RECUPERATOR REACTION
P_{vi} = INITIAL RECUPERATOR REACTION
P_{vf} = FINAL RECUPERATOR REACTION
RECOIL
COMPRESSION RATIO

Fig 8


2.3
2.00
1.75
1.5
1.3
EXPONENT OF COMPRESSION CURVE (L=1)
COUNTER RECOIL VELOCITY RATIOS FOR
VARIOUS COMPRESSION RATIOS $\frac{P_{A2}}{P_{A1}}$
V_2 = VELOCITY OF COUNTER RECOIL AT 60°
V_1 = VELOCITY OF COUNTER RECOIL AT 0°
Fig. 9
V_1 = FEET PER SECOND - HORIZONTAL ELEVATION
V_2 = FEET PER SECOND-60° ELEVATION

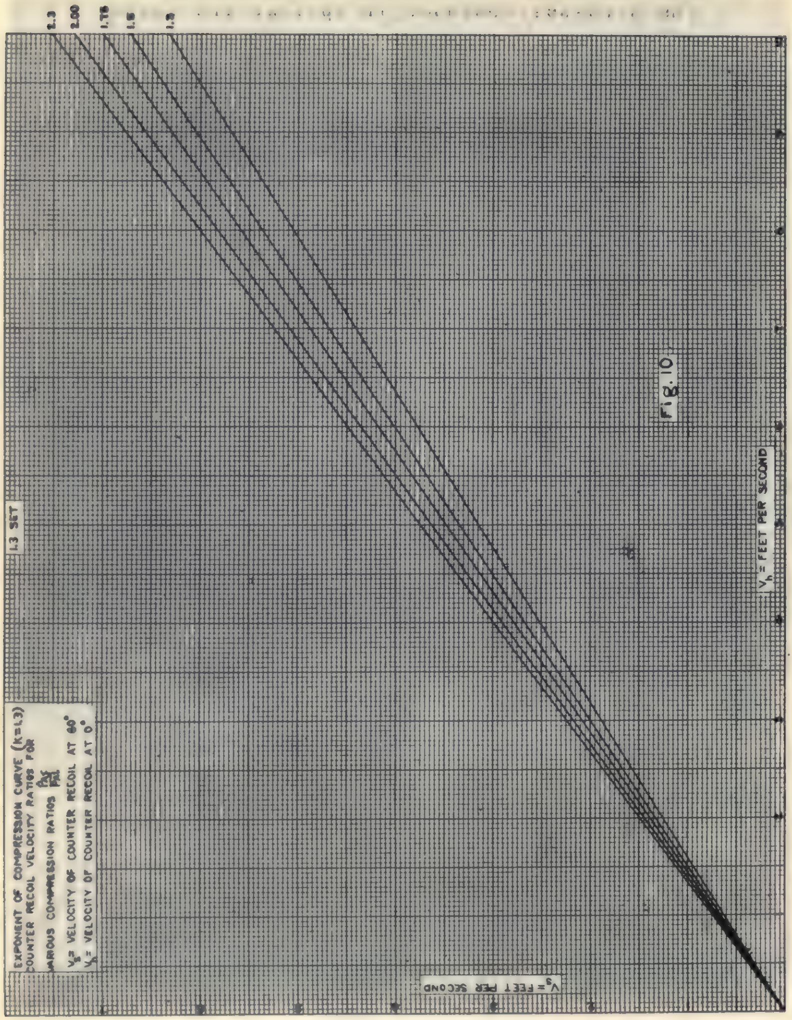

EXPONENT OF COMPRESSION CURVE (K=1.3)
COUNTER RECOIL VELOCITY RATIOS FOR
VARIOUS COMPRESSION RATIOS
V_s = VELOCITY OF COUNTER RECOIL AT 60°
V_h = VELOCITY OF COUNTER RECOIL AT 0°
1.3 SET
2.00
1.75
1.5
V_s = FEET PER SECOND
V_h = FEET PER SECOND

Fig. 10

Hence for large guns pneumatic recuperators have become almost universally employed.

The stresses computed in springs are based merely on their static loading. During the acceleration period of the gun, the spring coils adjacent to the attachment on the recoiling parts, necessarily are subjected to a very large acceleration, whereas those coils adjacent to their attachment on the cradle remain stationary. Due to the great resilience of a spring column, probably only a few of the front coils adjacent to the recoiling parts are subjected to any material acceleration, the spring not being capable of transmitting a force sufficient to accelerate the inner coils. Due to the very rapid acceleration during the first part of the powder period we have an impact or very suddenly applied loading on the spring which induces a compression wave, the peak of the wave being adjacent to the recoiling parts and the velocity of which depends upon the inertia per unit and elastic constant of the spring. It is possible that some of the failures in the service of recuperator springs are due to the dynamical aspects of the loading on the springs during the firing.

Since the inertia loading due to the powder acceleration comes practically on the front series of coils adjacent to the recoiling parts, the coils directly adjacent to the recoiling parts become more greatly compressed and correspondingly stressed. We should expect the front coils, therefore, to give the greatest trouble and this has been found the case in actual service.

Due to the complexity of the problem in actual calculations of the dynamic stresses in the spring no attempt will be made here to outline a procedure for such calculations, and only the static loading with suitable safety factors based on experience will be used in the preliminary design of counter

recoil springs. Let

- D = diam. of the helix of the coiled spring (in)
- R = radius of the helix of the coiled spring (in)
- d = diam. of the wire (in)
- f_s = max. allowable torsional fibre stress used (lbs/sq.in)
- N = torsional modulus of elasticity (lbs/sq. in)
- T = torque or total torsion at any cross section of the wire (in. lbs)

Considering any portion of a spring column subjected to a compressive load F (lbs), along the helical axis, we have at any section, through the wire,

$$\left.\begin{array}{ll}\text{A torsional load} & T = F\,R \\ \text{A shear} & S = P\end{array}\right\}$$

If we assume pure torsion at the section, the torsional fibre stress becomes,

$$f = f_s \frac{r}{r_o} \quad \text{(lbs/sq.in) where } r_o = \frac{d}{2} \quad \text{(in) hence}$$

$$T = F\,R = \int_0^{r_o} 2\pi r\,dr\, f_s \frac{r}{r_o}$$

$$= \frac{2 f_s \pi}{r_o} \int_0^{r_o} r^2\,dr = \frac{\pi f_s r_o^3}{2} = \frac{\pi f_s d^3}{16}$$

and therefore

$$F = \frac{\pi f_s d^3}{16 R} = \frac{\pi f_s d^3}{8 D} \quad \text{(lbs)}$$

Next consider the twist of any length of the wire 1. We have, for the torsional shear displacement of a circumferential annular of the wire,

$$\phi = \frac{f_s}{N} \text{ since } f_s = \phi\,N \quad \text{hence } \phi = \frac{r_o}{l} \text{ where } \theta = \text{the}$$

angle between two radius of the wire at two sections l distance apart. Therefore

$$\theta = \frac{f_s l}{N r_o} = \frac{2 f_s l}{N\ d} \text{ (radians)}$$

The relative displacement between the extremities of the helix for a load F, producing an extreme fibre stress f_s becomes

$\theta = R\ \theta = \dfrac{2 f_s R l}{Nd}$ but the length of the total wire of the helix, becomes, $l = 2\pi R n$ approx. $= \pi$ Dn where n = no. of coils. Hence

$$\theta = \frac{\pi f_s D^2 n}{Nd}$$

We have, therefore, the two fundamental spring formulas, for springs of circular cross section

$$F\ R = \frac{\pi f_s d^3}{8D} = \frac{\pi f_s d^3}{16R} \text{ (lbs)} \qquad (1)$$

$$\theta = \frac{\pi f_s D^2 n}{N\ d} \text{ (in)} \qquad (2)$$

The above formulas apply strictly only to closed coiled springs, no bending being considered; however, for a first approximation, they may be used for open coiled springs with sufficient accuracy for ordinary calculations.

For rectangular wire sections, we have semi-empircal formulas for the torsion, and deflections;

$$T = \left(\frac{a^2 b^2}{3a+1.8b}\right) f_s$$

$$\theta = \frac{4\ o\ J\ T\ l}{A^4 N}$$

where a_o = length of long side of rectangular section (in)

b_o = length of short side (in)

J = the polar moment of inertia of the rectangle.

l = length of wire (in)

A = cross section of the wire (sq.in)

Now

$$J = I_{xx}+I_{yy} = \frac{ab^3}{12} + \frac{ba^3}{12} = \frac{ab(a^2+b^2)}{12}$$

Hence for rectangular section spiral springs, we have,

$$F\ P = \left(\frac{a_o^2 b_o^2}{3a_o+1.8b_o}\right)\frac{f_s}{R} = 2\left(\frac{a_o^2 b_o^2}{3a_o+1.8b_o}\right)\frac{f_s}{D}\ (\text{lbs})(1')$$

$$\theta = \frac{10\pi JD^3 n}{A^4\ N}\ P$$

$$= \frac{166\pi D^2 n}{A^4 N}\ \frac{a_o^3 b_o^3 (a_o^2+b_o^2)}{(3a_o+1.8b_o)}\ f_s\ \ (\text{in})\quad (2')$$

If now, we let

a = deflection at assembled or battery height (in)

b"= displacement somewhat greater than the length of recoil (in)

F_{vi} = load at assembled height) initial recuperator reaction (lbs)

F_{ve} = load at solid height or at deflection corresponding to (a+b')

n = no. of effective coils

N = torsional modulus of elasticity (lbs/sq. in)

d = diam. of wire (in)

D = diam. of helix (in)

H_o= solid height of spring (in)

f_s= working max. fibre stress (lbs/sq.in)

then:

for circular springs:	for rectangular springs:
$F_{ve} = \frac{\pi f_s d^3}{8D}$ *(lbs) (1)	$F_{ve}=2\left(\frac{a_o^2 b_o^2}{3a+1.8b_o}\right)\frac{f_s}{D}$ (lbs) (1')

$$a+b'' = \frac{\pi f_s D^2 n}{N\, d} \text{(in)} \quad (2): \quad a+b' = \frac{1.66\pi D^2 n}{A^4\ N}\ \frac{a_o^3 b_o^3 (a_o^2+b_o^2)}{(3a_o+1.8b_o)} f_s$$

(in) (2')

$$\frac{a}{a+b''} = \frac{F_{vi}}{F_{ve}} \quad (3)$$

$$H_o = nd \quad (4)$$

In the four equations, above we are given f_s F_{vi} F_{ve} b' N and H_o

f_s F_{vi} b' N D and $\frac{F_{ve}}{F_{vi}}$

leaving the four unknowns, d D a and n or d a n and H_o. Therefore a complete solution is possible, and the proper size spring may be immediately arrived at.

ENERGY STORED IN SPRING The fibre stress on a helical spring is directly proportional to the axial load, that is $f = \frac{8\ D}{\pi d^3} F$ (lbs/sq.in) and the corresponding axial deflection, becomes, $\theta = \frac{\pi D^2 n}{Nd} f$ (in) hence the deflection of a helical spring loaded axially is directly proportional to the load, that is

$$\theta = \frac{8D^3 n}{Nd^4} F \quad \text{(in)}$$

The potential or resilient energy stored in a helical spring becomes,

$$A = \frac{F}{2}\theta = \frac{Nd^4}{16D^3 n}\theta^2 \quad \text{(in lbs)}$$

If the spring is to be stressed to a maximum allowable fibre stress f_s (lbs/sq.in) we have

$$A = \frac{\pi^2 D d^2 n}{16\ N} f_s^2 \text{ (in lbs)}$$

The volume of the material of the spring equals approximately

$$V = \frac{\pi}{4} d^2\ n\ \pi\ D = \frac{\pi^2}{4} D\ d^2\ n \text{ (cu.in)}$$

Hence the total energy in terms of the volume, is

$A = \frac{V}{4}\frac{f_s^2}{N}$ that is, the energy stored in a spring for a given max. allowable fibre stress and torsional modulus, is directly proportional to the volume and hence the weight of the spring. Thus with the same maximum stresses and same kind of material, the weight of the spring is directly proportional to the energy absorbed by the spring.

The weight of the spring in terms of the total energy stored in the spring, becomes

$W_s = \frac{4 w_s N A}{f_s^2}$ where W_s = total weight of the spring (lbs)

w_s = weight per cu.in. of the material of the spring (lbs/cu.in)

RATIO OF COMPRESSION WITH SPRING RECUPERATORS FOR MINIMUM WEIGHT OF COUNTER RECOIL SPRINGS.

For minimum weight of a set of counter recoil springs the compression ratio is definitely fixed.

Let F_{vi} = the initial recuperator reaction (lbs)

F_{vf} = the final recuperator reaction (lbs)

F_{ve} = the maximum solid load on the recuperator springs (lbs)

a = deflection of springs to assembled height in battery (in)

b = length of recoil (in)

b" = deflection of springs from assembled to solid height (in)

Since the load on the springs is proportional to the deflection we have immediately,

$$\frac{F_{ve}}{F_{vi}} = \frac{a+b''}{a}; \text{ and } F_{ve} = F_{vi}\left(1+\frac{b''}{a}\right)$$

The total energy stored in the spring column,

$$A = \frac{F_{ve}}{2}(a+b'') = \frac{F_{vi}}{2}\left(a+2b'+\frac{b''^2}{2}\right)(\text{in.lbs})$$

Since b" and F_{vi} are fixed conditions to be not in the design of the carriage, the only variable in the above energy expression is a. Therefore, for minimum weight,

$$\frac{dA}{da} = 0 \quad \text{hence} \quad \frac{d\left(a+2b''+\frac{b''^2}{a}\right)}{da} = 0$$

$$1 = \frac{b''^2}{a^2} = 0 \text{ and therefore } a = b''$$

The ratio of compression, becomes,

$$\frac{F_{ve}}{F_{vi}} = 2 = \frac{F_{vf}}{F_{vi}} \text{ (approx)}$$

Fortunately this ratio is nearly ideal for proper recuperation and hence satisfactory designed spring column with minimum weight may be used.

RECUPERATOR DIMENSIONS AND LIMITATIONS. With hydro pneumatic recuperators we have two or more cylinders, the recuperator cylinder and the air tank or cylinder. Let

b = length of recoil (in)

b = corresponding displacement in air cylinder

A_v = effective area of recuperator piston (sq. in)

A_a = cross section area of air cylinder (sq. in)

$m = \dfrac{p_{af}}{p_{ai}}$ = ratio of compression

$r = \dfrac{A_a}{A_v}$ = ratio of recuperator cylinders.

l = length of air volume in terms of cross section area of air cylinder (in)

$j = \dfrac{l}{b}$ = length of air volume in terms of recoil stroke

V_o = initial air volume (cu.in)

V_f = final air volume (cu.in)

Then $V_f = V_o - A_v b$

but

$$\frac{p_{af}}{p_{ai}} = m = \left(\frac{V_o}{V_f}\right)^k \quad \text{where } k = 1.1 \text{ to } 1.3$$

therefore,

$$\frac{V_o}{V_f} = m^{\frac{1}{k}} \qquad \text{that is } V_f = \frac{V_o}{m^{\frac{1}{k}}}$$

$$V_o\left(1 - \frac{1}{m^{\frac{1}{k}}}\right) = A_v b \qquad \text{hence} \qquad V_o = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} A_v b = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} A_a b'$$

which shows clearly, that the initial volume depends only upon the ratio of compression, the area of the recuperator cylinder and the length of recoil.

If now, we decrease the effective area of the recuperator piston, for a given recuperator reaction, we must increase the intensity of pressure in the recuperator cylinder, that is:

$K_{vi} = p_{vi} A_v$

$$\text{hence } V_o = \frac{K_{vi}b}{p_{vi}} \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} \qquad (6)$$

since $p_{vi} = p'_{ai} = p_{ai}$ approx,

$$V_o = \frac{K_{vi}b}{p_{ai}} \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} \qquad (6')$$

Now the size of the recuperator depends roughly on the initial volume V_o; hence, in pneumatic or hydro pneumatic systems, it is important to maintain as high air pressure as possible.

In recoil systems, where the recuperator and brake cylinder is one and the same as in the St. Chamond and Puteaux brakes, the effective area of the recuperator piston is that of the recoil piston.

Now the pressure during the recoil is limited to a given maximum consistent with the packing and therefore the effective area of the recoil piston is fixed. With large guns the recuperator reaction is relatively small as compared with the maximum recoil pressure, and therefore the intensity of the air pressure is small. Hence the recuperator volume and the size of the recuperator is large as compared with a separate recuperator system, using high recuperator pressure intensities.

Thus for large guns, or guns with low elevation, separate recuperator systems separate from the brake system usually gives a smaller recuperator brake forging.

Limitations of the ratio of compression "m".

The limitations of "m" are fairly fixed:

(1) The minimum "m" is based on a consideration of the proper functioning of counter recoil at all

elevations.

(2) The maximum "m" is based on a consideration of horizontal stability in the out of battery position for the recoil, as well as heating and rise of temperature caused by the compression of the air.

(1) With guns shooting at high elevation, the recoil must be shortened for clearance at high elevations and lengthened for stability at horizontal elevation. Thus high angle guns require a variable recoil, the ratio of short to long recoil being usually from one half to two thirds. The recuperator reaction at maximum elevation must be sufficient to bring the gun into battery with a moderate velocity in order that the time of counter recoil at maximum elevation may not be too long. This feature is of considerable importance. Raising the air pressure in the recuperator, though it will sufficiently accelerate the gun at maximum elevation, will give too great a velocity at horizontal recoil and thus endanger counter recoil stability. Thus in the initial design it is important that the initial volume is such that it will give the proper ratio of compression.

The mean recuperator reaction, or rather the recuperator reaction at the middle of the recoil, was shown in the discussion on counter recoil to be,

$$F_{vm} = n\,W_r + R_{s+p} + \frac{v_h^2}{v_h^2 - v_s^2}\,W_r[\sin\emptyset_m - n(1-\cos\emptyset_m)]$$

where v_h = the max. velocity a+ horizontal recoil (ft/sec)

v_s = the max. velocity at max. elevation (ft/sec)

R_{s+p} = total recoil packing friction

n = coefficient of friction from 0.1 to 0.2
For a preliminary design constant, we may assume,

$v_h = 3.5$ ft/sec. $v_s = 2.5$ ft/sec.

and taking a large value of $n = 0.3$ to compensate for R_{s+p}, we have

$$F_{vm} = 0.3W_r + 2W_r[\sin\phi_m - 0.3(1+\cos\phi_m)] \text{ (lbs)}$$

With a hydro pneumatic recoil system, we have roughly, $2.5(F_{vm}-F_{vi}) = F_{vf}-F_{vi}$ hence the minimum allowable ratio of compression, becomes,

$$m = \frac{F_{vf}}{F_{vi}} = \frac{2.5F_{vm}-1.5F_{vi}}{F_{vi}} = \frac{1.5(1.66F_{vm} - F_{vi})}{F_{vi}}$$

of course the ratio may be decreased by using lower values of v_s or higher values of v_h or both but the above assumed values give a satisfactory counter recoil at all elevations.

(2) The maximum value of m is based on the following considerations:-

(a) Horizontal stability, where a high final air pressure may exceed the allowable overturning force consistent with stability in the out of battery position.

(b) The maximum allowable c'recoil buffer pressure which limits the potential energy stored in the recuperator in the out of battery position.

(c) The allowable rise of temperature caused by the compression of the air.

With light mobile field carriages , stability is very often the determining factor for the maximum allowable ratio of compression. This is likely to especially occur when the mount elevates to very high angles and perfect horizontal

stability is required as in anti-aircraft material. If the resistance to recoil consistent with stability at horizontal recoil is small, and the initial recuperator reaction large, a high compression ratio will cause the total resistance to recoil in the out of battery position at horizontal recoil to be greater than the balancing stabilizing moment.

Obviously this critical condition will only occur with guns of high elevation and required to meet rigid horizontal stability limitations. In an ordinary recoil system as it is impossible not to have more or less throttling at the end of recoil, we must have the maximum allowable recuperator reaction a fraction of the total pull for minimum elevation of stability $\emptyset_i$.

That is $F_{vf} = 0.8[K_h + W_r(\sin\emptyset_i - 0.3\cos\emptyset_i)]$
Therefore the maximum allowable ratio of compression from a stability consideration, becomes

$$m_{max} = \frac{F_{vf}}{F_{vi}} = \frac{0.8[K_h + W_r(\sin\emptyset_i - 0.3\cos\emptyset_i)]}{F_{vi}} \quad \text{(min. elev.)}$$

$$= \frac{0.8(K_h - 0.3W_r)}{F_{vi}} = 0.75\,\frac{K_h}{F_{vi}} \quad \text{(horizontal elevation)}$$

Therefore, when F_{vi} is large and K_h small as with guns for high elevation and rigid stability requirements "m" becomes small and low ratio of compressions with corresponding larger recuperators are required. Very often in anti-aircraft material "m" becomes smaller than that required for proper counter recoil functioning at max. elevation. In such a case it is preferable to sacrifice horizontal stability somewhat and increase the horizontal resistance to recoil.

The previous formula may be expressed directly in terms of stability. If

W_s = weight of the total mount (lbs)

l_s = horizontal distance from spade to W_s (ft)

b_h = length of recoil at horizontal elev. (ft)

ϕ_i = min. angle of elevation.

We have for the max. compression ratio based on stability,

$$m = \frac{0.8[(W_s l_s - W_r b_h \cos\phi_i) + W_r(\sin\phi_i - 0.3\cos\phi_i)]}{F_{vi}}$$

and at horizontal recoil,

$$m = \frac{0.8[W_s l_s - W_r(b_h - 0.3)]}{F_{vi}} = 0.75 \left(\frac{W_s l_s - W_r b_h}{F_{vi}}\right)$$

In counter recoil systems using some form of a c'recoil regulator of a buffer type, we have a necessary geometrical limitation in the maximum area of the buffer. Thus in filling in types of buffers as in the Schneider and Filloux recoil systems as well as ordinary spear buffers which enter the piston rod at the end of c'recoil, the effective area must necessarily be considerably less than the area of the piston rod.

With spear buffers attached to the piston due to void considerations at the beginning of the re-recoil, we again are limited in a large effective buffer area. If

$P_{b\,max}$ = the max. average allowable buffer pressure (lbs/sq.in)

b = length of recoil (ft)

A_b = effective area of buffer

d_b = length of buffer c'recoil (ft)

R_p = total packing friction (lbs)

W_o = total potential energy of the recuperator (ft.lbs)

then, when the counter recoil brake comes into action towards the end of c'recoil, as with a spear buffer, we have,

$p_{b\ max} = \dfrac{W_o-(W_v \sin\phi+R_p)b}{A_b d_b}$ and where the counter recoil brake is effective throughout the counter recoil,

$$p_{b\ max} = \frac{W_o-(W_r \sin\phi+R_p)}{A_b} \quad \text{where } W_o = \frac{F_{vi}V_o}{k-1}\left(m^{\frac{k-1}{k}} - 1\right)$$

m = ratio of compression
V_o = initial volume of recuperator (cu.ft)
F_{vi} = initial recuperator reaction (lbs)
k = 1.1 or 1.3 depending whether air is in contact with oil or separated from it by a floating piston.

The expansions for p_b assume a constant buffer force during the buffer action. This however is not always the case and therefore the above expressions should be multiplied by a suitable constant to take care of the peak in the buffer pressure when the buffer pressure is not constant.

It is to be particularly noted that the peak buffer pressure may greatly exceed the average buffer pressure as obtained by the above expressions.

Combining the above expressions, we have

$$m = \left\{1 + \frac{[p_{b\ max}\ A_b d_b + (W_r \sin\phi+R_p)b](k-1)}{F_{vi}V_o}\right\}^{\frac{k}{k-1}}$$

This is a very important limitation for m and is inherent for all direct acting counter recoil buffer brakes. Values of $p_{b\ max}$ range as high as 8000 to 10000 lbs/sq.in. with short spear buffers but such pressures should not be tolerated on future designs.

<u>In general the c'recoil buffer pressure should be maintained as low as possible, thus simplifying the design of a counter recoil system; therefore, the lower value of m consistent with a satisfactory functioning of c'recoil at all elevations should be used.</u>

(3) Though the total energy dissipated in a re-

coil cycle must necessarily equal the initial recoil energy, it is important to distribute the energy in the parts of the system where radiation is most effective. If the energy is dissipated entirely in the throttling both on recoil and counter recoil we have a large mass of oil with corresponding radiating surface. With high compression ratios the air in the recuperator rises to a high temperature, which may cause injury to the packing and lubrication, and therefore it is important to maintain a low compression ratio and thus decrease the localized heating in the recuperator where radiation is the smallest.

As to the allowable rise of temperature to be permitted, depends greatly upon the type of packing to be used and the packing specification should state the allowable temperature rise.

The temperature T at the end of a recoil stroke, above the mean temperature T_m at the beginning of the stroke, may be obtained, from the relation,

$$\frac{T}{T_m} = \left(\frac{p_{af}}{p_{ai}}\right)^{\frac{k-1}{k}}$$

Assuming a ratio 2, and a mean temperature 25° centigrade, we have

$T = 298 \times 2^{0.23} = 349°$, when $k = 1.3$ and therefore the rise of temperature becomes, $T-T_m=51°C$ or $92°F$.

The temperature rise increases considerably with the ratio m, thus when $m = 2.5$, $T - T_m = 70°C$ or $158°F$.

RECUPERATOR DIMENSIONS AND LIMITATIONS. With hydro pneumatic recuperators we have two or more cylinders, the recuperator cylinder and the air tank or cylinder.

Let b = length of recoil

A_v = effective area of recuperator piston

A_a = cross section area of air cylinder

$m = \dfrac{p_{af}}{p_{ai}}$ ratio of compression

$r = \dfrac{A_a}{A}$ = ratio of recuperator cylinders.

l = length of air volume in terms of cross section area of air cylinder

$j = \dfrac{l}{b}$ = length of air volume in terms of recoil stroke.

Then, the initial volume becomes,

$$V_o = A_a l = A_v b \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} \quad \text{but since } \frac{A_a}{A_v} = r;\ \frac{l}{b} = j$$

hence

$$rj = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}$$

When a floating piston separates the oil and air, $k = 1.3$ (approx.) When the oil is constant with the air, $k = 1.1$ (approx.)

$$r = \frac{A_a}{A} = \frac{1}{j} \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}$$

$$r\, m^{\frac{1}{k}} - r - m^{\frac{1}{k}} = 0$$

$$m^{\frac{1}{k}} (r-1) = r$$

$$m^{\frac{1}{k}} = \frac{r}{r-1}$$

$$m = \left(\frac{r}{r-1}\right)^k$$

Tables for m and r for various air column lengths when k = 1.3 are given below:

r	r-1.66	$\frac{r}{r}$ - 1.66	log	1.3 log	m
3	1.34	2.239	.35005	.45507	2.851
3.5	1.84	1.902	.27921	.36297	2.307
4.	2.34	1.209	.23274	.30256	2.007
4.5	2.84	1.585	.20003	.26004	1.820
5.	3.34	1.497	.17522	.22779	1.690
5.5	3.84	1.432	.15594	.20272	1.595
6.	4.34	1.382	.14051	.18266	1.523

r	r-1.25	$\frac{r}{r}$ -1.25	log	1.3 log	m
3	1.75	1.714	.23401	.30421	2.015
3.5	2.25	1.556	.14201	.24961	1.777
4.0	2.75	1.455	.16286	.21172	1.628
4.5	3.25	1.385	.14145	.18389	1.527
5.	3.75	1.333	.12483	.16228	1.453
5.5	4.25	1.294	.11193	.14551	1.398
6.	4.75	1.263	.10140	.13182	1.355

r	$r-1$	$\frac{r}{(r-1)}$	$\log \frac{r}{r-1}$	$1.31 \log \frac{r}{r-1}$	m
3.00	2.00	1.5000	.17609	.22892	1.694
3.50	2.50	1.4000	.14613	.18997	1.549
4.00	3.00	1.3333	.12483	.16228	1.453
4.25	3.25	1.3077	.11611	.15094	1.416
4.50	3.50	1.2857	.10924	.14201	1.387
4.75	3.75	1.2667	.10278	.13361	1.360
5.00	4.00	1.2500	.09691	.12598	1.337
5.50	4.50	1.2222	.08707	.11319	1.298
6.00	5.00	1.2000	.07918	.10293	1.267

r	$r-.844$	$\frac{r}{r} - .844$	$\log \frac{r}{r} - .844$	$1.31\log\frac{r}{r-.844}$	m
3	2.156	1.391	.14333	.18633	1.536
3.5	2.656	1.318	.11992	.15590	1.432
4.0	3.156	1.267	.10278	.13361	1.360
4.5	3.656	1.231	.09026	.11734	1.310
5.0	4.156	1.203	.08027	.10435	1.271
5.5	4.656	1.181	.07225	.09393	1.242
6.	5.156	1.164	.06595	.08574	1.218

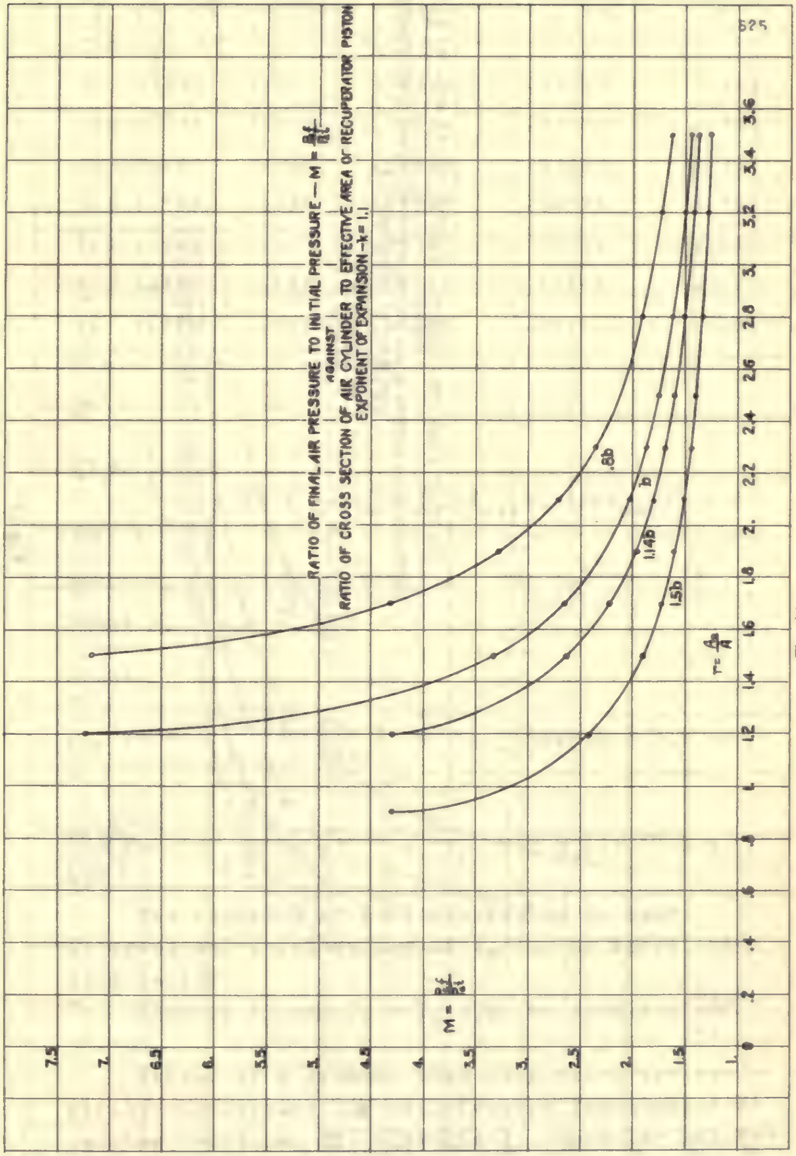

RATIO OF FINAL AIR PRESSURE TO INITIAL PRESSURE — M = pf/pi
AGAINST
RATIO OF CROSS SECTION OF AIR CYLINDER TO EFFECTIVE AREA OF RECUPERATOR PISTON
EXPONENT OF EXPANSION – k = 1.1
.8b
b
1.14b
1.5b
M = pf/pi
r = Aa/A

Fig. 11

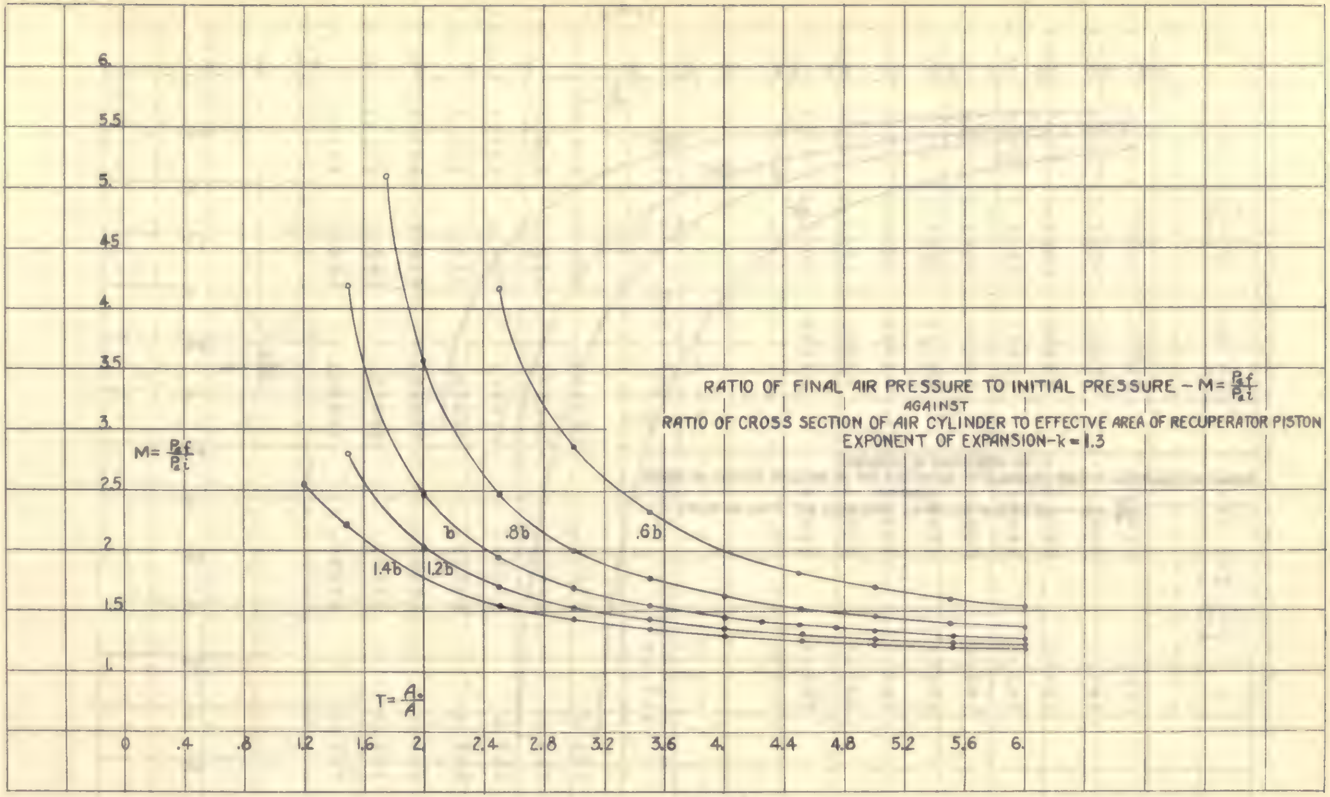

RATIO OF FINAL AIR PRESSURE TO INITIAL PRESSURE – M = Paf/Pai
AGAINST
RATIO OF CROSS SECTION OF AIR CYLINDER TO EFFECTVE AREA OF RECUPERATOR PISTON
EXPONENT OF EXPANSION – k = 1.3
M = Paf/Pai
T = A₀/A
.6b
.8b
b
1.2b
1.4b
6.
5.5
5.
4.5
4.
3.5
3.
2.5
2
1.5
1.
0
.4
.8
1.2
1.6
2
2.4
2.8
3.2
3.6
4.
4.4
4.8
5.2
5.6
6.

Fig. 12

r	r-.715	$\frac{r}{r}$-.715	log	1.3 log	m
3.	2.285	1.313	.11826	.15374	1.425
3.5	2.785	1.257	.09934	.12914	1.349
4.0	3.285	1.218	.09565	.11135	1.292
4.5	3.785	1.189	.07518	.09773	1.252
5.0	4.285	1.167	.06707	.08719	1.222
5.5	4.785	1.149	.06032	.07842	1.198
6.	5.285	1.135	.05500	.07150	1.179

$$\int_{V_o-\frac{3}{8}A_vb}^{V_o-\frac{4}{8}A_vb} p_v dV \geqq \frac{1}{2}m_r v_m^2 + W_r(\sin\emptyset_{max} + u\cos\emptyset_{max})\frac{b}{8} = E$$

where v_m = 2 ft/sec, roughly. Now $p_v V^k = p_{vi}\ V_o^k$;
hence $p_v = p_{vi}\ V_o^k \frac{1}{V}{}^k$

hence

$$p_{vi}\ V^k \int_{V_o-.375A_vb}^{V_o-0.5A_vb} \frac{dV}{V^k} \geqq E \qquad \text{where } k = 1.1 \text{ to } 1.3$$

$$\frac{p_{vi}V_o^k}{1-k}\ [(V_o-0.5A_vb)^{1-k} - (V_o-.375\ A_vb)]^{1-k} \geqq E$$

The solution of this expression is complicated and trial values of V_o may be substituted more easily.

Knowing V_o and $V_f = V_o - A_vb$, we have $m = (\frac{V_o}{V_f})^k$

Values of m greater than this value are entirely unnecessary for satisfactory functioning of counter recoil at all elevations. When the initial value of the recuperator reaction is made greater

than that required to hold the gun in battery, the necessary ratio of m decreases in the limit if m = 1, then

$$K_{vi} = p_{vi} A_v = \frac{4 m_r v_m^2}{b} + W_r(\sin\phi_{max} + u\cos\phi_{max})$$

Due to the uncertainty and variation of both packing and guide friction, an excess initial recuperator reaction is always used and thus even for very low values of "m" we usually have in modern artillery a surplus of potential energy in the recuperator.

GENERAL DESIGN LIMITATIONS.

SURVEY OF LIMITATIONS IN CARRIAGE DESIGN. The design limitations for a gun mount depend primarily of course on the particular use to be obtained from the gun and the general type of carriage to be used. Though each design is a problem by itself, it is however possible to derive and point out certain broad limitations that must be observed for a satisfactory design.

The fundamental requirements and limitations for the various classes of mounts are considerably different. The question of elevating, traversing, etc. certain more strictly to a given mount. However, certain broad limitations apply to the various classes of mounts and for good design these limitations must be always considered quite independent of the requirements for the particular service of the gun.

(1) For mobile mounts minimum weight and stability under firing conditions are primary limitations.

(2) For caterpillar mounts minimum weight and stability under firing

conditions are again primary limitations.

(3) For railway mounts, due to size and cost of parts, minimum weight consistent with stability is important but other factors such as clearance, method of loading, etc. have perhaps more influence on the design.

(4) For stationary mounts for defense work stability is easily secured and though it is highly desirable to keep the size and weight of parts as small as possible, the vital factors are accessibility, ease in loading and endurance.

LENGTH OF RECOIL AT MAXIMUM ELEVATION AND MAXIMUM RECOIL REACTION.

The strength of a gun carriage depends roughly on the maximum recoil reaction. Now the recoil reaction varies roughly inversely as the length of the recoil for a given recoiling mass and ballistics; therefore it is highly desirable, for lower stresses in the carriage, to maintain as long a recoil as possible. But at maximum elevation we are immediately limited by clearance of the gun striking the ground or platform. As the height of the trunnions and axis of the bore are fixed by stability at horizontal elevation clearance in traveling and accessibility for loading, the recoil at maximum elevation (as well as the maximum recoil reaction) becomes definitely limited.

Means for increasing the recoil and thereby diminishing the recoil reaction are as follows:

(1) By digging a pit under the gun.

(2) By placing the trunnions as far

as possible to the rear adjacent to the breech end of the gun and balancing the tipping parts by the use of a balancing gear.

(3) By raising the trunnions as the gun elevates, obtaining a low height of the trunnions above the ground when stability is required and a high position when stability is no longer a requirement and a long recoil is desired.

LENGTH OF RECOIL AT MINIMUM ELEVATION STABILITY.

As mentioned before, howitzers are designed for high angle fire, ranging roughly from 20 to 70 degrees. Therefore, stability is not of great importance up to 20 degrees elevation. At this elevation the moment arm of the overturning force , about the trail support becomes small, and therefore it is possible to considerably raise the trunnion and thereby lengthen the recoil at maximum elevation than with guns. Further for a given height of trunnions the length of recoil can be shortened for an elevation of 20° consistent with stability. Thus with howitzers, it is possible to maintain a constant recoil length for all elevations. This is of more or less advantage in simplifying the recoil system.

With a gun, the elevation ranges roughly from 0° to 50°. At 0° elevation the overturning moment about the spade support is a maximum, and the stabilizing moment a minimum. (See Chapter III). Hence a long recoil is, essential in order to reduce the recoil reaction and overturning moment.

The maximum horizontal recoil however is limited, due to the fact that at the end of recoil

though the overturning moment is decreased by lengthening the recoil, the stability moment is also decreased in the out of battery position due to the recoiling mass being displaced to the rear. Thus we arrive at an initial length of recoil where further increase causes a decreased stability.

If W_s = weight of carriage and mount together (lbs)

R_h = horizontal recoil reaction (lbs)

V_f = max. velocity of free recoil (ft/sec)

w_r = weight of recoiling parts (lbs)

h = height of axis of bore above ground (ft)

Then

$$R_h = \frac{0.47}{b}\frac{W_r}{g}V_f^2 \quad \text{(approx.)}$$

then $R_h\,h + W_r\,b = W_s l_s$ <u>at critical stability.</u> Now the actual overturning moment, becomes,

$$\frac{0.47W_r\,V_f^2\,h}{b\,g} + W_r\,b$$ and the corresponding stability moment $= W_s l_s$

If we differentiate the actual overturning moment with respect to b and equate to zero, we obtain, the maximum allowable horizontal recoil for a given recoiling weight, hence

$$d\left(\frac{0.47W_rV_f^2\,h}{d\,b} + W_r\,b\right) = -\frac{0.47W_rV_f^2\,h}{b^2 g} + W_r = 0$$

hence $b_{h\,max} = 0.121\,V_f\sqrt{h}$

Another limitation on the length of recoil at horizontal elevation, is due to the fact that as the recoil lengthens, the distance between the clip reactions decreases, and the clip reactions and the guide frictions become excessive in the out of battery position due to the overhanging weight of the recoiling parts. Such ex-

cessive guide friction caused by the moment of the overhanging weight combined with the recuperator reaction at the beginning of counter recoil may prevent satisfactory return into battery.

Further the bending moment at the rear clip reaction of the gun becomes excessive due to the large overhang in addition to the recoil pull on the gun lug. Thus the length of horizontal recoil is limited by the minimum allowable distance between clip reactions when the gun is out of battery. If, with this maximum recoil the mount is unstable, either the weight of the mount must be increased or outriggers reaching further out must be used. But for mobile mounts minimum weight is essential, hence extended outriggers or increase of trail length must be resorted to. As the gun elevates, stability increases and the recoil may be shortened consistent with clearance and stability.

With anti-aircraft guns, it is desirable to shoot from 0° to 80° since the piece must be interchangeable for field work if necessary. Therefore the limitations on anti-aircraft material are more pronounced and the change of length of recoil is greater from 0° to max. elevation than with other types of mounts.

RECOILING WEIGHT FOR MINIMUM WEIGHT OF GUN CARRIAGE.

The weight of carriage proper not including the recoiling mass is more or less proportional to the necessary strength required in the carriage. Now the strength of the carriage is roughly proportional to the maximum recoil reaction. Further the weight of a carriage depends upon the type or configuration of the mount. Hence, for any given type of carriage the weight is roughly proportional to the maximum recoil reaction. If, therefore, a given

type of carriage is designed to withstand a given recoil reaction, the higher the carriage is stressed, the smaller becomes the ratio of the weight of the carriage to the recoil reaction. Therefore the weight of efficiency for a particular type of mount is increased by decreasing the weight of the mount per given length of recoil. Obviously if a given type of mount was designed so that all its parts were stressed to the elastic limit for the maximum recoil reaction we would have the minimum possible weight for the given type of carriage.

Let w_c = weight of the carriage mount not including the recoiling mass.

R = the maximum recoil reaction.

w_c' = weight of the carriage mount proper when stressed to the elastic limit.

k = the weight constant for the carriage mount proper.

k' = the weight constant when stressed to the elastic limit.

Then

$$k = \frac{w_c}{R} \quad \text{and} \quad k' = \frac{w_c'}{R}$$

Obviously the weight efficiency in a given design pertaining to a given type of mount, becomes,

$$\text{weight eff.} = \frac{k}{k'} = \frac{w_c}{w_c'}$$

Now the weight efficiency varies considerably with the type of carriage used, certain types having considerably more dead weight than other types. Further the weight efficiency depends directly on the factor of safety recommended in the design.

A table for the constant "k" for various types of mounts is given below:

	Weight of System.	Recoiling Wt.	Max. Recoil Reaction.	Length of Recoil.	Weight of Mount not including Recoiling Wt.	Weight Constant of Carriage.
Carriage	W_s	W_r	R	L	W_c	K
3"Model of 1902.	2520	960	4923	45	1560	.317
75 m/m French M.1897.	2657	1050	5250	49	1607	.306
75m/m M.of 1916.	3045	911	12100	46 and 18	2134	.176
3.3" gun Carriage.	4372	1435	2100	45 and 30	2937	.146
3.8"How.Carriage,1915.	2040	935	13750	40 and 22	1105	.08
4.7" Gun, M.1906.	7420	2745	17500	70	5675	.324
4.7"How.Carriage,1908.	3988	1372	19430	52 and 24	2616	.135
155m/m How. Sohn.	7600	3498	39000	51.4	4100	.105
155m/mFilloux.	19860	9050	66000	43 and 71	10810	.164
8" Vickers, Mk.VII.	20048	9356	117300	52 and 24	10692	.091
240 m/m SCHNEIDER.	41296	15790	150000	46.	25526	.171

To give a further physical conception of the meaning of "k" we note from previous calculations that the 155 m/m Filloux is extra strong, most of the fibre stresses not exceeding 10,000 lbs. per sq. in. Comparing it with the 3.3 inch, a somewhat similar type of mount we would expect the 3.3 inch to be well stressed. This is actually the case. Two very similar types of heavy field trail carriages are the 8" Vickers and 155 m/m Schneider, both having the same type of trail. Both carriages are well designed, having in the various parts about the same maximum fibre stress. Therefore as we would expect the constant "k" is approximately the same. The 3" Model 1902 is not efficiently designed as compared with similar types such as the 75 m/m M.1916. We thus see that "k" when compared with types of similar carriages gives us a crude idea as to the efficiency of the design of the carriage itself.

Now the weight of the system is the recoiling weight plus the weight of the mount proper (i. e. the stationary parts), that is $w_s = w_r + w_c$

where w_s = the weight of system

w_r = recoiling weight

w_c = weight of stationary parts, or mount proper.

For a given type of mount, the weight of carriage may be assumed roughly proportional to the recoil reaction, that is, $w_c = k\,R$

Now from the principle of linear momentum, neglecting the small effect of the recoil reaction during the powder pressure period, and the air resistance, then $m\,v + \bar{m}\,4700 = m\,V$ where m and v = mass and muzzle vel. of projectile.

$\bar{m}\,4700$ = the momentum effect of the powder gases, hence

$$V = \frac{mv + \bar{m}\,4700}{m_r}$$

but $R = \frac{m_r V^2}{2b}$ approximately.

$$= \frac{(mv + \bar{m}\ 4700)^2}{2 m_r b}$$

hence $R = \frac{k}{w_r}$ where $k' = \frac{g(mv + \bar{m}\ 4700)^2}{2b}$

therefore $w_c = k\ R = \frac{k\ k'}{w_r}$

Now for minimum weight of the total system, recoiling parts together with carriage mount,

$\frac{dw_s}{dw_r} = 0$ that is $\frac{d(w_r + \frac{kk'}{w_r})}{dw_r} = 1 - \frac{kk'}{w_r^2} = 0$

hence $\frac{w_r^2 - kk'}{w_r^2} = 0$ or $w_r = \sqrt{kk'}$

where $k = \frac{w_c}{R}$ obtained from table

$$k' = \frac{g(mv + \bar{m}\ 4700)^2}{2b} = \text{ballistic constant}$$

To use the above formula in a new design we take the value of k from a similar well designed type of carriage, using a somewhat lower value of "k" according to the judgment of a designer in improving the weight efficiency of the mount proper over a similar previous design. Knowing the ballistics of the new mount, we find a very definite weight for the recoiling mass.

It is interesting to note that usually the strength curve of a gun may be considerably increased if the proper weight of recoiling mass consistent with minimum weight is used.

C H A P T E R VIII.

This chapter contains a discussion of some of the types of hydro-pneumatic recoil systems with calculations of characteristics of service designs.

It has been found desirable to print this chapter separately.

CHAPTER IX.

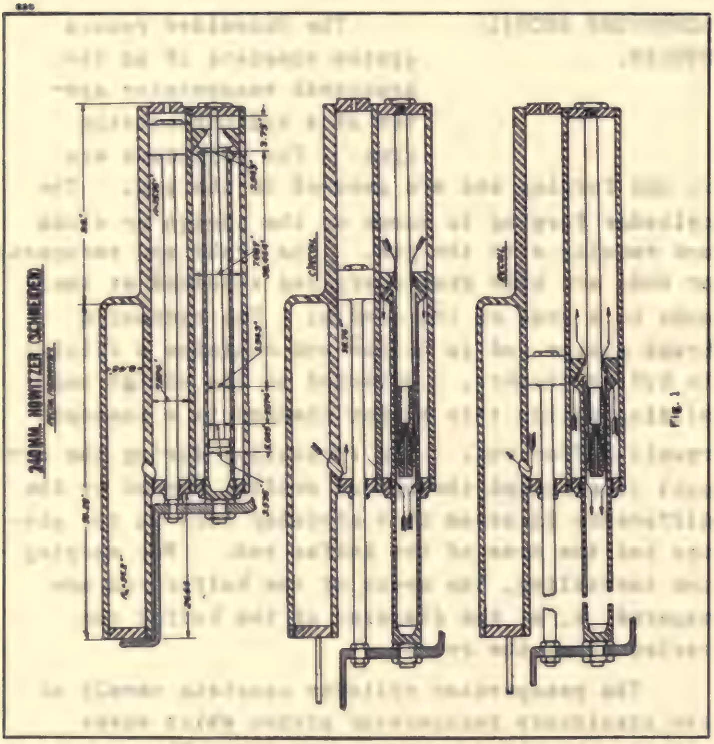
HYDRO-PNEUMATIC RECOIL SYSTEMS.

(Continued)

SCHNEIDER RECOIL SYSTEM.

The Schneider recoil system consists of an independent recuperator system of a hydro-pneumatic type. The cylinders are in one forging and are secured to the gun. The cylinder forging is known as the sleigh or slide and recoils with the gun. The brake and recuperator rods are held stationary and attached at their ends to a yoke on the cradle. The hydraulic brake piston rod is hollow and contains a filling in buffer chamber. Attached to the sleigh and sliding within this buffer chamber is a counter recoil buffer rod. The throttling during the recoil is effected through an orifice formed by the difference in areas of a circular hole in the piston and the area of the buffer rod. For varying the throttling, the areas of the buffer rod are tapered, i. e. the diameter of the buffer rod varies along the recoil.

The recuperator cylinder consists merely of the stationary recuperator piston which moves relative to the forging on recoil. The recuperator cylinder communicates by a large passage way to the air cylinder partly filled with air. The air cylinder is placed forward and is made shorter than the recuperator and brake cylinder. This is necessary in order that at maximum elevation the oil in the air cylinder covers the passage way communicating with the recuperator and air cylinders. It is very important in the initial


240MM. HOWITZER (SCHNEIDER)
Fig. 1

lay out of the Schneider recuperator system that at a maximum elevation the oil completely covers the communicating passage way in the air cylinder and the recuperator initial volume should be reckoned in the air tank beyond this oil covering. The passage way is made sufficiently large so that we have practically no throttling in the recuperator system.

During the recoil, figure (I), the brake throttling is effected primarily through an orifice formed by the counter recoil rod in a circular hole in the piston. The simultaneous compression of the air recuperator during the recoil takes place practically along an isothermal curve, due to the fact that oil and air are in direct contact in the recuperator. It has been found by careful computation, however, that an exponent equal to 1.1 gives a close approximation in the compression curve of the air and the compression of recoil in the brake cylinder. The buffer is filled by the pressure head in the recoil cylinder, the oil passing through fairly large orifices in the buffer head FF, the slide of the buffer head being away from the counter recoil buffer rod, see figure (I).

During the counter recoil the slide on the buffer head is pushed in contact with the buffer rod, and the apertures which filled the buffer chamber during the recoil are thereby closed and the throttling now takes place through new orifices of a very small magnitude. The buffer chamber having been completely filled during the recoil enables us to have a continuous regulation throughout counter recoil. The counter recoil throttling is effected through a constant orifice for over half of the counter recoil. We then have a tapering orifice until the gun nearly reaches the in battery position.

In the Schneider system the recoil is designed constant at all elevations or practically so, a slight variation taking place with the elevation. The recoil system is made to vary according to the stability slope at the minimum firing angle of elevation.

The primary advantages of the Schneider system are:

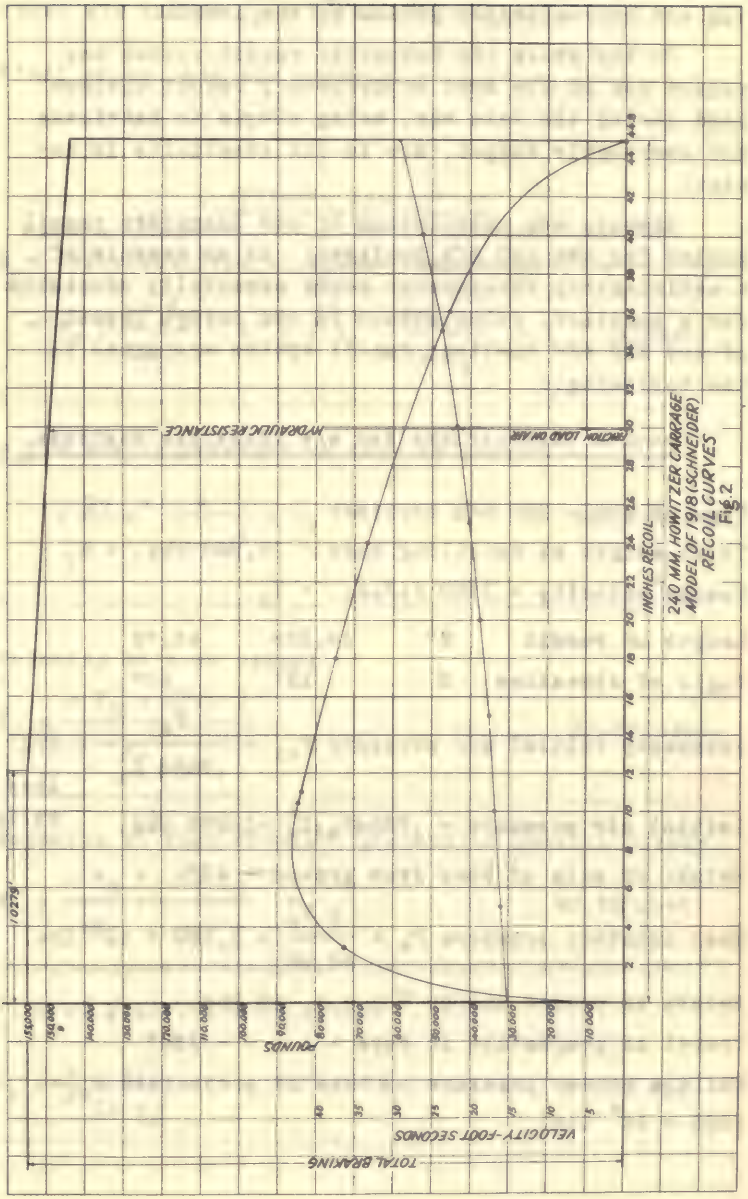
(1) An increased recoiling mass due to the recuperator sleigh containing the cylinders, recoiling with the gun and thereby decreasing the reaction on the carriage.

(2) The simplicity of the recoil mechanism, especially from a fabrication point of view.

The disadvantages of the Schneider system, are:

(1) due to the fact that the primary element of simplicity, the throttling effected through a simple tapering counter recoil rod, inherently prevents any possibility of a variable recoil.

(2) the massive sleigh or slide attached to the gun, though reducing the reaction on the carriage, lowers the center of gravity of the recoiling parts below the axis of the bore so that on firing a large load is thrown on the elevating arc. To offset this, on small caliber guns a counter weight has been mounted on top of the guns. On the larger caliber guns as in the 240 m/m howitzer, a brake clutch was introduced on the shaft of the elevating pinion which slipped during firing.



240 MM. HOWITZER CARRIAGE
MODEL OF 1918 (SCHNEIDER)
RECOIL CURVES
Fig. 2

Further the air cylinder, being necessarily placed forward of the recuperator brake cylinders with a long recoil gun requires a very long forging and corresponding guides on the cradle.

On the whole the Schneider recoil system has proved one of the most satisfactory recoil systems used during the late war, being simple to fabricate and thoroughly rugged, due to its simplicity in design.

Example and calculation of the Schneider recoil system for the 240 m/m Howitzer: As an example of a satisfactory recuperator brake especially adaptable for a howitzer, calculations in the design layout of the 240 m/m howitzer recoil system are given in the following:-

RECOIL CALCULATIONS 240 M/M SCHNEIDER HOWITZER.

Type of gun - 240 m/m howitzer

Total weight at recoiling mass = 15,790 lbs. = W_r

Muzzle velocity = 1700 ft/sec. = V_m

Length of recoil	B"	44,833	46.73
Angle of elevation	Ø	10°	60°

Intensity initial air pressure $P_{ai} = \dfrac{P_a}{.7854\ D_a^2} = 576$ lbs/sq.in.

Initial air pressure = $.7854 P_{ai} D_a^2 = 18800$ lbs.

Height of axis of bore from ground — 43"

Mean constant pressure $P_e = \dfrac{w\ V_o^2}{64.40} = 1,189 \times 10^3$ lbs.

Weight of powder charge $\bar{w}$ - - - 40 lbs.

Travel of projectile in bore - u - - - 160"

Maximum powder pressure on base of projectile P_m = 2005×10^3 lbs.

Maximum pressure on breech $P_b = 1.12\ P_m$

Initial air volume $V^{\circ} = 2970$ cu.in.

Final air volume $V_f = V^{\circ}$ cu. in $- A_e\ b'' = 1510$

Final or maximum air pressure $= p_{af} = p_{ai}\left(\frac{V_i}{V_f}\right)^{1.1}$

INTERIOR BALLISTICS.

e = twice abscissa at maximum pressure.

$$= u_o \left[\frac{27}{16} \frac{P_m}{P_e} - 1 \right] \pm \sqrt{\left(1 - \frac{27}{16} \frac{P_m}{P_e}\right)^2 - 1} \qquad 3.996\text{ft.}$$

P_{ob} = muzzle pressure on base of breech

$$= \frac{27}{4} e^2 \frac{u}{(e+u)^3} = P_b \qquad 622{,}000\text{ lbs.}$$

Velocity of free recoil

$$V_f = \frac{w\ V_m + 4700\ \bar{w}}{w_r} \qquad 50.25\text{ ft.sec.}$$

Velocity of free recoil - projectile leaving muzzle

$$V_o = \frac{w\ V_m + .5\bar{w}\ V_m}{w_r} \qquad 40.15\text{ ft.sec.}$$

Time of projectile to muzzle

$$t_1 = \frac{3}{2} \frac{u''}{12\ V_m} \qquad .01175\text{ sec.}$$

Time of expansion at free gases

$$t_2 = \frac{2(V_f - V_o)}{P_{ob}} = \frac{W_r}{32.2}$$ .01538 sec.

Free movement of gun while shot travels to muzzle

$$X'_1 = \frac{u''(w+.5\bar{w})}{12(W_r+w+\bar{w})} =$$ 31ft.

Free movement of gun during powder expansion

$$X_2 = \frac{P_{ob}}{W_r}\ \frac{gt_2^2}{3} + V_o t_2$$ .7179ft.

Total free movement of gun during powder pressure period

$$\Sigma = X_1 + X_2$$ 1.0279

Time of pressure period

$$T = t_1 + t_2$$ .02713 sec.

Total resistance to recoil in battery

$$K = \frac{m_r V_f^2 + m(b-E)^2}{2[b-E+V_f T - \frac{m}{2}\frac{T^2}{m_r}(b-E)]}$$ Variable recoil.

---

where K = total resistance to recoil during powder period (lbs)

b = length of recoil (ft)

E = free displacement of recoil during powder period (ft)

T = total powder period (sec)

$$m = \frac{c\ w_r}{d} \cos \emptyset \text{ stability slope}$$

c = constant of stability

d = distance from line through center of gravity of recoiling parts parallel to bore to center of pressure exerted on spades.

$$m_r = \frac{w_r}{g} = \frac{15790}{32.2} = 490$$

$$m = \frac{C\ W_r \cos \emptyset}{d} = \frac{c w_r}{h} \text{ (approx.)} = \frac{0.85 \times 15790}{3.58} =$$

3760

E = 1.0279 ft.

T = .02713 sec.

$$b = \frac{44.833}{12} = 3.736 \text{ ft.}$$

Hence

$$K = \frac{490 \times \overline{50.25}^2 + 3760(2.708)^2}{2(2.708+50.25\times.02713 - \frac{3760}{2} \times \frac{\overline{.02713}^2}{490} \times 2.708)}$$

$$= \frac{1264660}{8.13} = 155000 \text{ lbs. (approx)}$$

Total resistance to recoil out of battery

$$k = K - m(b - E + \frac{Kt^2}{2m_r})$$

$$= 155000 - 3760(3.736-1.028+ \frac{155000 \times \overline{.0271}^2}{2 \times 490})$$

$= 155000 - 3760 \times 2.824 = 144{,}000$ lbs.

CALCULATION OF THE VARIATION OF THE REACTION AIR PRESSURE IN THE RECOIL.

Initial air volume = 2970 cu.in.

Initial air pressure = 576 lbs/sq.in)

Length of recoil (10° elevation) = 44.8 inches.

Length of recoil (60°elevation)=46.73

Effective area of recuperator piston = 35.766 lbs.

Effective area of hydraulic piston = 31.2

$$\frac{\text{Final Pressure}}{\text{Initial pressure}} = \frac{(\text{Initial volume})^{1.1}}{(\text{Final volume})}$$

Final volume = initial volume = area at recuperator piston × length of recoil.

$$\therefore \text{ Final pressure (10° elevation)} = 576\left(\frac{2970}{2970 - 35.766 \times 44.8}\right)^{1.1}$$

$$= 576\left(\frac{2970}{1368}\right)^{1.1} = 576 \times 2.345 = 1350 \text{ lbs/sq.in.}$$

Final pressure (60° elevation) =

$$576\left(\frac{2970}{2970 - 35.766 \times 46.73}\right)^{1.1} =$$

$$576\left(\frac{2970}{1299}\right)^{1.1} = 576 \times 2.49 = 1434 \text{ lbs/sq.in.}$$

For 40" Recoil

$$\text{Final pressure} = 576\left(\frac{2970}{2970 - 35.766 \times 40}\right)^{1.1} =$$

$576 \times 2.065 = 1189$ lbs/sq.in.

1189 × 35.766 = 42525 (Plot these values above friction)

For 35" Recoil

Final pressure = $576 \left(\frac{2970}{2970 - 35.766 \times 35}\right)^{1.1}$ =

576 × 1.815 = 1045 lbs/sq.in.
1045 × 35.766 = 37375

For 30" Recoil

Final pressure = $576 \left(\frac{2970}{2970 - 35.766 \times 30}\right)^{1.1}$ =

576 × 1.643 = 946 lbs/sq.in.
946 × 35.766 = 33835

For 25" Recoil

Final pressure = $576 \left(\frac{2970}{2970 - 35.766 \times 25}\right)^{1.1}$ =

576 × 1.483 = 854 lbs/sq.in.
854 × 35.766 = 30544

For 20° Recoil

Final pressure = $576 \left(\frac{2970}{2970 - 35.766 \times 20}\right)^{1.1}$ =

576 × 1.35 = 788 lbs/sq.in.
778 × 35.766 = 27825

For 15" Recoil

Final pressure = $576\left(\frac{2970}{2970 - 35.766 \times 15}\right)^{1.1}$ =

576 × 1.22 = 702 lbs/sq.in.
702 × 35.766 = 25107

For 10" Recoil

Final pressure = $576\left(\frac{2970}{2970 - 35.766 \times 10}\right)^{1.1}$ =

576 × 1.155 = 665 lbs/sq.in.
665 × 35.766 = 23784

For 5" Recoil
Final pressure = 576 $\left(\dfrac{2970}{2970 - 35.766 \times 5}\right)^{1.1}$ =

576 × 1.072 = 617 lbs/sq.in.
617 × 35.766 = 22067

Calculation of Velocity Curve

(During Powder Pressure Period)
Point #1. Coordinates V_o and X_o

$$V_o = V_{fo} - \frac{Kt_o}{m_r}$$

$$X_o = X_{fo} - \frac{Kt_o^2}{2m_r}$$

When the projectile leaves the muzzle,
K = 155000 total resistance to recoil
u = travel of projectile in bore = 160"
v_o = muzzle velocity = 1700 ftsec.
w = weight of shell = 353
$\bar{w}$ = weight of powder charge = 40

W_r = weight of recoiling parts

$$m_r = \frac{15790}{32} = 490$$

$$V_{fo} = \frac{(w + .5\bar{w})V_o}{W_r}; \qquad t_o = \frac{3}{2}\frac{u}{V_o}$$

$$V_{fo} = \frac{(353 + .5 \times 40 \times 1700)}{15790} = 40.15$$

$$t_o = \frac{3 \times 160}{2 \times 12 \times 1700} = .01175$$

$$\therefore V_o = 40.15 - \frac{155000 \times .01175}{490} = 40.15 - 3.701 =$$

V_o = 36.449 ftsec.

$$X_{fo} = \frac{w + .5\bar{w}}{W_r} u = \frac{(353 + .5 \times 40)160}{15790 \times 12}$$

$$X_{fo} = .32$$

$$\therefore X_o = .32 - \frac{155000 \times .01175^2}{2 \times 490}$$

$$= .32 - .0221 = .2979 \text{ ft.} = 3.57 \text{ inches.}$$

Point #2.
Maximum restrained recoil velocity and corresponding orifice.

$$\left.\begin{aligned} V_m &= V_{fm} - \frac{Kt_m}{m_r} \\ X_m &= X_{fm} - \frac{Kt_m^2}{2m_r} \end{aligned}\right\} \left\{\begin{aligned} t_m &= T - \frac{K(T-t_o)}{P_{ob}} \\ t_m &= .02713 - \frac{155000(.02713-.01175)}{622000} = t_m = .02329 \end{aligned}\right.$$

$$V_{fm} = V_{fo} + \frac{P_{ob}}{m_r}(t_m-t_o)1 - \frac{P_{ob}(t_m-t_o)}{4m_r(V_f-V_{fo})}$$

$$V_{fm} = 40.15 + \frac{622000}{490}[.02329 - .01175]1- \frac{62000(.02324 - .01175)}{4 \times 490(50.25-40.15)} = 40.15+9.328$$

$$V_{fm} = 49.478$$

$$V_m = 49.478 - \frac{15500 \times .02329}{490} = 42.118 \text{ ft.sec.}$$

$$X_m = X_{fm} - \frac{Kt_m^2}{2m_r}$$

$$X_{fm} = X_{fo} + [V_{fo} + \frac{P_{ob}}{m_r}(t_m - t_o) - \frac{(t_m - t_o)^2}{6m_r(V_f - V_o)}(t_m - t_o)$$

$$X_{fo} = \frac{w + .5\bar{w}}{W_r} \quad x \ u = .32 \text{ (see Point \#1)}$$

$$X_{fm} = .32 + [40.15 + \frac{622000}{490}(.02329 - .01175) -$$

$$\frac{(.02329 - .01175)^2}{6 \times 490(50.25 - 40.15)}](.02329 - .01175)$$

$$= .32 + .632 = .952 \text{ F t} = 11.42 \text{ in.}$$

$$X_m = .952 - \frac{155000 \times .02329^2}{2 \times 490} = .952 - .0855 =$$

$$.866\text{ft.} = 10.39 \text{ in.}$$

Point #3 (At end of the powder period)

$$V_r = V_f - \frac{KT}{m_r}$$

$$V_r = 50.25 - \frac{155000 \times .02713}{490} = 50.25 - 8.55 =$$

$$V_r = 41.7\text{ft.sec.}$$

$$X_r = X_f - \frac{KT^2}{2m_r}$$

$$X_r = 1.0279 - \frac{155000 \times .02713^2}{2 \times 490}$$

$$1.0279 - .1155 = .9124\text{ft} = 10.94\text{in.}$$

Velocity Curve (during retardation period)

$$V_x = \sqrt{\frac{2[K - \frac{m}{2}(b + X - 2X_r)(b - x)]}{m_r}}$$

For x = 1.5 feet.

$$V_x = \sqrt{\frac{2[155000-\frac{3760}{2}(3.89+1.0279-2\times.9124)](3.89-1.5)}{490}}$$

37.4 ft.per sec.

For x = 2 feet

$$V_x = \sqrt{\frac{2[155000-\frac{3760}{2}(3.89+1.0279-2\times.9124)](3.89-2)}{490}} = 33.6$$

For x = 3 feet

$$V_x = \sqrt{\frac{2[155000-\frac{3760}{2}(3.89+1.0279-2\times.9124)](3.89-3)}{490}} = 22.8$$

For x = 3.73(total recoil)

Velocity = 0

Calculation of Guide and Packing Frictions.

$$\text{Guide friction } R_g = \frac{2nBd_b}{l} = \frac{2nKd_b}{l} \quad \text{approx.}$$

$u = .15$

$K = 155000$

$d_b = 15.5'$(in) iistance from center of gravity to resultant pull.

$l = 37 + 48 = 85''$(in) mean distance between clip reaction.

$$\therefore \text{ Guide friction} = \frac{2\times.15\times154725\times15.5}{85} = 8450 \text{ lbs}$$

Stuffing Box Friction

Recuperator stuffing box
Diam. = 2.169

- Rear sleeve - .5"+.875" contact
- Inner packing ring - .787
- Gland - .87

$$\left.\begin{array}{l}\text{Recoil stuffing box} \\ \text{diam.} = 4.728\end{array}\right\} \begin{array}{ll}\text{Rear sleeve} - & .75+.5 \\ \text{Inner packing ring} & .787 \\ \text{Inner gland} - & .866\end{array}$$

(Spring pressure + 0.1 pressure)(.75 diam. × π × .09 × length of contact) Formula.

Spring pressure from drawing $\dfrac{1058}{.785(6.4375^2-5.3437^2)} =$

$\dfrac{1058}{10.124} = 104$ lbs/sq.in.

Oil pressure in recuperator = 576 + 1350

$\dfrac{\text{Initial Final}}{2} = 963$ lbs/sq.in.

Oil pressure in recoil

$= \dfrac{2222+1670}{2} = 1946$ lbs/sq.in.

Recuperator stuffing box diam. = 2.169 length of contact(dermatine)=.787

Friction = .75×2.169×3.14×(963+104)×.09×.787=375.

Recoil stuffing box diam. =4.728 Length of contact = .787

Friction = .73×4.728×3.14×(1946+104)×.09×.787=1572

Total stuffing box friction = Recoil stuffing box friction + Recuperator stuffing box friction

= 1572+375=1947 lbs.Total stuffing box friction.

Total friction = guide + stuffing box.

$$\begin{array}{r} = 8450 \\ 1947 \\ \hline 10397 \text{ lbs.}\end{array}$$

Calculation of Throttling Areas.

$$w_x = \frac{C\ A^{\frac{3}{2}} \sqrt{\dfrac{2[K-\frac{m}{2}(b+X-2X_r)](b-x)}{m_r}}}{13.2\sqrt{K-p_a-R_t+W_r \sin \emptyset}}$$

But

$$\sqrt{\frac{2[K-\frac{m}{2}(b+X-2X_r)](b-x)}{m_r}} = V_x$$

$$W_x = \frac{C\ A^{\frac{3}{2}}\ V_x}{13.2\sqrt{K-p_a-R_t+W_r \sin \emptyset}}$$

$p_a = p_{ai} \times A_r$(approx)=initial pressure x effective area of piston
$= 576 \times 35.766 = 20600$
$C = 1.39$ (constant)
$A = 35.766$
$W_r \sin \emptyset = 15790 \times .0848 = 15550$
R_t=guide friction + stuffing box friction = 10,000 lbs.
$K = 155000$
V_x= take the values as calculated for vel. curve.

From calculations:
when $x = 3.57"$
$V = 36.449$ ft.sec.
$w_x = .061 \times 36.449 = \underline{2.223 \times 2 = 4.446}$

When $x = 10.39$ in.
$V = 42.118$ ft.sec.
$w_x = .061 \times 42.118 = \underline{2.5691 \times 2 = 5.1382}$

When $x = 10.94$ in.
$V = 41.7$ ftsec.
$w_x = .061 \times 41.7 = \underline{2.5437 \times 2 = 5.0874}$

When $x = 1.5$ ft. or 18 in.
$V_x = 37.4$ ftsec.

$$W_x = \frac{1.39 \times 35.766^{\frac{3}{2}}}{13.2\sqrt{154725 - 20600 - 10397 + 15550}} \quad (37.4)$$

$= .061 \times 37.4 = 2.2814$ sq.in. $\times 2$ rods $= 4.5628$

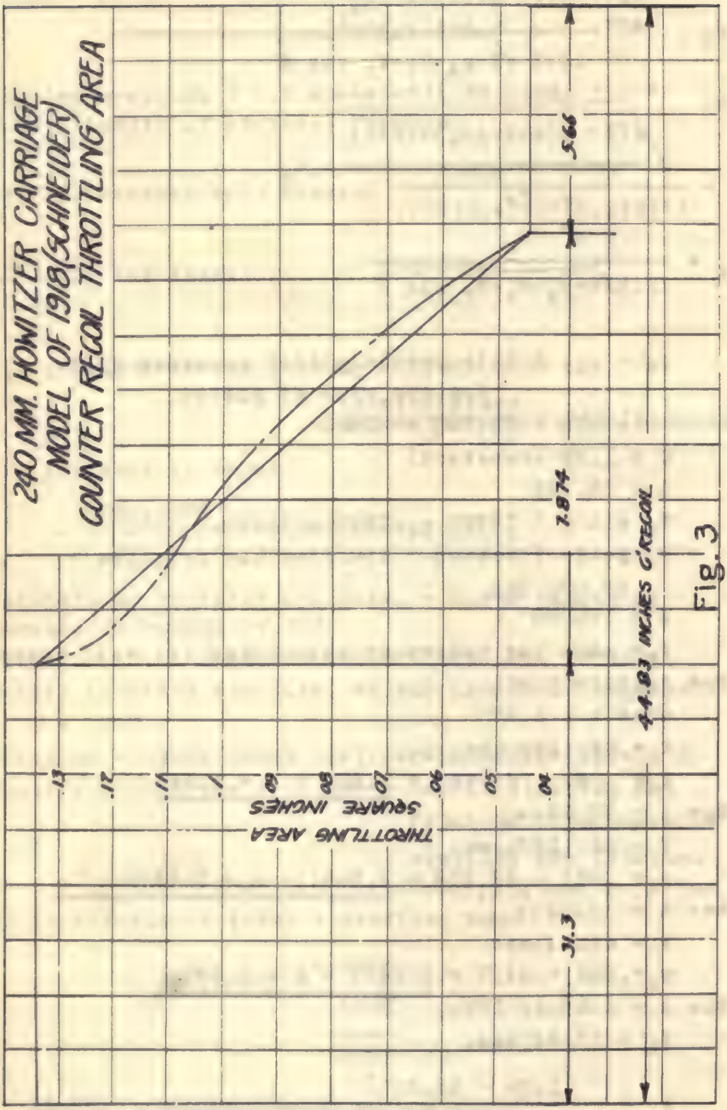

240 MM. HOWITZER CARRIAGE
MODEL OF 1918 (SCHNEIDER)
COUNTER RECOIL THROTTLING AREA
THROTTLING AREA
SQUARE INCHES
.13
.12
.11
.1
.09
.08
.07
.06
.05
.04
5.66
7.874
31.3
44.83 INCHES C'RECOIL

Fig. 3

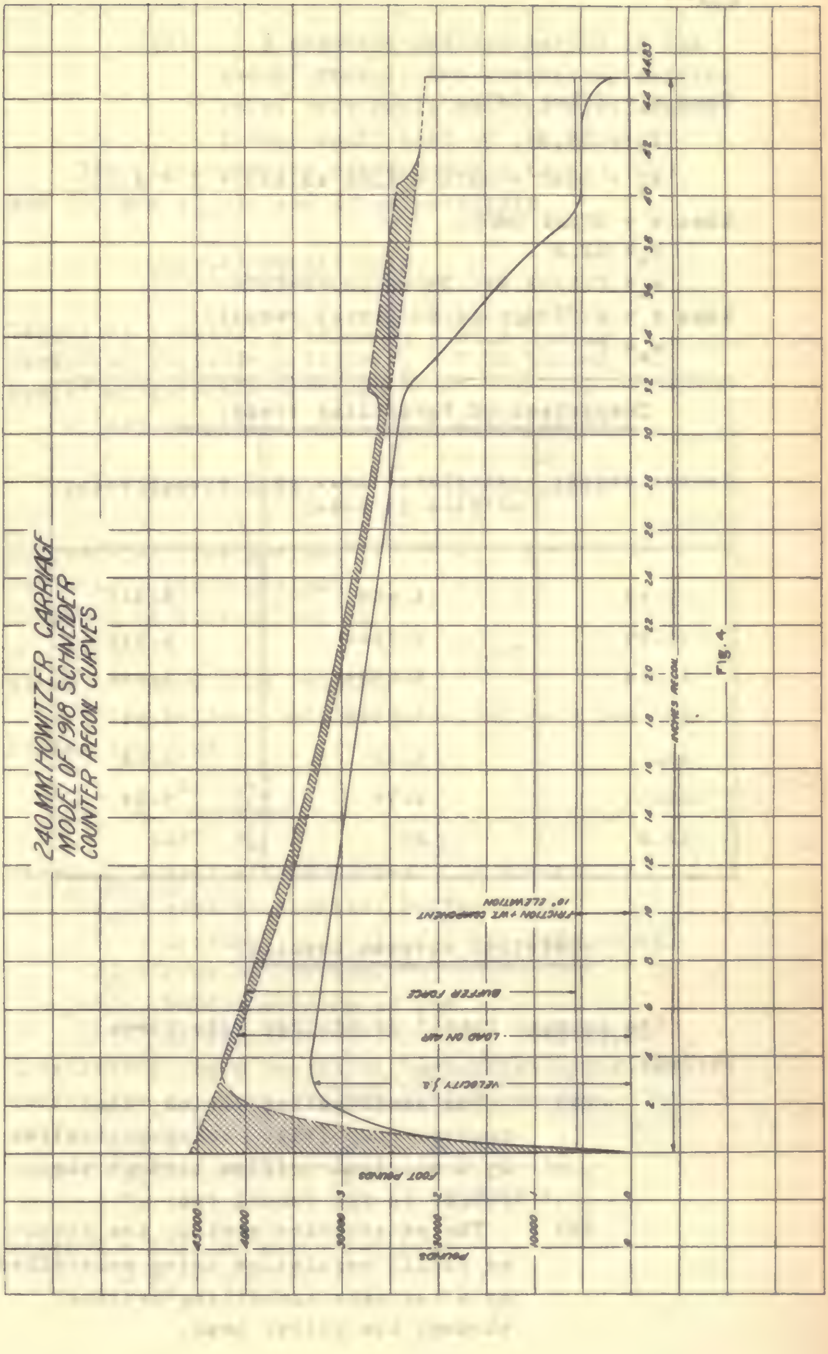

240 MM. HOWITZER CARRIAGE
MODEL OF 1918 SCHNEIDER
COUNTER RECOIL CURVES
INCHES RECOIL
FRICTION + WT. COMPONENT
10° ELEVATION
BUFFER FORCE
LOAD ON AIR
VELOCITY f.s.
FOOT POUNDS
POUNDS

Fig. 4

When x = 2ft.or 24in.

V_x = 33.6

w_x = .061 × 33.6 = 2.05 sq.in. × 2 = 4.10

When x = 3ft.or 36in.

V_x = 22.8

w_x = .061×22.8=1.28 sq.in.×2=2.76

When x = 3.75ft.or 44.8in.(total recoil)

w_x = 0

Comparison of Throttling Areas.

Inches Recoil	Calculated Area of Orifice (2 rods)	French Value
3.57	4.446	4.413
10.39	5.1382	5.129
10.94	5.0874	5.084
18.	4.5628	4.54
24.	4.10	4.08
36.	2.76	2.69
44.8	0.	0.

SCHNEIDER COUNTER RECOIL.

The counter recoil is divided into three periods:

(1) The accelerating period, the counter regulation being controlled by a constant orifice through the buffer in the recoil rod.

(2) The retardation period, the counter recoil regulation being controlled by a variable throttling orifice through the buffer head.

(3) A constant orifice period at the end of recoil, the throttling orifice being very small and the displacement a very small part of the recoil.

The displacements corresponding to (1), (2) and (3) are l_o, l_b and l_o' respectively.

Counter Recoil Data.

Length of constant orifice l_o	= 31.3 inches
Length of variable orifice l_b	= 7.85 inches
Length of constant orifice at end of recoil l_o'	= 5.68 inches
b = Total c'recoil	44.83 inches

Constant orifice period	0.7 b
Variable orifice period	0.175 b
Constant orifice at end of c'recoil	0.125 b

where b = length of recoil

There being 2 recoil brakes, we have for the buffer reaction:

$$B_x' = \frac{2c^2 A_b^3 v_x^2}{175\ a_x^2}$$

where A_b = area of one buffer = 9.859

a_o = area of constant buffer orifice = .0664 sq.in.

a_o' = area of constant buffer orifice at end of recoil = .022 sq.in.

Considering the c'recoil at horizontal elevation, during the constant orifice period, we have

$$\log\left(A - \frac{c_o v_2^2}{w_o^2}\right) \quad \log\left(A - \frac{c_o v_1^2}{w_o^2}\right) - \frac{2c_o(x_2 - x_1)}{2.3026\ m_r w_o^2}$$

where A = load on air - friction

$= F_v - \Sigma R$

$$\text{and } C_o = \frac{2c^2 A_b^2}{175} = \frac{2.78 \times 940}{175} = 15.$$

$\Sigma R = 6290$ lbs.

$m_r = 490$

Δx in.	Total x in.	L	$\frac{cv^2}{a^2}$	$\frac{2c(x^2-x^1)}{2.3\ ma^2}$	V	Buffer force #
.4	.4	39700	$3415v^2$	.198	1.16	4550.
.8	1.2	37860	"	.386	2.67	24300.
.8	2.	36860	"	.386	3.05	31700.
1.	3.	35810	"	.493	3.3	37100.
1.	4.	34910	"	.493	3.23	35500.
1.	5.	33910	"	.493	3.18	34400.
1.	6.	33160	"	.493	3.13	33500.
1.	7.	32510	"	.493	3.09	32900.
5.	12.	28710	"	2.5	2.91	29000.
5.	17.	25710	"	2.5	2.745	25720.
5.	22.	22710	"	2.5	2.5	22700.
5.	27.	20710	"	2.5	2.46	20750.
4.3	31.3	18910	"	2.13	2.36	18950.

Beginning of Variable Orifice.

After this period the unbalanced force was assumed constant.

$$\phi\, l_b = \tfrac{1}{2} M_r (V_0^2 - V_1^2)$$

$$\phi = 245\left(\frac{5.3-.25}{.655}\right) = 1880. \ \# = \text{unbalanced force}$$

$$V_x = \sqrt{\frac{\frac{1}{2} m_r V_0^2 - \phi x}{\frac{1}{2} m_r}}$$

Knowing V_x solve for w_x for various parts then

$$B_x' = \frac{2c^2 A_b^2 v^2}{175\ w_x^2} = \phi + F_v - \Sigma R$$

Inches Recoil.	x in	1880x	$\frac{1300-1800x}{245}$	V	Total lbs. Buffer Forces
32.3	.0833	157.	4.67	2.16	20500
33.3	.1666	360.	3.94	1.99	20200
34.3	.25	470	3.38	1.84	9900
35.3	.333	627.	4.75	1.66	19500
36.3	.416	780.	2.12	1.45	19000
37.3	.5	940.	1.47	1.215	18500
39.15	.656	1230.	.286	.52	18000
40.				.5	17500
					17000
42.				.5	16500
44.83				.0	16000

Point	X Feet	4580x	3215-4580x	$\frac{3215-4580x}{245}$	Vel. f.s.	Load on Air	W
1	.0833	382	2833	11.5	3.39	28400	116
2	.1666	762	2453	10.	3.16	27600	108
3	.25	1145	2070	8.45	2.91	26800	.102
4	.333	1525	1690	6.9	2.63	26000	.092
5	.416	1900	1315	5.38	2.32	24600	.083
6	.5	2290	925	3.78	1.945	24400	.065
7	.656	3000	215	8.76	.94	21000	.044

S C H N E I D E R

X = any interval

Ø = unbalanced force

M_r = mass recoiling parts

V_o = max. velocity of c'recoil

V_x = velocity at any point

V_t = velocity at beginning of period l_c

l_o = length of constant orifice period in feet

l_b = length of variable orifice period in feet
l_c = length of final period for c'recoil in feet
P_a = load on air in lbs.
R = Total friction

$$\frac{2K^2A^3V^2}{175\ W_x^2} = \text{total buffer force}$$

b = length of c'recoil (ft)

Period l_o

$$P_a - R - \frac{2K^2A_o^3V^2}{175A_x^2} = 0 \quad \text{acceleration} = 0$$

Assume velocity of 3.5 ft. per second and solve for orifice W_x

---

Period l_c

$$P_a - R - \frac{2K^2A_b^3V^2}{175\ W_x^2} = 0$$

Assume velocity of 1 ft. per sec. and solve for W_x

Period l_b

$$\varnothing\ l_b = \tfrac{1}{2}\ m_r(V_o^2 - V_1^2)$$

$$\varnothing = \tfrac{1}{2}\ m_r\ \frac{(V_o^2 - V_1^2)}{l_b}$$

$$\varnothing\ x = \tfrac{1}{2}\ m_r(V_o^2 - V_x^2)$$

$$\therefore V_x = \sqrt{\frac{\tfrac{1}{2}\ m_rV_o^2 - \varnothing x}{\tfrac{1}{2}\ m_r}}$$

Knowing V_x solve for W_x for various points

$$\phi = \frac{2K^2A^3V^2}{175\ W_x^2} - P_a = R \quad \text{Solve for } W_x$$

ST. CHAMOND RECOIL.

ST. CHAMOND RECOIL SYSTEM. The type of St.Chamond brake here discussed, consists of three cylinders; a hydraulic brake cylinder, a recuperator cylinder containing the floating piston which separates the air and oil, together with a regulator valve for throttling the oil between the hydraulic and recuperator cylinders, a third cylinder serving as a part of the air reservoir and therefore communicating with the recuperator cylinder air volume, the remainder of the third cylinder being used for storing oil for the brake mechanism.

One of the peculiar features of this type is the regulated spring valve where the main throttling occurs. The valve functions somewhat as a pressure regulator or governor, since if the pressure falls, the spring reduces the valve opening thereby increasing the throttling drop and the pressure in the hydraulic cylinder. The pressure in the recoil cylinder,(i.e. the hydraulic pressure) is the sum of the air pressure, plus the floating piston friction drop, plus the throttling drop through the regulator valve. At short recoil the air pressure is necessarily small compared with the throttling drop. The resistance to recoil is large and therefore the recoil pressure large. This requires a large throttling drop and the air pressure becomes necessarily small compared with the throttling drop. The large throttling drop requires a very small valve opening, with a large pressure reaction against the valve. To balance this reaction a very stiff spring is required. Such spring characteristics have been ad-

mirably met by the use of Belleville washers. At long recoil the resistance to recoil is small, therefore the throttling drop is small, requiring a large orifice area. Since the pressure in the recoil cylinder is small together with a large orifice opening, a weak spring with large deflection is desirable. Such spring characteristics are best met with an ordinary spiral spring. Hence, at long recoil, low elevation, a spiral spring functions alone, while at short recoil maximum elevation the belleville and spiral spring function in parallel. The regulator is so designed that at low elevation only the spiral spring functions.

To modulate or regulate the velocity of counter recoil to a low velocity, the pressure in the recoil cylinder is lowered just sufficiently to balance the total friction during counter recoil. At the end of counter recoil the recoil cylinder pressure is reduced to zero and the recoiling mass is brought to rest by the total friction alone. To reduce the pressure during the first part of counter recoil throttling through a constant orifice is effected in a separate passage way or channel leading from the recuperator to the recoil cylinder. At the end of counter recoil additional throttling around a buffer rod and its chamber, is effected reducing the pressure in the recoil cylinder to zero or nearly so.

DESCRIPTION OF THE OPERATION OF THE ST. CHAMOND RECOIL. Referring to figure (10) is shown a schematic diagram of the operation of the St. Chamond recoil system for both recoil and counter recoil.

Recoil:- During the recoil a flow or stream of oil passes by the regulator valve from the hydraulic to the recuperator (oil side)cylinder. The pressure p of the oil against the recoil piston is re-

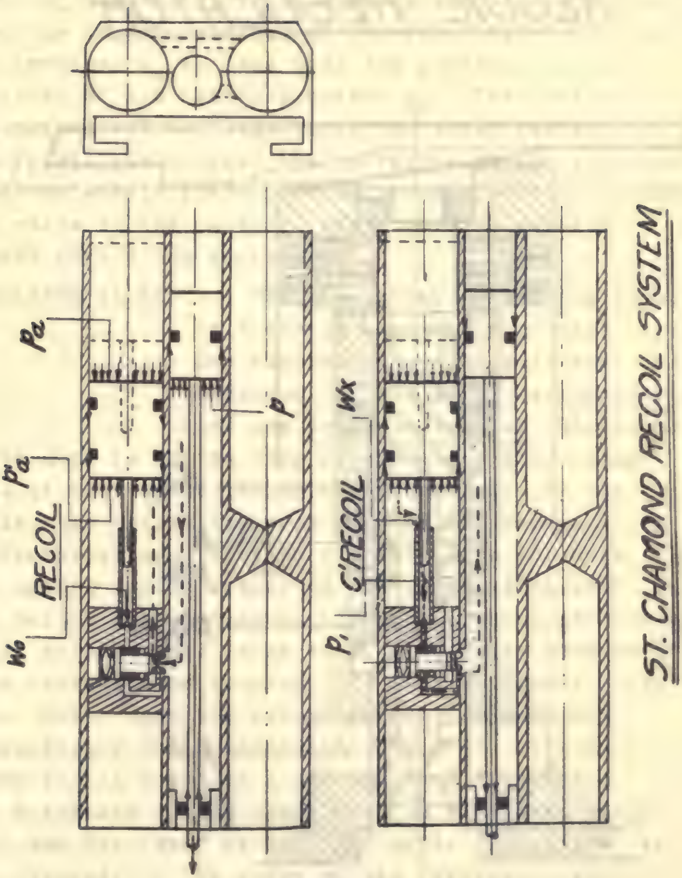


Fig. 5

RECOIL REGULATOR

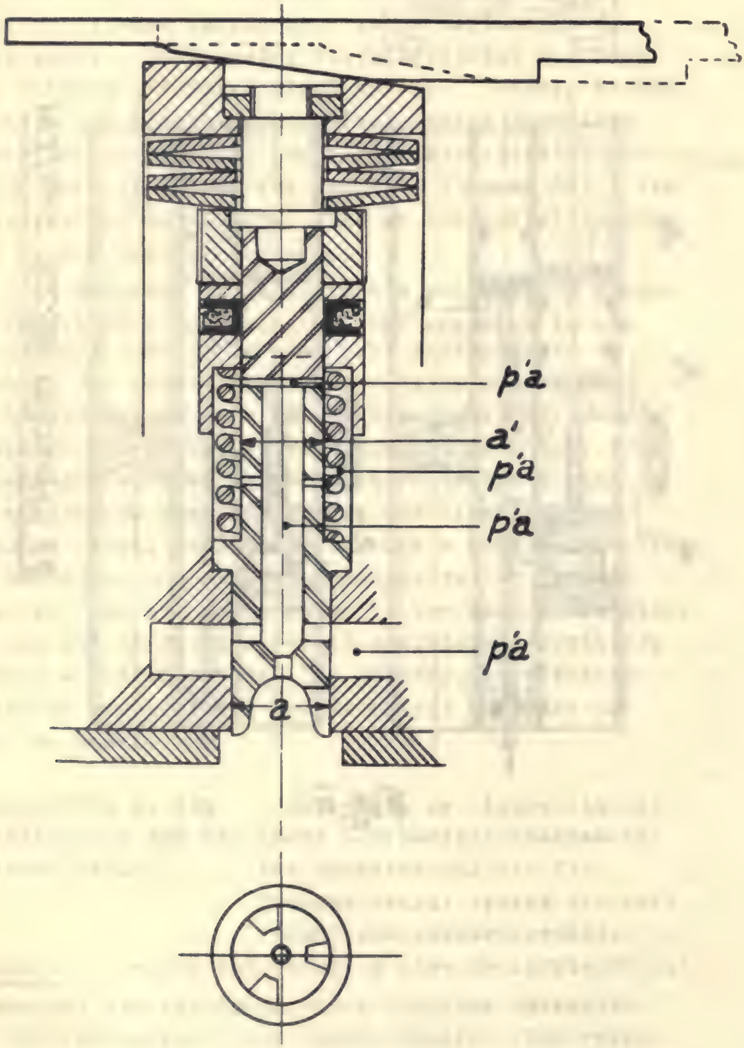


Fig. 6

duced by throttling through the regulator to a pressure (p_a') against the oil side of the floating piston. Due to the friction of the floating piston the air pressure p_a is less than the pressure on the oil side of the floating piston p_a'. The tension in the recoil rod is balanced by the total pressure on the recoil piston plus the hydraulic piston friction plus the stuffing box friction in the recoil cylinder. The valve in the counter recoil orifice remains closed during the recoil.

REGULATOR VALVE. The throttling during the recoil is controlled by the regulator valve. See figure (11). The regulator valve consists of two parts: an upper stem and the lower valve stem. The lower valve stem is seated very carefully on a circular seat at the top of the entrance channel. As the valve lifts, the throttling area becomes the vertical circumferential area between the valve and its seat. The spiral spring reacts on the lower valve stem. The Belleville washers at the top of the upper stem, react only on that valve stem. The upper stem rests in a valve box or housing. To move the upper valve stem (other than the slight deflection possibly compressing the Bellevilles) the whole housing or valve box is moved by a cam as shown in diagram. The diameters of the upper part of the lower valve stem and the lower part of the upper stem, (that is the diameter of the stems of the regulator valve) are the same. At short recoil the reaction of the Belleville on the upper stem is transmitted by the mutual reaction between the upper and lower stems at their surface of contact.

The valve opening and consequent throttling drop of pressure depends upon the deflection of the spiral springs or Belleville washers, the spring reaction balancing the hydraulic reaction on the valve. Neglecting the small dynamic reaction, the

hydraulic reaction on the valve is the product of the intensity of pressure in the recoil cylinder and the base of the regulator valve, minus the product of the intensity of pressure in the recuperator cylinder and the effective area on the upper part of the regulator valve. At long recoil, since the lower valve stem comes in contact with the upper stem, the effective area on the upper part of the valve is obviously equal to the area at the base of the valve. Hence the hydraulic reaction at long recoil is merely the product of the difference in pressures between the recoil and recuperator cylinders and the area at the base of the valve. At short recoil the upper stem of the regulator is brought down by the cam at its top, until its lower surface is in contact with the top surface of the lower valve stem. The effective area, therefore, on the upper part of the regulator valve equals the difference in areas between the area at the base of the lower valve stem and the area at the upper end of the lower valve stem, or the area of the upper valve stem; the two latter being always equal. Hence the hydraulic reaction at short (or intermediate recoil for the greater part of recoil) equals the product of the recoil intensity of pressure and the base of the valve, minus the product of the recuperator intensity of pressure and the difference in areas between the base and middle stem of the valve, when upper and lower stems are in contact. At long recoil the hydraulic reaction is balanced above by the spiral spring reaction. At short or intermediate recoil the hydraulic reaction is balanced by the combined reaction of the Belleville washers and the spiral spring though the latter is negligible compared with the former.

COUNTER RECOIL. The regulator valve is closed during counter recoil. The oil flow during counter recoil, therefore, is different from that in recoil. The valve is seated,

but the oil is allowed to pass through a very small hole in its center. This orifice is constant throughout the whole of counter recoil. There is another channel for the oil leading from the bottom of the buffer chamber in the regulator body. This oil passes through a ball valve. As the floating piston returns to its initial position at the end of counter recoil, the regulator rod enters the buffer cavity, thus obstructing entrance of oil to this cavity. This rod is tapered so that when it has fully entered the cavity there is no clearance between the rod and the entrance, and the oil in returning to the recoil cylinder must all pass through the central opening in the valve. By means of this regulation it is possible to allow the gun to return to its "in battery" position quickly, but its final movement is so controlled that there is no shock.

The throttling areas in the counter recoil channels are so designed as to cause sufficient throttling to lower the pressure in the recoil cylinder that it may practically balance the total friction, during the counter recoil. At the end of counter recoil this friction alone brings the recoiling mass to rest when it reaches the battery position.

GENERAL THEORY OF THE ST. CHAMOND BRAKE. Figure (11) shows the regulator valve stem for both long and short recoil.

Let R_v = reaction on base of throttling valve

p = intensity of pressure in recoil cylinder

p_a = intensity of air pressure.

p'_a = intensity of pressure in recuperator cylinder (i. e. on oil side of floating piston)

a = entrance area of valve or effective area at base of valve.

a_1 = area of valve stem

S_b = spring constant of belleville washer

S_s = spring constant of spiral springs

C = effective circumference at base valve

h = lift of valve from initial opening

h_s = the initial compression of the spiral valve spring at initial opening

h_b = the initial compression of belleville washer at initial opening

w = throttling area

v = velocity of flow through entrance area "a"

V = velocity of recoil

A = effective area of recoil piston

d = density of oil

The hydraulic reaction, at long recoil becomes $R_v - p_a'a$, and at short recoil, we have, the value $R_v - p_a'(a-a_1)$. The belleville washer reaction, becomes $R_b=S_b(h_b+h)$ and the spiral spring reaction, becomes, $R_s = S_s(h_s+h)$. Hence at long recoil, we have $R_v-p_a'a = S_s(h_s+h)+F$ (1)

and at short recoil, we find

$R_v-p_a'(a-a_1)=S_s(h_s+h)+S_b(h_b+h)+F$ (2)

that is, $R_v-p_a'(a-a_1)=S_sh_s+S_bh_b +h(S_s+S_b)+F$

Now at intermediate recoil the upper valve stem is separated from the lower valve stem by a distance h_o when the latter is just about to leave its seat. If h_o' is the separation between the two stems before recoil and if e = the initial lift of lower valve stem required to clear the valve, then $h_o = h_o' = e$

Hence at intermediate recoil, we have

$R_v-p_a'(a-a_1)=S_s(h_s+h)+S_b(h-h_o+h_b)+F$, that is

$R_v-p_a'(a-a_1)=S_sh_s+S_b(h_b-h_o)+h(S_s+S_b)+F$ (3)

Where F is the valve stem friction and will be neglected, let

$C_o=S_sh_s$ and $C_o' = S_sh_s+S_bh_b$

$C_o''=S_sh_s + S_b(h_b-h_o)$

The reaction against the base of throttling valve, in terms of the pressure at the entrance to valve, becomes,

$$R_v - p_1 a = \frac{dav^2}{g} \qquad (4)$$

where p_1 = the pressure at a mid section in the entrance channel of the valve.

Further

$$\frac{p_1}{d} + \frac{v^2}{2g} = \frac{p}{d} + h_t \qquad (5)$$

Neglecting the friction and accelerating head, h_t as small we have, therefore

$p_1 = p - \dfrac{dv^2}{2g}$ which gives the pressure in the entrance channel in terms of the recoil pressure (i.e. the pressure against the hydraulic piston) hence

$$R_v = p_1 a + \frac{dav^2}{g} = pa - \frac{dav^2}{2g} + \frac{dav^2}{g}$$

or

$$R_v = pa + \frac{dav^2}{2g} = (p + \frac{dv^2}{2g})a \qquad (6)$$

Therefore at long recoil, we have

$$(p + \frac{dv^2}{2g})a = C_o + S_s h + p_a' a \qquad (7)$$

and at short recoil we have,

$$(p + \frac{dv^2}{2g})a = C_o' + (S_s + S_b)h + p_a'(a - a_1) \qquad (8)$$

$$(p + \frac{dv^2}{2g})a = C_o'' + (S_s + S_b)h + p_a'(a - a_1) \qquad (9)$$

Considering now the main throttling through the circumferential section, around the <u>effective</u>

circumference of the valve, we have, for the __effective__ throttling area,

$$w = \frac{ch}{K_o} \quad \text{where } K_o = \frac{1}{0.775} \quad \text{Contraction factor of orifice,}$$

the corresponding pressure drop through the valve becomes,

$$\underline{P} = \frac{K_o^2 A^2 V^2}{175(c^2 h^2)}$$

Further $av = AV$. hence $v^2 = (\frac{A}{a})^2 V^2$

hence

$$h = \frac{[p + \frac{d}{2g}(\frac{A}{a})^2 \frac{V^2}{144}]a - C_o - p_a' a}{S_s} \quad \text{long recoil (10)}$$

$$h = \frac{[p + \frac{d}{2g}(\frac{A}{a})^2 \frac{V^2}{144}]a - C_o' - p_a'(a - a_1)}{S_s + S_b} \quad \text{at short recoil (11)}$$

and

$$h = \frac{[p + \frac{d}{2g}(\frac{A}{a})^2 \frac{V^2}{144}]a - C_o'' - p_a'(a - a_1)}{S_s + S_b} \quad \text{at intermediate recoil (12)}$$

Considering only the main throttling, or rather designing the recoil flow channels to have throttling as compared with the throttling through the regulator valve, we have

$p = \underline{P} + p_a$ hence we have the three fundamental equations for the recoil pressure, in terms of the velocity of recoil and the pressure in the recuperator cylinder:

$$p = \frac{K^2 S_s^2 A^2 V^2}{175 C^2 [(p + \frac{d}{2g}(\frac{A}{a})^2 \frac{V^2}{144}]a - C_o - p_a' a)^2} + p_a' \quad (13) \text{ at long recoil}$$

$$p = \frac{K^2(S_s+S_b)^2A^2V^2}{175C^2[(p+\frac{d}{2g}(\frac{A}{a})^2\frac{V^2}{144}]a-C'-p_a'(a-a_1)^2} + p_a' \quad (14)$$

at short recoil.

$$p = \frac{K^2(S_s+S_b)^2\ A^2V^2}{175C^2[(p+\frac{d}{2g}(\frac{A}{a})^2\frac{V^2}{144}a-C_o''-p_a'(a-a_1)]^2} + p_a' \quad (15)$$

at intermediate recoil.

where the units are obviously, p_a' and p in lbs. per sq.in.

V in ft. per sec.

$A_1 a_1$ and a_1 in sq. in.

d in lbs. per cu.ft.

If further $J_o = \frac{1}{2}\,\frac{d}{y}\,\frac{A^2}{a} \times \frac{1}{144}$ then equations (13), (14) and (15) reduce to the simpler form

$$p-p_a' = \frac{K^2S_s^2A^2V^2}{175C^2[(p-p_a')a+J_oV^2-C_o]^2} \quad (16)$$

at long recoil

$$p-p_a' = \frac{K^2(S_s+S_b)A^2V^2}{175\ C^2[(p-p_a')a+p_a'a+J_oV^2-C_o']^2} \quad (17)$$

at short recoil

$$p-p_a' = \frac{K^2(S_s+S_b)A^2V^2}{175C^2[(p-p_a')a+p_a'a+J_oV^2-C_o'']^2} \quad (18)$$

at intermediate recoil

To compute p for any given displacement and corresponding recuperator pressure and recoil velocity, we find the solution in the form of a cubic equation.

The solution is as follows:

From equations (16), (17) and (18),

$$p-p_a' = \frac{\dfrac{K^2S_s^2A^2V^2}{175C^2a^2}}{(p-p_a')+(\dfrac{J_oV^2-C_o}{a})^2}$$

or

$$p-p_a' = \frac{\dfrac{K(S_s+S_b)^2A^2V^2}{175C^2a^2}}{[(p-p_a')+(\dfrac{p_a'a+J_oV^2-C_o'}{a})]}$$

let

$$B = \frac{K^2A^2V^2S^2}{175C^2a^2} \quad \text{or} \quad = \frac{K^2A^2V^2(S_s+S_b)^2}{175\ C^2a^2}$$

$$p-p_a = Z$$

and $\dfrac{J_oV^2-C_o}{a}$ or $\dfrac{p_a'a_1+J_oV^2-C_o'}{a} = m$

Then from the above equations, we have

$$Z = \frac{B}{(Z+m)^2} \quad \text{or} \quad B=Z^3+2Z^2m+Zm^2$$

To eliminate the 2nd degree term, substitute,

$$Z = X - \frac{2}{3}m \quad \text{hence} \quad Z^2=X^2 - \frac{4}{3}mX + \frac{4}{9}m^2$$

and

$$Z^3 = X^3 - 2mX^2 + \frac{4}{3}m^2X - \frac{8}{27}m^3$$

Expanding, we find

$$B = X^3-2mX^2+\frac{4}{3}m^2X - \frac{8}{27}m^3+2mX^2 - \frac{8}{3}m^2X$$

$$+\frac{8}{9}m^3 + m^2X - \frac{2}{3}m^3$$

$$= X^3 - \frac{1}{3}m^2X - \frac{2}{27}m^3$$

Further let $N^3 = \dfrac{2}{27} + \dfrac{B}{m^3}$ then $X^3 - \dfrac{1}{3}m^2X-N^3m^3 = 0$

Solving by Cardan's method

$$X = (3\sqrt{\frac{N^3}{2}+\sqrt{\frac{N^6}{4}-\frac{1}{730}}} + \sqrt{\frac{N^3}{2}-\sqrt{\frac{N^6}{4}-\frac{1}{730}}})\ m$$

$$X = Z + \frac{2}{3}m = p - p_a + \frac{2}{3}m$$

hence

$$p = \left(\sqrt[3]{\frac{N^3}{2} + \sqrt{\frac{N^6}{4} - \frac{1}{730}}} + \sqrt[3]{\frac{N^3}{2} - \sqrt{\frac{N^6}{4} - \frac{1}{730}}} - \frac{2}{3}\right)m + p_a'$$

During the greater part of recoil except at the very beginning and towards the very end of recoil it has been found by actual calculations, that the term $\frac{1}{730}$ becomes negligible in comparison with $\frac{N^6}{4}$ and may be omitted without appreciable error.

The above equation, therefore reduces to the simple form

$$p = m(N - \frac{2}{3}) + p_a'$$

Another and far simpler method for the computation of p established by Mr. McVey, consists in the construction of a table, with assumed values of $p - p_a'$.

The table is based on the two following equations: (neglecting dynamic head as small)

$$\left.\begin{array}{ll} (p-p_a')A+p_a'A_1 = C_o'+(S_s+S_b)h & \text{short recoil} \\ (p-p_a')A=C_o+S_s h & \text{long recoil} \end{array}\right\} \quad (a)$$

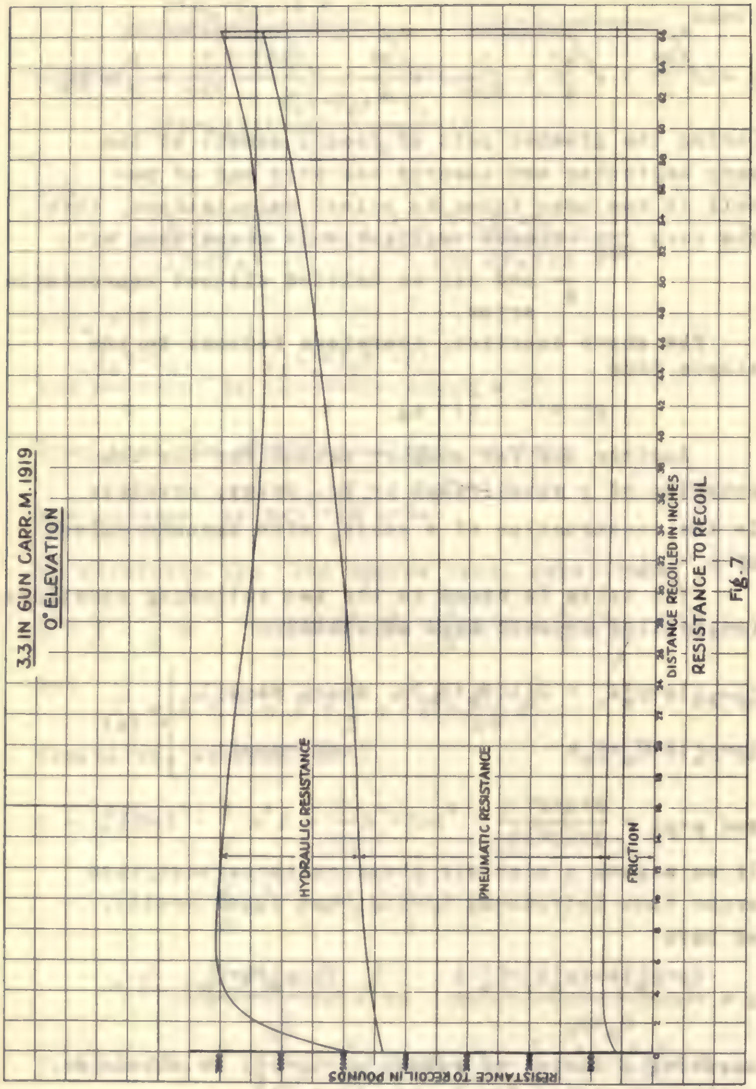
$$\text{and } p-p_a' = \frac{K^2A^2V^2}{175C^2h^2} \qquad (b)$$

If we assume a mean air pressure throughout, (the error thus introduced having been found small), we have

$$h = \frac{(p-p_a')A+(p_a'A_1-C_o')}{S_s+S_b} \quad \text{or} = \frac{(p-p_a')A-C_o}{S_s}$$

Assuming a series of values of $(p-p_a')$ we obtain a series of values for h, now from (b)

$$V = \frac{13.2\ ch}{KA}\sqrt{p-p_a'}$$



RESISTANCE TO RECOIL

FIG. 7

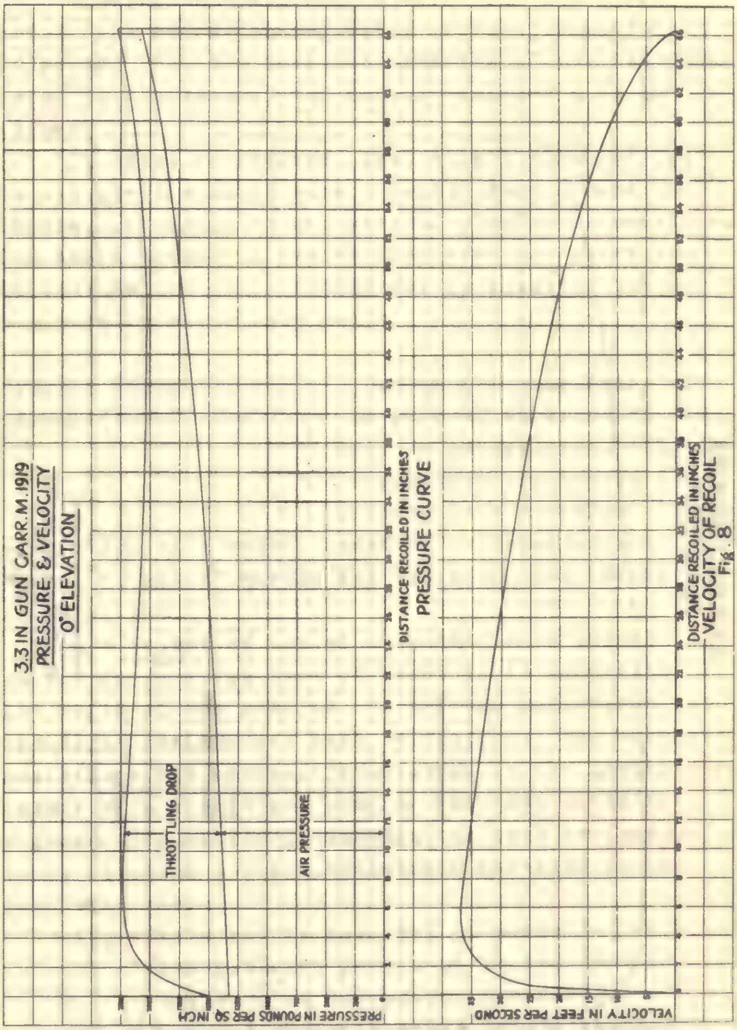
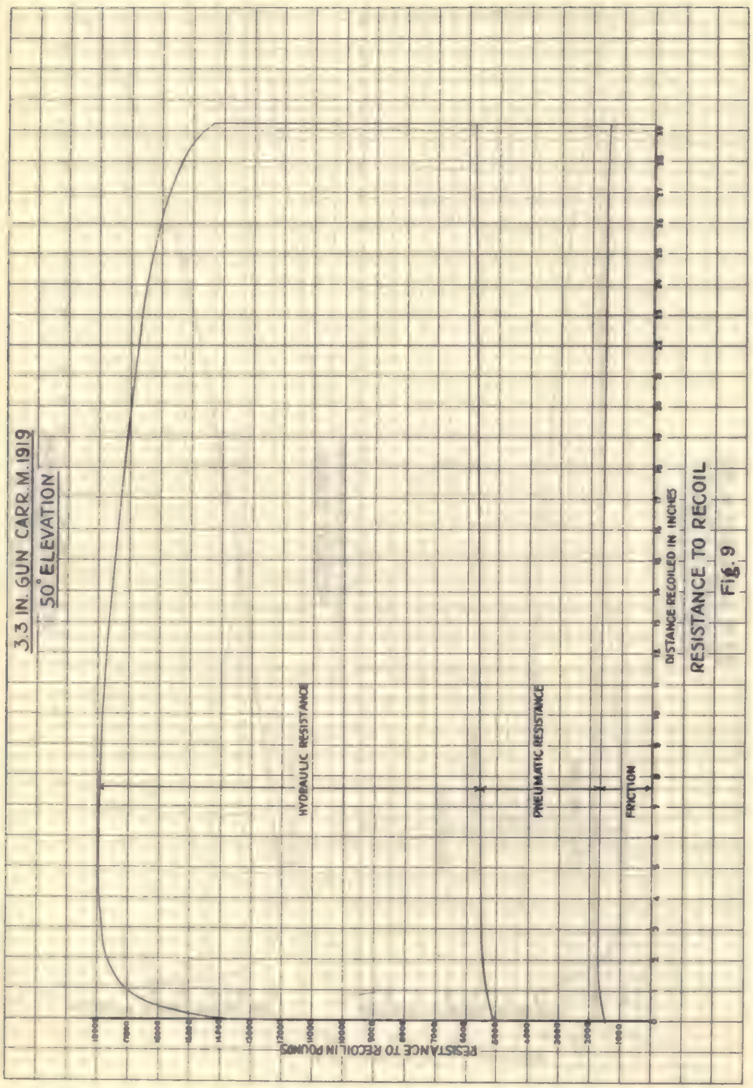

3.3 IN GUN CARR. M. 1919
PRESSURE & VELOCITY
0° ELEVATION
THROTTLING DROP
AIR PRESSURE
PRESSURE IN POUNDS PER SQ. INCH
DISTANCE RECOILED IN INCHES
PRESSURE CURVE
VELOCITY IN FEET PER SECOND
DISTANCE RECOILED IN INCHES
VELOCITY OF RECOIL

Fig. 8



RESISTANCE TO RECOIL

Fig. 9

from which a series of values can be established for corresponding values of $p-p_a'$ and h.

Knowing the retarded velocity for any given point, the corresponding value of $(p-p_a')$ and h can be picked from the table and knowing p_a' for the given point in the recoil, the recoil pressure p is obtained.

It is to be noticed, that substitution of (b) in (a) gives a cubic with a second degree term, as before. Thus no direct simple solution is possible. The table method is recommended even for short recoil since the error introduced by assuming the air constant is relatively small.

GENERAL PROCEDURE FOR CALCULATION OF RECOIL. Due to the complexity of the general equation of recoil no mathematical solution is possible, except by expanding into a series. Such a solution of a recoil equation is known as the "point by point" method and has been used before in this text.

The object of actual computation of recoil curves for a given type of mount is to ascertain the ratio of the peak to the average resistance to recoil at maximum and zero elevation. The average resistance may be readily obtained in the preliminary layout of a design and knowing the peak ratio for a given type of mount, enables the peak resistance to be obtained and the consequent stresses in the carriage. Let

V_{fn} = free recoil velocity at point "n" (i.e. the velocity generated in the recoiling mass by the powder pressure).

V_{rn} = corresponding retarded recoil velocity

R_n = total friction, stuffing box and guide friction.

$\emptyset$ = angle of elevation of gun.

Now, the end pressures at the beginning and end of

recoil, becomes $p = p_a' + \frac{C_o}{a}$ for long recoil

$$p = p_a'\left(\frac{a - a_1}{a}\right) + \frac{C_o'}{a} \text{ for short recoil}$$

Since $\emptyset = 0$ at long recoil, we have for the resistance to recoil, $K = p_n A + R_n$ for long recoil

$K = p_n A + R_n - W_r \sin\emptyset$ for short or intermediate recoil

Long recoil:

For 1st point long recoil,

$$V_{r_1} = V_{f_1} - \left(\frac{p_{ao}' + \frac{C_o}{a} + R_o}{m_r}\right) \Delta t$$

and knowing V_{r_1}

$$p' = \left(\sqrt[3]{\frac{N^3}{4} + \sqrt{\frac{N^6}{4} - \frac{1}{730}}} + \sqrt[3]{\frac{N^3}{2} - \sqrt{\frac{N^6}{4} - \frac{1}{730}}} - \frac{2}{3}\right)m + p_{a_1}'$$

where

$$N = \sqrt[3]{\frac{2}{27} + \frac{B}{m^3}}$$

$$B = \frac{K^2 A^2 V_{r_1}^2 A_s^2}{175 C^2 a^2} \; ; \quad m = \frac{J_o V_{r_1}^2 C_o}{a}$$

For 2nd point long recoil,

$$V_{r_2} - V_{r_1} - (V_{f_2} - V_{f_1}) - \left(\frac{p_1 A + R_1}{m_r}\right) \Delta t$$

and knowing V_{r_2}

$$p^2 = \left(\sqrt[3]{\frac{N^3}{2} + \sqrt{\frac{N^6}{4} - \frac{1}{730}}} + \sqrt[3]{\frac{N^3}{2} - \sqrt{\frac{N^6}{4} - \frac{1}{730}}} - \frac{2}{3}\right)m + p_{a_2}'$$

where as before

$$N = \sqrt[3]{\frac{2}{27} + \frac{B}{m^3}}$$

$$B = \frac{K^2 A^2 V_{r_2}^2 S_s}{175 C^2 a^2} \; ; \quad m = \frac{J_o V_{r_2}^2 - C_o}{a}$$

After a very few intervals the valve opens sufficiently so that the term

$\frac{1}{730}$ may be omitted, then for point "n" at long recoil,

$$V_n - V_m = (V_{rn} - V_{rm}) - \frac{(p_m A + R_n)}{m_r} \quad \Delta t$$ and knowing V_n

$p_n = m(N - \frac{2}{3}) + p'_a$ Obviously after the powder pressure period $V_{fn} - V_{fm} = 0$ and we have the simple dynamic equation of recoil.

Short Recoil:

The procedure for the calculation of the velocity and pressure curves for short recoil is exactly similar as for long recoil.

For 1st point short recoil,

$$V_{r_1} = V_{f_1} - \left(\frac{p'_{a_1}\left(\frac{a-a_1}{a}\right) + \frac{C'_o}{a} + R_1 - W_r \sin\emptyset}{m_r} \right) \Delta t$$

and knowing V_{r_1}

$$p_1 = \left(\sqrt[3]{\frac{N^3}{2} + \sqrt{\frac{N^6}{4} - \frac{1}{730}}} + \sqrt[3]{\frac{N}{2} - \sqrt{\frac{N^6}{4} - \frac{1}{730}}} - \frac{2}{3} \right) m + p'_{a_1}$$

where

$$N = \sqrt[3]{\frac{2}{27} + \frac{B}{m^3}}$$

$$B = \frac{K^2 A^2 V^2 (S_s + S_b)^2}{175 C^2 a^2} \quad ; \quad m = \frac{p'_{a_1} a + J_o V^2 - C'_o}{a}$$

for 2nd period short recoil,

$$V_{r_2} - V_{r_1} = V_{f_2} - V_{f_1} - \left(\frac{p' A + R_1 - W_r \sin\emptyset}{m_r} \right) \Delta t$$

and knowing V_{r_2}, p_2 can be obtained by a solution of the previous cubic equation. The greater number of points of recoil excepting a few points at the beginning and end may be solved with sufficient accuracy by the expression,

$p_n = m(N - \frac{2}{3}) + p'_a$ where, as before,

$$N = \sqrt[3]{\frac{2}{27} + \frac{B}{m^3}} \qquad B = \frac{K^2A^2V^2(S_s+S_b)^2}{175C^2a^2}$$

and

$$m = \frac{p'_{an}a_1 + J_oV^2 - C'_o}{a}$$

Calculation from constructed table of $(p-p'_a)$, h, and V_r

The procedure here is exactly similar as above; each preceding interval establishes a new retarded velocity which from the table establishes a new recoil pressure. This recoil pressure substituted in the dynamic equation in turn establishes the retarded velocity at the end of the interval under consideration.

Judgment must be used in the proper increments of time to be used. The closer the intervals the more accurate the velocity and pressure curves. At the beginning and end, the time intervals should be taken smaller. During the major part of recoil the time intervals can be fairly large. As a check during the powder period the retarded velocity should be roughly 0.9 of the free velocity of recoil.

CALCULATION OF THE VARIOUS FRICTION COMPONENTS DURING RECOIL.

In the calculation of the vertical pressure and retarded velocity curves for the St. Chamond brake, the frictions vary as a function of the pressure. At long recoil the pressure variation is small and we are not in great error in assuming constant friction: with short recoil, however, a peak value is obtained and with it a change in friction.

The frictional resistance opposing recoil are:

(1) Guide friction which is function of the total pull.

(2) Stuffing box friction which is a function of the recoil pressure.

(3) Recoil piston friction which also is a function of the recoil pressure.

(1) The guide friction during recoil has been previously expressed by the following equations:

$$R_g = \frac{2(p_b e + Bb) + W_r \cos\emptyset N}{m} n$$

where p_b is the powder reaction on the breech

c is the perpendicular distance between the axis of the bore and a line through the center of gravity of the recoiling parts parallel to the axis of the bore.

$B = pA$, the hydraulic reaction of the recoil piston

n = coefficient of friction, from 0.15 to 0.20

b = distance down of the line of pull from the center of gravity of recoiling parts

$N = x_1 - x_2 + n(y_2 - y_1)$

$M = x_1 + x_2 - n(y_1 + y_2)$

where x_1 and y_1 are the coordinates of the front clip reaction and x_2 and y_2 are the coordinates of the rear clip reaction having axis and origin through the center of gravity of the recoiling parts.

Considering the somewhat inaccuracy of a "point by point" method of computation, it is believed the following formula is sufficiently accurate,

$$R = \frac{2nBb + nW_r \cos\emptyset(x_1 - x_2)}{C - 2nr}$$

or when $x_1 - x_2$ is small,

$$R = \frac{2nBb}{l - 2nr} \quad \text{where } l = x_1 - y_2$$

where r is the mean distance from the center of gravity of the recoiling parts to the line of action of the guide frictions.

(2) The packing friction formulas have been already considered in more or less detail in Chapter VIII. The stuffing box friction,

$$R_s = C_1' + C_2' \, p$$

where

$$C_1 = \pi d_r p_o (bf + af_1)$$

$$C_2 = \pi d_r (bf + af + a_1 f_1)$$

(3) The hydraulic piston friction,

$$R_p = C_1'' + C_2'' \, p$$

$$C_1' = \pi d p_o (bf + af_1)$$

$$C_2' = d(bf + af + a_1 f_1)$$

From the above formulae

p = recoil pressure in lbs. per sq.in.

p_o = intensity of pressure caused by Bellevilles or packing springs in lbs. per sq.in

R_b = belleville or packing spring reaction on annular area of packing spring at assembled load in lbs.

d_r = diam. of piston rod in inches.

d_o = outer diam. of stuffing box packing ring in inches.

d = diam. of recoil cylinder in inches.

d_1 = inner diam. of piston packing ring in inches.

b = width of leather contact of packing in inches.

f = corresponding coefficient of friction = .05

$\frac{a}{2}$ = silver contact of flap of one flange of packing ring in inches.

f_1 = coefficient of silver friction = .09

Then p_o becomes,

for (1) $p = \dfrac{4R_b}{\pi(d_o^2 - d_r^2)}$ and for (2) $p = \dfrac{4R_b}{\pi(d^2 - d_1^2)}$ -

In summing up the component frictions, we have

$$R = \frac{nW_r \cos \phi (x_1 - x_2)}{C-2nr} + \frac{2nAb}{C-2nr} p$$

$$= R_1 + K_2 p$$

$$R_s = C_1' + C_2' p$$

$$R_p = C_1'' + C_2'' p$$

hence $R = (C_1' + C_1'' + K_1) + (C_2' + C_2'' + K_2) p$

$$= C_1 + C_2 p$$

showing the total friction resisting recoil is a linear function of the pressure in the recoil cylinder.

Floating piston:

The oil pressure in the recuperator cylinder during the recoil is greater than the air pressure by the drop of pressure caused by the floating piston friction. In the previous recoil equations, the recuperator oil pressure has been used in place of the air pressure. To compute this pressure knowing the air pressure, it is only necessary to compute the floating piston friction drop. In the discussion of the floating piston in Chapter VIII, we have $R_f = C_1 + C_2 p_a$ where

$C_1 = \pi d[(bf + af_1)(p_o + p_o') + 2O_1 f_1 p_q]$

$C_2 = \pi d[2(bf + af_1) + 2a_1 f_1$

For symbols see discussion of floating piston in Chapter VIII. All dimensions may be expressed in inches and pressures in lbs. per sq.in. in place of the center of gravity system as used previously. The resulting friction R_f is therefore in lbs.

The drop due to friction, becomes,

$$p_a' - p_a = \frac{R_f}{A_a}$$

where A_a is the area of the floating piston or recuperator cylinder in sq. inches.

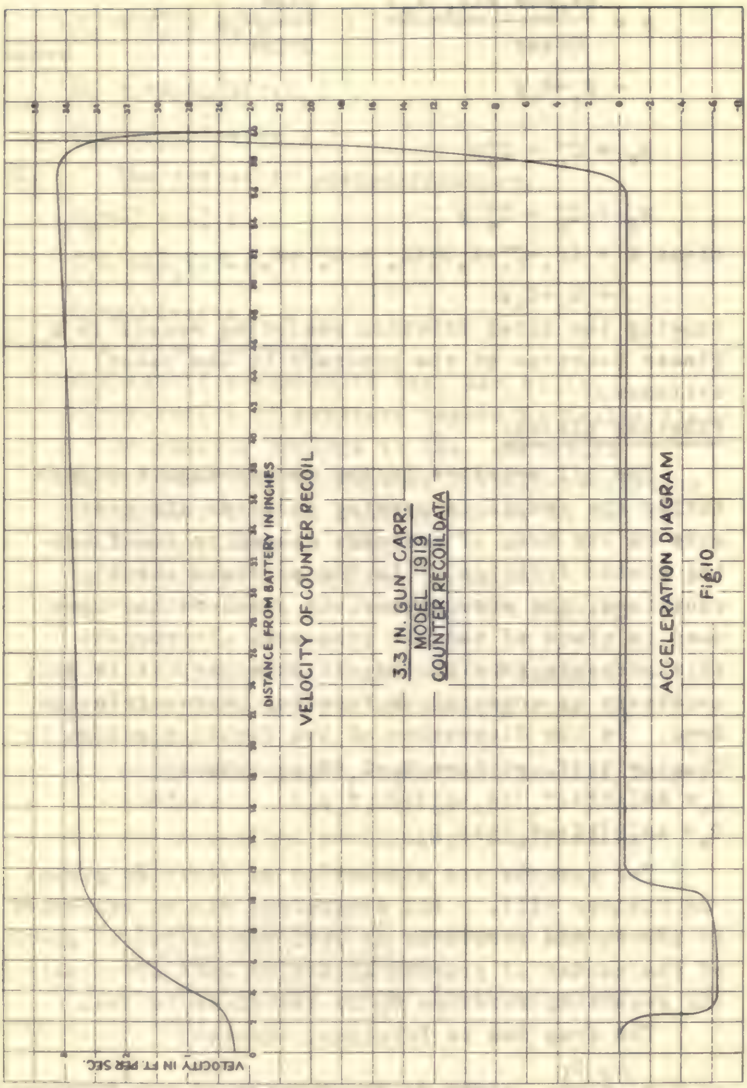

DISTANCE FROM BATTERY IN INCHES
VELOCITY OF COUNTER RECOIL
3.3 IN. GUN CARR.
MODEL 1919
COUNTER RECOIL DATA
ACCELERATION DIAGRAM
VELOCITY IN FT. PER SEC.

Fig.10

GENERAL THEORY OF COUNTER RECOIL. Counter recoil is divided into two periods, (1) the first period or constant orifice period and (2) the second period or buffer period where the main retardation takes place. The second period is the critical period in the design of a counter recoil system, since with field carriages the stabilizing force of counter recoil is relatively small, therefore too rapid retardation of the recoiling mass will cause the mount to be unstable on counter recoil. Let

A = effective area of recoil piston

K_o = contraction factor of constant orifice

K_t = contraction factor for variable orifice.

w_o = area of constant orifice

w_x = variable area of buffer throttling

R = total friction

p_a' = recuperator oil pressure

$p_a' = p_a'A$ equivalent recuperator pressure on recoil piston

$C_o = \dfrac{K_o^2 A^3}{175}$ = throttling drop constant for constant orifice.

$C_x = \dfrac{K^2 A^3}{175}$ = throttling drop constant for buffer orifice.

$R_o = W_r \sin\emptyset + R$ = resistance constant.

For 1st Period of Counter Recoil:

Considering the motion of the recoiling mass, from the initial displacement of out of battery, we have

$$pA - W_r \sin\emptyset - R = m_r v \frac{dv}{dx}$$

but

$$p = p_a' - \frac{K_o^2 A^2 V^2}{175\, w_o^2}$$

Assuming the throttling drop is entirely through the constant orifice during the first period

of recoil. Hence $p_a'A - \frac{K_o^2A^3V^2}{175w_o^2} - W_r\sin\phi - R = m_r v\frac{dv}{dx}$

or

$$p_a' - \frac{C_oV^2}{w_o^2} - R_o = m_r v\,\frac{dv}{dx}$$

Since p_a' is a function of x, the equation is not possible to integrate directly, but by dividing the constant orifice period into several increments, and taking a constant air pressure equal to the mean air pressure for the interval, we get a very close approximation of the true velocity of counter recoil by the following solution,

$$dx = \frac{m_r\ v\ d_v}{(p_a'-R_o) - \frac{c_ov^2}{w_o^2}}$$

Integrating, we have

$$x_2 - x_1 = -\frac{m_rw_o^2}{2C_o}\left\{\log[(p_a'-R_o) - \frac{C_ov_2^2}{w_o^2}] - \log[(p_a'-R_o) - \frac{C_ov_1}{w_o^2}]\right\}$$

Substituting for the base 10, we have

$$\log[(p_a'-R_o) - \frac{C_ov_2^2}{w_o^2}] = \log[(p_a'-R_o) - \frac{C_ov_1^2}{w_o^2}] - \frac{2C(x_2-x_1)}{2.3m_rw_o^2}$$

From this equation, knowing the velocity at the beginning of any arbitrary interval and with the mean recuperator pressure we can obtain the velocity at the end of the interval. It will be found that fairly large intervals may be assured with considerable accuracy, providing the air pressure does not vary greatly.

The velocity curve for the first period should be continued from the out of battery position to $x = b-d$, where d= the length of the counter recoil corresponding to the buffer length.

Let v_b = velocity of counter recoil at entrance to buffer.

<u>For 2nd Period of Counter Recoil.</u>

The recoil displacement is d, and the initial

velocity v_b. In order to be assured that the c'recoil is completely checked, the counter recoil energy of the recoiling mass at entrance to buffer should be dissipated in a distance somewhat less than d, from 0.7 to 0.9d, depending upon the design constant of the recoil system and gun. Let k = the proportional distance of d that the recoil energy is to be dissipated along.

For counter recoil stability the minimum force during the buffer period is obviously obtained by using a constant force during the entire period.

There are two methods consistent with counter recoil stability:

(1) When the total friction is small compared with the overturning force permissible with counter recoil stability, by bringing the recoiling mass to rest into battery with the friction alone.

(2) When the total friction is greater than the overturning force permissible with counter recoil stability, by bringing the recoiling mass to rest into battery by a force equal to the total friction minus a recoil pressure exerted on the recoil piston.

In method (1) obviously, for a given kinetic energy of the recoiling mass at entrance into buffer, the recoil displacement during the buffer action is fixed.

Method (1)

We have for the required recoil displacement during the buffer action:

$$d = \frac{\frac{1}{2} m_r v_b^2}{kR} \qquad \text{where } k = 0.7 \text{ to } 0.9$$

where R is the total recoil friction (guide, stuffing box and piston friction).

The length of the buffer in the recuperator cylinder becomes,

$$d' = \frac{A}{A_a} d$$

The velocity curve is evidently a parabolic curve against displacement, that is

$$R = -m_r v \frac{dv}{dx}$$

$$\int_{b-d}^{x} R dx = -m_r \int_{v_b}^{v} v \, dv$$

$$R(x-b+d) = \frac{m_r}{2}(v_b^2 - v^2)$$

hence

$$v \sqrt{v_o^2 - \frac{2R}{m_r}(x-b+d)}$$

Since it is assumed that p = 0, we have

$$p' - \frac{K_o^2 A^2 V^2}{175 \; w_o^2} - \frac{K_1^2 A^2 V^2}{175 w_x^2} = 0$$

hence

$$w_x = \frac{K_1 AV}{13.2} \sqrt{\frac{1}{p_a' - \frac{K_o^2 A^2 V^2}{175 w_o^2}}}$$

substituting for v, we have w_x in terms of the displacement x from the out of battery position,

$$w_x = K_1 A \, w_o \sqrt{\frac{v_b^2 - \frac{2R}{m_r}(x-b+d)}{175 p_a' w^2 - K^2 A^2 [v^2 - \frac{2R}{m_r}(x-b+d)]}}$$

where b = length of recoil in ft. and d = length of buffer recoil in ft.

x = counter recoil displacement from out of battery position in ft.

The throttling drop

$\frac{K_o^2 A^2 v^2}{175 w_o^2}$ through the constant orifice has been found by calculation to be small as compared with the throttling drop due to the buffer. Therefore a simplification in the calculation may be made by omitting this term, hence

$$w_x = \frac{K_1 A\, v}{13.2}\sqrt{\frac{1}{p_a'}}$$

and substituting for v, we have

$$w_x = \frac{K_1 A}{13.2}\sqrt{\frac{m_r v_b - 2R(x-b+a)}{m_r p_a'}}$$

approx. which gives the required throttling area in terms of the displacement of counter recoil. (x is measured from out of battery position in ft; b = recoil displacement in ft. and d = recoil buffer displacement in ft).

Method 2

If h = height of center of gravity of recoiling parts above ground (practically from ground to axis of bore) and w_s = weight of entire carriage and gun and C_s' = distance of w_s from wheel contact, then critical stability (at 0° elevation) we have, $(R-pA)h=W_s l_s'$ using a factor of 0.8, we have

$$R - p\,A = 0.8\,\frac{W_s l_s'}{h}$$

where R is the total friction, but now a function of the recoil pressure, let $R = C_1 + C_2 p$ then

$$C_1 + P(C_2 - A) = 0.8\,\frac{W_s l_s'}{h}$$

or

$$p = \frac{0.8\,\frac{W_s l_s'}{h} - C_1 h}{h(C_2 - A)}$$

where C_1 = guide friction assumed independent of p + that part of packing friction independent of p.

C_2 = that part of packing friction dependent upon p. The counter recoil velocity curve, becomes during the buffer recoil,

$$v = \sqrt{v_b^2 - \frac{2(R-p\ A)}{m_r}(x-b+d)}$$ and for the pressure p in the recoil cylinder, we have

$$p_a' - \frac{K_o^2 A^2 v^2}{175 w_o^2} - \frac{K^2 A^2 v^2}{175 w_o^2} = p$$

hence

$$w_x = \frac{K_1 A\ v}{13.2} \sqrt{\frac{1}{(p_a'-p) - \frac{K_o^2 A^2 v^2}{175\ w_o^2}}}$$ or in terms of the displacement x,

$$w_x = K_1 A\ w_o \sqrt{\frac{v_b^2 - \frac{2}{m_r}(R-pA)(x-b+d)}{175(p_a'-p)w_o^2 - K_o^2 A^2 \left[v_b^2 - \frac{2(R-p\ A)}{m_r}(x-b+d)\right]}}$$

Neglecting the constant orifice throttling drop, we have the following approximate formula

$$w_x = \frac{K_1 A v}{12.2} \sqrt{\frac{1}{p_a'-p}}$$

It should be carefully noted that if v_b is allowed to become too great it may be found very difficult to prevent the final check of counter recoil without shock sure with even prompt throttling by the regulator the kinetic energy ot the gun may overcome the the opposing friction and cause bumping.

DESIGN FORMULAE ST. CHAMOND RECOIL SYSTEM. In the preliminary design of a St. Chamond recoil system, we must consider the following.

(1) The proper weight of recoiling mass with given ballistics and allowable recoil at maximum elevation for minimum weight of the total mount, gun and carriage.

(2) The length of recoil at zero elevation consistent with stability.

(3) The total resistance to recoil at short recoil maximum elevation and at long recoil, zero elevation.

(4) An estimation of the guide friction and packing frictions for both recoil and counter recoil.

(5) The recuperator reaction required to hold the gun in battery at Maximum elevation.

(6) Limitations of recuperator dimensions.

(7) The calculation of initial air pressure and air volume, final air pressure and air volume. From this the equivalent air column length.

(8) The calculation of strength of cylinders and proper thickness between walls.

(9) The layout of the recuperator forging distance of center lines of cylinders with respect to axis of bore, location of trunnions, etc.

(10) The calculation for maximum and minimum throttling areas.

(11) The calculation for entrance channel area to regulator valve, regulator dimensions and channel areas around and leading from the regulator orifice.

(12) The reactions on regulator valve corresponding to deflections at maximum and minimum opening and the design of spiral springs and belleville washers.

(13) The design of cam mechanism for changing the initial opening to regulator valve for decreasing the recoil on elevation.

(14) The design of packing for float-

ing piston, recoil piston and stuffing box.

(15) The design of the counter recoil and chamber, throttling grooves and constant orifice with its channel leading from the inside end of the buffer to the recoil cylinder

(16) The layout of gauge and pump details and all other details.

(1) Proper weight of recoiling mass:

From "General design Limitations" we have $w_r = \sqrt{kk'}$ where

$$k = \frac{w_c}{R}$$

and

$$k' = \frac{g(mv+\bar{m}\,4700)^2}{2b}$$

m = mass of projectile
$\bar{m}$ = mass of charge
b = length of short recoil in feet
R = recoil reaction in lbs.
g = acceleration due to gravity (ft/sec)
w_c = weight of carriage excluding recoiling mass in lbs.

k may be obtained from table in Chapter VII or the ratio

$\frac{w_c}{R}$ may be computed from a similar well designed type of carriage, using a somewhat lower value of "b" according to the judgment of the designer in improving the weight efficiency of the mount proper over a similar previous design.

(2) Length of recoil at zero elevation,

From pressure curves obtained experimentally it was found that the resistance to recoil at zero elevation is practically constant throughout the recoil.

Let b = length of horizontal recoil consistent with stability in feet

$V_f = \dfrac{wv+\bar{w}4700}{w_r}$ = free velocity of recoil.

w = weight of projectile in lbs.

$\bar{w}$ = weight of charge in lbs.

W_r = weight of recoiling parts in lbs.

v = muzzle velocity (ft/sec)

$V_r = 0.9\ V_f$ (approx.) = velocity of restrained recoil.

u = travel of projectile up bore in feet

A = recoil constrained energy = $\frac{1}{2}m_r V_r^2$

E = recoil displacement during powder period

$= 2.24\ \dfrac{(w+0.5\bar{w})}{w_r}\ u$

C = constant of stability = $\dfrac{\text{Overturning moment}}{\text{Stability moment}}$

W_s = weight of total mount, gun and carriage

l_s = moment arm of w_s about spadepoint.

d = perpendicular distance from spade point to line of action of the total resistance to recoil.

Usually $\emptyset = 0$, $\cos\emptyset = 1$ and $d = h$ = height of axis of bore above ground.

then

$$b=E+\frac{1}{W_r\cos\emptyset}\left(W_s l_s \pm\sqrt{(W_s l_s)^2-\frac{2A\,W_r d\cos\emptyset}{C}}\right)$$

and when $\emptyset = 0$, we have,

$$b = E + \frac{1}{W_r}\left[W_s l_s \pm\sqrt{(W_s l_s)^2-\frac{2AW_r h}{C}}\right]$$

Ordinarily the constant of stability will be assumed at $C = 0.85$.

For rough estimates, especially where the length of recoil is comparatively long,

$$b = \frac{-B\pm\sqrt{B^2-4AC}}{2A} \quad \text{where } A = \frac{C_s\ W_r\cos\emptyset}{d};$$

$$B = \frac{C_s W_s l_s}{d} \quad ; \quad C = \frac{1}{2} m_r V_f^2$$

(3) Resistance to recoil, short and long recoil,

For design calculations, Bethel's formula for the total resistance to recoil is sufficiently accurate. Let w = weight of projectile, lbs.

W_r = weight of recoiling mass, lbs.

M = travel up bore in inches

d = diam. of bore, inches.

v = muzzle velocity of projectile (ft/sec)

V_f = max. free velocity of recoil (ft/sec)

b_s = length of short recoil at max. elevation in ft.

b_h = length of long recoil at zero elevation in ft.

$\bar{w}$ = weight of powder charge

Now for the free velocity recoil, we have

$$V_f = \frac{\bar{w}\, 4700 + wv}{W_r}$$

then at maximum elevation and short recoil, we have

$$K_s = 1.05 \left[\frac{\bar{w}_r V_f^2}{2g} \; \frac{1}{b_s + (.096 + .0003d) m \frac{V_f}{v}} \right]$$

(where 1.05 accounts for the peak effect due to throttling) and at horizontal elevation, long recoil, we have

$$K_h = \frac{\bar{w}_r V_f^2}{2g} \; \frac{1}{h_h + (.096 + .0003d) m \frac{V_f}{v}}$$ the peak effect being zero.

(4) Guide and packing frictions - Recoil and Counter Recoil:-

In the calculation of guide friction during the recoil consideration must be given to the pinching action at the clips due to the pull being

usually below the center of gravity of recoiling parts. Failure to consider this fact will give erroneous results for the friction at high elevation. Also the recuperator must be designed not only for the weight component at max. elevation but the friction just out of battery. Since at the end of counter recoil we have the full air pressure acting on the recoil piston, the pinching action and corresponding guide friction being a factor of importance. Let

B = the total braking including the packing friction of recoil piston and stuffing box, lbs.

d_b = distance down from center of gravity of recoiling mass of line of action of B in inches.

pA = the recoil reaction, lbs.

n = coefficient of guide friction

x_1 and x_2 = coordinates in direction of bore of clip reactions measures respectively from center of gravity of recoiling parts in inches.

$l = x_1 + x_2$ = distance between normal clip reaction, in inches.

R_g = total guide friction.

From a similar previous design, a value of b and l may be assumed.

From Chapter IV, we have

$$R_g = \frac{2nB^d b + nW_r \cos \emptyset (x_1 - x_2)}{l}$$ and for a first approximation

assume $x_1 = x_2$ then

$$R_g = \frac{2nB^d b}{l}$$

but $K = B + R_g - W_r \sin \emptyset$

$$B = \frac{(K + W_r \sin \emptyset) l}{l + 2 n b}$$

Knowing K' and assuming l and b, we have
$B = pA + R_s + R_p$ where R_s = stuffing box friction
R_p = recoil piston friction.

Since the design and estimation of packing friction is in greater part based on previous empirical data, the width of packing and corresponding dimensions being entirely an empirical matter, we must estimate the proper value from data on previous satisfactory packing used on other guns.

Now in general the packing friction may always be represented by the following equation:-
$R = C_1 + C_2 p$. where C_1 is the friction component caused by the springs or Belleville washers. Since the object of the Bellevilles is to compensate for the deficiency of the oil or air pressure normal to the cylinder due to Poisson's ratio, if we know the maximum pressure and assume dimensions for the packing we may compute C_1 as well as C_2 and thus estimate the friction at any other pressure.

Maximum pressures normal to cylinder should be taken as follows:

	p_n	k
Hydraulic piston	$0.88\ p_{max.}$	0.88
Stuffing box	$0.86\ p_{max.}$	0.86
Floating piston:		
Air side	$1.20\ p_{a\ max.}$	1.20
Oil side	$1.38\ p_{a\ max.}'$	1.38

Knowing the maximum pressure normal to the cylinder "p_n" we have,

$$p_{max.} = \pi\, d[.05\, b + .09(a + \frac{a_1}{2})]p_n \text{ approx.}$$

PHYSICAL CHARACTERISTICS OF PACKING
ST. CHAMOND TYPE
RECOIL MECHANISM

Column key: 1 = Assembled load, pounds; 2 = Annular area, sq. in.; 3 = Spring pressure ∥ to cyl. %; 4 = Spring pressure ⊥ to cyl. %; 5 = Initial fluid pressure %; 6 = Maximum fluid pressure %; 7 = Total metal pack. press. ⊥ to cyl.; 8 = Total max. pack. press. ⊥ to cyl.; 9 = Width of packing; 10 = Effective diameter

75 MM. MOD. 1916 — BELLEVILLES (NEW)

	1 Assembled load pounds	2 Annular area sq. in.	3 Spring pressure ∥ to cyl. %	4 Spring pressure ⊥ to cyl. %	5 Initial fluid pressure %	6 Maximum fluid pressure %	7 Total metal pack. press. ⊥ to cyl.	8 Total max. pack. press. ⊥ to cyl.	9 Width of packing	10 Effective diameter
HYDRAULIC PISTON	1700.	1.36	1290.	940.	895	5120.	1660.	4680	.41	1.095
STUFFING BOX	1340.	1.34	1000.	730.	995	5120.	1456	4470.	.41	1.024
FLOATING P. - AIR	2290	2.66	861.	628.	1095	1220	1428	1520.	.41	2.75
OIL	4290	2.66	1610.	1175.	995	1320.	1910.	2140.	.41	2.75
INT.	396	.356	1110.	810.	995	1220	1536	1700.	.16	.55
GREASE	740	3.692	200.	200.	995	1220	1195.	1420		2.75

75 MM. MOD. 1916 — HELICAL (OLD)

	1 Assembled load	2 Annular area	3 Spring pressure ∥ to cyl. %	4 Spring pressure ⊥ to cyl. %	5 Initial fluid pressure %	6 Maximum fluid pressure %	7 Total metal pack. press. ⊥ to cyl.	8 Total max. pack. press. ⊥ to cyl.	9 Width of packing	10 Effective diameter
HYDRAULIC PISTON	442.	1.58	286.	209.	935.	5120.	834.	3945.	.394	1.085
STUFFING BOX	341.	1.33	256.	187.	995.	5120.	813.	3880	.394	1.024
FLOATING P. - AIR	631.	2.39	261.	207.	1095.	1220	1007.	1098.	.394	2.75
OIL	631	2.39	264.	207.	995	1320	930	1170.	.394	2.75
INT.	525.	1.44	356	259.	995.	1220.	965.	1150.	.394	1.16
GREASE		4.85	1220	1220	995	1220.	1220	1510		2.75

4.7 IN. MOD. 1918 — BELLEVILLES

	1 Assembled load	2 Annular area	3 Spring pressure ∥ to cyl. %	4 Spring pressure ⊥ to cyl. %	5 Initial fluid pressure %	6 Maximum fluid pressure %	7 Total metal pack. press. ⊥ to cyl.	8 Total max. pack. press. ⊥ to cyl.	9 Width of packing	10 Effective diameter
HYDRAULIC PISTON	2805	4.38	638.	467.	1400	4900	1490.	3750.	.38	3.625
STUFFING BOX	1710.	3.01	568.	415.	1400	4500.	1436.	3700.	.53	1.75
FLOATING P. - AIR	5120	6.68	767.	560.	1600.	2250	1653.	2200.	.53	4.75
OIL	9460	6.68	1420.	1040.	1400.	2250.	2060.	2750.	.53	4.75
INT.	322.	.755	426.	211.	1400.	2250.	1330.	1950.	.25	1.00
GREASE	2120	10.92	125.	125.	1400.	2250	1625.	2375.		4.75

3.3 IN. MOD. 1919 — BELLVILLES (NEW)

	1 Assembled load	2 Annular area	3 Spring pressure ∥ to cyl. %	4 Spring pressure ⊥ to cyl. %	5 Initial fluid pressure %	6 Maximum fluid pressure %	7 Total metal pack. press. ⊥ to cyl.	8 Total max. pack. press. ⊥ to cyl.	9 Width of packing	10 Effective diameter
HYDRAULIC PISTON	2520	2.20	1136.	830.	1100.	5500	1630.	4040.	.4469	2.25
STUFFING BOX	1890.	1.87	1010.	738.	1100.	5500	1540.	4750.	.4469.	1.22
FLOATING P. - AIR	1920.	2.65	810	592.	1208.	1800.	1470.	1900.	.4095	2.75
OIL	3880	2.65	1620.	1180.	1100.	1900.	1380.	2570.	.4095	2.75
INT.	617.	1.48	417.	305.	1100.	1800.	1110.	1620.	.1969	1.18
GREASE		4.85	1480.	1480.	1100.	1800.	1480.	2420		2.75

3.3 IN. MOD. 1919 — HELICAL (OLD)

	1 Assembled load	2 Annular area	3 Spring pressure ∥ to cyl. %	4 Spring pressure ⊥ to cyl. %	5 Initial fluid pressure %	6 Maximum fluid pressure %	7 Total metal pack. press. ⊥ to cyl.	8 Total max. pack. press. ⊥ to cyl.	9 Width of packing	10 Effective diameter
HYDRAULIC PISTON	625.	2.20.	264.	207.	1100.	5500	1010.	4220	.1969	2.25
STUFFING BOX	449.	1.87	240.	175	1100.	5500	980.	4190.	.1969	1.22
FLOATING P. - AIR	790.	2.65	298.	218.	1208.	1800.	1100	1530.	.1969	2.75
OIL	705	2.65	266	194.	1100.	1900.	1600.	1580.	.1969	2.75
INT.	526.	1.48	357.	261.	1100.	1800	1063.	1573.	.1969	1.18
GREASE		4.85	1480	1480	1100.	1800.	480.	2420		2.75

4.7 IN. MOD. 1906 — BELLEVILLES

	1 Assembled load	2 Annular area	3 Spring pressure ∥ to cyl. %	4 Spring pressure ⊥ to cyl. %	5 Initial fluid pressure %	6 Maximum fluid pressure %	7 Total metal pack. press. ⊥ to cyl.	8 Total max. pack. press. ⊥ to cyl.	9 Width of packing	10 Effective diameter
HYDRAULIC PISTON	1990.	2.8	710.	519.	600.	3820	960.	3500.	.532	2.75
STUFFING BOX	1180.	1.66	625.	457.	600.	3820	880.	3240.	.532	1.38
FLOATING P. - AIR	2735	5.5	494	361.	700.	1150.	870.	1200.	.532	4.
OIL	5470	5.8	995.	727.	600.	1250.	1460.	1640.	.532	4.
INT.										
GREASE	1147.	11.78	125.	125.	700.	1150.	825.	1275.	.532	4.

Fig. 11

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where d = diam. of piston rod or cylinder in inches.
b = width of leather contact of packing
a = total depth of one silver flange of packing cup in inches.
a_1 = total depth of outer silver flange.

From the above equation, we have

$C_1+C_2\ p_{max.} = \pi d[.05\ b+.09(a+\frac{a_1}{2})]\ p_n$ where $p_{max.}$ is the maximum fluid pressure, Further,

$$C_2 p_{max} = \pi\ d\ 0.73[.05b+.09(a+\frac{a_1}{2})]p_{max.}$$ hence

$$C_1 = \pi\ d[.05b+.09(a+\frac{a_1}{2})](p_n - 0.73\ p_{max})$$

$$C_2 = \pi\ d\ 0.73[.05b+.09(a+\frac{a_1}{2})]$$

As a guide for suitable values of a, a_1 and b with given maximum fluid pressures, the following table has been constructed of values used in certain experimental recuperators.

75 m/m Model of 1916 MI.

	a	a_1	b	p_{max} lbs/sq. in.
Recoil piston	0.14"	0.18"	0.19"	5120
Stuffing box	0.14"	0.18"	0.19"	5120
Floating piston	0.09"	0.18"	0.29"	1270

3.3" Model of 1919.

	a	a_1	b	p_{max} lbs/sq. in.
Recoil piston	0.137"		0.233"	5500
Stuffing box	0.137"		0.233"	5500
Floating piston	0.137"		0.194"	1850

4.7" Model of 1906

	a	a_1	b	p_{max} lbs/sq.in.
Recoil piston	0.128"	0.128"	0.335"	3820
Stuffing box	0.128"	0.128"	0.335"	3820
Floating piston	0.128"	0.200"	0.335"	1200

4.7" Model of 1918.

	a	a_1	b	p_{max} lbs/sq.in.
Recoil piston	0.156"	0.218"	0.218"	4500
Stuffing box	0.156"	0.218"	0.218"	4500
Floating piston	0.125"	0.218"	0.35"	2300

The dimensions a, a_1 and especially b increase somewhat with the diameters of the cylinders (that is with the caliber of gun) as well as with the fluid pressure. Let C_1' and C_2' be the packing friction constants for the stuffing box.

C_1'' and C_2'' be the packing friction constants for the recoil piston

then $R_s + R_p = (C_1' + C_1'') + (C_2' + C_2'')p$

$$= C_{o_1} + C_{o_2} p$$

Therefore, the recoil reaction, becomes, for any pressure p_1

$$pA = \frac{(K + W_r \sin\emptyset)l}{l + W\ n\ d_b} - (C_{o_1} + C_{o_2} p)$$

If we assume the maximum recoil pressure p_{max} corresponding to maximum elevation $\emptyset_{max}$ we have

$$C_{o'} + C_{o''} = \pi\ d[.05b + .09(a + \frac{a_1}{2})]p_n$$

where $p_n = k\ p_{max}$, k being obtained from the previous table. Therefore the effective recoil piston area, becomes

$$A = \frac{(K+W_r \sin\phi_{max})l}{(l+2\ n\ d_b)p_{max}} - k\ \pi\ d[.05\ b+.09(a+\frac{a_1}{2})]$$

In general p_{max} = 4500 lbs. per sq.in. but as we shall see in 6, with guns of low elevation and with reasonable horizontal stability, the max. pressure may be necessarily smaller than the packing limit pressure of 4500 lbs. per sq. inch. The assumed max. pressure for calculation of packing friction and in (5) the recuperator reaction is at this stage a question of experience.

The guide friction when the gun is in battery becomes,

$$R_g = \frac{2n\ B_r d_b}{l+2nr}$$

where $B_v = p_a'A-(R_s+R_p)$ i.e. the tension of the rod in battery

r = distance down to mean friction line

l = distance between clip reaction in battery

n = coefficient of friction = 0.15 to 0.2

b = distance from center of gravity of recoiling parts to line of action of B_v

Further the value of R_s+R_p in battery becomes,

$$R_s+R_p=(C_1'+C_1'')+(C_2''+C_2'')p_a'$$
$$=C_{o_1}+C_{o_2}p_a'$$

To compute the drop of pressure across the floating piston friction, we have

$$R_f=+C_1'''+C_2''' \frac{(p_a+p_a')}{2} \quad \text{approx.}$$

while $p_a-p_a' = \dfrac{R_f}{A_a}$

hence

$$p_a = \frac{2C_1'''+(C_1'''+2A_a)p_a'}{2A_a-C_2}$$

$$p_a' = \frac{(2A_a-C_2)p_a-2C_1}{C_2+2A_a}$$

which gives the air pressure in terms of the recuperator pressure (for recoil computations) or

the recuperator pressure in terms of the air pressure in terms of the air pressure (for counter recoil computations).

In a preliminary layout, we are not greatly in error in assuming either

$$\left.\begin{aligned} p_a - p_a' &= \frac{C_1''' + C_2''' p_a}{A_a} \\ \text{or} \qquad & \\ p_a - p_a' &= \frac{C_1''' + C_2''' p_a'}{A_a} \end{aligned}\right\} \text{approx. drop due to floating piston.}$$

(5) <u>The recuperator reaction required to hold the gun in battery</u>,

To ensure a sufficient margin for the holding of the gun in battery and overcoming the friction in battery, an excess of 20% to 30% is used over the minimum recoil reaction, hence

$K_v = p_{ai} A = k(W_r \sin \phi_{max} + R_g + R_s + R_p)$ (1)

now

$$R_g = \frac{2nK_v d_b}{1+2\,nr}\ ;\ k = 1.2 \text{ to } 1.3,\ n = 0.15$$

and $R_s + R_p = C_1 + C_2 \dfrac{K_v}{A}$

where $C_1 = 0.15\pi(d_r+d)(d_r+d)[.05b + .09(a + \frac{a_1}{2})]p_{max}$

$$C_2 = 0.73\pi(d_r+d)[.05b + .09(a + \frac{a_1}{2})]$$

and for a trial value, p_{max} = 4500 lbs. per sq.in.

$$A = \frac{P_s}{4500}$$

d = diam. of recoil piston

d_r = diam. of piston rod

d_b = distance down from center of gravity of recoiling parts to line of action of K_v

Substituting in (1), we have,

$$K_v = kW_r \sin\phi_{max} + \frac{2nk\ K_v d_b}{1+2\,nr} + C_1 + C_2\frac{K_v}{A}$$

$$\text{hence } K_v = \frac{kW_r smn\emptyset_{max} + C_1}{1 - \frac{2nk\ d_b}{1+2nr} - \frac{C_2}{A}} \qquad (2)$$

(6) Limitations of Recuperator Dimensions:

In the design of a recuperator layout, we must consider the proper ratio of area of recuperator cylinder to effective area of piston, the limitations of areas based on this ratio and on the maximum allowable packing pressure in the recoil cylinder as well as on the difference between the horizontal pull and recuperator reaction at maximum elevation. If now,

w_h = max. area of orifice at horizontal recoil in sq. in.

A = effective area of recoil piston in sq.in.

A_a = area of recuperator cylinder or floating piston in sq. in.

V = max. recoil velocity in ft. per sec.

P_h = horizontal pull in lbs.

p_s = pull at maximum elevation in lbs.

K_v = recuperator reaction at maximum elevation in lbs.

$K = \frac{1}{0.773}$ = reciprocal of orifice contraction constant.

w_c = channel or port area at cross section beyond regulator valve in recuperator cylinder in sq. in.

W_s = area of channel section through diameter of regulator valve in sq. in.

a = entrance area to regulator valve.

d = diam. of regulator valve.

r = ratio of $\frac{\text{Floating Piston Area}}{\text{Effective Area of Recoil Piston}}$

Now the maximum throttling area w_h is the limiting throttling area, since all constant port

or channel areas in the recuperator should bear given ratios with respect to this area and must be sufficiently large as compared with w_h so that there is no appreciable throttling through them, or loss of head due to friction or acceleration.

The following table gives ratios of channel or port areas in the recuperator with respect to the maximum throttling area w_h and the area of the recuperator cylinder or floating piston.

Model	w_h	w_c	$C_1 = \frac{w_c}{w_h}$	$C_2 = \frac{w_c}{A_a}$	$C_3 = \frac{w_s}{w_c}$
4.7"-M.1906	0.380	1.61	4.235	0.126	0.608
4.7"M.1918	0.854	3.67	4.300	0.207	0.640
3.3"-M.1918	0.267	1.11	4.160	0.186	0.470
75mm-M.1916	0.175	0.78	4.470	0.120	0.371
			4.300	0.160	

From the above table, the following design constants will be used based on satisfactory layouts:

$$C_1 = \frac{w_c}{w_h} = 4.3 \qquad C_2 = \frac{w_c}{A_a} = 0.16$$

Now

$$w_h = \frac{w_c}{C_1} = \frac{C_2}{C_1} A_a = r \frac{C_2}{C_1} A = .0373 \ r \ A$$

Considering the throttling through the regulator orifice, we have

$$w_h^2 = \frac{K^2 A^3 V^2}{175(p_h - K_v)} \qquad (1)$$

and substituting for $w_h = .0373 \ r \ A$, we find

$$r^2 = \frac{4.11 \ AV^2}{p_h - K_v} \text{ that is } r = 2.62 \ V \sqrt{\frac{A}{p_h - K_v}} \qquad (2)$$

Hence, the ratio of the recuperator area to effective area of recoil piston varies as the square root of the effective area of the recoil piston,

and for minimum recuperator area we must have minimum effective area of recoil piston.

That is
$$A_a = \frac{2.03\ V}{\sqrt{p_h - K_v}} A^{\frac{3}{2}} \tag{3}$$

Hence the recuperator area always varies as the $\frac{3}{2}$ power of the effective area of the recoil piston.

Now for minimum weight of the recuperator forging it is important that the cylinder areas be made as small as possible. Therefore the recoil piston area in general is limited by the maximum allowable pressure in the recoil cylinder consistent with the packing pressure limitations.

If the packing limiting pressure is taken at 4500 lbs. per sq. in., then
$$A = \frac{p_s}{4500}$$

Substituting for A, we have
$$r = .039V \sqrt{\frac{p_s}{p_h - K_v}} \tag{4}$$

Now r is limited by the following considerations:

(a) In an ordinary layout, the initial volume of the air, may be represented as the sum of air column in the recuperator cylinder plus the air column in the air cylinder, that is

$$V_o = A_a(k_1 + k_2)b \tag{5}$$

where k_1 = $\frac{\text{initial air column length in recuperator}}{\text{length of recoil}}$

and k_2 = $\frac{\text{air column length in air cylinder}}{\text{length of recoil}}$

k_1 may be obtained from the fol-

lowing table:

	b	b'_{max}	$\frac{b}{b'_{max}}$	$k_1 = \frac{b'_{max}}{b}$
4.7"-M.1918	40	28.27	1.415	0.707
4.7"-M.1906	70	56.50	1.240	0.807
75mm-M.1916	46	43.08	1.070	0.937
3.3"-M.1918	60	47.67	1.260	0.795
			1.246	0.811

Therefore we may assume K_1=0.8 and k_2 will be assumed k_2= 0.7, hence $k_1 + k_2 = 1.5$

The initial volume as shown in (7) may be represented in the recoil piston displacement and the ratio of the final and initial air pressures.

If $m = \frac{p_{af}}{p_{ao}}$ and k = (1 to 1.41 use 1.3)

then

$$V_o = Ab \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} \qquad (6)$$

when

$$m = 1.5 \quad \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = 3.73$$

$$m = 2.0 \quad \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = 2.42$$

$$m = 3.0 \quad \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = 1.75$$

Using a ratio of m = 1.5, and combining Eq. 4 and 5, we have 3.73 Ab=1.5A_ab hence

$$\frac{A_a}{A} = r = 2.5$$

If a lower value of $m = \frac{p_{af}}{p_{ai}}$ is used, then $r > 2.5$ where as with a higher ratio, $r < 2.5$

If we continue increasing m for the minimum permissible value of r we soon arrive at the limitation where the maximum possible recoil of the floating piston limits the ratio r, since $A_{a_{min}} b'_{max} = A\ b$. Then

$$r_{min} = \frac{A_{a_{min}}}{A} = \frac{b}{b'_{max}} = 1.25$$

On the other hand to obtain a value of $r_{min} = 1.25$, would require a high value of m, approx. = 3.0, and the temperature rise of the air would be excessive and very injurious to the floating piston packing.

Further, at horizontal recoil, where a stability slope for the total resistance to recoil is highly desirable, we have the minimum throttling drop due the small value of the pressure in the recoil cylinder at horizontal recoil. The peak effect of the throttling drop is thereby reduced, and since the pressure in the recoil cylinder is the throttling drop plus the recuperator pressure, a large increase in the final air pressure over the initial will overbalance the decrease in the throttling drop towards the end of recoil. Therefore, a large value of $m = \frac{p_{af}}{p_{ai}}$ must result in a rise of the total resistance to recoil towards the end of recoil which is entirely inconsistent with the requirements for horizontal stability

If, therefore, at horizontal recoil, we limit the ratio $m = \frac{p_{af}}{p_{ai}}$ to that value which gives us a constant resistance throughout the recoil, we have

$$p_{ai} + \frac{K^2 A^2 V^2}{175\ w_h^2} = p_{af} \qquad (7)$$

neglecting the slight throttling drop at the end

of horizontal recoil. Substituting for w_h

$$w_h = .0373\ r\ A \qquad K = \frac{1}{0.773}$$

and

$$p_{af} = m\ p_{ai}$$

we have,

$$0.145(m-1)r^2 = 6.87\ \frac{V^2}{p_{ai}} \qquad (8)$$

From the following table, knowing V and p_{ai} and from the above equation $(m-1)r^2$, we can readily obtain r or corresponding value of m.

		3 CYLINDERS			2 CYLINDERS	
r	r^2	$\frac{r^2}{6.88}$	(m-1)	$\frac{V^2}{p_{ai}}$	m-1	$\frac{V^2}{p_{ai}}$
1.5	2.75	.3273	1.147	.3787	1.547	.5063
1.6	2.56	.3724	1.015	.3780	1.345	.5001
1.7	2.89	.4204	.910	.3824	1.188	.4994
1.8	3.24	.4714	.824	.3884	1.064	.5018
1.9	3.61	.5252	.752	.3950	.960	.5041
2.0	4.00	.5819	.694	.4038	.880	.5120
2.1	4.41	.6416	.645	.4138	.810	.5197
2.2	4.84	.7041	.599	.4218	.750	.5280
2.3	5.29	.7696	.560	.4310	.700	.5287
2.4	5.76	.8380	.527	.4416	.653	.5472
2.5	6.25	.9093	.497	.4519	.612	.5565

As a check, we may obtain the ratio m consistent with horizontal stability from another point of view.

At the end of recoil, we have, neglecting a small throttling drop, $p_{af}A = p_h$ whereas the initial reaction, we have, $p_{ai}A = K_v$ hence

$m = \frac{p_{af}}{p_{ai}} = \frac{p_h}{K_v}$ which gives immediately the maximum ratio for m or the minimum ratio for r, consistent with the require-

ments for horizontal stability.

Again the maximum value of $m = \frac{p_{af}}{p_{ai}}$ depends further upon the heating limitation (i. e. the permissible rise of temperature of the air in the recuperator forging). Since the question of heating depends upon the various factors, as radiation through the cylinder walls, the frequency of firing, etc. we must assume by experience for the given type of gun, the maximum allowable temperature use during the compression of the recuperator air. Thus, using a ratio $m = 2$, we have

$$\frac{T}{T_m} \left(\frac{p_{af}}{p_{ai}}\right)^{\frac{k-1}{k}} = 2^{0.23}$$

Assuming a mean temperature at 25° Centigrade, we find

$T = 298 \times 2^{0.23} = 349°, (k=1.3)$ therefore, the rise of temperature during a recoil stroke becomes, $T - T_m = 51°C$ or $92°F$. This rise of temperature is not excessive. If the rapidity of fire is great the mean temperature will rise. The quantity of heat generated is the work done on the air divided by the mechanical equivalent of heat. Now if the mean temperature has risen to its constant maximum value, then the heat generated during the firing stroke, must be dissipated by radiation through the cylinder walls during the period of loading, the temperature gradient varying, decreasing during the process of radiation through the cylinder walls.

Thus we see we have two aspects for the maximum ratio of m and the corresponding minimum value of r:

(1) To maintain at best, a constant resistance to recoil throughout the recoil at horizontal elevation, rather than a rise in the overturning force at the end of recoil which would be entirely opposite

to the requirements for the proper stability slope at horizontal recoil. This limitation for r is of special importance when $p_h - K_v$ is small, we have r_{min} determined by the equation,

$$(m-1)r^2 = 6.87 \frac{V^2}{p_{ai}}$$ where we may obtain m d r for various values of $(m-1)r^2$ from the previous table.

(2) On the other hand, when $p_h - K_v$ is large as with guns of low elevation and where we have a good margin of stability, the peak effect in the throttling becomes larger or the ratio

$\frac{p_h}{K_v}$ is larger, therefore, a higher value of m can be used. In such a case we become limited by the rise in temperature of the air during the firing.

(a) When (2) becomes the limitation we may use a higher value of m and therefore a lower value of r.

(b) The expansion of the oil varies considerably with temperature, different viscosities, the rapidity or frequency and continuity of fire, etc. Therefore the floating piston will have various initial displacements, resulting in different initial air pressures and most important in unsatisfactory functioning of the buffer on counter recoil, the buffer action starting at various different points

on counter recoil, unless the ratio is made sufficiently large, since with a large bulk of oil, temperature changes and consequent expansion is less.

(c) Due to the wear of the packing on the floating piston it has been customary to limit the maximum velocity of the floating piston to not over 12 ft. per sec. though it is believed that the packing may be designed to withstand a surface velocity of 20 ft. per sec.

Therefore, in conclusion, from a consideration of (a), (b) and (c) r_{min} ordinarily should be limited to: $r_{min}=1.3$ to 2.0

In difficult designs, however, the proper minimum value of r should be determined more from a consideration of aspects (1) and (2) in (a) rather than (b) and (c).

Limitations for the maximum value of r:

For very large values of r, slight changes in the quantity of oil due to leakage, will have a marked effect on the relative initial positions of the floating piston and recoil piston. Further it would be difficult to gauge slight variations in the quantity of oil due to the relatively small motion of the floating piston which moves the gauge rod.

But most important from a point of economy in the weight of recuperator forgings and very often in a satisfactory layout, a too large value of r becomes prohibitive.

We limit the max. value of r to, $r_{max}=3.5$ Hence the design limitations for r becomes for an ordinary layout with 3 cylinders $r \Rightarrow 1.8$ and $r \Rightarrow 3.5$. When only two cylinders are used we have

the following considerations:

(1) If the length of the recuperator air cylinder has the same total length as the recoil cylinder, we have $k_1=0.8$ and $k_2=0$, hence

$$V_0 = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} Ab = 0.8\, A_a b \text{ hence } r = \frac{1}{0.8} \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}$$

If $m = 2$ a heating limit on the ratio of compression, we have

$$\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = 2.42 \quad \text{for } k = 1.3$$

and

$$r = \frac{2.42}{0.8} = 3.025$$

On the other hand (in consideration of a constant recoil reaction for stability at horizontal elevation) if we decrease m to $m = 1.3$, we have

$$r = \frac{3.73}{0.8} = 4.67$$ which gives a very bulky recuperator with too small relative displacement of the floating piston as compared with the recoil displacement.

Therefore, if only two cylinders are to be used and of the same length we are peculiarly limited by bulk and a small movement of the floating piston as compared with the recoil piston, on the one hand, while with a decrease in the ration r, the final air pressure is increased and overbalances the peak of the throttling plus the initial air, thereby giving a rise in the recoil reaction towards the end of recoil at horizontal elevation.

Therefore as a compromise, if two cylinders must be used of the same length we may take $r = 3.5$ and $m = 1.8$ and r is to be considered

constant for this combination. Thus we see by the use of two cylinders only and of the same length, the ratio cylinders become excessive if moderate compression ratios are maintained, whereas for moderate cylinder ratios we must maintain high compression ratios which cause undue heating and a rise of the recoil reaction at horizontal elevation.

A more satisfactory combination for two cylinders only, is by use of longer recuperator cylinder than recoil cylinder. This is usually feasible especially for guns, where the tube is long. A satisfactory approach to three cylinders may be had by the use of a sufficient overhang of the recuperator cylinder.

The air column in the overhang can be reasonably assumed at 0.5 the horizontal recoil. Therefore in the equation $V_0 = A_a(k_1 + k_2)b$. We may assume as before $k_1 = 0.8$

$$k_2 = 0.5$$

$$= 1.3$$

hence $k_1 + k_2$

Hence with a ratio of pressure expansion in the recuperator $m = 1.5$, we have $3.73\ Ab = 1.3 A_a b$ hence $\frac{A_a}{A} = r = 2.87$. By increasing the ratio of expansion we may limit the minimum r to: $r_{min} = 2.5$. From the above it is evident that though it is not feasible to use a recuperator air cylinder as long as the combined length of a separate recuperator and air cylinder, on the other hand the minimum ratio of r is greater than with an ordinary layout and with only two cylinders, the total weight of the forging may exceed that of these cylinders. Hence if two cylinders are to be tried in place of three, preliminary separate layouts for the two combinations should be worked out in view of minimum weight and

satisfactory layout before either plan is adopted. Therefore, in a design layout we start with

$$r = .0302\,V\sqrt{\frac{p_s}{p_h - k_v}}$$ which determines the ratio $\frac{A_a}{A}$ providing it falls within the limits r_{min} and r_{max}. Hence the recoil area, becomes,

$$A = \frac{p_s}{4500}$$

<u>Anti-aircraft Guns:</u>

Anti-aircraft guns are the most difficult to design without having excessive bulky recuperator forgings, $p_h - K_v$ becomes small, since K_v is larger to hold the gun in battery at maximum elevation and p_h is small to satisfy stability at zero elevation, further p_s is large, therefore <u>r is in general large, usually 3 or above.</u>

If r exceeds 3.5 using 3 cylinders, we must increase $p_h - K_v$ either by reducing K_v and then allowing the gun to return slower into battery at maximum elevation and with a smaller margin of excess battery reaction of the recuperator at maximum elevation or preferably increase p_h at the slight sacrifice of stability at zero elevation, - in this case we have

$$p_h = K_v + .000912V^2\,\frac{p_s}{3.5^2}$$

It will be rarely found, however that r exceeds 3.5.

<u>Howitzers:</u>

With Howitzers, we again meet the condition of a large K_v but since horizontal stability is not a consideration, p_h may be relatively large, and therefore $p_h - K_v$ still may be maintained large.

r is generally medium or small at the sacrifice of horizontal stability.

Guns:

With guns, r is in general small, usually from 2 to 3. If r decreases below, 2.0 to 1.8 using 3 cylinders, or 2.5 using 2 cylinders with a longer recuperator cylinder than the recoil, the effective area of the recoil piston is now determined by the formula:

$A = 0.2425 \frac{r^2}{V^2}(p_h - K_v)$ and the maximum pressure in the recoil cylinder, becomes, $p_{max} = \frac{p_s}{A}$.

Howitzers and Guns on Same Carriage:

When Howitzers and guns are adopted for the same carriage, K_v must be large for the howitzer and p_h small for the horizontal stability of the gun. Therefore $p_h - K_v$ is small, p_s large and we meet exactly similar conditions as with anti-aircraft guns.

Hence, for this combination, r is in general large, usually 3 or above. If r exceeds 3.5, p_h must be increased or K_v decreased, these values being connected by the relation

$$p_h = K_v + .000912V^2 \frac{p_s}{(3.5)^2}$$

(7) Calculation for air pressures and volumes:

From (5) we have for the initial recuperator reaction

$$K_v = \frac{k\, W_r \sin \phi_{max} + C_1}{1 - \frac{2nkK_v d_b}{1+2\,nr} - \frac{C_2}{A}}$$

where k = 1.2 to 1.3 p_{max} may be assumed at 4500 lbs. per sq. in.

$$C_1 = 0.15\pi(d_r+d)[.05b+.09(a+\frac{a_1}{2})]p_{max}$$

$$C_2 = 0.73\pi(d_r+d)[.05b+.09(a+\frac{a_1}{2})]$$

$A = \frac{P_s}{4500}$ = effective area of recoil piston

a, a_1 and b are contact lengths of the packing in inches.

d_b= distance down from center of gravity of recoiling parts to line of action of K_v in inches.

l = distance between clip reactions in inches

n = 0.15

r = distance down from center of gravity of recoiling parts to mean friction line in inches.

For guns of low elevation and reasonable stability where r falls below, r_{min} we have

$$A = 0.2425\,\frac{r^2}{V^2}(p_h - K_v) \quad \text{and } p_{max} = \frac{P_s}{A}$$

$$ld = \sqrt{0.7854A+d_r^2}$$

$A = 0.2425\,\frac{r^2}{V^2}(p_h-K_v)$ and the maximum pressure in the recoil cylinder is thereby reduced to:

$p_{max} = \frac{P_s}{A}$ and the area of the recuperator cylinder becomes, $A_a=r_{min}A$. With an ordinary layout using 3 cylinders, r_{min}=2 and with 2 cylinders and an overhang, r_{min}=2.5. Knowing A_a, we have for the inside diameter of the recuperator,

$$D_a = \sqrt{\frac{A_a}{0.7854}}$$

To determine the diameter of recoil cylinder, we must know the area of the piston rod, A_r. Then

$D = \sqrt{\frac{A+A_r}{0.7854}}$ where A_r is determined as follows:

If B is the total hydraulic braking including the joint frictions at the stuffing box and recoil piston, we have $B+R_g=K+W_r \sin\emptyset$ where

$$R_g = \frac{2nBd_b}{1+2nr}$$

$$B\left(1+\frac{2nd_b}{1+2nr}\right)=K+W_r \sin\emptyset$$

$$\therefore\quad B = \frac{K+W_r \sin\emptyset}{1+\dfrac{2nd_b}{1-2nr}}$$

The maximum stress in the rod is at a section at the lug, at maximum acceleration of the recoiling parts and at maximum elevation. If

T_L = the tension in the rod at the lug

$$T_L=B+\frac{W_e}{W_r}(p_b-B)$$

$$= \frac{K+W_r \sin\emptyset_{max}}{1+\dfrac{2nd_b}{1+2nr}}\left(1-\frac{W_e}{W_r}\right)+p_b\frac{W_e}{W_r}$$

where W_e = weight of rod and recoil piston

p_b = total maximum powder pressure on base of breech.

If f_{max} is the maximum allowable working fibre stress in the rod, we have

$$A_r = \frac{T_L}{f_{max}}$$

and $A_a = rA$. Now with guns of low elevation, K_v is small and p_h relatively large, hence the difference p_h-K_v is large and p_s is small. Therefore r becomes small.

In (7) tables and a chart has been constructed giving values of m and r for different air column lengths, the air columns being expressed

in terms of a ratio of the length of air column divided by the length of recoil, that is

$$j = \frac{l}{b} \quad \text{where } l = \text{length of air column,} \quad b = \text{length of recoil.}$$

A maximum limitation of m, based on a moderate temperature rise of the air during the recoil and a constant reaction throughout the recoil at horizontal elevation (i. e. no increase of the recoil reaction at the end of recoil,) will be taken at 1.8. Evidently for different air column lengths, we will have different minimum values of r, corresponding to a maximum value of m = 1.8.

The longer the air column lengths, the lower the ratio r.

If r falls below the minimum allowable value of r(i.e.the r corresponding to m = 1.8) for a given air column length, r becomes a constant, and the area of the recoil piston must be increased according to the formula:- then

$$p'_{ai} = \frac{K_v}{A} \text{ lbs. per sq.in. initial recuperator pressure intensity.}$$

Hence the initial air pressure becomes,

$$p_{ai} = p'_{ai} + \frac{C_1''' + C_2''' p'_{ai}}{A_a} \quad \text{and } p_{af} = m\, p_{ai}$$

$$C_1''' = 1.12\pi d_o[.05b + .09(a + \frac{a_1}{2})]p_{af}$$

$$C_2''' = 1.46\pi d_a[.05b + .09(a + \frac{a_1}{2})]$$

Next to determine the proper ratio of $m = \dfrac{p_{af}}{p_{ai}}$ we must consider the following:

The initial volume V_o is expressed by either member of the equation,

$$Ab \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = A_a l \quad \text{where } k = 1.3,$$

$m = \frac{p_{af}}{p_{ai}}$

b = length of recoil

l = length of air column, reduced to an equivalent cross section A_a

Since $r = \frac{A_a}{A}$, we have

$$\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = r \frac{l}{b} = rj, \text{ where } j = \frac{l}{b}$$

The following tables give the relation of m and r for various values of j.

$m = \frac{p_{ax}}{p_{ai}}$ $\qquad k = 1.3 \qquad \frac{1}{k} = .77$

$$\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = 0.8\ r$$

m	r
1.2	9.575
1.4	5.476
1.6	4.116
1.8	3.442
2.0	3.002
2.2	2.746
2.4	2.546
2.6	2.401
2.8	2.282
3.0	2.188
3.2	2.111
3.4	2.047
3.5	2.018

Values of m and r for j = 1.1

r	1.1r	1.1r -1	$\frac{1.1r}{1.1r-1}$	$\log\frac{1.1r}{1.1r-1}$	$K\log\frac{1.1r}{1.1r-1}$	m
1.8	1.98	.98	2.020	.30535	.39696	2.494
2.	2.2	1.2	1.833	.26316	.34211	2.198
2.2	2.42	1.42	1.704	.23147	.30091	1.999
2.4	2.64	1.64	1.61	.20603	.26888	1.857
2.6	2.86	1.86	1.54	.18686	.24291	1.750
2.8	3.08	2.08	1.48	.17056	.22173	1.666
3.2	3.52	2.52	1.4	.14520	.18876	1.544
3.4	3.74	2.74	1.36	.13513	.17567	1.499
3.5	3.85	2.85	1.35	.13066	.16986	1.479

Values of m and r for j = 1.3

r	1.3r	1.3r-1	$\frac{1.3r}{1.3r-1}$	$\log\frac{1.3r}{1.3r-1}$	$K\log\frac{1.3r}{1.3r-1}$	m
1.8	2.34	1.34	1.746	.24204	.31465	2.064
2.0	2.60	1.60	1.625	.21085	.27411	1.880
2.2	2.86	1.86	1.538	.18696	.24305	1.850
2.4	3.12	2.12	1.472	.16791	.21828	1.653
2.6	3.38	2.38	1.420	.15229	.19798	1.577
2.8	3.64	2.64	1.379	.13956	.18143	1.519
3.0	3.90	2.90	1.345	.12972	.16734	1.470
3.2	4.16	3.16	1.316	.11926	.15504	1.429
3.4	4.42	3.42	1.292	.11126	.14464	1.395
3.5	4.55	3.55	1.282	.10789	.14026	1.381

Values of m and r for j = 1.5

r	1.5r	1.5r-1	$\frac{1.5r}{1.5r-1}$	$\log\frac{1.5r}{1.5r-1}$	$K\log\frac{1.5r}{1.5r-1}$	m
1.8	2.70	1.70	1.588	.20085	.26111	1.824
2.0	3.00	2.00	1.500	.17609	.22892	1.694
2.2	3.30	2.30	1.435	.15685	.20391	1.599
2.4	3.60	2.60	1.385	.14145	.18389	1.527
2.6	3.90	1.345	1.345	.12872	.16734	1.470
2.8	4.20	3.20	1.313	.11826	.15374	1.425
3.0	4.50	3.50	1.286	.10924	.14201	1.387
3.2	4.80	3.80	1.263	.10140	.13182	1.355
3.4	5.10	4.10	1.244	.09482	.12327	1.328
3.5	5.25	4.25	1.235	.09167	.11917	1.316

Values of m and r for j = 1.7

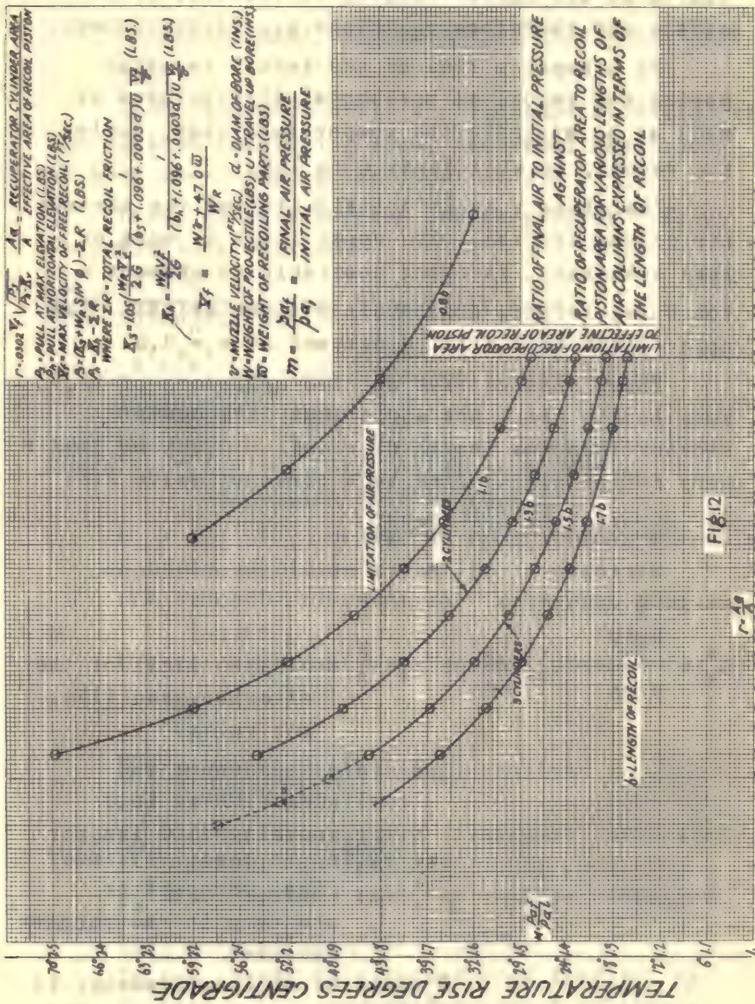
r	1.7r	1.7r-1	$\frac{1.7r}{1.7r-1}$	$\log\frac{1.7r}{1.7r-1}$	$K\log\frac{1.7r}{1.7r-1}$	m
1.8	3.06	2.06	1.485	.17184	.22339	1.672
2.	3.4	2.4	1.4166	.15124	.196612	1.572
2.2	3.74	2.74	1.3649	.13510	.17563	1.498
2.4	4.08	3.08	1.3246	.122084	.15870	1.441
2.6	4.42	3.42	1.2923	.11139	.144807	1.395
2.8	4.76	3.76	1.2659	.10205	.13266	1.357
3.2	5.44	4.44	1.22522	.08820	.11466	1.302
3.4	5.78	4.78	1.2092	.08249	.107237	1.280
3.5	5.95	4.95	1.2020	.07997	.103961	1.2704

The values in these tables have been plotted, with the chart, fig.(12).

The ordinates in this chart give values of m corresponding to values of r in the abscissa. Each curve represents the relation of m and r for a satisfactory layout, with a given air column, the air column length being expressed as a ratio with respect to the length of recoil.

The air column lengths are taken at 0.8b, 1.1b, 1.3b, 1.5b and 1.7b where b = length of recoil.

Values of m and r for air column lengths intermediate between these values, may be easily ob-


RATIO OF FINAL AIR TO INITIAL PRESSURE
AGAINST
RATIO OF RECUPERATOR AREA TO RECOIL PISTON AREA FOR VARIOUS LENGTHS OF AIR COLUMNS EXPRESSED IN TERMS OF THE LENGTH OF RECOIL
LIMITATION OF RECUPERATOR AREA TO EFFECTIVE AREA OF RECOIL PISTON
LIMITATION OF AIR PRESSURE
2 CYLINDERS
3 CYLINDERS
b = LENGTH OF RECOIL
0.8b
1.1b
1.3b
1.5b
1.7b
FIG. 12
TEMPERATURE RISE DEGREES CENTIGRADE

tained by interpolation.

Solving for the proper value of r, and assuming an air column length, we immediately obtain m and therefore p_{af} since p_{ai} is now known.

To prevent a rise of the recoil reaction during the recoil at horizontal displacement as well as to minimize the temperature rise, during the recoil, we will limit m to 1.8.

Therefore r is definitely limited for various air column lengths. Its upper limit is more or less arbitrary, it being desirable to prevent a too bulky forging and obtain minimum weight. The upper limit of r will be assumed at r = 3.5.

Thus, when we have but two cylinders of the same length where the air column is somewhat shorter than the length of recoil = 0.8b, we find r very definitely limited to a constant value 3.5.

(8) Strength of Cylinders:

Strength of cylinders should be based on maximum pressures. As shown in Chapter IV.

$$D_1^2 = \frac{p_t+p}{p_t-p} D_o^2$$

where $p_t = \frac{3}{8}$ x elastic limit of the material used (lbs. per sq.in)

p= maximum pressure in cylinder (lbs/sq in.)

= p_{max} usually 4500 lbs.per sq.in. in recoil cylinder

= p_{af} final recuperator pressure in recuperator cylinder

D_1=outside minimum diameter in inches.

D_o= inside diameter which is given in inches,

Thickness between cylinders should be

$$w = \frac{pd+p_{af}d_a}{1.8p_t}$$

where w = minimum allowable thick-

ness between cylinders (inches)

d = diam. of recoil cylinder in inches

d_a= diam. of air cylinder in inches

(9) Calculation of maximum and minimum throttling areas.

Since all port areas are constant multiples of the maximum throttling area, the exact deviation of this area is of prime importance.

From (6) we find, for the throttling through the regulator orifice,

$$w_h^2 = \frac{K^2 A^3 V^2}{175(p_h - K_v)} \quad \text{(max. throttling area)}$$

$$w_s^2 = \frac{K^2 A^3 V^2}{175(p_s - K_v)} \quad \text{(min. throttling area)}$$

where w_h = the max. throttling area usually at horizontal recoil (sq.in)

w_s = the min. throttling area at max. elevation (sq.in)

A = the effective area of the recoil piston (sq.in)

V = the max. restrained velocity = $0.9\ V_f$ approx.

$K = \frac{1}{0.773}$ the throttling constant

p_h = the minimum pull, usually at horizontal recoil (in lbs)

p_s = the maximum pull, at maximum elevation.

(10) Layout of Recuperator Forging:

In the layout of a recuperator forging, we must decide as to the arrangements of cylinders. Depending upon the value of "r"= $\frac{A_a}{A}$, we have three possible arrangements:

(1) Three cylinders, the two recuperator cylinders symmetrical with the recoil cylinder.

(2) Two cylinders, the recuperator cylinder having an overhang with respect to the recoil cylinder.

(3) Two cylinders, the recoil and recuperator cylinder being of the same overall length.

From the chart giving values of $m = \frac{p_{af}}{p_{ai}}$ for values of $r = \frac{A_a}{A}$ for different air column lengths, we see that the values of "r" for the curves giving lengths of various air columns are limited on the one hand by the maximum value of $m = 1.8$ consistent with stability at zero degrees elevation and normal use of temperature in the recuperator, thus giving the various minimum limiting values of "r" for various air columns, while on the other hand the maximum allowable value of $r = 3.5$ depends upon proper counter recoil functioning and layout considerations. If now r is obtained by the formula,

$$r = .0309 \quad V\sqrt{\frac{p_s}{p_h - K_v}}$$

The possible lengths of air columns consistent with the limitations are determined.

If r from the above equation is low, then we must have longer air column lengths and therefore usually three cylinders, whereas if r is large, short air column lengths are possible and two cylinders may be used. With arrangement (3), r becomes practically constant and unless r from the above equation falls in the neighborhood of 3.5, it will be necessary to increase the effective area of the recoil cylinder with a consequent larger recoil cylinder.

Having decided upon the arrangement and number of cylinders from a consideration of the proper air column consistent with "r" we have now to obtain the exterior dimensions of the forging.

Exterior Dimensions:

The primary exterior dimensions of importance are:

(1) A cross section of the recuperator, giving the location of the piston rod with respect to the center line of the bore, the axis of the several cylinders, and the position of the guides, thus determining the external contour of the cross section of the forging.

(2) A longitudinal section of the recuperator, giving the overall length, location of the trunnions, elevating arc, etc.

In a satisfactory exterior layout, the following points must be observed:

(a) The center of gravity of the recoil parts should be made in a vertical plane through the axis of the bore and at a minimum perpendicular distance below the axis of the bore consistent with a satisfactory layout.

(b) The center line of the entrance channel to the regulator valve, (that is for the passageway between the recoil and recuperator cylinders) should pass through the center of the recuperator cylinder. Preferably the center line of the entrance channel should be in a horizontal

plane.

(c) If d_a' is the diameter of the connecting channel cross section, and D the diameter of the recoil cylinder, the distance between the center line of the recuperator cylinder and recoil cylinder must not exceed $D = \frac{d_a}{2}$ inches. To meet condition (a) the recoil axis is usually nearer to the axis of the bore.

(d) To overall lengths of the recuperator forging may be estimated roughly from the following table:

	l	b	$\frac{l}{b}$
4.7"-M.1906	94"	70"	1.34
4.7"-M.1918	68.75"	40"	1.72
3.3"-M.1918	86"	60"	1.43
75 m/m-M.1916	72.83"	46"	1.58
		4	6.07
			1.52

Therefore ordinarily the total length of recuperator forging will be taken at 1.5 the length of max. recoil. It should be shorter if practicable.

(e) Without a balancing gear, for guns of moderate elevation, the trunnions should be located in the horizontal direction at the center of gravity of the tipping parts plus one-half weight of projectile and charge

when the gun is in battery. More or less error in the location of the trunnions as respects the center of gravity of the tipping parts in the vertical plane perpendicular to the axis of the bore will not effect the balance, unless the angle of elevation is considerable, and the center of gravity of the tipping parts is considerably above or below the trunnions. Therefore, in order to prevent a reversal of the reaction on the elevating arc and pinion during recoil and counter recoil, it is highly desirable with guns of moderate elevation to locate the trunnions on or below the center of gravity of the recoiling parts which are usually below the axis of the bore.

When a balancing gear is introduced, as is sometimes necessary when the gun fires at high elevation, the trunnions are placed axially or in a longitudinal direction, farther to the rear in order to have as long a recoil as possible at max. elevation. Further with a proper design of the balancing gear location of the trunnions with respect to the center of gravity of the tipping parts in a direction perpendicular to the axis of the bore is no longer so restricted except that in order to avoid reversal of stresses on the elevating arc it is desirable to locate the trunnions slightly below the center of gravity of the recoiling parts, but the distance must be quite small or the arc reaction will become large.

In some designs it may be necessary to locate the center of gravity of the recoiling parts above the bore, and the powder pressure couple will then be in the opposite direction.

If $P_b e$ is the powder pressure couple, and K the resistance to recoil and S the distance down from the axis of the bore to the center of the trunnions, in order that there be no reversal of stress on the elevating arc, we must have

$$K(S+e) = > P_b e \quad \text{hence } S = > \frac{(P_b - K)e}{K}$$

In determining the final values of e and S, the weight components, out of battery and conditions existing in counter recoil must be considered.

(f) With guns above 155 m/m, two separate recoil systems symmetrically placed above and below the gun should be used. The gun should recoil in a sleeve and the trunnions should be located slightly below the axis of the bore.

<u>Interior Dimensions:</u>

The primary interior dimensions of importance, are:

(1) The port area or channel leading from the regulator towards the floating piston in the recuperator cylinder should bear a constant ratio to the <u>maximum</u> opening of the valve which occurs for <u>minimum</u> pull, usually at horizontal elevation.

If w_c = the constant channel area from the regulator valve

w_h = the max. recoil orifice.

Then $w_c = 4.3\ w_h$

(2) The area of the channel or port connecting the recoil and recuperator

cylinder, w_a should have the following relation with respect to w_h, that is, w_a = 3.5 to 4.3 w_h

(3) The entrance channel to the regulator valve from the recoil cylinder a, which is also the area at the base of the regulator valve, should be:

$a = k\,w_h$ where the limits of h are 2.3 to 3.5

If we pass a cross section of the recuperator through the center of the regulator valve, the channel area on either side of the valve, that is

w_c -(the vertical section through the axis of the valve normal to the recuperator axis enclosed within the area w_c)= w_c'

and

$$w_c' \Rightarrow \frac{w_c}{2}$$

If h represents the depth of the section w_c and d_a the diameter of the regulator valve at its base, we have roughly,

$w_c - d_a h_c \geqq 0.5\, w_c = 0.55\, w_c$ approx. Hence

$$d_a = 0.45 \frac{w_c}{h_c} = 1.935 \frac{w_h}{h_c} \quad \text{and } a = 2.94 \frac{w_h^2}{h_c^2}$$

Now in a suitable layout $h_c = 0.2\, D_a$ where D_a = the diameter of the recuperator cylinder, hence

$$a = 73.5 \frac{w_h^2}{D_a^2}$$

(4) The length of the buffer chamber is based on a consideration of counter recoil.

If d_b = the recoil length during the buffer action

d_b' = the length of the buffer

From a consideration of counter recoil,

$$d_b(0.15\,\bar{W}_r+R_p)=\frac{\frac{1}{2}m_rV^2}{0.8}$$ If V = 3.5 ft. per sec. as a maximum value, we have $d_b=\dfrac{0.238W_r}{0.15W_r+R_p}$

and

$$d_b' = \left(\frac{0.238W_r}{0.15W_r+R_p}\right)\frac{A}{A_a} = \text{min. length of buffer}$$

where $R_p=0.15\,\pi(d+d_r)[.05b+.09(a+\frac{a_1}{2})]p_{max}$.

and

$p_{max}=4500$ usually. The buffer chamber should be made $d_b''=1.2$ to $1.3\ d_b'$

(11) Regulator Dimensions:

Referring to fig.(11) let,

a = area at base of regulator valve (sq.in)

a_1 = area of upper and lower valve stem (sq. in)

d_a = diameter of regulator valve at base (in)

d_{a_1} = diameter of regulator valve at stem (in)

c = effective circumference at base of valve (in)

From (10) we find that, $a = 73.5\,\dfrac{w_h^2}{D_a^2}$

and

$d_a = 9.675\,\dfrac{w_h}{D_a}$ where w_h = maximum throttling opening(sq.in)

D_a = diameter of recuperator (sq.in)

Now $d_{a_1}=0.6d_a$ approx. and $a_1=0.7854d_{a_1}^2=0.2825d_a^2$ hence $a_1=0.36a$.

The opening of the valve is the effective lift multiplied by the effective circumference of the valve at the valve seat. Extension guides or "flaps" to ensure proper seating of these valves reduce the effective circumference at the valve seat. It is customary to use three flaps of a circumferential length each, equal to the arc of 60° angle, decreased by two millimeters on either side, making the linear length of flap atthe circumference equal to the arc of 60° minus 4 millimeters. Hence

$$c = \frac{\pi d_a}{2} + \frac{12}{25.4}$$
$$= 0.3925 d_a + 0.4725$$

In the throttling or lower valve and its stem equalizing pressure ports should be bored within. In the stem itself, the inside diameter or diameter of the vertical port should be
$d'_{a_1} = 0.5$ to $0.6\ d_{a_1}$

Four equalizing holes just above the seat in the regulator valve, in a horizontal plane, meeting at a common opening at the center should be inserted. From the center opening there should be a very small vertical opening leading to the recoil cylinder, this acting as a pressure equalizer between the recoil and recuperator cylinders. The opening, however, should be made negligible as compared with the throttling opening and small as compared with the counter recoil constant orifice.

(12) Reactions on Regulator Valve:

Let P_b = reaction of Belleville washer on regulator valve (in lbs)
R_s = reaction of spiral spring on regulator (in lbs)
p = pressure in recoil cylinder (lbs/sq. in)
p'_a = pressure in recuperator (lbs/sq.in)
a = area at base of valve (sq.in)
a_1 = area of valve stem (sq.in)
h = lift of valve from initial opening (in)
h_o = lift of valve from seat of initial opening (in)
c = effective circumference at base of valve in inches.
S_b = spring constant of Belleville washer (lbs/in)
S_s = spring constant of spiral springs (lbs/in)

h_b=initial compression of Belleville washer at initial opening (in)

h_s=initial compression of spiral spring at initial opening (in)

Then, at short recoil, or intermediate recoil, we have $pa - p'_a(a-a_1) = R_b + R_s$ (approx) hence

$$(p-p'_a)a + p'_a a_1 = R_b + R_s \qquad (1)$$

and at long recoil, we have

$$(p-p'_a)a = R_s \text{ (approx)} \qquad (2)$$

Further, we have the following lifts of the valve,

$$h = \frac{K\,A\,V}{13.2c\sqrt{p_s - p'_a}} \qquad (3) \text{ At short recoil}$$

$$h = \frac{K\,A\,V}{13.2c\sqrt{p_h - p'_a}} \qquad (4) \text{ At long recoil}$$

where p_s and p_h are the values of p at short and long recoil respectively,

$$K = \frac{1}{0.773}$$

The spiral spring should be designed on the following basis:

(1) The maximum compression should be taken at from 2/3 to 3/4 the solid load of the spring.

(2) The initial compression should be taken at from 1/4 to 1/3 the solid load on the spring. Hence, using the maximum limits, the compression from free to solid height $h_{fs}=2h$ (5) and therefore

$$h = \frac{\pi f_s D_s^2}{2Nd} n \qquad (6)$$

where f_s= max. allowable torsional fibre stress (lbs/sq.in) (Usually = 120,000 lbs.per sq.in)

D_s = diam. of helix in inches.
d_s = diam. of wire
N = torsional modulus (taken at 12,000,000 lbs/sq.in)
n = number of coils of the spring

From previous design layouts, the total height of spring column, at assembled height should not exceed $\frac{D_a}{2}$ inches. Hence the solid height H_o becomes, $\frac{D_a}{2} - h = H_o$ but $H_o = d_s(n+1)$ hence

$$d = \frac{0.5D_a - h}{n+1} \qquad (7)$$

Combining (6) and (7) we have

$$\frac{\pi f_s D_s^2}{2N\,h}\,n = \frac{0.5D_a - h}{n+1} \qquad (8)$$

The load at assembled height = $\frac{4}{3}\,R_s$

hence

$$R_s = \frac{3\pi f_s d_s^3}{32D} \qquad (9)$$

Combining with (7), we have

$$R_s = \frac{3\pi f_s}{32D}\left(\frac{0.5D_a - h}{n+1}\right) \qquad (10)$$

and with,

$$\frac{\pi f_s\ D_s^2 n}{2Nh} = \frac{0.5D_a - h}{n+1}$$ (Eq.8) we may determine n and D. The solution may be simplified by assuming

$\frac{0.5D_a - h}{n+1} = \frac{0.4D_a}{n}$, as a first approximation, we then have

$$\left.\begin{aligned} R_s &= .01885\ \frac{f_s D_a^3}{n^3 D_s} \\ \text{and}\ \ \frac{\pi f_s D^2 n^2}{Nh} &= 0.8\ D_a \end{aligned}\right\}$$

Solving for D, d, and n, we have

$$D_s = 2.62 \sqrt{\frac{N^{\frac{3}{2}}\ h^{\frac{3}{2}}\ R_s}{f_s^{\frac{5}{2}}\ D_a^{\frac{3}{2}}}} \quad \text{inches}$$

If we assume N=10,500,000 lbs/sq.in.
f_s=120,000 lbs/sq.in.
then

$$D_s = 0.216 \sqrt{\frac{h^{\frac{3}{2}}\ R_s}{D_a^{\frac{3}{2}}}}$$

If D is too great for a satisfactory layout, we may increase the height of the spring column slightly or let the maximum working load on the spring move closely approach the load at maximum compression.

Solving for the diameter of wire "d" and the number of coils "n", we have

$$d = 1.503 \sqrt[3]{\frac{R_s D_s}{f_s}}$$ and if f_s=120,000 lbs per sq. in. then

$$d = .0305 \sqrt[3]{R_s D_s}$$

Test pressures are usually at double the service pressure, hence the material will be strained up to 3/4 the elastic limit.

(13) Design of Cam Mechanism and Layout:

Briefly, the action of the cam is to control the motion of the upper valve stem which reacts against the Belleville washers. At long recoil the valve displacement (i. e. the displacement of the upper valve stem) is sufficient so that the lower valve stem. is never brought into contact with the upper stem, and the lower stem is controlled entirely by the spiral spring. At an intermediate

recoil, the lower stem is brought ultimately in contact with the upper stem and the valve is controlled by the compound characteristics of the two stems. As the upper stem initial position is brought closer and closer to the lower valve stem, the valve opening depends more on the characteristics load deflection slope of the Belleville washers. Finally at short recoil, where the upper valve stem is brought into initial contact with the lower valve stem and the displacement of the cam is zero, the valve opening depends practically on the Belleville characteristics alone, the effect of the spiral spring being negligible. It is to be noted that the throttling at intermediate recoils approximates that if a constant orifice, with however the characteristic peak effect in the braking with a constant orifice eliminated. The throttling, therefore depends upon the displacement of the valve and the characteristic load deflection curves of the Belleville and spiral springs.

Let g = ratio of cam movement to valve movement (usually taken at 5)

X = distance valve should lift to engage Bellevilles (in)

h_s = initial compression of spiral springs (in)

h_o = clearance of valve (in)

h = lift of valve (in)

h_b = initial compression of the Bellevilles (in)

S_b = change in load per unit deflection of the Belleville washers, i.e. the Belleville spring characteristic (lbs)

S_s = change in load per unit deflection of the spiral spring, i.e. the spiral spring characteristic (lbs)

Then at an intermediate recoil, the reaction of the spiral spring, becomes, $R_s = S_s h_s + S_s(h+h_o)$ (lbs) The reaction of the Belleville washers becomes $R_b = S_b h_b + S_b(h+h_o-X)$ (lbs) while the hydraulic reaction becomes, $(p-p_{ai})a + p_{ai}\ a_1$ (lbs)

where p = the intensity of pressure in the recoil brake cylinder (lbs/sq.in)

p_{ai} = the intensity of pressure in the recuperator (lbs/sq.in)

a = area at base of valve (sq.in)

a_t = area of valve stem (sq.in)

Then for equilibrium of the valve,

$$S_s(h_s+h+h_o)+S_b(h_b+h+h_o-X)-[(p-p_{ai})a+p_{ai}a_t]=0$$

Therefore, for the distance of valve lift to engage Bellevilles, is

$$X=\frac{1}{S_b}\left\{S_s(h_s+h_o+h)+S_b(h_b+h_o+h)-[(p-p_{ai})a+p_{ai}a_t]\right\}$$

The variation of the length of recoil against elevation may be made in any arbitrary way, but, however, the following method is usually employed.

In general, assume the length of recoil that of horizontal recoil from $\emptyset_o$ to $\emptyset_t$ degrees, (usually from 0° to 20° elevation), then, decrease the recoil proportionally with the elevation, (i.e. from 20° to max. elevation, the recoil length decreases uniformly to short recoil at maximum elevation).

Thus if,

b = length of an intermediate recoil

$\emptyset$ = corresponding angle of elevation

b_h = length of recoil at horizontal elevation

b_s = length of recoil at maximum elevation

$\emptyset_m$ = maximum elevation

$\emptyset_i$ = initial elevation where the recoil is shortened

then

$$b=\frac{b_h-b_s}{\emptyset_m-\emptyset_i}(\emptyset_m-\emptyset)+b_s \text{ (ft)}$$

The resistance to recoil corresponding to the length of recoil "b" is given by:

$$K=1.03\left[\frac{W_rV_f^2}{2g}\;\frac{1}{b_s+(.096+.0003d)\frac{uV_f}{v}}\right]$$

where w = weight of projectile (lbs)

W = weight of powder charge (lbs)

W_r = weight of recoiling parts (lbs)

u = travel up bore (inches)

d = diam. of bore (in)

v = muzzle velocity (ft/sec)

V_f = max. free velocity of recoil (ft/sec)

and

$$V_f = \frac{wv + W4700}{W_r} \quad \text{(ft/sec)}$$

For a rough approximation, $K = \dfrac{0.47 W_r V_f^2}{gb}$ (lbs)

The required recoil braking is given by

$$B = \frac{K + W_r \sin\emptyset}{1 + \dfrac{2ue_b}{1-2ur}} - R_p \quad \text{or} \quad B = \frac{(K + W_r \sin\emptyset)l}{l + 2ue_b} - R_p$$

approximately

where l = distance between guide clips (in)

e_b = distance from center line of bore to center line of brake cylinder (in)

r = mean distance to guide contact (in)

R_p = brake cylinder packing friction (lbs)

For the lift of the valve, we have

$$h = \frac{.098 A^{\frac{3}{2}} V_r}{c\sqrt{B - K_v}} \quad \text{(in)}$$

where A = the effective area of recoil piston (sq.in)

K_v = recuperator reaction (lbs)

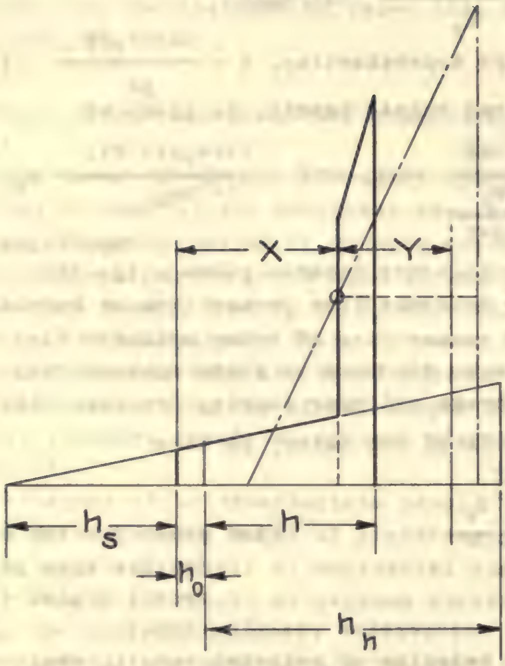
V_r = velocity of retarded recoil, about $0.9V_f$ (ft/sec)

c = effective circumference of lower stem (in)

we may also express the lift in terms of the pressures, then

$$h = \frac{.098 A\, V_r}{c\sqrt{p - p'_{ai}}} \quad \text{(in)}$$

where $p = \dfrac{B}{A}$ = the pressure intensity in the brake cylinder (lbs/sq.in)



SPRING & CAM MOTION (X) DIAGRAM

Fig. 13

$$p_{ai} = \frac{K_v}{A} = \text{the initial recuperator pressure intensity (lbs/sq.in)}$$

(14) Counter Recoil Design:

The function of the counter recoil buffer is to reduce the pressure in the recoil cylinder to a very low value practically zero. The recoiling parts are therefore brought to rest by the combined packing and guide friction in a displacement corresponding to the buffer length in the recuperator cylinder. For a preliminary design layout, the entrance velocity into the buffer may be taken at a counter recoil velocity of 1 meter or 3.28 ft/sec, but preferably less than this. To allow for a margin in variation of counter recoil friction the buffer displacement will be reduced in the counter recoil to 0.7 its actual value. Then, we have

$$0.7d_b(0.15W_r + R_p') = \frac{1}{2}\frac{W_r}{32.16}\overline{3.28}^2 \quad \text{hence}$$

$$d_b = \frac{0.238W_r}{0.15W_r + R_p'} \qquad \text{(ft)}$$

The corresponding displacement of the buffer in the recuperator, is

$\dfrac{A}{A_a} d_b$ where A = effective area of recoil piston
A_a = cross section area of recuperator cylinder

The length of the buffer rod will be made about 20% greater. Hence for the length of the buffer rod, we have

$$d_b' = 1.2 \frac{0.238W_r}{0.15W_r + R_p'}\frac{A}{A_a} \quad \text{(ft)}$$

The length of buffer chamber is usually constructed from 20 to 30% greater than the buffer rod, hence $d_b'' = 1.2$ to $1.3\ d_b'$ (ft) for length of buffer chamber.

The maximum allowable counter recoil velocity at horizontal elevation should not exceed 3.5 ft/sec. The counter recoil velocity for a satisfactory design ranges from 2.5 to 3.5 ft/sec. The velocity used in counter recoil should be such that with the expression

$$0.7\ d_b(0.15W_r+R_p') = \frac{1}{2}\frac{W_r}{32.16}v^2$$

d_b ranges from 1/4 to 1/3 the short recoil b_s

The packing friction for the recoil may be expressed by the relation, $R_p = C_1 + C_2 p$ (lbs) where p = lbs/sq.in. in the recoil cylinder. On counter recoil during the buffer action p = 0 approx. hence $R_p' = C_1$ approx. Now C_1 is that part of the packing friction due to the Belleville compression of the packing and is designed for the maximum recoil pressure p_{max} (lbs/sq.in)

If D_r = outside diameter of packing ring, (in)
d_r = diameter of rod (in)
a = depth of silver flange of packing (in)
a' = depth of outer silver flange (in)
b = packing contact (in)

then

$$R_p' = C_1 = \pi(D_r+d_r)[.05b+.09(a+\frac{a'}{2})]0.15\ p_{max}\ \text{(lbs)}$$

The guide friction on counter recoil may be taken at $R_g' = 0.15$ to $0.2\ W_r$ (lbs)

For the total recoil friction, we have

$R_f = R_p' + R_g'$

<u>Constant Orifice Opening</u>: <u>at max. elevation,</u>

$$w_o' = \frac{K\ A^{\frac{3}{2}}\ V_{\emptyset m}}{13.2\sqrt{p_a'A - R_f - W_r \sin\emptyset_m}} \quad \text{(sq.in)}$$

<u>at horizontal elevation,</u>

$$w_o'' = \frac{K\ A^{\frac{3}{2}} V_{\emptyset i}}{13.2\sqrt{p_a'A - R_f}} \quad \text{(sq.in)}$$

where $p_a' = \frac{p_{ai}+p_{af}}{2}$ = mean air pressure (lbs/sq.in)

p_{ai} = initial air pressure (lbs/sq.in)
p_{af} = final air pressure (lbs/sq.in)
The orifice to be used should be taken the mean of w_o' and w_o'', hence

$$w_o = \frac{w_o'+w_o''}{2} \text{ (sq.in)}$$

and

$$\begin{cases} V_{\phi i} = 2.5 \text{ to } 3 \text{ (ft/sec)} \\ V_{\phi f} = 2\phi_o 2.5 \text{ (ft/sec)} \end{cases}$$

The buffer entrance area should be

$$w_b = \frac{KAV}{13.2} \sqrt{\frac{1}{p_a' - \frac{.00894}{w_o^2} A^2 V^2}} \text{ (sq.in)}$$

where V = 2.5 to 3.5 ft/sec.

DESIGN PROCEDURE FOR ST. CHAMOND RECOIL

Given:

Diam.of bore(inches) d =	4.134	inches
Muzzle velocity(ft/sec) v =	1500	ft/sec.
Wt.of charge(in lbs) $\bar{w}$ =	3.25	lbs.
Travel of shot up bore(inches) u =	80	inches
Max.angle of elevation ϕ_m =	80	degrees
Min.angle of elevation ϕ_i =	0	degrees
Max.powder pressure on base of breech(lbs/sq.in) P_b =	24000	lbs/sq.in.
Length of recoil at max. elevation (ft) b_e =	2.5	feet

Length of recoil at 0° elevation (ft) 3.75 ft.

$\bar{w}$ = weight of charge (lbs) 3.25 lbs.
w = weight of projectile (lbs) 33. lbs.
W_r = weight of recoiling parts (lbs) 1260. lbs.
W_s = Probable weight of total mount 2700. lbs.
Weight of trail 300. lbs.

3000. lbs.

STABILITY LIMITATIONS.

Max.free recoil velocity (ft/sec)

$$V_f = \frac{wv + \bar{w}\,4700}{W_r} = \frac{33.\times1500+4700\times3.25}{1260} =$$

51.50 ft/sec.

Max.recoil reaction R(approx) =

$$0.45\,\frac{W_r}{g}\,\frac{V_f^2}{b_s} = 0.45\,\frac{1260 \times \overline{51.50}^2}{32.2\times2.3} =$$

18,700 lbs.

Height of axis of bores above ground(assumed) (ft) h = 3. ft.

Max.allowable horizontal recoil (ft)

$$b_h \max = \sqrt{\frac{V_f^2 h}{2g}} = \sqrt{\frac{\overline{51.50}^2\times3}{2\times32.2}} = 11.1 \text{ ft.}$$

Max.velocity of constrained recoil(ft/sec)

$V_r = 0.9\ V_f$ (approx) = 0.9 × 51.5 = 46.4 ft.

Recoil constrained energy (ft/lbs) A =

$\frac{1}{2}\frac{W_r}{g}V_r^2 = \frac{1260 \times \overline{46.4}^2}{2 \times 32.2} = 42{,}000$ ft/lbs.

Recoil displacement during powder period (ft)

$E_r = 2.24\left(\frac{w+0.5\bar{w}}{W_r}\right) u = 2.24\left(\frac{33+0.5\times3.25}{1260}\right)\frac{80}{12} =$

0.41 ft.

Constant of stability (assumed)

$C = \frac{\text{Overturning moment}}{\text{Stability moment}} = 0.96$

Horizontal distance from spade point to line of action of W_s (ft) l_s = $\frac{2700\times81+300\times34}{3000} = 6.35$ ft.

$\emptyset_s$ = angle of stability — 20°

---

d = moment arm of overturning force =

$h_t \cos\emptyset + ds - l \sin\emptyset = 36\times0.9397+7.5-81\times0.3420 =$

13.60 in.

Horizontal recoil consistent with stability (ft)

$$b_h = \frac{W_s l_s + W_r E\cos\emptyset + \sqrt{(W_s l_s + W_r E\cos\emptyset)^2 - 4W_r\cos\emptyset\left(W_s l_s E - \frac{dA}{c}\right)}}{2W_r\cos\emptyset} = \frac{3000\times6.35\times41 - \frac{42000\times1.13}{.96}}{2\times1260\times0.9397}$$

---

b_h max 3.74 ft used

$b_h$max.3.75 = 3.74 ft.

APPROXIMATE DIMENSION OF RECUPERATOR FORGINGS:

Max. resistance to recoil(at max. elevation) (lbs) $K_s = \frac{0.45}{b}\frac{W_r}{g}V_f^2 =$	$\frac{0.45}{2.6}\frac{1260}{32.2}\ \overline{51.5}^2 =$ 18700 lbs.
Min. resistance to recoil(at horizontal elev.)(lbs) $K_h = \frac{0.47}{b}\frac{W_r}{g}V_f^2 =$	$\frac{0.47}{3.75}\frac{1260}{32.2} \times \overline{51.5}^2 =$ 13100 lbs.
Max.pull(max.elev.) (lbs) $P_s = K_s + W_r \sin \emptyset - \Sigma R$; $\Sigma R = W_r \sin \emptyset_m$ (approx) $\therefore P_s = K_s =$	18700 lbs.
Min.pull(0° elev.) (lbs) $P_h = K_h - 0.3W_r =$	
Initial recuperator reaction (lbs) $K_v = 1.3W_r(\sin\emptyset_m + 0.3 \cos \emptyset_m) =$	$1.3\times1260(.9848+.05\times0.1736)=$ 1700

Ratio of recuperator cylinder area	
Effective area of recoil piston $r = \frac{A_a}{A} = .039V_r\sqrt{\frac{P_s}{P_h-K_v}} =$	$0.039\times46.35\sqrt{\frac{18700}{12730-1700}} =$ 2.35
From chart - assume air column =	1.36
r_{min} =	2.5
Total weight of recoil piston and rods (lbs) W_e =	30 lbs.
Effective area of recoil piston (sq.in) If $r > r_{min}$ $A = 0.243\frac{r_{min}}{V_r^2}(P_h-K_v) =$	
Corresponding max. pressure (lbs/sq.in) $P_{max} = \frac{P_s}{A} =$	
Approx.max.tension rods at horizontal (lbs) $T_L=K_s+W_r\sin\emptyset_m+P_b\frac{W_e}{W_r} =$	$18700+1260\times0.3420+24000\times13.45\frac{.30}{1260} =$ 27000 lbs.
Assumed max.fibre stress (lbs/sq.in) $f_{max}=\frac{2}{3}$ elastic limit =	$\frac{2}{3}$ 60,000 = 40,000 lbs/sq.in.

Area of recoil rod (sq.in) $a_r = \frac{T_L}{f_{max}} =$	$\frac{27000}{40000} = .676$
Diam. of recoil rod $d_r = 1.127 \sqrt{a_r} =$	$1.127 \sqrt{.676} = .925$ in. use 1 inch.
Total area of recoil cylinder (sq.in) $A_r = A + A_a =$	$4.16 + 0.676 = 4.836$ sq. in.
Inside diam. of recoil cylinder (inches) $D_r = 1.127 \sqrt{A_r} =$	$1.127 \sqrt{4.836} = 2.48$ inches.
Area of recuperator cylinder (sq.in) $A_a = rA =$	$2.5 \times 4.16 = 10.40$ sq. in.
Inside diam. of recuperator cylinder(in) $D_a = 1.127 \sqrt{A_a} =$	$1.127 \sqrt{10.40} = 3.63$ inches.

COMPUTATION OF PACKING FRICTIONS.

Recoil friction

Width of leather contact of packing (assumed)(in) b = 0.18 in. to 0.25 in.=	0.21 inches
Depth of one silver flange of packing cup (in) a = 0.14 in.to 0.16 in. =	0.14 inches.

Depth of outer silver flange $a' = 0.18$ to 0.22 in.=	0.18 inches
Constant spring component of total packing friction (lbs) $C_1 = \pi(D_r+d_r)[.05b+.09(a+\frac{a_1}{2})0.15\,p_{max}] =$	$\pi(2.48+1)0.05\times0.21+0.09(0.14+0.09)0.15\times4500 =$ 230 lbs.
Pressure constant for fluid pressure component of total packing friction (lbs) $C_2 = \pi(D_r+d_r)[.05b+.09(a+\frac{a_1}{2})]0.73 =$	$\pi(2.48+1)[0.05\times0.21+0.09(0.14+0.09)]0.73 =$ 0.250
Total recoil packing friction (lbs) $R_p = C_1 + C_{2p} =$	$230+0.250\times4500 = 1350$ lbs.
Floating Piston Friction	
Constant spring component of floating piston friction $C_1' = 1.12\pi D_a[.05b+.09(a+\frac{a_1}{2})]$ $p_{af}' = Gp_{af}'\,G =$	$1.12\pi\times3.63[0.05\times0.21+0.09(0.14+0.09)]p_{af}' =$ $0.4P_{af}$
Pressure constant for fluid pressure component of total packing friction (lbs) $C_2' = 1.46\pi D_a[.05b+.09(a+\frac{a_1}{2})] =$	$1.46\times\pi\times3.63\times0.0312 = 0.52$

CALCULATION OF THE DIMENSIONS OF THE RECUPERATOR FORGING:

Max. resistance to recoil (max. elevation) (lbs)

$$K_s = 1.05\left(\frac{W_r V_f^2}{2g}\right)\frac{1}{b_s + (.096 + .0003d)\dfrac{uV_f}{v}} = 1.05\,\frac{1260\times\overline{51.50}^2}{64.4}\times\frac{1}{2.5+(.096+0.0003\times4.134)\dfrac{80\times51.5}{1500}} = 19{,}700 \text{ lbs.}$$

Min. resistance to recoil (horizontal elevation) (lbs)

$$K_h = \frac{W_r V_f^2}{2g}\,\frac{1}{b_h + (.096 + .0003d)\dfrac{uV_f}{v}} = \frac{1250\times\overline{51.5}^2}{64.4}\,\frac{1}{3.75+(0.096+0.0003\times4.134)\dfrac{80\times51.5}{1500}} =$$

12900 lbs.

Maximum recoil packing friction (lbs) $P_{max} = C_1 + C_2 p_{max}$ (see packing friction) = $230+0.250\times4500=1360$ lbs.

($p_{max} = 4500$ or $\frac{K_s}{A}$ approx.)

Distance between clip reactions (inches)(assumed) l =	60 inches
Distance from center of gravity of recoiling parts to mean friction line (inches) r' =	6.5 inches
Coefficient of guide friction, n=0.1 to0.2 =	0.15
Distance from center of gravity of recoiling parts to axis of piston rod(inches) d_b =	7.5
Pull at max. elevation (lbs) $P_s = \dfrac{K_s + W_r \sin\phi_m}{1+\dfrac{2nd_b}{1-2nr}} - R_p \max$ =	$\dfrac{19700+1260\times0.9848}{1+\dfrac{2\times0.15\times7.5}{60-2\times0.15\times6.5}} - 1360 = 18800$
Pull at horizontal elevation (lbs) $P_h(1+\dfrac{C_2}{A}) = \dfrac{K_h}{1+\dfrac{2nd_b}{1-2nr}}$ = C_1 hence P_h =	$P_h(1+\dfrac{0.25}{4.16}) = \dfrac{12900}{1+\dfrac{2\times0.15\times7.5}{60-2\times0.15\times6.5}} - 230$ = 11,550 lbs.
Excess recuperator battery reaction constant (recuperator constant) K=1.1 to 1.3 =	1.2

Recuperator reaction in battery at max. elevation $K_v = \dfrac{k(W_r \sin\emptyset_m + C_1)}{1-k\left(\dfrac{2nd_b}{1+2nr} + \dfrac{C_2}{A}\right)} =$	$\dfrac{1.2(1260\times0.9848+230)}{1.1.2\left(\dfrac{2\times0.15\times7.5}{60+2\times0.15\times6.5} + \dfrac{.25}{4.16}\right)} = 1980$
Max. restrained recoil velocity(ft/sec) $V_r = 0.92\ V_f =$	47.40 ft/sec.
Ratio of $\dfrac{\text{Recuperator area}}{\text{Effective recoil piston area.}} =$	$2.625\ V_r \sqrt{\dfrac{P_s}{P_{max}(P_h-K_v)}}$ $= 2.625\times47.4\sqrt{\dfrac{18800}{4500(11550-1980)}} = 2.6$
If $r < r_{min}$(see chart and assume air column)	51.6 inches
Effective area of recoil piston(sq.in) $A = \dfrac{P_s}{4500} =$	$\dfrac{18800}{4500} = 4.18$ sq.in.
Area of recuperator cylinder (sq.in) $A_a = r\ A =$	$2.6\times4.18 = 10.87$ sq.in.
If $r < r_{min}$(see chart and assume air column)	

Effective area of recoil piston

$A = 0.2425 \dfrac{r^2 min}{V_r^2}(P_h - K_v) =$

If $r > 3.5$ (Two short cylinders - see chart)

Effective area of recoil piston(sq.in)

$A = \dfrac{P_s}{4500}$

Area of recuperator cylinder (sq.in)

$A_a = 3.5\ A$

Horizontal recoil pull

$P_h = K_v + .000912 V_r^2 \dfrac{P_s}{12.25} =$

Where length of Air column is assumed

Length of air column in terms of length of max. recoil (assumed)

$j = \dfrac{l_a}{b_h} =$ 1.445

Ratio of $\dfrac{\text{final air pressure}}{\text{initial air pressure}}$

$= (\dfrac{P_{af}}{P_{ai}})$

$M = (\dfrac{ri}{rj-1})^{1.3}$ (see chart of tables) = 1.5

Initial recuperator pressure (lbs/sq.in)

$p'_{ai} = \frac{K_v}{A} =$ $\frac{1980}{4.18} = 473$ lbs/sq.in.

Final recuperator pressure(lbs/sq.in)

$p'_{af} = p'_{ai}$ (approx) = $1.5 \times 473 = 710$ lbs/sq.in.

Initial air volume (cu.in)

$V_o = A_a l_a =$ $10.87 \times 51.6 = 560$ cu.in.

When ratio of final to initial air pressure is assumed.

Assume

$m = \frac{p_{af}}{p_{ai}} = \frac{p'_{af}}{p'_{ai}} = 1.5 =$ 1.5

Initial air volume(cu. in)

$V_o = \left(\frac{m^{0.77}}{m^{0.77} - 1}\right) A_s\, b_h =$ $\left(\frac{1.5^{0.77}}{1.5^{0.77} - 1}\right) 10.87 \times 45 =$

560 cu.in.

Length of air column (inches)

$l_a = \frac{V_o}{A_a} =$ $\frac{560}{10.87} = 51.8$ in.

Initial recuperator pressure (lbs/sq.in)

$p'_{ai} = \frac{K_v}{A} =$ $\frac{1980}{4.18} = 473$ lbs/sq.in.

Final recuperator pressure	
$p'_{af} = 1.5 p'_{ai}$ (approx) =	1.5 4.73 = 710 lbs/sq.in.

INITIAL AND FINAL AIR PRESSURE AND AIR VOLUME.

Initial recuperator pressure (lbs/sq.in)	
p'_{ai} =	473 lbs/sq.in.
Floating piston friction(initial)(lbs)	
$C'_1 + C'_2\, p'_{ai}$ =	$.4 \times 710 + 0.52 \times 473 =$ 530 lbs.
Drop of pressure across floating piston(lbs/sq.in)	
$p_{ai} = p'_{ai} + \dfrac{C'_1 + C'_2\, p'_{ai}}{A_a}$ =	$473 + \dfrac{530}{10.87} = 520$ lbs/sq.in.
Final air pressure (lbs/sq.in)	
$p_{af} = m p_{ai}$ =	$1.5 \times 520 = 780$ lbs/sq.in.
Final air volume (cu.in)	
$V_f = V_o - A\, b_h$ =	560 - 4.18 45 = 370 cu.in

Average drop of pressure across floating piston (lbs-sq.in)

$$P_a = p_a' - p_a = \frac{C_1 + C_2(p_{af} + p_{ai})0.5}{A_a} = \frac{.4 \times 780 + 0.52(780+520)0.5}{10.87}$$

= 620 lbs.

Strength of cylinders

Test Pressure 2 X Service pressure.

Area of recoil cylinder =

$A_r' = A_r + a_r$ = $4.18 + 0.79 = 4.97$ sq.in.

Diameter of recoil cylinder (inches) D_r =

$1.127\sqrt{A_r}$ = $1.127\sqrt{4.97} = 2.51$ inches.

Diameter of recuperator cylinder (inches)

$D_a = 1.127\sqrt{A_a}$ = $1.127\sqrt{10.87} = 3.71$ inches.

Max. allowable fibre stress for cylinders (lbs/sq.in)

$P_t = \frac{3}{8}$ elastic limit = $\frac{3}{8}\ 60000 = 22500$ lbs/sq.in.

Min. outside diam. of recoil cylinder (inches)

$$D_{ro} = D_r\sqrt{\frac{P_t + p_{max}}{P_t - p_{max}}} = 2.51\sqrt{\frac{22500 + 4500}{22500 - 4500}}$$

= 3.07 inches.

Min. outside diam. of recuperator or air cylinder

$$D_{ao} = D_a \sqrt{\frac{p_t + p_{af}}{p_t - p_{af}}} = 3.71 \sqrt{\frac{22500}{22500} + \frac{810}{810}} =$$

3.84 inches
use 3.96 inches.

Min. width between adjacent cylinders (inches)

$$w = \frac{p_{max} D_r + p_{af} D_a}{1.5\ p_t} = \frac{4500 \times 2.305 + 810 \times 3.71}{1.5 \times 22500}$$

$= 0.393$

Calculation of max. and min. throttling areas

Max. throttling area (at horizontal recoil) (sq.in)

$$w_h = \frac{.098 A^{\frac{3}{2}} V_r}{\sqrt{P_h - K_v}} = \frac{0.098 \times \overline{4.18}^{\frac{3}{2}} \times 47.4}{(11550-1980)^{\frac{1}{2}}}$$

$= 0.374$ sq.in.

Min. throttling area at max. elevation (sq.in)

$$w_s = \frac{.098 A^{\frac{3}{2}} V_r}{\sqrt{P_s - K_v}} = \frac{0.098 \times \overline{4.18}^{\frac{3}{2}} \times 47.4}{(18800-1980)^{\frac{1}{2}}}$$

$= 0.282$ sq.in.

LAYOUT OF RECUPERATOR FORGING, PORT AND CHANNEL AREAS.

No. of cylinders = 3

Overall length of forging (inches) $l_f = 1.5\ b_h$ =	$1.5 \times 45 = 67.5$ in.
Diameter of recoil cylinder(inches) D_r =	2.51 in.
Diameter of recuperator air cylinder(inches) D_a =	3.71 in.
Length of air column (inches) l_a =	51.6 in.
Area of connecting channel between recoil and recuperator cylinder (sq.in) $w_a = 3.5$ to $4.3\ w_h$ =	$4.02 \times 0.374 = 1.5$ sq.in.
Diam. of connecting channel (inches) $d'_a = 1.127\sqrt{w_a}$ =	$1.127\sqrt{1.5} = 1.38$ in.
Max. depth of recuperator cylinder below recoil cylinder (inches) $D' = D_r - \frac{d'_a}{2}$ =	$2.51 - \frac{1.38}{2} = 1.82$ in.
Constant channel area from regulator valve (sq.in) $w_c = 4.3\ w_h$ =	$4.3 \times 0.374 = 1.606$ in.
Depth of constant channel area w_c(inches) $h_c = 0.2\ D_a$ =	$0.2 \times 3.71 = 0.742$
Extreme area to regulator valve (sq.in) $a = 73.5\ \frac{w_h^2}{D_a^2}$ =	$73.5\ \frac{\overline{0.374}^2}{\overline{3.71}^2} = 0.747$ sq. in.

Diameter of entrance channel (inches) $d_a = 9.675\ \frac{w_h}{D_a} =$	$= 9.675\ \frac{0.374}{3.71} = 0.975$ in.

DESIGN OF REGULATOR.

Area at base of regulator valve (sq.in) $a = 73.5\ \frac{w_h^2}{D_a^2} =$	0.747 in.
Diam. of regulator valve at base (inches) $d_a = 9.675\ \frac{w_h}{D_a} =$	0.975 in.
Diam. of upper and lower valve stem (inches) $d_{a_1} = 0.6\ d_a =$	$0.6 \times 0.975 = 0.585$ in.
Cross section area of upper and lower valve stem(sq.in) $a_1 = 0.36 =$	$0.36 \times 0.747 = 0.269$ (sq.in)
Diam. of inside port in valve stem $d_a' = 0.5$ to $0.6\ d_{a_1} =$	$0.55 \times 0.585 = 0.322$ (sq.in)
Guides or flaps at base of valve use =3 subtends are 60°- 2mm on either end. Length of one flap at base of valve $c' = 0.524\ d_a - 0.1573 =$	$0.524 \times 0.975 - 0.1573 = 0.354$ in.
Effective circumference at base of valve (throttling area) (inches) $c = 1.571\ d_a + 0.4725 =$	$1.571 \times 0.975 + 0.4725 = 2.004$ in.

Load on spring and Bellevilles at short recoil (max. elevation) (lbs) $R_b+R'_s=(p-p'_{ai})a+p'_{ai}a_1$ =	$(4500-473)0.747+473\times0.269$ = 3140 lbs.
Load on spiral spring at long recoil -(0° elevation)(lbs) $R_s=(p-p'_{ai})a$ =	$(\frac{11550}{4.18}-473)\ 0.747$ = 1710
Lift of valve (inches) short recoil(max. elevation) $h' = \frac{.098A^{\frac{3}{2}}\ V_r}{c\ \sqrt{p_s K_v}} - \frac{W_s}{C}$ =	$\frac{0.282}{2.004} = 0.1405$ in.
Lift of valve(inches) long recoil(0° elevation) $h'' = \frac{.098\ A^{\frac{3}{2}}\ V_r}{c\sqrt{p_h-K_v}} = \frac{W_h}{c}$ =	$\frac{0.374}{2.004} = 0.1862$ in.
Load at solid height on spiral spring (lbs) $R_{sc} = \frac{4}{3}\ R_s$ =	$\frac{4}{3}\ 1710 - 2280$

Spiral regulator spring			
Max.torsional fibre stress(lbs/sq.in) $f_s = \left.\begin{matrix}100{,}000\\120{,}000\\140{,}000\end{matrix}\right\} =$			
Torsional modulus (lbs/sq.in) $N = \left.\begin{matrix}11{,}500{,}000\\10{,}500{,}000\\10{,}000{,}000\end{matrix}\right\} =$			
Diam.of helix spiral spring regulator valve (inches) $D_s = 2.62\sqrt{\dfrac{N^{\frac{3}{2}} h''^{\frac{3}{2}} R_s}{f_s^{\frac{3}{2}} D_a^{\frac{3}{2}}}} =$			
If $N = 10{,}500{,}000$ $f_s = 120{,}000$ (lbs/sq.in) $D_s = 0.216\sqrt{\dfrac{h''^{\frac{3}{2}} R_s}{D_a^{\frac{3}{2}}}} =$			
Diam. of wire of spiral spring (inches) $d = 1.503\sqrt[3]{\dfrac{R_s D_s}{f_s}} =$			
If $f_s = 120{,}000$ lbs/sq.in. $d = .0305\sqrt[3]{R_s D_s}$			

COUNTER RECOIL DESIGN - BUFFER DESIGN

CONSTANT ORIFICE AND PORT AREAS.

Packing friction at end of counter recoil (lbs) $R_p'=0.15\pi(D_R+d_R)[.05b+.09(a+\frac{a_1}{2})]p_{max}=C_1=$	230 lbs.
Recoil length during buffer action (ft) $d_b=\dfrac{0.238W_r}{0.15W_r+R_p'}=$ $(d_b=\frac{1}{4}\text{ to }\frac{1}{3}b_s)$	$\dfrac{0.238\times1260}{.015\times1260+230}=0.715\text{ ft.}=$ 8.6 in.
Length of buffer rod (ft) $d_b'=1.2\dfrac{0.238W_r}{0.15W_r+R_p'}\dfrac{A}{A_a}=$ $\dfrac{1.2\,d_b}{r}=$	$\dfrac{1.2\times.715}{2.6}=.33\text{ ft.}=$ 3.96 in.
Length of buffer chamber (ft) $d_b''=1.2\text{ to }1.3\,d_b'=$	$1.25\times3.96=4.95$ in.
Min. allowable counter recoil velocity (ft/sec) at max. elev. $v_{\emptyset_m}=$	
Max. allowable counter recoil velocity (ft/sec) at horizontal elev. $v_{\emptyset_i}=2.5-3.5$ ft/sec=	2.5 ft.sec.

Total counter recoil friction (max.elevation)(lbs)

$$R_t=C_1+C_2\left[\frac{W_r(\sin\emptyset_m+0.3\cos\emptyset_m)}{A}\right]+0.3W_r\cos\emptyset_m=$$

$$230+.25\left[\frac{1260(.9848+.3\times.1736)}{4.18}\right]+.3\times1260\times.1736 = 290 \text{ lbs.}$$

Total counter recoil friction(min. elevation) (lbs)

$$R_t=C_1+C_2\left(\frac{0.3W_r}{A}\right)+0.3W_r = $$

$$230+.25\left(\frac{0.3\times1260}{4.18}\right)+.3\times1260=700 \text{ lbs.}$$

Recuperator mean pressure (lbs/sq.in)

$$p_a' = \frac{p_{af}+p_{ai}}{2} = \frac{473+710}{2} = 590$$

Required constant counter recoil orifice at max.elevation

$$w_o' = \frac{KA^{\frac{3}{2}}V_{\emptyset m}}{13.2\sqrt{p_a'A-R_t-W_r\sin\emptyset_m}} = \frac{1.25\times\overline{4.18}^{\frac{3}{2}}\times2.5}{13.2\sqrt{590\times418-290-1260\times.9848}} =$$

(K=1.25) .0645 sq.in.

Required constant counter recoil orifice at 0° elevation (sq.in)

$$w_o'' = \frac{KA^{\frac{3}{2}}\ V_{oi}}{13.2\sqrt{p_a'A-R_t}} = \frac{1.25\times\overline{4.18}^{\frac{3}{2}}\times3.5}{12.2\sqrt{4.73\times4+18-700}}$$

$$=.0765 \text{ sq.in.}$$

$$W_o = \frac{w_o'' + w_o'}{2} = \frac{0.0645+0.0765}{2} = 0.0705$$

Entrance buffer area
(sq.in)

$$w_b = \frac{KAV}{13.2}\sqrt{\frac{1}{p_a' - \frac{.00894A^2v^2}{w_o^2}}} = \frac{1.33\times4.18\times3.5}{13.2}\sqrt{\frac{1}{590 - \frac{.00894\times\overline{4.18}^2\times\overline{3.5}^2}{(.0705)^2}}}$$

K = 1.33(approx)
v = 3.5 ft/sec(approx)

= 1.00 sq.in.

Layout entrance area of buffer, with required depth in groove. Decrease depth of groove to zero at end of buffer d_b'.

Deflection from free to solid height of spiral spring (in)

h_{so} = 2h" = 2×0.1862 = 0.373 in.

Spiral spring constant
(lbs.per in.)

$$s_s = \frac{R_{sc}}{h_{so}} = \frac{2280}{.373} = 6110$$

This spiral spring will be too bulky for practical purposes. Therefore we will let the Belleville spring washers take care of all conditions at different elevations and design the spiral spring strong enough to keep the valve closed when gun is in battery.

Spiral spring reaction
at short recoil

$R_s' = S_s(h'+h_o) =$ 0

h_o=.0197 (initial lift)(in.)

Load on Belleville at short recoil(max.elev) (lbs)

$R_b = \{(p - p'_{ai})a + p'_{ai}a'\} - R_s =$ (4500−473)0.747−0= 3010 lbs.

Load at solid height on Belleville washers(lbs)

$R_{bo} = \frac{3}{2} R_b =$ $\frac{3}{2} \times 3010$ - 4520 lbs.

Deflection from free to solid height of Bellevilles (in)

$h_{bo} = 3h' =$ $3 \times 0.1405 = 0.422$

Belleville spring constant (lbs.per in)

$S_b = \frac{R_{bo}}{h_{bo}} =$ $\frac{4520}{0.422} = 10700$

---

h_b=initial compression of Belleville washers.

n = no. of Belleville spring washers =

$\frac{0.422}{.071} = 6$

h_b=initial compression = 115

h_o=valve clearance .0197

DESIGN OF CAM MECHANISM AND LAYOUT.

Ratio of $\frac{\text{Cam Movement}}{\text{Valve movement}}$

g = 5

(taken usually at 5)

In general, assume the length of recoil at horizontal recoil constant from $Ø_1$ to $Ø_1$ degrees,(usually from 0° to 20° elevation); then, decrease the recoil proportionally with the elevation, that is:- 20°

Length of intermediate recoil (ft)

$$b = \frac{b_h - b_s}{\emptyset_m - \emptyset_i}(\emptyset_m - \emptyset\ ) + b_s = \qquad \frac{45-30}{60}(80-\emptyset)+30 =$$

K''_s Total resistance to recoil:

$$\frac{0.47 W_r V_f^2}{gb} = \qquad \frac{0.47 \times 1260 \times \overline{51.5}^2}{32.2_b} =$$

$$\frac{48800}{b'} = \frac{58700}{b''} = K_s$$

P_{s_2}=Total recoil pressure =

$$\frac{k_s + 1260\sin\emptyset}{1.1} - 210 =$$

---

h = lift corresponding to required throttling opening (inches)

$$= \frac{.098 A^2 V_r}{c\sqrt{P-K_v}} = \frac{0.178 V_r}{(p-p_{ai})^{\frac{1}{2}}} = \frac{0.178 \times 47.4}{(p-p_{ai})^{\frac{1}{2}}} = \frac{8.45}{(p-p_{ai})^{\frac{1}{2}}} = h$$

$\emptyset$	b''	k_s	$\sin\emptyset$	$K_s + W_r \sin\emptyset$	P_s	$p_s - k_v$	$(p-p_{ai})$	$(p-p_{ai})^{\frac{1}{2}}$	h
20	45.00	13050	.3420	13430	12000	10000	2390	48.89	.1725
25	43.75	13400	.4226	13930	12470	10470	2500	50.00	.1680
25	41.25	14200	.5736	14920	13370	11370	2710	52.06	.1624
50	37.50	15650	.7660	16620	14900	12900	3080	55.50	.1520
65	33.75	17400	.9063	18540	16640	14640	3500	59.16	.1425
75	31.25	18750	.9659	19970	17930	15930	3800	61.64	.1370
80	30.00	19550	.9848	20790	18690	16690	3980	63.09	.1338

Linear motion of cam rod against elevation. Considering spiral spring reaction negligible.

$$X = h_s + h + h_s - \frac{(p-p_{ai})a + p_{ai}a_1}{S_b} = 0.175 + h - \frac{(p-p_{ai})0.747 + 127}{10700}$$

a	b	c	d	e	f	g	x	Max. of cam rod inches
Ø	b"	h corrected	h+ 0.175	$\frac{(p-p_{ai})}{0.747}$	e+127	$\frac{f}{10700}$	d−g	
20	45.00	.1867	.3612	1785	1910	.1784	.1828	.911
25	43.75	.1800	.3550	1865	1990	.1860	.1690	.845
35	41.25	.1110	.3460	2030	2160	.2020	.1440	.720
50	37.50	.1600	.3350	2300	2430	.2270	.1080	.540
65	33.75	.1500	.3250	2610	2740	.2560	.0690	.345
75	31.25	.1450	.3200	2840	2970	.2780	.0420	.210
80	30.00	.1405	.3155	2970	3100	.2900	.0255	.127

105 M/M HOWITZER
75 M/M GUN (Double Charge)

MOUNTED ON SAME CARRIAGE.

Given:

	75 m/m Gun Normal Super	105 m/m How.
d = diameter of the bore (in)	2.953in.	4.134in.
v = muzzle velocity (ft/sec)	1500 2175	1500
$\bar{w}$ = weight of charge (lbs)	1.40lbs. 3.00lbs.	3.25lbs.
u = travel of shot up bore (in)	109.50in.	80.00in.
$Ø_m$ = max. angle of elevation	80°	
$Ø_i$ = min. angle of elevation	0°	
w = weight of projectile (lbs)	15lbs.	33lbs.
P_b = max. powder pressure on base of breach (lbs/sq.in.)	34,000	24,000
b_s = length of recoil at max. elevation	1.3ft.	2.5ft.
b_h = length of recoil at 0° elevation	2.4ft. 3.75ft.	3.75ft.

WEIGHT OF GUN AND CARRIAGE.

Similar Guns	W	V	E=Muzzle Energy	W_g	E/wg	W_t	w= X% wt.
75 mm.French	16	1700	716000	1050	705	2657	39
75 mm.U.S.	16	1700	716000	750	956	3045	25
75 mm.British	16	1700	716000	995	720	2945	29
3.8 How.M. 1906	50	900	378000	432	876	2040	22.6
4.7 Gun M. 1906	60	1700	2690000	2688	1000	8068	33.6
4.7 How. M. 1908	60	900	755000	1056	716	3988	27.
6" How.M. 1908	120	900	1510000	1925	785	7582	25.7
155 m/m How. (Sch)	95	1420	2970000	2745	1080	7600	36.5
155 m/m Gun (Fil.)	95	2300	8400000	8795	960	25600	34.5
155 m/mHow. (St.Cham)	95	1520	3400000	3040	1120	7700	25.3
8" How.VI	200	1300	5250000	6652	790	19100	35.0
8" How.VII	200	1525	7200000	7730	933	20050	38.7

Average E/wg of 888

E/wg=1000

E/wg=888

$$W_g = \frac{1}{2}\frac{W}{g}V^2\frac{1}{1050} = \text{normal gun } \frac{15\times\overline{1500}^2}{64.4\times1050} = 525\#$$

$$\text{super gun } \frac{15\times\overline{2175}^2}{64.4\times1000} = 1100\# \quad 1240$$

$$\text{howitzer } \frac{33\times\overline{1500}^2}{64.4\times1000} = 1150\# \quad 1290$$

$\underline{W_r \text{ gun}}$ 1100+30 = <u>1130#</u>

$\underline{W_r \text{ How.}}$ 1180+30 = <u>1180#</u>

} Average 1155# 1260#

Using highest % of W_r to W_t (39%) $W_t = \frac{1155}{397} =$ 2970#

$W_c = W_t - W_r = 2970 - 1155 =$ 1815#

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W_r Weight of recoiling parts 1230# and 1260# for gun and howitzer respectively are the minimum weight that could be used on account of stresses.

The condition being to get the minimum weight. These values are used:

W_r = 1230# gun

W_r = 1260# how.

- - - - - - - - - - -

	75m/m normal	Gun super	105 m/m How.
V_f max. free velocity = $\frac{wv+4700\bar{w}}{W_r}$			
$= \frac{15\times1500+4700\times1.4}{1230}$ = ft/sec.	23.60		
$= \frac{15\times2175+4700\times3}{1230}$ = ft/sec		38.00	
$= \frac{33\times1500+4700\times3.25}{1260}$ = ft/sec.			51.50
K_s Resistance to recoil at 80° elevation			
$= 1.05\left[\frac{W_r}{2g} V_f^2 \times \frac{1}{b_s+(.096+.0003d)\frac{uV_f}{v}}\right]$			

$$= 1.05\left[\frac{1230\times\overline{23.60}^2}{64.4} \times \frac{1}{1.5+(.096+.0003\times2.953)\frac{109.5\times23.60}{1500}}\right]$$

$$=1.05\left[10640\times\frac{1}{1.5(.096+.0009)1.724}\right]$$

$$=1.05\left(10640\times\frac{1}{1.5+.167}\right)=\frac{1.05\times10640}{1.662}=6720$$

$$=1.05\left[\frac{1230\times\overline{38.00}^2}{64.4}\times\frac{1}{1.5+.0969\left(\frac{1095.5\times38}{2175}\right)}\right]$$

$$\frac{1.05\times27600}{1.645}= \qquad \underline{17600}$$

$$=1.05\left[\frac{1260\times\overline{51.50}^2}{64.4}\times\frac{1}{2.5+(.096+.0003\times4.134)\frac{80\times51.5}{1500}}\right]$$

$$\frac{1.05\times51900}{2.761}= \qquad \underline{19700}$$

h = height of axis of bore above ground. Assumed 36"

b_h = max. allowable horizontal recoil = $\sqrt{\frac{V_f^2 h}{2g}}$

$$=\sqrt{\frac{\overline{23.60}^2}{64.4}\times3} \qquad - - - - - - 51", 82", 1004"$$

V_r = max. velocity of constrained recoil $.9V_f$ (app)

21.20, 34.20, 46.35

A = recoil constrained energy = $\frac{W_r V_r^2}{2g}$

$$\frac{1230\times\overline{21.20}^2}{64.4} \qquad 8580$$

$$\frac{1230\times\overline{34.20}^2}{64.4} \qquad 22400$$

$$\frac{1260\times\overline{46.35}^2}{64.4} \qquad 42000$$

E = recoil displacement during powder period.

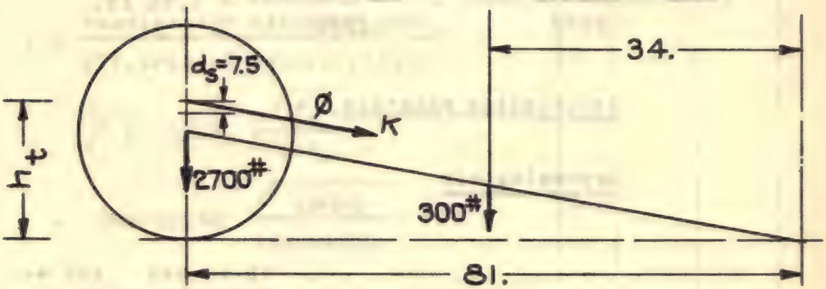
$$E = 2.24\left(\frac{W+0.5\overline{w}}{W_r}\right)\frac{u}{12} \text{ in ft.}$$

$$= 2.241\left(\frac{15+.5\times1.4}{1230}\right)\frac{109.5}{12} = \frac{2.24\times15.7\times109.5}{1230\times12} = .261\text{ft.}$$

$$= 2.24\left(\frac{15+.5\times3.}{1230}\right)\frac{109.5}{12} = \frac{2.24\times16.5\times109.5}{1230\times12} = .275 \text{ ft.}$$

$$= 2.24\left(\frac{33+.5\times3.25}{1260}\right)\frac{80}{12} = \frac{2.24\times34.63\times80}{1260\times12} = .41$$

STABILITY



$$l_s = l_s = \frac{2700\times81+300\times34}{3000} = 76.25\text{in.} = 6.35\text{ ft.}$$

c = constant of stability = $\dfrac{\text{overturning moment}}{\text{stabilizing moment}}$ = .96

$Ø_s$ = angle of stability 20°

d = moment arm of overturning force

$h_t \cos Ø + ds = l_s \sin Ø = 36\times0.9397+7.5-81\times0.3420$ =

13.60 in. = 1.13 ft.

b_h = length of recoil at the angle of stability

$$= \frac{W_s l_s + W_r E \cos Ø \pm \sqrt{(W_s l_s + W_r E \cos Ø)^2 - 4W_r \cos Ø \left(W_s l_s E - \frac{dA}{c}\right)}}{2W_r \cos Ø}$$

$$b_h = \frac{3000\times 6.35+1260\times 0.41\times .09397 \pm \sqrt{(W_s l_s + W_r E\cos Ø)^2 - 4\times 1260\times .9397(3000\times 6.35\times .41)\left(\frac{42000\times 1.13}{.96}\right)}}{2\times 1260\times 0.9397}$$

$$= \frac{19{,}540 \pm \sqrt{(19{,}540)^2 - 4740(7800-49500)}}{2370}$$

$$= \frac{19.540 \pm \sqrt{110{,}000{,}000}}{2370} = \frac{19.540 \pm 10490}{2370} = 3.75 \text{ ft.}$$

RECUPERATOR FORGINGS.

Approximately

	75 mm Gun Normal	75 mm Gun Super	105 mm How.
K_g = maximum resistance to recoil = $\frac{.45}{b}\frac{W_r}{g}V_f^2 = \frac{.45}{1.5}\frac{1230}{32.2}\overline{23.60}^2$	6380		
$\frac{.45}{1.5}\frac{1230}{32.2}\times\overline{38.}^2$		16500	
$\frac{.45}{2.5}\frac{1260}{32.2}\times\overline{51.5}^2$			18700
K_h = min. resistance to recoil = $\frac{0.47}{b_h}\frac{W_r}{g}V_f^2 = \frac{0.47}{b_h}\frac{W_r}{g}V_f^2 =$			
$\frac{0.47}{3.75}\frac{1230}{32.2}\times\overline{23.60}^2$	2660		

	75mm Gun Normal	75mm Gun Super	105 mm How.
$\frac{0.47}{3.75}\ \frac{1230}{32.20} \times \overline{38}^2$		6740	
$\frac{0.47}{3.75} \times \frac{1260}{32.2} \times \overline{51.5}^2$			13100
P_s = max. pull = K_s approximately	6380	16500	18700
P_h = min. pull = $K_h - 0.3\ W_r$	2290	6370	12730
K_v = initial recuperator reaction= $1.3W_r(\sin Ø_m + .3\cos Ø_m)$			
$= 1.3 \times 1260(.9848 + .3 \times .1836)$			
$= 1.3 \times 1260 \times 1.037$			1700
$r = \dfrac{\text{recuperator cylinder area}}{\text{eff.area of recoil piston}} =$			
$\frac{A_a}{A} = .039V_r \sqrt{\frac{P_s}{P_h - K_v}}$			
$= .039 \times 23.60 \sqrt{\frac{19000}{2290-1700}} =$			
$.945 \sqrt{\frac{19000}{590}} =$		5.35	
$r = .039 \times 38 \sqrt{\frac{1900}{6370-1700}} =$			
$.039 \sqrt{\frac{19000}{4670}} = 2.9$			
Assume $r = 3$			
$3 = .945 \sqrt{\frac{19000}{P_h - 1700}}$			
$10.1 = \frac{19000}{P_h - 1700}$			

$$P_h = \frac{19000+17150}{10.1}$$

$$P_h = 3560$$

- - - - - - - - - - -

$$m = \frac{P_{af}}{P_{ai}} = \frac{\text{final air pressure}}{\text{initial air pressure}} \text{ (generally) } 1.5$$

e = length of air column from chart 1.25 b

A = effective area of recoil piston =

$$\frac{P_s}{4500} = \frac{19000}{4750} = 4.00 \text{ sq.in.}$$

(Usually packing is designed to stand a pressure of 4500 to 5000 lbs)

P_s = max. pressure corresponding to r = 3

$P_m = \frac{19000}{4} = 4750$ lbs.

W_e = total weight of recoil piston and rods, 30 lbs.

T_L = max. tension on the rods at 0° elev. $= K_s + W_r \sin\phi_m + P_b \frac{W_e}{W_r}$

$19300 + 13.45 \times 24000 \times \frac{30}{1260} = 27000$

F_{max} = assumed fibre stress = $\frac{1}{2}$ elastic limit = $\frac{70000}{2} = 35000$ lbs/sq.in.

A_r = area of the recoil rod = $\frac{27000}{35000} = .772$ sq.in.

d_r = diameter of recoil rod = $1.127\sqrt{a_r} = 1.127\sqrt{.772}$

+.99in. make 1 in.

A_r = total area of recoil cylinder = 4. + .781 = 4.781 sq.in.

D_r = inside diameter of recoil cylinder = $1.127\sqrt{A_r} = 1.127\sqrt{4.781} = 2.46$ in.

$A_r = rA = 3 \times 4 = 12$ sq.in.

$D_a = 1.127\sqrt{A_a} = 1.127\sqrt{12} = 3.9$ in. diameter of floating piston.

CALCULATION OF PACKING FRICTION.

b = 0.18 in. to 0.25 in. use 0.21 in.

a = depth of outer silver flange of packing cup 0.14 in. to 0.16 in., use 0.14 in.

a' = depth of outer silver flange 0.18" to 22", 0.18"

c_1 = constant spring comp. of total packing friction

$$c_1 = \pi(D_r+d_r)[.05b+.09(a+\frac{a'}{2})]0.15\ P_{max}$$

$$= \pi(2.46+.781)[.05\times.21+.09(.14+.09)].15\ P_{max}$$

$$= 10.2(.0105+.0207).15P_{max}$$

$$=10.2\times.0312\times.15\times4750$$

c' = 226

$$C_r = \pi(D_r+d_r)[.05b+.09(a+\frac{a'}{2})].73$$

$$= \pi(2.46+.781)[.05\times.21+.09(.14+.09)].73$$

$$= 10.2\times.0312\times.73$$

C_2 = .232

R_p = total recoil packing friction = c_1+c_2p (p= lbs/sq.in)

226+.232 × 4750   1326 lbs.

FLOATING PISTON FRICTION

c_1' = cast(spring constant) of floating piston =

$$1.12\ \pi D_a[.05b+.09(a+\frac{a_1}{2})]\ p'_{af} = G\ P'_{af}$$

$$= 1.12\times\pi\times3.9[.05\times.21+.09(.14+.09)]p'_{af} = GP'_{af}$$

$$= 1.12\times\pi\times3.9\times.0312P'_{af}$$

$= .428P'_{af} = GP'_{af}$ (in lbs) P'_{af} = final air pressure

c_2' = pressure constant for fluid pressure comp. of total packing friction.

$$= 1.46\pi D_a[.05b+.09(a+\frac{a_1}{2})]$$

$$= 1.46\pi\times D_a\times.0312 = 1.46\times\pi\times3.9\times.0312$$

$$= .558$$

DESIGN OF RECUPERATOR

	75m/m Normal	Gun Super	105m/m How.
	6720	17600	19700

$$K_s = 1.05\left[\frac{W_r V_f^2}{2g}\,\frac{1}{b_s + (.096 + .0003d)\frac{uV_f}{v}}\right]$$

K_h = min. resistance to recoil

$$= \frac{W_r V_f^2}{2g}\,\frac{1}{b + (.096 + .0003d)\frac{uV_f}{v}}$$

$$= \frac{1230 \times \overline{23.60}^2}{64.4} \times \frac{1}{b_h + (.09b + .003 \times 2.953)\frac{109.5 \times 23.60}{1500}}$$

$$= 10640\,\frac{1}{b_h + .167} = (b_h = 2.4 \text{ ft.} = 29 \text{ in.}) \quad \text{----} \quad 4140$$

$$= \frac{1230 \times \overline{38}^2}{64.4}\,\frac{1}{3.75 + .098\left(\frac{109.5 \times 38}{2175}\right)}$$

$$= \frac{27600}{3.925} = \qquad 7030 \text{ lbs.}$$

$$= \frac{1260 \times \overline{51.5}^2}{64.4}\,\frac{1}{3.75 + .267}$$

$$= \frac{1260 \times \overline{51.5}^2}{64.4 \times 4.017} \qquad 12{,}900 \text{ lbs.}$$

$$R_{max} = c_1 + c_2\,P_{max} \qquad P_{max} = \frac{K_s}{A}$$

$$226 + .232\,\frac{6720}{4} \qquad 616 \quad 1236 \quad 1366$$

1 = distance between clip reactions 60in.

r' = distance from center of gravity of recoiling parts to mean friction line. 6.5in.

n = coefficient of guide friction (.1 to .2) .15

d_b = distance from center of gravity of recoiling parts to axis of piston rod 7.5in.

PULL ON THE ROD

	75 m/m Normal	Gun Super	105 m/m How

P_s=pull at max. elevation

$$= \frac{K_s + W_r \sin\emptyset}{1 + \frac{2nd_b}{1-2nr}} - R_{max}$$

$$= \frac{6720+1230\times.9848}{1+\frac{2\times.15\times7.5}{60.2\times.15\times6.5}} - 616$$

$$\frac{6720+1230\times.9848}{1.+.0388}$$

$\frac{7930}{1.0388} - 616$	7044		
$= \frac{17600\times1210}{1.0388} - 1236$		16900	
$= \frac{19700+1240}{1.0388} - 1366.$			18800

P_h=pull at horizontal elevation in lbs.

3560

$$P_h \left(1+ \frac{C_2}{A}\right) = \frac{K_h}{1+\frac{2nd_h}{1-2nr}} - C_1$$

$$P_h(1+\frac{.232}{4})=\frac{K_h}{1.0388}-226$$

$$P_h=(\frac{K_h}{1.0388}-226).945$$

$(3560\times1.0385+226)1.0388=K_h=4140$

$$P_h=(\frac{7030}{1.0388}-226)\times.945 \quad\text{-----------} \quad 6180$$

$$P_h=(\frac{12900}{1.0388}-226)\times.945 \qquad 11550$$

R = excess recuperator battery reaction constant 1.2

(1.1 to 1.3)

K_v= recuperator reaction in battery at max. elevation

$$K_v=\frac{R(W_r\sin\emptyset_m+c_1)}{1-R(\frac{2nd_b}{1+2nr}+\frac{C_2}{A})}=\frac{1.2(1260\times.9848+226)}{1-1.2(\frac{2\times.15\times7.5}{60+2\times.15\times6.5}+\frac{.232}{4})}$$

$$\frac{1.2\times 1466}{1-1.2(.0363+058}=\frac{1760}{1-.1128}=\frac{1760}{.8872}$$

$\underline{K_v = 1980}$

$V_r = .92V_f=.92\times23.6=21.7=V_r$; 35.00 = 47.40

$$r=\frac{A_a}{A}=2.625\ V_r\sqrt{\frac{P_s}{P_p(P_h-K_v)}}=2.625\times21.7\sqrt{\frac{18800}{4750(3560-1980)}}$$

$$=57\sqrt{\frac{18800}{4750\times1580}}=57.\times.05$$

$\underline{r = 2.85}$

$$r_{min} = 2.625\ V_r \sqrt{\frac{18800}{4750(6180-1980)}} = 91.6 \sqrt{\frac{1880}{4750\times4200}}$$

$\underline{r_{min} = 2.79}$

$$m = \frac{P_{af}}{P_{ai}} = 1.5 = m$$

$l = 1.3b$

$$A\ \frac{P_s}{4750} = \frac{18800}{4750} = 3.96$$

$A_a = rA = 2.85\times3.96 = 11.30$ sq.in. $= A_a$

P_h = min. pull on the rod

V_r = velocity of recoil corresponding

$\underline{R = 1.295}$

W_r

A

$$W_r^2 = (.0373\ rA)^2 = \frac{R^2 A^3 V_r^2}{175(P_h - K_v)} = .00139 r^2 A^2$$

$$r^2 = \frac{R^2 A\ V_r^2}{175\times.00139(P_h - K_v)} = \frac{\overline{1.295}^2}{.2435}\ \frac{AV_r^2}{(P_h - K_v)}$$

P_p = pressure the packing should withstand

$$A = \frac{P_s}{P_p}$$

$$r^2 = 6.9\ \frac{P_s V_r^2}{P_p(P_h - K_v)}$$

$$r = 2.625\ V_r \sqrt{\frac{P_s}{P_p(P_h - K_v)}}$$

\- - - - - - - - - - -

j = length of air column in terms of max. recoil length $= \dfrac{l_a}{b_h} = \dfrac{1.3b}{b}$

$j = 1.3$

$m = \left(\frac{rj}{rj-1}\right)^{1.3}$ from the chart 1.5

P_{ai} = initial air pressure (lbs/sq.in) = $\frac{K_v}{A} = \frac{1980}{3.96}$

P_{ai} = 500 lbs/sq.in. (approx.)

$P'_{af} = m\ P_{ai} = 1.5 \times 500 = 750 \times P_{af}$ approx.

V'_o = initial air volume = $A_a \times l_a = 11.30 \times 45 =$

662 cu.in. = V_o

$$V_o = \left(\frac{m^{0.77}}{m^{0.77} - 1}\right) A\ b_h$$

INITIAL AND FINAL AIR PRESSURE AND

AIR VOLUME.

$P_{ai} = 500$

R_1 = floating piston friction initial

$= C'_1 + C'_2\ P_{ai} = .428 \times P'_{af} + .558 \times 500$

$= .428 \times 700 + .558 \times 500$

$= 321 + 279$

$R_1 = 500$

$$P_{ai} = P'_{ai} + \frac{C'_1 + C'_2 P'_{ai}}{A_a} = 500 = \frac{500}{11.30} = 500 + 44.2$$

P_{ai} = 544 lbs/sq.in. P_{ai} = 540 lbs/sq.in.

$P_{af} = m\ P_{ai} = 1.5 \times 544 = 816$ P_{af} = 810 lbs/sq.in.

V_o =662 sq.in.

V_f = final air volume = $V_o - Ab_h = 662 - 3.96 \times 45 = 662 - 178$

V_f = 484 cu.in.

P_a = average drop of pressure across floating piston = $C_1 + C_2 (P_{ai} + P_{af}) 0.5$

$$= .428 \times \frac{540+810}{2} + .558 \times \frac{540+810}{2}$$

$$= (.428+.668)675 = .986 \times 675$$

P_a = 665 lbs.

W_c = 30 lbs.

T_L = tension on the rod $= K_s + W_r \sin\emptyset + P_b \dfrac{W_e}{W_r}$

$$= 188000 + 1260 \times .9845 + 13.45 \times 24000 \times \frac{30}{1260}$$

$$= 18800 + 1240 + 7700$$

T_L = 27750 lbs.

F_{max} = assume fibre stress 1/2 elastic limit =

$$\frac{65000}{2} = 32500 \text{ lbs/sq.in} = F_{max}$$

A_r = area of the recoil rod $= \dfrac{27750}{32500} = .853$ sq.in.$= a_r$

d_r = diameter of recoil rod $1.127\sqrt{.853} = 1.04$in$= d_r$

A'= area of the recoil cylinder = 3.96+1.04 =
500 sq.in.= A'

$D_r = 1.127\sqrt{5} = 2.52$in.$= D_r$

$D_a = 1.127\sqrt{11.30} = 3.78$in.$= D_a$ diameter of air cylinder
W_c = 30 lbs.

STRENGTH OF CYLINDERS:

Test pressure = 2 × service pressure.

P_t = max. allowable fibre stress for cylinders =
3/8 elastic limit = 3/8 × 60,000 = 22500 lbs/sq.in.$= P_t$

R_{ao} = min. outside diameter of recuperator

$$= R_a \sqrt{\frac{P_t + P_{af}}{P_t - P_{ai}}} = 1.89 \sqrt{\frac{22500+810}{22800+810}} = 1.89\sqrt{\frac{23300}{21700}}$$

$$= 1.958$$

D_{ao} = 3.92 in.

$$R_{ro} = 1.26\sqrt{\frac{22500+4750}{22500-4750}} = 1.26\sqrt{\frac{27250}{17750}} = 1.26\sqrt{1.535}$$

$=1.561$

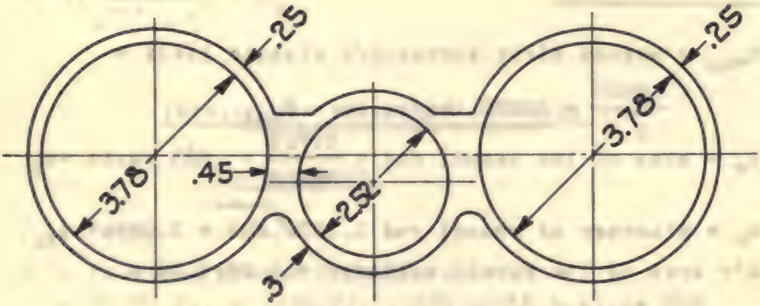
$D_{ro} = 3.122$ in.

W = min. width between adjacent cylinders

$$= \frac{P_{max}D_r + P_{af} \times D_a}{1.5P_t} = \frac{4750 \times 2.52 + 810 \times 3.78}{1.5 \times 22500}$$

$$= \frac{11950+3060}{33750} = .445 \text{ in.}$$

$W = .445$



MAXIMUM AND MINIMUM THROTTLING AREA.

	75 m/m Gun		105m/m
	Normal	Super	How.

W_h = maximum throttling area (at 0°_s elev.)

$$= \frac{.098A^{\frac{3}{2}}V_r}{\sqrt{P_h - K_v}}$$

$$= \frac{.098 \times \overline{3.96}^{\frac{3}{2}} \times V_r}{\sqrt{P_h - 1980}}$$

$$= \frac{.772V_r}{\sqrt{P_h - 1980}}$$

$$= \frac{.772\times21.7}{(3560-1980)^{\frac{1}{2}}} = \frac{16.72}{39.75} \qquad .422 \text{ sq.in.}$$

$$= \frac{.772\times35.}{(6180-1980)^{\frac{1}{2}}} = \frac{27}{64.81} \qquad 417 \text{ sq.in.}$$

$$= \frac{.772\times47.4}{(11550-1980)^{\frac{1}{2}}} = \frac{36.60}{97.83} \qquad .374 \text{ sq.in.}$$

W_h = .422 sq.in.

W_s = minimum throttling area (at 80° elev.)

$$= \frac{.772\ V_r}{\sqrt{P_s - 1980}} = \frac{16.72}{(7044-1980)^{\frac{1}{2}}} = \frac{16.72}{71.16}$$

.237 sq.in.

$$\frac{27}{(16900-1980)^{\frac{1}{2}}} = \frac{27}{122.2} \qquad .221 \text{ sqin.}$$

$$\frac{36.6}{129.7} = \frac{36.60}{(18800-1980)} = \qquad 282 \text{ sq.in}$$

$$= \frac{.221+.282}{2} = .25 \text{ sq.in} = W_s$$

LAYOUT OF RECUPERATOR FORGING

l_f = overall length of forging = 1.5 × b_h = 1.5 ×45 = 67.5 in.

D_r = diameter of recoil cylinder 2.52 in.

D_a = diameter of air cylinder 3.78 in.
l_a = length of air column 58.5 in.
W_r = area of connecting channel between recoil and recuperator cylinder.

$W_a = 3.5$ to $4.3\ W_h$

$= 4 \times .422 =$

$W_a = 1.70$ sq.in. $d_a' = 1.468$ in.

D' = maximum depth of recuperator cylinder below recoil cylinder =

$$D_r - \frac{d_a'}{2}$$

$= 2.52 - .734$

$D' = 1.786$ in.

W_c = const. channel area from regulator valve = $4.3\ W_h = 4.3 \times .422$

$W_c = 1.814$

h_o = depth of const. channel area, W_c in inches.

$h_o = 0.2 D_a = .2 \times 3.78 = .756$ in. $= h_o$

a = extreme area of regulator valve =

$$73.5\ \frac{W_h^2}{D_a^2} = \frac{73.5 \times \overline{.422}^2}{(3.75)^2}$$

a = .933 sq.in.

d_a = diameter of entrance channel $- 9.675\ \dfrac{W_r}{D_a} =$

$$0.675\ \frac{422}{3.75} = 1.09 \text{ in.} = d_a$$

DESIGN OF REGULATOR.

a = area at base of regulator valve $= 73.5\ \dfrac{W_h^2}{D_a^2} =$

$$\frac{73.5 \times \overline{.422}^2}{(3.75)^2} - .93 \text{ sq.in} = a$$

d_a = diameter of a, $9.875\ \dfrac{W_h}{D_a} = 9.675\ \dfrac{.422}{3.75} = 1.09 = d_a$

d_{a_1} = diameter of upper and lower valve stem $= 0.6 d_a =$

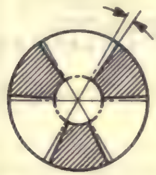
$.6 \times 1.09 = 0.655 = d_{a_1}$

a_1 = cross section of stem = $0.36a = .36 \times .93$ =

$.335 \text{ sq.in.} = a_1$

d'_{a_1} = diameter of inside foot of valve stem = $.5$ to $.6\ d_{a_1}$

$= .55 \times .655 = .36 \text{ in.} = d'_{a_1}$



c' = length of flap at base of scale = $1.571\ d_a - .4725$

$1.571 \times 1.09 - .4725 = 1.237$ in.

c = effective circumference = $1.571 \times 1.09 + .4725$

$2.184 \text{ in.} = c$

R_t = reaction on the valve at short recoil

$R_b + R_s = (p - P'_{ai})a + P_{ai}a_1 = (4750-500).93 + 500 \times .335$

$= 4250 \times .93 + 500 \times .335$

$= 3950 + 170$

$R_t = 4120$

SPIRAL SPRING DESIGN FOR REGULATOR VALVE.

f_s = maximum torsional fiber stress 120,000 lbs/sq.in.

N = torsional modulus 10,500,000 lbs/sq.in.

d_s = let diameter of the spiral be 4 diameter of the wire

$$d_s^3 = \frac{8PD}{\pi f}, \quad d^2 = \frac{32P}{\pi f} = \frac{10.16P}{f}$$

$d_s = 3.19 \sqrt{\frac{P}{f}} = 3.19\left(\frac{496}{120000}\right)^{\frac{1}{2}} = 3.19 \times .0643$

$\underline{d_s = .205 \text{in.}}$

D_s = diameter of helical spring $4 \times d = \underline{D_s = .82 \text{in.}}$

f = deflection per coil =

$$\frac{\pi f_s D_s^2}{G d_s} = \frac{\pi \times 120000 \times \overline{.82}^2}{10{,}500{,}000 \times .205}$$

f = .118

$\frac{496}{.118}$ = 4200 lbs. per inch of deflection required

$\frac{248}{.213}$ = 1162 spring const.

$\frac{4200}{1162}$ = 3.61 effective coils

n = no. of coils = 3.61 + 1 = 4.61 $\underline{\text{use 4.5 coils}}$

R_s = load on spiral springs at 0° elev. = $\frac{P_h}{A} - P_{ai}$

$= \left(\frac{3560}{3.96} - 500\right).93 = 400 \times .93$

$\underline{R_s = 372. \text{lbs.}}$

h" lift of valve at long recoil = $\frac{W_r}{c}\ \frac{.422}{2184}$ = .193

$\underline{h" = .213 \text{ inches}}$ Valve seat clearance = $\underline{.02}$

.213in.

h" = lift at short recoil = $\frac{W_s}{c} = \frac{25}{2.184}$ = .1144

$\underline{h" = .1144 \text{in.}}$

R_{sc} = load at solid height of apiral spring $\frac{4}{3} \times 372$ =

496 lbs.

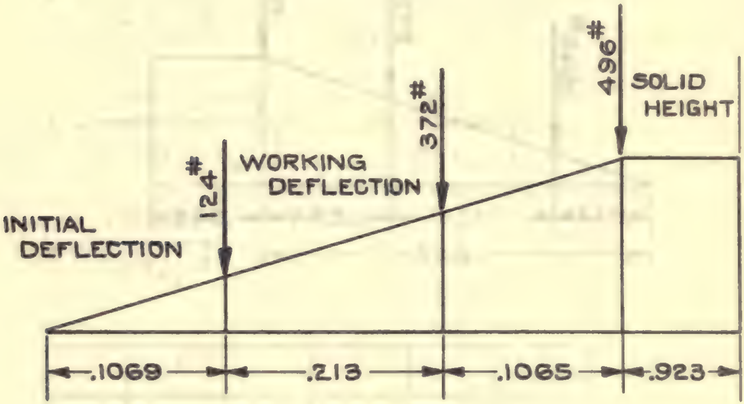
h_{sc} = deflection from free to solid height of spiral = 2h" = 2×.213

$\underline{h_{sc} = .426 \text{in.}}$

S_s = spiral spring const. $\frac{R_{sc}}{h_{sc}} = \frac{496}{426} = \underline{1162 \# = S_s}$

R_s' = spiral spring reaction at short recoil =
$S_s(h''+.02) = 1140(.1144+.02)$

$\underline{R_s' = 153 \text{ lbs.}}$



R_b = load on Belleville at short recoil

$= (P-P'_{ai})a+a'P_{ai}-R_s'$

$= (\frac{18800}{3.96} - .500).93 - .355 \times 500$

$= (4750-500).93+177.5$

$\underline{R_b = 3770 \#}$

R_{bo} = load at solid height of Belleville washers

$= \frac{3}{2} \ R_b = \frac{3}{2} \times 3770$

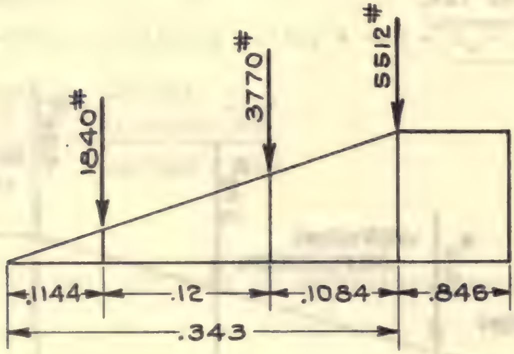
$\underline{R_{bo} = 5650 \#}$

h_{bo} = deflection of Belleville from free to solid height $3h' = 3 \times .1144$

$\underline{h_{bo} = .343 \#}$

S_b = Belleville spring const. = $\frac{5512}{.343}$ = 16080

S_b = 16080



DESIGN OF CAM MECHANISM AND LAYOUT.

g = ratio of cam movement to valve movement usually 5.

X = distance valve should lift to engage Bellevilles

S_b = working deflection

h_s = initial comp.of spiral spring

h_o = clearance of valve

h = lift of valve

h_b = initial compression of the Bellevilles.

$$X = \frac{1}{S_b}\left\{S_s(h_s+h_o+h)+S_b(h_b+h_o+h)-\left[(P-P_{ai})a+P_{ai}a'\right]\right\}$$

$R_s+R_b=(P-P_{ai})a+P_{ai}A_1$

$R_s=S_s h_s+S_s(h+h_o)$ (Stems of two springs are in contact)

$R_b=S_b h_b+S_b(h+h_o)$

$$\left.\begin{array}{l} 1162\times.1065+11.62(h+.02) \\ +16080\times.1144\times16080(h+.02) \end{array}\right\} (P-P_{ai}).93+500\times.355$$

$$124 + 1162h + 23 + 1840 + 16080h + 322 = (p - p_{ai}).93 + 177.2$$

$$17242h + 2132 = (p - p_{ai}).93$$

$$h = \frac{(P - P_{ai}) - 2290}{18500} \qquad (1)$$

$$W = \frac{.098AV_r}{(P - P_{ai})^{\frac{1}{2}}} = \frac{.098 \times 3.96 \times V_r}{(P - P_{ai})^{\frac{1}{2}}} = ch = 2.184\ h$$

$$h = \frac{.098 \times 3.96 \times V_r}{2.184(P - P_{ai})^{\frac{1}{2}}} = .178\frac{V_r}{(P - P_{ai})^{\frac{1}{2}}}$$

$$h = \frac{.178V_r}{(P - P_{ai})^{\frac{1}{2}}}$$

a	b		c		
$P - P_{ai}$	a-2290	h	$(P - P_{ai})^{\frac{1}{2}}$	$h(p - p_{ai})^{\frac{1}{2}}$	V_r
2660	360	.0194	51.58	1.00	5.60
3000	700	.0378	54.77	2.07	11.62
3250	950	.0512	57.01	2.92	16.40
3500	1200	.0648	59.16	3.83	21.50
3750	1450	.0781	61.24	4.78	26.90
4000	1700	.0917	63.25	5.80	32.60
4250	1950	.1050	65.19	6.85	38.50
4500	2200	.1188	67.08	7.95	44.60

LENGTH OF RECOIL

80° Elevation

Normal

V_r = 21.70 corresponding $(P - P_{ai})$ from curve 3500

$P_s = (3500 + .500)3.96 = 4000 \times 3.96 = 15820 = P_s$

$$R_{max} = c_1 + c_2 p$$

$$=226+.232\times4000=226+930$$

$$R_{max}=1160 \text{ lbs.}$$

$$P_s = \frac{K_s + W_r \sin\emptyset}{1+\frac{2nd_s}{1-2nr}} - R_{max}$$

$$15820 = \frac{K_s + 1230\times.9848}{1+.0388} - 1160$$

$$16450=K_s+1210-1200$$

$$K_s = 16440 = 1.05\left[\frac{W_r V_f^2}{2g}\,\frac{1}{b_s+(.096+.0003d)\frac{uV_f}{v}}\right]$$

$$16440 = \frac{1.05\times10640}{b+.167} = \frac{11200}{b+.167}$$

$$16440b+2750=11200$$

$$b = \frac{8450}{16440} = .513$$

$$b = 6.17\text{in.}$$

Super

V_r = 35 corresponding $(p-p_{ai})$=4100 lbs.

$$P_s = (4100-500)\times3.96=4600\times3.96$$

$$P_s = 18200\#$$

$$= \frac{K_s + W_r \sin\emptyset}{1+\frac{2nd_s}{1-2nr}} - R_{max}$$

$$R_{max} = C_1+C_2p = 226+.232\times4600=226+106$$

$$R_{max} = 1300 \text{ lbs.}$$

$$P_s = 18200 = \frac{K_s+1210}{1.0388} - 1300$$

$$18880=K_s+1210-1350$$

$$K_s = 19020 \text{ lbs.} = 1.05 \left[\frac{W_r V_f^2}{2g}\right.$$

$$= \frac{1.05\times 27600}{b+.145}$$

$$19020b+2760=29000$$

$$b = \frac{26240}{19020} = 1.380$$

$$b = 16.55 \text{in.}$$

<u>Howitzer</u>

V_f=47.30 ft/sec. $(P-P_{ai})$=4620 lbs.

$$P_s = 5120 \times 3.96 = 20250 \text{ lbs.} =$$

$$\frac{K_s+W_r\sin\emptyset}{1+\dfrac{2nd_b}{1-2nr}} - R_{max}$$

$$R_{max} = C_1+C_2P = 226+.232\times 5120 = 226+1190$$

$$R_{max} = 1420 \text{ lbs.}$$

$$P_s = 20250 = \frac{K_s-1240}{1.0388} - 1420 = 21000 = K_s-1240-1470$$

$$K_s = 21230 \text{ lbs.} =$$

$$21230 = \frac{1.05\times 51900}{b+.267}$$

$$21230b+5630=54500$$

$$b' = \frac{48800}{2123} = 2.3 \text{ feet}$$

$$b'' = 27.6 \text{ inches.}$$

$$b = \frac{b_h-b_s}{\emptyset_m-\emptyset_i}(\emptyset_m-\emptyset)+b_s = \frac{45-30}{60}(80-\emptyset)+30 =$$

$$\tfrac{1}{4}(80-\emptyset)+30$$

$$b'' = \frac{1}{4}(80-\emptyset)+30$$

$$K_s = \frac{1.05\times 51900}{b+.267} = \frac{54500}{\frac{b''}{12}+.267} = \frac{655000}{b''+3.21}$$

$$K_s = \frac{655000}{b''+3.21}$$

$$P_s(1+\frac{C_2}{A}) = \frac{K_s+W_r\sin\emptyset}{1+\frac{2nd_s}{1-2nr}} - C_1$$

$$P_s(1+\frac{.232}{3.96}) = \frac{K_s+1260\sin\emptyset}{1.0388} - 226$$

$$P_s = \frac{K_s+1260\sin\emptyset}{1.098} - 212.$$

$$h = \frac{.178\ V_r}{(P-P_{ai})^{\frac{1}{2}}} = \frac{.178\times 47.30}{(P-P_{ai})^{\frac{1}{2}}} = \frac{8.43}{(P-P_{ai})^{\frac{1}{2}}}$$

$$h = \frac{.178\ V_r}{(P-P_{ai})^{\frac{1}{2}}}$$

$\emptyset^\circ$	20	25	35	50	65	75	80
b''	45	43.75	41.25	37.5	33.75	31.25	30
K_s	13600	13950	14710	16050	17720	19000	19700
$\sin\emptyset$	.3420	.4226	.5736	.7660	.9063	.9659	.9848
$K_s+W_r\sin\emptyset$	14030	14480	15430	17020	18860	20220	20940
$\frac{K_s W_r \sin\emptyset}{1.098}$	12800	13200	14100	15500	17200	18900	19100
P_s	12600	13000	13900	15300	17000	18700	18900
(P_s-P_{ai})	10600	11000	11900	13300	15000	16700	16900
$\frac{P_s-K_v}{3.96}$	2680	2780	3000	3360	3790	4220	4270

$(P-P_{ai})^{\frac{1}{2}}$	51.77	52.73	54.77	57.97	61.56	64.96	65.35
h	.1628	.1596	.1536	.1452	.1368	.1297	.1286

$$X = \frac{1}{S_b}\left\{S_s(h_s+h_o+h)+S_b(h_b+h_s+h)-[(P-P_{ai})a+P_{ai}A_t]\right\}$$

$$= \frac{1}{16080}\left\{1162(.1065+.02+h)+16080(.1144+.02+h)-[(p-p_{ai})\right.$$

$$\left. .93+500\times.355]\right\}$$

$$= \frac{1}{16080}\left(1162h+147+16080h+2150[(\frac{P_s-1980}{3.96})\ .93+178]\right.$$

$$= \frac{1}{16080}(17240h+2300-.235P_s-290) = \frac{1}{16080}(17240h-235P_s$$

$$+2010$$

$$= 1.072h-(.00001463P_s-.1251)$$

Ø	h	1.072h	P_s	$\frac{1.463}{105}P_s$	b	X	5X
20°	.1628	.1749	12600	.1842	.0591	.1158	.580
25°	.596	.1715	13000	.1900	.0649	.1060	.530
35°	.1536	.1648	13900	.2034	.0783	.0870	.435
50°	.1452	.1559	15300	.2390	.1139	.0420	.210
65°	.1368	.1469	17000	.2490	.1264	.0205	.103
75°	.1297	.1392	18700	.2735	.1484	.0008	.004
80°	.1286	.1380	18900	.2765	.1514	.0000	.000

Counter Recoil

Buffer, constant orifice and port Areas.

R_p' = packing friction at end of counter recoil C_t = 226 lbs.

d_b = recoil length during buffer action =

$$\frac{0.238W_r}{.15W_r+R_b'}$$

$$= \frac{.238\times 1260}{15\times 1260+226} = \frac{300}{189+226} = \frac{300}{415}$$

d_b = .723 ft = 8.7 in.

d_b' = length of buffer rod = $1.2 \times d_b \times \frac{A}{A_a} = 1.2\frac{d_b}{r}$

$$= \frac{1.2 \times .723}{2.9} = .3 \text{ ft} = d_b = 3.6 \text{ in.}$$

Length of buffer chamber = 1.2 to 1.3 d_b'

d_b'' = 4.5 in.

V_ϕ = allowable counter recoil velocity 2.5 to 3.5 ft/sec.

R_t = total counter recoil friction - max. elev.

$$= C_1 + C_2\left[\frac{W_r(\sin\phi+0.3\cos\phi_m)}{A}\right]+0.3W_r\cos\phi_m$$

$$= 226+.232\left[\frac{1260(9848+.3\times .1736)}{3.96}\right] +.3\times 1260 \times .1736$$

$$= 226 + .232(.2615 \qquad)+66$$

R_t = 290 lbs. Max. elevation

$$C_1+C_2\left(\frac{.3W_r}{A}\right) +.3W_r = 226+96+378$$

R_t' = 700 lbs. Min. elevation.

P_a' = max. recuperator pressure = $P_a' = \frac{P_{ai}+P_{ap}}{2}$

$$= \frac{500+750}{2} = 625$$

W'_o = c'recoil orifice at 80° elevation

$$= \frac{KA^{\frac{3}{2}}V\phi_m}{13.2\sqrt{P'_aA-R_t-W_r\sin\phi_m}} \qquad (K=1.25)$$

$$= \frac{1.25\times\overline{3.96}^{\frac{3}{2}}\times 2.5}{13.2\sqrt{625\times3.96-290-1260\times9848}} = \frac{24.6}{13.2\sqrt{970}} = \frac{24.6}{411}$$

$$\underline{\underline{W'_o = .06 \text{ sq.in}}}$$

$$W''_o = \frac{KA^{\frac{3}{2}}V\phi_i}{13.2\sqrt{P'_aA-R'_t}} = \frac{34.5}{13.2\sqrt{2470-700}} = \frac{34.5}{13.2\times38.35}$$

$$= \frac{34.5}{506}$$

$$\underline{\underline{W''_o = .0682 \text{ sq.in.}}}$$

$$W_o = \frac{.06+.0682}{2} = \underline{\underline{.0641 \text{ sq.in.}=W_o}}$$

$$W_b = \text{entrance buffer area} = \frac{KAV}{13.2}\sqrt{\frac{1}{P'_a - \frac{0.00894A^2V^2}{W_o^2}}}$$

$$= \frac{1.33\times3.96\times3.5}{13.2}\sqrt{\frac{1}{625 - \frac{.00894\times\overline{3.96}^2\times\overline{3.5}^2}{(.0641)^2}}}$$

$$1.395\sqrt{\frac{1}{625-418}} = \frac{1.395}{14.4} = .097 \text{ sq.in.}$$

Lay out entrance area of buffer, with required depth of groove, decrease depth of groove to zero at end of buffer d'_b.

C H A P T E R X.

RAILWAY GUN CARRIAGES.

TYPES OF MOUNTS. For coast defense or other use of heavy artillery, it has been accepted that mobility is of great importance.

Materiel in permament emplacements is more readily subjected to attack. Further with long coast lines it is impracticable to supply enough permament batteries for adequate protection. By introducing heavy mobile artillery we increase the protection and develop the advantage of concentrated fire at any one point when needed.

Railway artillery meets the demand for mobility in a very satisfactory degree. Very heavy weights, as occur with large caliber guns and their corresponding mounts, are most readily transported by rail. Hence there has been a tendency of development along two lines; first, a mobile railway carriage that is entirely self contained and fired directly from the rail and (2) a mobile mount, transported by rail but set up on a semi-fixed emplacement. For extreme mobility the first is most useful, wherein for coast defense work the second plan offers many advantages.

Railway carriages have been developed along the following lines. in their methods of firing.

(1) Sliding carriage type with no recoil mechanism , the carriage merely sliding back during the recoil along special constructed rails or guides, trucks being disengaged.

(2) Railway carriages with a recoil system. the whole carriage in addition recoiling on special ways on

rails, the trucks being disengaged, or the trucks being engaged and the secondary recoil being directly along the rails.

(3) Fixed or platform mounts. With light railway artillery, the car is held stationary by suitable outriggers and we have usually a barbette type of mount, mounted on the car. With heavier types, the girder which supports the tipping parts is placed on a large pintle bearing with sometimes additional support at the tail of the girder with a circular way or track for all round or sufficient traverse. In this latter type the trucks must be disengaged and the main girder run on to the permament emplacement.

The sliding carriage type (1), was developed successfully in France and was considered satisfactory during the late war. This mount, however, is subjected to the direct firing stresses with consequent requirements for a very heavy girder and trunnion support. It has on the other hand the advantage of doing away with a recoil system. At best, however, it can be regarded merely as an emergency type of carriage that might be developed under great stress of war pressure and not suitable for use against moving targets.

In railway carriages of type (2), we have virtually a double recoil system. However, since the recoil is designed for stationary service as well, or for the condition at max. elevation where the secondary recoil is small, the maximum reactions at the beginning of the recoil are the same as in a stationary mount, with a single constant recoil. When the trucks are disengaged a specially built track must be laid, and the

girder slides back on friction shoes, which are lowered to engage with the track. Mounts of this type are illustrated in our 14" railway mount ME. When the trucks are not disengaged and the secondary recoil takes place on the track, the bearing reactions of the truck wheels must be suitably designed to sustain the additional firing load and the trucks must be suitable braked to resist the secondary recoil, and bring the mount to rest after the firing. When a built up track, trucks disengaged, is used the successive firings must necessarily take place along the tangent of the track, whereas firing directly from the rails, permits the use of a curved or Y track, and considerable traversing is thus possible by the firing taking place at different points on the curved track. With railway carriages of type (2) very little traversing is possible on the mount itself and therefore the track must be laid very closely in the direction of firing. In railway carriages of type (2), we are greatly limited by road clearance. For clearance, the trunnions must therefore be in the traveling position in a low position. On firing however, at maximum elevation, the recoil becomes limited. To provide for a suitable recoil at maximum elevation the trunnions are raised and a balancing gear throwing the trunnions to the rear may also be introduced.

With fixed or platform mounts of type (3), the special features are the methods of erection on to a semi permament emplacement and the disengagement from the traveling condition of the mount. We may have a center turn table which serves for the pintle in traversing and the tail of the girder is supported by a suitable circular guide which balances the overturning moment and thus releases the otherwise bending or overturning moment on the pintle bearing.

With this type of mount large traversing is completely possible.

SPECIAL FEATURES IN THE DESIGN.

Recoil System:

(1) The recoil should be simple and rugged.

(2) A constant recoil or approximately constant for all elevations should be used.

(3) A constant resistance to recoil is satisfactory since questions of stability are not usually of prime consideration, and the recoil is thus simplified.

(4) The counter recoil should be simple, an ordinary spear buffer being usually satisfactory although other control may be sometimes necessary. Here again counter recoil stability is no longer a consideration and high velocities of counter recoil are not objectionable provided there is no shock at end of counter recoil.

(5) With very large guns used at high elevations, high pressure pneumatic recuperator systems should be used in place of spring columns, since the weight and bulk of springs become excessive.

(6) Sleeve guides for the gun have been found most suitable and the various pulls should be so far as possible symmetrically spaced about the axis of the bore, thus reducing the bearing reactions in the sleeve and making it also possible to keep

the center of gravity of the recoiling parts close to the axis of the bore.

Tipping parts:

(1) The cradle should be of the sleeve type thus reducing the bearing pressures over guides and clips.

(2) The recoil and recuperator can be strapped on with suitable shoulders for bearing surface to take up the recoil load from the cylinders.

(3) The trunnions should be located near line through the center of gravity of the recoiling parts and parallel to the axis of the bore. This reduces the elevating reaction during the pure recoil to merely that due to the moment effect of the recoiling parts out of battery.

(4) Great effort should be made to locate the center of gravity of the recoiling parts as near the axis of the bore as possible either by symmetrically distributing the recoil rods and attachments or if necessary introducing counter balancing weights. Thus the whipping action during the powder period is reduced with a corresponding reduction in the elevating arc reaction during the powder period.

(5) With high angle fire guns or howitzers, the trunnions may be thrown to the rear, and balancing

gear introduced, thus making long recoil possible. Another plan for accomplishing the same results is to raise the trunnions before firing.

(6) The trunnion bearings should be supported on springs during traveling, though compressed so we have solid contact during firing.

(7) To reduce the friction during the elevating process, ball or roller bearings should be introduced in the trunnion bearings, or in an inner trunnion should be introduced of smaller radius than the main trunnion for reducing friction on rotating the tipping parts.

LIMITATIONS IN BRAKE LAYOUT. With heavy artillery mounts, either railway or for permament or mobile emplacements, counter recoil stability is not a consideration.

On the other hand we are limited to a maximum allowable buffer pressure in the counter recoil. With counter recoil systems which come into action towards the end of counter recoil, practically the entire potential energy of the recuperator must be dissipated by the buffer over a relatively short displacement. Now since the potential energy of the recuperator is a considerable fraction of the energy of recoil, we see that the buffer reaction is of a magnitude comparative with the brake resistance during the recoil. Further the effective area of the c'recoil buffer, due to constructive limitations, is necessarily considerably smaller than the effective area of the recoil brake. Hence the buffer pressures with a short c'recoil buffer, become very great. This is especially pronounced with a short buffer and high

angle fire gun where the unbalanced recuperator energy is necessarily great, when the gun c'recoils at low elevations. As to the limiting allowable buffer pressures, no hard and fast rule can be made, but it is certain that the buffer pressures in many of our mounts are rather too high for light construction, requiring heavy and strong buffer chambers.

With recoil brakes having a continuous rod extending through both ends of the cylinder, the effective area of the buffer must be necessarily very small and the stroke of the buffer short due to the fact that during the recoil it is important that the void displacement be not too great. Hence this type of brake with continuous rod and enlargement for c'recoil buffer ram, has inherently excessive buffer pressures. It is very important with such mounts to maintain a minimum recuperator energy, that is to use the minimum recuperator reaction combined with a low ratio of compression, consistent with proper c'recoil at maximum elevation.

To reduce the buffer pressure, the c'recoil regulator should be effective throughout the recoil, and the effective area of the buffer should be as large as possible. This actually has been obtained constructively in our 16 inch railway mount, the buffer area being equal to that of the recoil brake and c'recoil regulation taking place throughout the recoil. The buffer pressures are therefore comparable with the brake pressures during recoil.

DESIGN LAYOUT OF RECOIL SYSTEMS. Assuming a preliminary layout has been made, the weight and the ballistics of the gun given, we may estimate from previous mounts, the probable weight of the recoiling parts and tipping parts.

Therefore, we will assume the following data given or estimated from previous mounts:

W_r = weight of recoiling parts (estimated)(lbs)
d = diameter of bore (in)
v = muzzle velocity (ft/sec)
w = weight of projectile (lbs)
$\bar{w}$ = weight of charge (lbs)
p_{bm} = maximum powder pressure (lbs/sq.in)
b = mean length of recoil (ft)
ϕ_m = maximum angle of elevation
ϕ_i = minimum angle of elevation
u = travel up the bore of the projectile (ft)

Calculation of E and T:

From the principle of Interior Ballistics, we have,

$$P_m = \frac{\pi}{4} d^2 p_{bm} = \text{max. total pressure on breech (lbs)}$$

$$P_e = \frac{wv^2}{2gu} = \text{average force on projectile (lbs)}$$

$$e = u\left[\left(\frac{27}{16}\frac{P_m}{P_e} - 1\right) \pm \sqrt{\left(1 - \frac{27}{16}\frac{P_m}{P_e}\right)^2 - 1}\,\right] = \text{twice the travel of projectile to max. pressure in gun (ft)}$$

$$P_{ob} = \frac{27}{4} e^2 \frac{u}{(e+u)^3} P_m = \text{total pressure on breech when shot leaves muzzle. (lbs)}$$

$$V_f = \frac{wv + 4700\bar{w}}{W_r} = \text{max. velocity of free recoil (ft/sec)}$$

$$V_{fo} = \frac{(w+0.5)\ v}{W_r} = \text{velocity of free recoil when shot leaves muzzle (ft/sec)}$$

$$T_o = \frac{3}{3}\frac{u}{v} = \text{time of travel of shot to muzzle (sec)}$$

$$T_{to} = \frac{2(V_f - V_{fo})}{P_{ob}}\ \frac{W_r}{g} = \text{time of free expansion of gears (sec)}$$

$T = t_o + t_{1o}$ = total powder period (sec)

$X_{fo} = \left(\frac{w+0.5\,\bar{w}}{W_r}\right)u$ = free displacement when shot leaves muzzle (ft)

$X_{f'o} = \frac{P_{ob}}{W_r}\, g \,\frac{(T-t_o)^2}{3} + V_{fo}(T-t_o)$ = free displacement of recoil during free expansion of gas (ft)

$E = X_{fo} + X_{f'o}$ = free recoil during powder period, ft.

Resistance to Recoil

Knowing E and T we may immediately calculate the total resistance to recoil for any elevation, from the formula:-

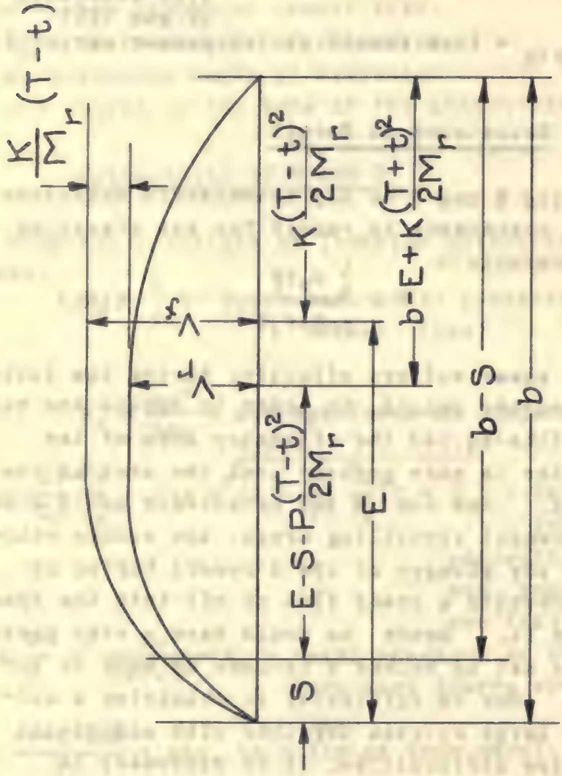
$$K = \frac{\frac{1}{2}\, m_r V_f^2}{b - E + V_f T} \qquad \text{(lbs)}$$

With spear buffers effective during the latter part of counter recoil, in order to reduce the buffer pressure (lbs/sq.in) the effective area of the spear buffer is made greater than the area of the recoil rod. Now due to the relatively small area of the c'recoil throttling areas, the sudden withdrawal of the plunger of the c'recoil buffer on firing, prevents a ready flow of oil into the space vacated by it. Hence we would have a very great resistance set up unless a by-pass or void is introduced. Due to difficulty in obtaining a sufficiently large by-pass together with additional constructive difficulties, it is customary to partially fill the recoil brake cylinder leaving a void in the cylinder.

To calculate the void displacement, fig. (1) let

A = effective area of recoil piston (sq.ft.)

A' = effective area of recoil piston on c'recoil plunger side (sq.ft)



$\frac{K}{M_r}(T-t)$
V_s
V_r
S
$E-S-P\frac{(T-t)^2}{2M_r}$
E
$-K\frac{(T-t)^2}{2M_r}$
$b-E+K\frac{(T+t)^2}{2M_r}$
b-S
b

Fig. 1

d_b = length of buffer or plunger (ft)

S = length of void displacement (ft)

t_s = time of recoil through the void (sec)

then, we have $A(d_b-S)-A'd_b = 0$

hence

$$S = \frac{(A-A')d_b}{A} \quad \text{(ft)}$$

The resistance to recoil, with a void, becomes

$$K = \frac{\frac{1}{2} m_r V_f^2}{b-E+V_f(T-t_s)}$$

To compute t_s we proceed as follows:

(1) If the void displacement is less than,

$$S \quad \frac{(w+\frac{\bar{w}}{2})u}{W_r} \quad \text{(ft)}$$

$$\text{then } u' = \frac{W_r S}{w+\frac{\bar{w}}{2}} \quad \text{(ft)}$$

$$t_s = \frac{e}{a}\left(2.3 \log \frac{2u'}{e} + \frac{u'}{e} + 2\right) \text{(sec)}$$

$$a = \frac{(e+u)v}{u}$$

e is obtained from the previous inertia ballistic calculations.

(2) When the void displacement is given by,

$$S \quad \frac{(w+\frac{\bar{w}}{2})u}{W_r}$$

then

$$S = X_{fo}+[X_{fo}+\frac{P_{ob}}{m_r}(t_s-t_o)-\frac{(t_s-t_o)^2}{6m_r(V_f-V_{fo})}](t_s-t_o) \quad \text{(ft)}$$

where

$$X_{fo} = \left(\frac{w+\frac{\bar{w}}{2}}{W_r}\right)u; \quad V_f = \frac{wv+4700\,\bar{w}}{W_r}$$

$$V_{fo} = \left(\frac{w + \frac{\bar{w}}{2}}{W_r}\right)v; \quad t_o = \frac{3}{2}\frac{u}{v}$$

Since the above expression is a cubic equation in t_s, we may more conveniently solve it by substituting trial values for t_s until we obtain the approx. value of s.

In an approximate design if we assume no void, K may be calculated immediately without the computation of E and T, from the formula:

$$K = \frac{1}{2} m_r V_f^2 \frac{1}{\left[b + (.096 + .0003d)\frac{uV_f}{v}\right]} \text{ where } V_f = \frac{\bar{w}4700 + wv}{W_r}$$

d = diam.of bore (in)
v = muzzle velocity of projectile (ft/sec)
b = length of recoil (ft)
u = travel of shot up bore (in)
w = weight of shell
$\bar{w}$ = weight of charge
m_r and W_r = mass and weight of recoiling parts.

Estimation of Pulls:-Recuperator and Brake.

Cylinder preliminary layout.

If

R_g = guide friction (lbs)
$\emptyset_m$ = max. angle of elevation
$\emptyset_i$ = initial angle of elevation
R_p = total packing friction (lbs)
B = total braking resistance (lbs)
P_b = brake cylinder pull (lbs)
F_{vi} = initial recuperator reaction (lbs)
m = ratio of compression (assumed from 1.3 to 1.7)
$F_{vf} = m\ F_{vi}$ = final recuperator reaction (lbs)
l = length of cradle and gun sleeve (in)
e_b = distance from center of gravity of recoiling parts to center of pulls

n = coefficient of guide friction = 0.15 approx.

x_1 and x_2 = coordinates of front and rear clip reactions from center of gravity of recoiling parts (in)

A = effective area of recoil brake piston

A_v = effective area of recuperator piston

a_r = area of recoil brake piston rod

a_v = area of recuperator piston rod

Then $K = B+R_p+R_g-W_r\sin\emptyset$. As a first approximation, we will neglect R_p and assume, $R_g = n\,W_r\cos\emptyset$, then $B = K+W_r(\sin\emptyset-n\cos\emptyset)$ (lbs). For the initial recuperator reaction, $F_{vi} = 1.3\,W_r(\sin\emptyset+n\cos\emptyset)$ (lbs) and since $B = P_h+F_{vi}$, the total braking (lbs) we have for the initial hydraulic pull, $P_h = K-W_r(0.3\sin\emptyset+2.3\,n\cos\emptyset)$ (lbs). In a preliminary design, the following are working pressures, consistent with the packings:-

$p_{h\,max}$ = 3500 to 4500 lbs/sq.in. brake cylinder.

p_{vi} = 1000 to 1500 lbs/sq.in. recuperator cylinder.

Further let f_m = max. allowable fibre stress in the various piston rods. Hence for the recoil brake, we have, for "n" cylinders

$$A_r = \frac{1.2\,P_h}{n\,f_m}\,; \quad A = \frac{P_h}{np_{n\,max}} \quad \text{(sq.in)}$$

(the factor 1.2 is to allow for the acceleration of the rod during the powder period), and the diam. of a recoil cylinder, becomes,

$$D\,a = \sqrt{\frac{A+a_r}{0.785}} \quad \text{(in)}$$

and the diam. of a brake rod,

$$d_r = \sqrt{\frac{a_r}{0.785}}$$

The recuperator dimension, for "n" cylinders becomes,

$$a_v = \frac{F_{vf}}{nf_m}\,; \quad A = \frac{F_{vi}}{np_{vi}}$$

where $F_{vf} = mF_{vi}$ = 1.5 to 1.7 F_{vi}

The diam. of a recuperator cylinder, becomes,

$$D_v = \sqrt{\frac{A_v + a_v}{0.7854}}$$ and the diam. of the recuperator rod

$$d_v = \sqrt{\frac{a_v}{0.785}}$$

From these dimensions a preliminary layout of the brake and recuperator cylinders may be made, and the positions of the center lines of the various pulls located with respect to the axis of the bord or center of gravity of the recoiling parts. If now,

e_h = distance from center of gravity of recoiling parts to line up action of hydraulic brake pull (in)

e_v = distance from center of gravity of recoiling parts to line of action of recuperator reaction (in)

e_b = distance from center of gravity of recoiling parts to line of action of resultant pull (in)

then

$$e_b = \frac{F_{vi}\ e_v + P_h e_h}{B} \text{ (in) where } B = F_{vi} + P_h$$

Calculation of packing friction.

To estimate the packing friction, we must assume the diameters of cylinders and rods, as approximated from the previous calculations: then

$R_p = \Sigma\ .05\ \pi d\ w_p\ p_{max}$ where d = diam. of the various rods and cylinders

w_p = corresponding width of the packing

p_{max} = max. pressure in the various cylinders

The component packing frictions for the recuperator and brake cylinders, consist of the stuffing box and piston frictions respectively.

For the brake cylinder,

$R_{ph} = \Sigma\ .05\ \pi(d_r W_r + D\ w_d) p_{h\ max}.$

where

d_r = diam. of brake rod

D = diam. of brake cylinder

W_r = width of stuffing box packing

w_d = width of piston packing

For the recuperator cylinder $R_{pv} = \Sigma\ .05\ \pi\ (d_v w_v + D_v W_v) p_v$ max.

where d_v = diam. of recuperator rod

D_v = diam. of recuperator cylinder

w_v = width of stuffing box packing

W_v = width of piston packing

then $R_p = \Sigma\ R_{ph} + \Sigma R_{pv}$ = total packing friction

If P_h = the total hydraulic reaction

P_h' = the total tension or pull in the brake rods

F_v = the total recuperator reaction

F_v' = the total tension or pull in the recuperator rods

then $P_h' = P_h + \Sigma R_{ph}$; $F_v' = F_v + \Sigma R_{pv}$

Guide Friction

We may now estimate, more exactly, the guide friction. We have two cases,

(1) When the resultant pulls are symmetrically balanced around the axis of the bore

(2) When the resultant pull is off set from the axis of the bore.

In (1) we have simply $R_g = nW_r \cos \emptyset$ (lbs)

In (2) we have

$$R_g = \frac{2n(B+R_p)e_b + nW_r \cos\emptyset (x_1 - x_2)}{1+2\ n\ e_b} \quad \text{(lbs)}$$

where n = 0.15

x_1 and x_2 are the front and rear clip reaction coordinates with respect to the center of gravity of the recoiling parts.

l = distance between clip reactions and length of sleeve in cradle.
e_b = distance down from bore to resultant line of action of mean total pull $(B+R_p)$. In general, however, we may neglect R_p as small compared with B, and $2\,n\,e_b$ as small compared with l, then,

$$R_g = \frac{2n(K+W_r\sin\emptyset)e_b+nW_r\cos\emptyset(x_1-x_2)}{l} \quad \text{(lbs)}$$

The term $n\,W_r\cos\emptyset(x_1-x_2)$ is usually small compared with $2n(K+W_r\sin\emptyset)e_b$ and further very often we may assume $x_1 = x_2$ approx., hence,

$$R_g = \frac{2n(K+W_r\sin\emptyset)e_b}{l} \text{(lbs)}$$

which is usually sufficiently accurate for ordinary calculations.

It is to be particularly noticed, that when the pulls are offset from the axis of the bore, the guide friction increases on elevating which is exactly opposite to the condition of symmetrically and balance pulls about the axis of the bore, when $R_g = nW_r\cos\emptyset$.

Initial Recuperator reaction,

The required initial recuperator reaction is given by the following formula:

$$F_{vi} = R_{pv} + \frac{W_r\left[\sin\emptyset_m + \dfrac{n\cos\emptyset_m(x_1-x_2)}{l+2\,n\,e_r}\right]}{1 - \dfrac{2e_v\,n}{l+2\,ne_r}}$$

where $R_{pv} = \Sigma\ .05\,\pi(d_v w_v + D_v W_v)p_{vi}$

p_{vi} = assumed initial recuperator pressure

$$= \frac{1.3(\sin\emptyset_m + n\cos\emptyset_m)W_r}{A_v}$$

A_v = assumed effective area of recuperator piston
d_v = diam. of recuperator rod (in)
D_v = diam. of recuperator cylinder (in)
w_v = width of stuffing box packing (in)
W_v = width of piston packing (in)
l = length of sleeve or distance between guide reactions (in)
e_v = distance from center of gravity of recoiling parts to resultant line of action of F_v

e_r = mean distance from center of gravity of recoiling parts to guides (= 0, for sleeve cradles)
x_1 and x_2 = coordinates of front and rear clip reactions from center of gravity of recoiling parts in battery (in)
n = coefficient of guide friction (=0.15)
$\emptyset_m$ = angle of max. elevation

The above formula is complicated and the following formula is usually sufficiently accurate and takes into consideration as well the pinching action between the guides and clips,

$$F_{vi} = \left(\frac{W_r \sin\emptyset_m + R_p}{1 - \frac{2ne_v}{l}}\right)$$

$$= k\left(\frac{W_r \sin\emptyset_m + R_p}{1 - \frac{2ne_v}{l}}\right)\text{(lbs) where } k = 1.1 \text{ to } 1.2$$

when e_v is small as with symmetrically balanced recuperator pulls, then $F_{vi} = k[W_r(\sin\emptyset_m + n\cos\emptyset_m) + R_p]$ where $k = 1.1$ to 1.2

If we include R_p with $n\,W_r\cos\emptyset$, we may increase k, and we have the elementary formula as before used, $F_{vi} = 1.3(W_r\sin\emptyset_m + 0.3\cos\emptyset_m)$ (lbs)

Counter Recoil Buffer or Regulator Design

Counter recoil regulators may be divided into two general types,

(1) Systems which are effective only during the latter part of counter recoil.

(2) Systems which fill themselves during the recoil and are effective throughout the counter recoil.

In type (1) we have a short spear buffer or plunger entering the buffer chamber towards the end of recoil. Type (1) buffers may be further subdivided into:-

(a) Plungers attached to a continuous recoil rod, the recoil rod passing through a stuffing box at either end of the piston.

(b) Ordinary spear buffers without a continuous recoil rod.

In the design of a counter recoil system, we are primarily limited to a maximum allowable buffer pressure, counter recoil stability in heavy artillery being of no great importance since the stability limit on a counter recoil is usually as great as on recoil. Since, however, a considerable part of the recoil energy becomes at the end of recoil stored in the recuperator, we have this energy absorbed in the counter recoil, by the counter recoil regulator in a short buffer displacement, with a consequent large total buffer reaction. We are limited in the counter recoil brake usually to a smaller effective area than in the recoil brake; consequently the buffer pressures become, due to constructive limitations, very large. Hence it is highly desirable to maintain as low a buffer pressure as possible.

With any form of spear buffer of type (1), to reduce the buffer pressure, the effective area

of the buffer plunger should be as large as possible and the length of buffer as long as possible.

In the design of a spear buffer of type (1) we have the following limitations:-

(1) The diameter of the buffer, should not exceed a value, that due to the sudden withdrawal of the buffer, the void displacement in the recoil brake should not be greater than the free recoil displacement during the powder period E.

(2) The length of the buffer should not exceed a value that during the counter recoil before the buffer enters its chamber the buffer chamber should be completely filled.

Let A = effective area of recoil piston (sq.ft)

A'= effective area of recoil piston on counter recoil plunger side (sq.ft)

L_b=length of plunger or buffer (ft)

A_b = effective area of buffer (sq.ft)

d_b =diam. of buffer chamber

D = diam. of recoil brake cylinder

d_r = diam. of recoil brake rod

Now $A = 0.7854(D^2-d_r^2)$; $A'=0.7854(D^2-d_b^2)$ sq.ft.

$A_b=0.7854(d_b^2-d_r^2)$ (sq.ft) type (1)(a) buffer.

$A_b=0.7854\, d_b^2$ (sq.ft) type (1) (b) buffer.

Now for condition or limitation (1), we have

$$E = \frac{A-A'}{A} L_b \quad \text{(ft)}$$

or

$$A' = \frac{A(L_b-E)}{L_b} \quad A(1-\frac{E}{L_b})\text{sq.ft.}$$

In terms of the diameters, we have

$$D^2-d_b^2 = (D^2-d_r^2)(1-\frac{E}{L_b})$$

hence $d_b = \sqrt{D^2 \frac{E}{L_b} + d_r^2 (1 - \frac{E}{L_b})}$ which gives us the limiting value of d_b. It is interesting to note that when $E = 0$, $d_b = d_r$ or in other words when the diameter of the buffer a plunger is made equal to that of the rod, no void is required in the recoil cylinder.

From the above expression, we note that increasing the length of the buffer decreases the diameter of the buffer and thereby increases the buffer pressure.

On the other hand the c'recoil energy is absorbed over a greater distance with a longer buffer, thus reducing the total buffer reaction, and it is probable, that this cause more than effects the slight increase of the buffer pressure due to the decrease of the buffer diameter. Further the value of d_b is very often entirely limited by constructive considerations alone; hence a long buffer is highly desirable.

In a type (1) (a) buffer due to the relatively large value of d_b required to give a sufficient buffer area, the length of the buffer depends entirely on the limitations (1). This type of buffer will be considered in detail later.

For the limitation (2), with a continuous rod, we have a void produced at the end of recoil on the buffer side of the recoil piston. To compute this void, we have, with an initial void in the battery position AE, for the void on the buffer side of the recoil piston at the end of recoil, or the out of battery position.

$\text{Void}_c = A_r b - A(b - E) \text{ (cu.ft)}$

where A_r = area of the recoil cylinder (sq.ft)

Therefore, $\text{Void}_c = (A_r - A)b + AE$

$= a_r b + AE \text{ (cu.ft)}$

Now in the c'recoil, the spear buffer chamber is evidently not filled until the void displacement has been over run, and this displacement becomes, $X_a = \frac{a_r b + AE}{A_r} = b - L_b$

hence $L_b = b - \dfrac{a_r b + AE}{A_r}$

$$= 0.8[b(1-\frac{d_r^2}{D^2}) = (\frac{D^2-d_r^2}{D^2})E]$$

Since $\frac{AE}{A_r}$ is small, for a close approximation, the buffer length should not exceed

$$L_b = 0.8b(1-\frac{a_r}{A_r}) = 0.8b(1-\frac{d_r^2}{D^2}) \quad \text{ft.}$$

The mean buffer pressure may now be computed, knowing the potential energy of the recuperator.

The potential energy of the recuperator is given by either of the following expressions:

$$W_o = \frac{F_{vi}V_o}{A_v(k-1)}(m^{\frac{k-1}{k}} - 1)(\text{ft.lbs}) \quad (k=1.3 \text{ approx.})$$

or

$$W_o = \frac{F_{vf}V_f - F_{vi}V_o}{A_r(k-1)} \qquad (\text{ft.lbs})$$

where $V_f = V_o - A_v b$

A_v = effective area of the recuperator piston (sq.ft)

V_o = the initial volume (cu.ft)

F_{vi} = the initial recuperator reaction (lbs)

$m = \dfrac{F_{vi}}{F_{vi}}$ = the ratio of compression

Then, the mean buffer pressure, becomes

$$P_b = \frac{W_o - (W_r \sin\emptyset + R_o)b}{A_b L_b} \quad (\text{lbs/sq.ft})$$

where R_o=total packing and guide friction (lbs)

b = length of recoil (ft)

$$L_b = 0.8(1-\frac{d_r^2}{D^2}) \quad (\text{ft})$$

$$A_b = 0.7854\, d_b^2 \ )\text{sq.ft})$$

$$d_b = \sqrt{D^2\frac{E}{L_b} + d_r^2(1-\frac{E}{L_b})} \quad (\text{ ft.})$$

In type (1) (a) buffer, where we have a continuous rod and enlargement back of the piston for the c'recoil plunger, in order to have a sufficient effective buffer area, the diameter of the plunger must be necessarily large as compared with a spear buffer. Therefore, to maintain a void displacement in the recoil not exceeding the free recoil displacement during the powder pressure period, we must have a very short buffer. Hence if

A = effective area of recoil piston (sq.ft)

A'=effective area of recoil piston on c'recoil plunger side (sq.ft)

L_b = length of plunger or buffer (ft)

A_b = effective area of the buffer

then

$$E = \frac{(A-A')L_b}{A}$$

hence

$$L_b = \frac{AE}{A-A'} \quad \text{ft}$$

If further d_r = diam. of recoil brake rod

D = diam. of recoil brake cylinder

d_b = diam. of buffer chamber

we have $d_b = C_b D$ where C_b depends upon constructive considerations

$$\therefore L_b = \frac{(D^2-d_r^2)E}{[(1-C_b^2)D^2-d_r^2]}$$

and

$$A_b = 0.7854(d_b^2-d_r^2)-0.785\ (C_b^2\ D^2-d_r^2)$$

Now to reduce the buffer pressure it is desirable to make L_b as long as possible and A_b as small as possible. To do this we must make d_r as small as possible as compared with D. This requires a large effective area for the recoil brake.

Hence in type (1) buffer we may reduce the buffer pressure by reducing the recoil brake pressure. If W_o = the total potential energy of the recuperator we have

$$W_o = \frac{F_{vi}\ V_o}{A_v(k-1)}\left(m^{\frac{k-1}{k}} - 1\right) \text{ (ft. lbs) } (k=1.3 \text{ approx.})$$

where

$m = \frac{F_{vf}}{F_{vi}}$ = the ratio of compression

A_v = effective area of the recuperator piston (sq.ft)

V_o = initial volume of the recuperator (cu.ft)

then for the mean buffer pressure, we have

$$p_b = \frac{W_o-(W_r \sin\emptyset + R_p)b}{A_b L_b}$$

Now

$$A_b L_b = 0.785\ \frac{(D^2-d_r^2)(C_b^2 D^2-d_r^2)}{(1-C_b^2)D^2-d_r^2}$$

If we assume C_b = 0.7 roughly, we have $A_b L_b = 0.785(D^2-d_r^2)E$

hence

$$p_b = \frac{W_o-(W_r \sin\emptyset + R_p)b}{0.785(D^2-d_r^2)E}$$

where

b = length of recoil (ft)

E = free displacement in the recoil during powder period (ft)

W_o = potential energy of the recuperator (ft. lbs)

R_o = total friction (lbs)

since

$$0.785(D^2-d_r^2) = \frac{P_h}{p_h}$$

we have $p_b = [W_o+(W_r \sin\emptyset + R_o)]\ \frac{p_h b}{P_h E}$

where

P_h = total hydraulic brake pull (lbs)

p_h = assumed intensity of pressure in hydraulic cylinder (lbs/sq.in)

Therefore, to decrease the buffer pressure, with a type (1) (a) c'recoil regulator:

(1) Lower the max. pressure in hydraulic brake cylinder during the

recoil.

(2) Decrease the length of recoil

(3) Decrease the potential energy in the recuperator.

We see that the above expression is fixed by the free recoil displacement E during the powder period.

BY PASS PIPES USED WITH LARGE SPEAR BUFFERS. In order to lower the buffer pressure on counter recoil, when the c'recoil regulation is by a short spear buffer or plunger, it is often necessary to increase the diameter of the plunger materially over that of the rod.

By the introduction of a by pass and valve (closing on the counter recoil)leading from the buffer side of the recoil cylinder to the outer end of the void chamber of the buffer, the pressure back of the recoil piston (on the buffer side) can be effectively lowered without a full void by being required in the recoil cylinder to take care of the sudden withdrawal of the buffer plunger during the first part of the recoil.

Let w_a = required area of the by pass pipe

T = total powder period (sec)

t_s=time of travel through void during the recoil (sec)

E = recoil displacement during powder period (ft)

A = effective area of recoil piston (sq.ft)

A'=effective area of recoil piston on plunger side (sq.ft)

S = recoil displacement during void (ft)

L_b = length of buffer (ft)

p_m' = mean pressure in the rear of the recoil piston (lbs/sq.in)

p' = max. pressure in the rear of the recoil piston (lbs/sq.in)

Now the total quantity of oil that must pass through the by pass pipe, becomes, $Q = A(L_b-S)-A'L_b$ (cu.ft)

After the gun has recoiled the void displacement, the void back of the piston, i.e. the plunger side of the recoil piston, becomes gradually filled with the further recoil. The pressure in this rear chamber however is zero until the chamber becomes completely filled. If X_s is the displacement in the recoil when this chamber is just filled, obviously, $A(X_s-S)=A'X_s$ hence

$$X_s = \frac{AS}{A-A'}$$

Let t_{xs} = the corresponding time in the recoil.
We have two cases:

(1) When $X_s < E$:

(2) Where $X_s > E$.

For case (1), t_{xs} and X_s are connected by the equation,

$$X_s=X_{fo}+[V_{fo}-\frac{P_{ob}}{m_r}(t_{xs}-t_o)-\frac{(t_{xs}-t_o)^2}{6m_r(V_f-V_{fo})}](t_{xs}-t_o)\text{(ft)}$$

(approx)

from which by trial values we may estimate t_{xs}
For case (2) we may compute t_{xs} from

$$t_{xs} = T + \frac{m_r(V_r-V_{xs})}{K} \text{ where } V_{xs} = \sqrt{V_r^2 - \frac{2K(X_s-E)}{m_r}}$$

$V_r = V_f$ approx. = max. recoil velocity (ft.sec)

K = total resistance to recoil

T = total time of powder period

To calculate the mean pressure in the chamber back of the recoil piston,-

$$p_m' = \frac{Dv_m^2}{2g}$$ where D = density of the fluid= 53 lbs/cu.ft.

v_m = the mean velocity in the pipe

and $$v_m = \frac{Q}{w_a(t_b-t_{xs})}$$ ft/sec.

where $Q = A(L_s-S)-A'L_b$

w_a = area of pipe (sq.ft)

t_b-t_{xs} = time of travel through the recoil displacement L_b-X_s

Now t_b is the time for the recoil displacement L_b, hence

$$t_b = T + \frac{m_r(V_r-V_b)}{K}$$

where

$$V_b = \sqrt{V_f^2 - \frac{2K(L_b-E)}{m_r}}$$

To calculate the maximum pressure in the chamber back of the piston,-

$$p' = \frac{D(A-A')V_{xs}^2}{2gw_a^2} \text{ lbs/sq.ft.}$$

where, when $x_s < E$,

$V_{xs} = V_f$ (approx) the maximum velocity of free recoil (ft/sec)

where $X_s > E$

$$V_{xs} = \sqrt{V_f^2 - \frac{2K(X_s-E)}{m_r}}$$

From the constrained velocity curve, we may calculate p' during the displacement (L_s-X_s). Since $p'=p_m$ approx. we may assume p' constant and use the previous expression for p_m. It is important here to note that the recoil throttling must be modified to maintain a constant pull on the brake. $p_h=pA=p'A'$ (lbs)

$$p-p'=\frac{D\ A^2V^2}{2gC^2w_x^2} \quad \text{(lbs/sq.ft)}$$

Combining these two expressions, we may solve for the required modified recoil throttling area w_x (sq.ft) in terms of the known values p' and P_b.

It is important that the recoil brake function at least at the end of the powder. We place, therefore,

$$S = E, \text{ then } X_s = \frac{AE}{A-A'}$$

$$\text{and } p' = \frac{D[A(L_b-E)-A'L_b]^2}{2gw_a^2(t_b-t_{xs})^2} \text{ where } t_{xs}-T=m_r(V_r-V_{xs})$$

$$V_{xs} = \sqrt{V_r^2 - \frac{2K(X_s-E)}{m_r}}$$

$$\text{and } t_b = T+m_r(V_r-V_b)$$

$$V_b = \sqrt{V_r^2 - \frac{2K(L_s-E)}{m_r}}$$

usually p' should not exceed a few hundred lbs/sq.in. and the value of w_a and A' should therefore be corresponding. In such a case no material effect in the recoil throttling is obtained and a modification of the grooves is unnecessary.

DESIGN OF SIDE FRAME GIRDERS. The loading on the girders and the corresponding stresses depends upon the method proposed for firing. These methods may be classified as follows:

(1) Firing from semi fixed base plate, with a large pintle bearing and the girders extending to the rear supported at their end by an outer circular track. The horizontal and a part of the vertical reaction is transmitted to the pintle base plate, the horizontal reaction being taken up by a vertical spade extending below into the ground from the base plate and the vertical load being balanced by the upward reaction of the ground on the base plate. No balancing <u>moment</u> is assumed to be exerted by the ground on the base plate. This assumption, makes it possible to readily determine the upward normal reaction of the outer circular track. We have, therefore,

with this method of loading the horizontal and vertical reaction at the pintle bearing and a vertical reaction at the tail of the girder balancing the trunnion reaction due to firing. This loading should be considered at both horizontal and maximum elevation.

(2) Firing from the pintle base plate assumed bolted down to a concrete base. In this method no outer track for supporting the tail of the girder is necessary. We have therefore at the pintle bearing a horizontal and vertical reaction, together with a bending couple balancing the firing reactions at the trunnions. This loading should be considered at both horizontal and maximum elevation.

(3) Firing from a special layed track, the mount recoiling in translation on this track. By this method the vertical load is somewhat distributed by several shoes brought down in contact with the track. The horizontal component due to firing at the trunnions is balanced by the total sliding friction equal to the weight of the mount plus the vertical firing component times the coefficient of track friction and the inertia resistance of the mass below the trunnions to acceleration. Though the horizontal reaction on the trunnions is theoretically slightly reduced due to the acceleration of the cradle in which the gun recoils, we may practically consider that the total firing load is brought on to the

trunnions, since the acceleration of the total mount backwards is relatively small and the mass of the cradle quite negligible as compared with the large mass of the main girders, trucks, etc. below the trunnions. The vertical component due to firing at the trunnions is balanced by the upward reactions on the various shoes. Finally the couple produced by the horizontal reaction at the trunnion and the resultant of the inertia resistance and the shoe frictions, is balanced by a couple produced by the vertical reaction at the trunnions and the resultant normal or vertical reactions of the track or guides on the various shoes.

This requires a uniformity increasing upward reaction on the various shoes towards the rear. The loadings should be considered at horizontal and maximum elevation.

(4) Firing directly from trucks riding or recoiling back on the rails. This loading is similar in characteristics to (3) except now the supporting reactions are concentrated at the truck pintles. Again the loadings should be considered at horizontal and maximum elevations.

When a girder is designed to meet all four requirements in the methods of firing, we have for the two elevations, eight types of loading to be considered as applied to the girder. Knowing then the loads brought on to the girde, we have, the following points to consider in the layout of the girder as regards its strength.

(1) The proper flange area to carry the requisite bending at a section of given depth.

(2) The proper depth of girder for all other sections.

(3) The proper cross section of the webs for carrying the total shear.

(4) The proper pitching of the rivets for carrying the longitudinal shear.

(5) A careful study of web reinforcements or stiffeners.

(6) The distribution and design of cross beams or transoms connecting the two girders.

(7) The detailing and design of the pintle bearing.

(8) The reinforcement in the web required for the elevating pinion bearing.

Reactions between tipping parts and girder trunnion reactions:

$$2H = K\cos\phi + E\cos\theta_e' \quad \text{(lbs)} \quad (1)$$

$$2V = K\sin\phi - E\sin\theta_e' + W_t \quad \text{(lbs)} \quad (2)$$

and for the elevating gear reaction,

$$E = \frac{Ks + P_b e}{j} \quad \text{In battery (lbs)} \quad (3)$$

$$E = \frac{Ks + W_r b\cos\phi}{j} \quad \text{Out of battery (lbs)} \quad (4)$$

where

H and V = the horizontal and vertical components of the trunnion reaction (lbs)

K = total resistance to recoil (lbs)

E = elevating gear reaction (lbs)

j = radius from trunnion axis to line of action of elevating gear reaction - with rack and pinion = radius of each (in)

θ_e' = angle between j and the vertical.

S = perpendicular distance from line through center of gravity of recoiling parts and parallel to bore to center of trunnions (in.)

e = perpendicular distance from axis of bore to center of gravity of recoiling parts.

With a balancing gear introduced between the tipping parts and girders, we must modify the trunnion reaction to include this reaction. The elevating gear reaction is not changed, since the moment of the tipping parts about the trunnions is always balanced by the balancing gear in the battery position of the recoiling parts.

Since it is usually customary to locate the trunnions along a line through the center of gravity of the recoiling parts parallel to the bore, S =0, and therefore,

$$E = \frac{P_b e}{j} \quad \text{in battery}$$

$$E = \frac{W_r b \cos \phi}{j} \quad \text{out of battery}$$

Now since e is usually made very small, $\frac{P_b e}{j}$ and $\frac{W_r b \cos \phi}{j}$ may be neglected as compared with H and V. Hence, we will assume the elevating gear reaction to be negligible, and we have the total firing load brought onto the girders at the trunnion. Then

$$\left.\begin{aligned} 2H &= K \cos \phi \\ 2N &= K \sin\phi + W_t \end{aligned}\right\} \text{ approx. reaction between tipping parts and girder.}$$

Reactions between base plate and girder.

Considering the reactions on the base plate, if it is considered that the ground can offer no bending resistance as in assumption or method (1)

REACTION OF BASE PLATE ON GIRDER

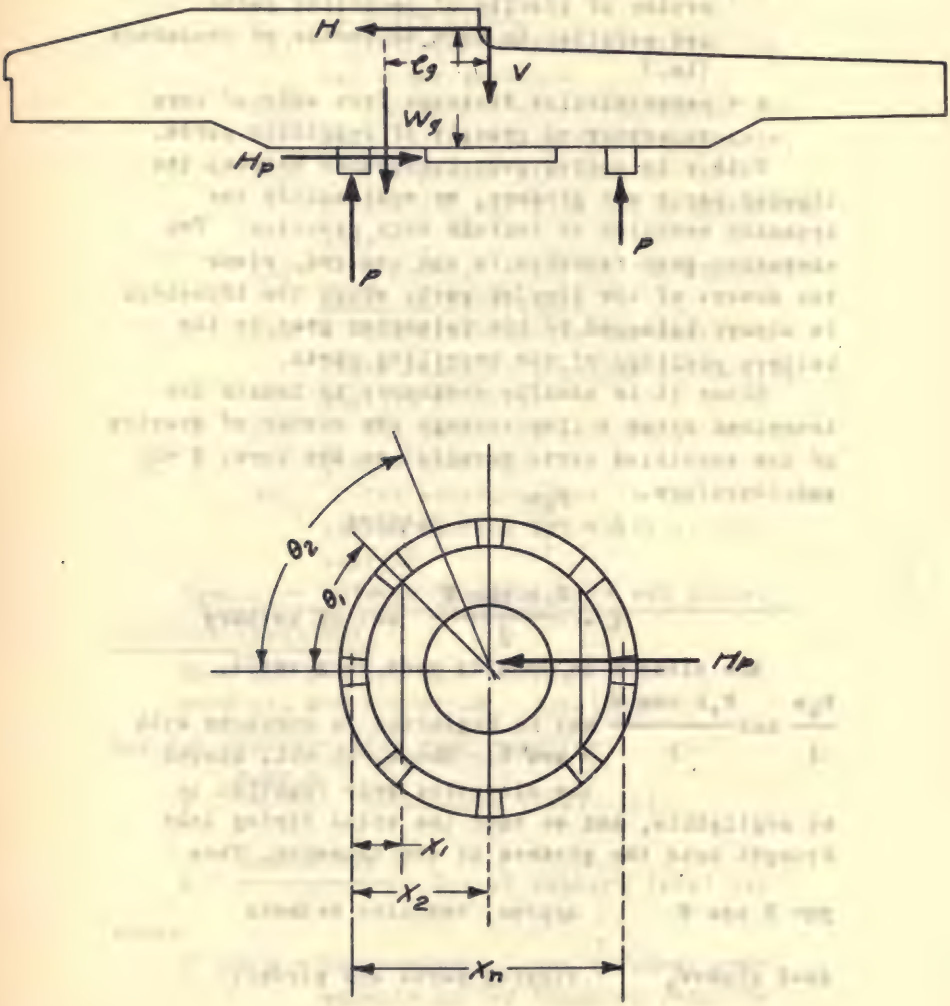
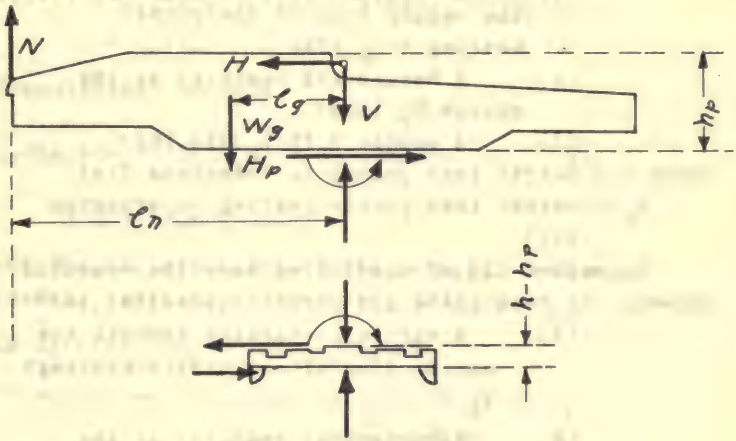


Fig. 2

METHOD (1) OF LOADING



METHOD (2) OF LOADING

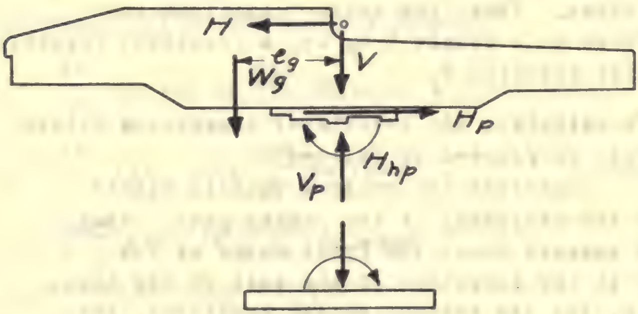


Fig. 3

of loading we have the reaction between the base plate and girder as equivalent to:-

(1) A vertical reaction through the center line of the pintle bearing = V_p (lbs)

(2) A horizontal reaction at the pintle H_p (lbs)

(3) A couple $H_p(h-h_p)$(in.lbs)

where h = height from ground to trunnions (in)

h_p = height from pintle bearing to trunnion (in)

In method (2) of loading we have the reaction between the base plate and girder equivalent to:-

(1) A vertical reaction through the center line of the pintle bearing = V_p

(2) A horizontal reaction at the pintle H_p

(3) A couple resisting the overturning moment = $H\ h_p$

Constructively, only the horizontal reaction is taken up at the pintle bearing, the vertical or normal reactions being taken up at the traversing rollers. Thus, the roller reactions are equivalent to a couple $H\ h_p$ and a resultant vertical or normal reaction V_p

To calculate the individual traversing roller reactions we proceed as follows:

Consider the rollers equally spaced around the periphery of the roller path. Then, taking moments about the front outer or end roller in the direction of the axis of the bore, we have, for the various roller reactions, see fig.(2).

Assuming "n" chords passing through a pair of rollers and perpendicular to the axis of the bore projected in a horizontal plane, then,

$p_1=k(x_1+y)\ p_2=k(x_2+y)$ -----$p_n=k(x_n+y)$

Taking moments about the front roller

$k[2x_1(x_1+y)+2x_2(x_2+y)$ ----$2x_{n-1}+y=+x_n(x_n+y)]$

$= H\ h_p+V_p r$

Simplifying, we have

$ky(2x_1+2x_2$---$2x_{n-1}+x_n)+k(2x_1^2+2x_2^2$---$2x_{n-1}^2+x_n^2)$

$= H\ h_p+V_p r$

and for the summation of the vertical reactions,

$ky+Wk(x_1+y)+2k(x_2+y)$---$2k(x_{n-1}+k(x_n+y)=V_p$

$2k\ ny+k(2x_1+2x_2+$----$2x_{n-1}+x_n)=V_p$

To solve, we note that, $A(ky)+B(k)=H\ h_p+V_p r$

$C(ky)+D(k)=V_p$

where $A=(2x_1+2x_2$----$2x_{n-1}+x_n)$

$B=(2x_1^2+2x_2^2$----$2x_{n-1}^2+x_n^2)$

$C = 2n$

$D=(2x_1+2x_2$----$2x_{n-1}+x_n)$

Knowing $x_1 x_2$ ----x_n we may readily obtain p_0, p_1---p_n

To compute $x_1 x_2$ -----x_n for the rollers, we have for the angle to the various chords,

$$\theta_1 = \frac{2\pi}{n} \text{ radians or } \frac{360}{n} \text{ degrees}$$

$$\theta_2 = 2\frac{2\pi}{n} \text{ rad. or } 2\frac{360}{n} \text{ degrees}$$

$$\theta_n = \frac{n}{2}\frac{2\pi}{n} \text{ rad. or } \frac{n}{2}\frac{360}{n} \text{ degrees.}$$

therefore $x_1=r(1-\cos\theta_1)$ (in)

$x_2=r(1-\cos\theta_2)$ (in)

$x_n=r(1-\cos\theta_n)$ (in)

where r = radius to the center line of the roller path.

From the previous equations we may now compute P_o, P_1---P_n(lbs), the individual roller reactions.

The previous formulae, assume contact between each roller and the roller track under maximum firing conditions. If the roller path has a small diameter, we may have the condition, when, only the rear roller is brought into contact, the overturning moment on the girder being balanced by a couple exerted by the base plate an upward reaction at the rear roller contact and a downward reaction at the front circular clip contact. If the circular clip has a radius approx. equal to that of the roller path, then we have for the max. roller reaction $H_p h_p + V_p r = 2p_{max} r$

hence

$$P_{max} = \frac{H_p h_p + V_p r}{2r}$$

where r = radius of the roller path (in)

P_{max} = max. roller reaction (lbs)

V_p=max. upward reaction at pintle (lbs)

<u>External forces exerted on the girder during firing:</u>

The external force or the girders are shown in plates A and B for the four methods of loading.

In method (1) of loading, we have the reactions of the tipping parts H and V, the reaction of the base plate H_p and V_p together with the couple $H_p(h-h_p)$ and the reaction of the outer track on the tail of the girder N. Further we must include the total weight of the girder which though actually distributed we will assume concentrated at its center of gravity at horizontal lg from the axis of the trunnions.

Taking moments about the pintle bearing,
$H h_p + H(h-h_p) - Nl_n = 0$ hence

$$N = \frac{Hh}{l_n} \text{ (lbs)}$$

where $H = K \cos Ø$ and h = the height of the trunnions from the ground (in).

Knowing N we may compute for the strength of the tail of the girder, for method (1) of loading.

In method (2) of loading since we are detailing the strength of the girder in the region of the trunnion and pintle reactions, we must take the actual components of the reaction into consideration. These consist of the trunnion and elevating arc reactions of the tipping parts, that is the reactions H,V, and E and the reaction of the base plate consisting of the various roller reactions and the horizontal reaction of the pintle as shown in "Reaction of Base Plate on Girder" diagram.

In method (3) of loading, where the mount slides back on a special constructed track, we have for the reactions on the girder.

(1) The H and V components of the trunnion reaction of the tipping parts.

(2) The inertia resistance of the girder, resisting the acceleration of the girder acting at the center of gravity of the girder =

$$m_r \frac{d^2x}{dt^2}$$

(3) The weight of the girder acting at its center of gravity W_g

(4) The normal reactions of the track shoes N_a and N_b

(5) The frictional or tangential components of the track shows $n(N_a+N_b)$

In calculating the stresses on the various portions of the girder we must of course consider both the weight and inertia as distributed forces, but for dealing with the overall reactions, we may assume their resultant effect as concentrated force passing through the center of gravity of the girder.

When the trucks are entirely disengaged in this method of firing, we have,

$$H-n(N_a+N_b)-m_g \frac{d^2x}{dt^2} = 0 \quad \text{and } N_a+N_b=V+W_g$$ when the trucks are not disengaged but hang from the girder, we must consider both their weight and inertia reaction, hence if W_{tk} = weight of trucks (lbs)

M_{tg} = mass of truck

we have,

$$H-n(N_a+N_b)-(m_g+m_{tk})\frac{d^2x}{dt^2} \quad \text{and } N_a+N_b=V+W_g+W_{tk}$$

To compute N_a take moments about N_b (see fig. (4).

$$N_a(l_a+l_b)+H\ h-Vl_b-Vl_b-W_g(l_b-l_b)-m_g \frac{d^2x}{dt^2}(h-h_g)=0$$

hence

$$N_a = \frac{Vl_b+W_g(l_b-l_g)+m_g \frac{d^2x}{dt^2}(h-h_g) - H\ h}{l_a+l_b} \text{ (lbs)}$$

and for N_b taking moments about N_a, we have

$$N_a(l_a+l_b+m_g \frac{d^2x}{dt^2}(h-h_g)-H\ h-Vl_a-W_g(l_a+l_g=0$$

hence

$$N_b = \frac{Hh+Vl_a+W_g(l_a+l_g)-m_g \frac{d^2x}{dt^2}(h-h_g)}{l_a+l_b} \text{ (lbs)}$$

In method (4) of loading, we have the mount recoiling back directly on the rails, and the trucks react on the girder with reactions H_a, N_a and H_b, N_b, at the truck pintles a and b. The

tipping parts react on the girder with components H and V at the trunnions. In addition we have the inertia resistance

$m_g \frac{d^2x}{dt^2}$ resisting the acceleration and the weight of the girder both acting through the center of gravity of the girder.

For a horizontal motion back along the rails, we have

$$H-(H_a+H_b)-m_g\frac{d^2x}{dt^2}=0$$

and normal to the rails, $V+W_g-(N_a+N_b)=0$

To calculate H_a and H_b the horizontal components of the truck reaction we must consider the trucks separately. In firing directly from the rails the trucks are usually braked.

If W_{tk} and M_{tk} = weight and mass of either truck

W_w and m_w = weight and mass of a pair of wheels

$I=m_wk^2$ = moment of inertia of a pair of wheels about the center line of the axle.

d = diameter of a car wheel

k = radius of gyration of a pair of car wheels (=0.7 d approx.)

N_w = normal reaction at base of car wheel

N_s = normal reaction of brake shoe on wheel per pair of wheels

f_w = coefficient of rail friction

f_s = coefficient of brake shoe friction

R_w=tangential force exerted by rail on base of car wheel

Now for the motion along the rails, we have,

$$H_a-\Sigma R_w=m_{tk}\frac{d^2x}{dt^2}$$

Considering the rotation about the center of gravity of a single wheel we have,

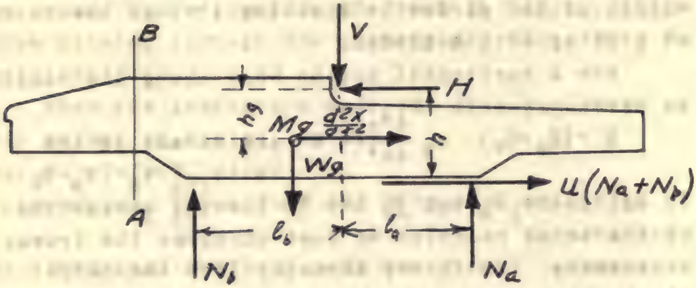
$$(R_w-N_sf_s)\frac{d}{2}=\frac{2m_wk^2}{d}\frac{d^2x}{dt^2}\quad\therefore\; R_w=N_sf_s+\frac{4m_wk^2}{d^2}\frac{d^2x}{dt^2}$$

hence

$$H_a=\left(m_{tk}+n\frac{4m_wk^2}{d^2}\right)\frac{d^2x}{dt^2}+\Sigma N_sf_s$$

where n = no. of pair of wheels per

METHOD (3) OF LOADING



METHOD (4) OF LOADING

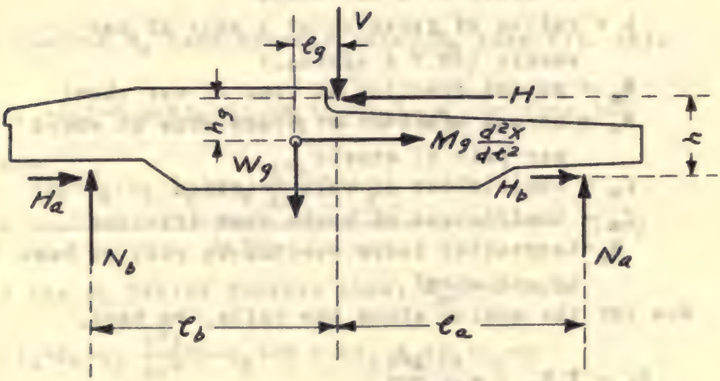


Fig. 4

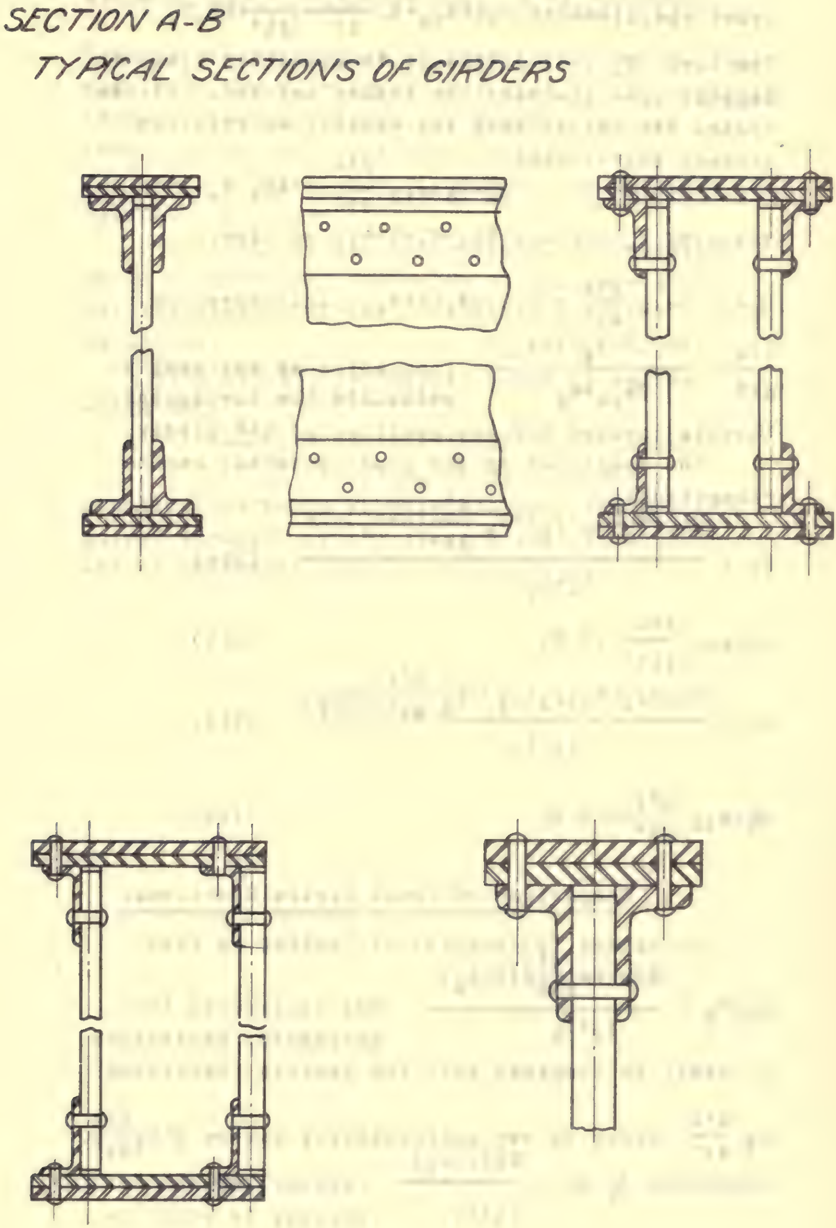


Fig. 5

truck and likewise $H_b=(m_{tk}+n\,\frac{4m_w k^2}{d^2})\frac{d^2x}{dt^2}+\Sigma N_s f_s$

The term $\Sigma N_s f_s$ is difficult to calculate since it depends upon how hard the brakes are set. If the brakes are set to skid the wheels, no rotation occurs, and we have

$$H_a=H_b=m_{tk}\frac{d^2x}{dt^2}+\Sigma N_w f_w$$

Assuming $f_w=0.2$ and $\Sigma N_w=W_g+V+W_{tk}$ we have,

$H_a+H_b=2m_{tk}\frac{d^2x}{dt^2}+0.2\,(W_g+V+W_{tk})$ and therefore

$\frac{d^2x}{dt^2}=\frac{H-0.2(W_g+V+W_{tk})}{2m_{tk}+m_g}$ from which we may easily calculate the horizontal inertia loading for any position of the girder.

The reactions at the truck pintles, become respectively,

$$N_a=\frac{Vl_b+W_g(l_b-l_g)+m_g\frac{d^2x}{dt^2}(h-h_g)-Hh}{l_a+l_b}\quad\text{(lbs)}$$

$$H_a=m_{tk}\frac{d^2x}{dt^2}+0.2\,N_a\quad\text{(lbs)}$$

$$N_b=\frac{Hh+Vl_1+W_g(l_a+l_g)-m_g\frac{d^2x}{dt^2}(h-h_g)}{l_a+l_b}\quad\text{(lbs)}$$

$$H_b+m_{tk}\frac{d^2x}{dt^2}+0.2\,N_b\quad\text{(lbs)}$$

Comparison of Truck Pintle Reactions.

In method (3) and (4) of loading we find

$N_b-N_a=\frac{2Hh-2m_g\frac{d^2x}{dt^2}(h-h_g)}{l_a+l_b}$ Now in general the horizontal resistance is small as compared with the inertia resistance $m_g\frac{d^2x}{dt^2}$ Hence we may approximately assume $H=m_g\frac{d^2x}{dt^2}$

therefore $N_b-N_a=\frac{2H(h-h_g)}{l_a+l_b}$ Further, we are not greatly in error in

assuming $h_g = \frac{h}{2}$ then we have, $N_b - N_a = \frac{Hh}{l_a + l_b}$

That is the difference of load thrown on the rear and front truck respectively equals the horizontal trunnion reaction times the height from the trunnions to the horizontal center line through the truck pintles, and divided by the distance between the trucks.

Obviously as the gun elevates H decreases, while V increases; therefore at max. elevation the loadings on the trucks are more nearly equalized.

With railway carriages, since at maximum elevation

$\frac{h}{l_a + l_b} H$ is relatively small compared with N_a or N_b, for all practical purposes we may consider that the required strength of the girders must be equally strong on either side of the trunnions.

CHAPTER XI.

GUN LIFT CARRIAGE.

Single recoil systems where the recoiling mass does not translate in recoil parallel to the axis of the bore, appear in various types of mounts. Illustrations of such types may be found in our model 1897 Barbette mount, where the gun and top carriage form a single recoiling mass, recoiling up an inclined plane. Railway carriages especially in France have been used, where the recoiling mass, (gun and top carriage) recoil on a gravity plane mounted on the car. The object of the inclined plane is to return the piece by gravity into battery. Carriages with no recoil except the sliding back of the gun and top carriage as a single mass on rails have also been extensively used, the resistance to recoil being merely the friction offered by the rails or slides.

CHARACTERISTICS OF INCLINED PLANE CARRIAGES. Due to the fact that the recoil is not along the axis of the bore, during the powder period, a component of the total powder force normal to the inclined plane or slides is introduced. This component therefore introduces large stresses in the carriage, the component increasing with the elevation. The excessive stresses thus introduced at high elevation, prohibits the use of this type of mount for firing at high elevations especially for large calibers. The type of mount is useful for where the elevation is not great. With large size howitzers this type of mount would necessarily produce a very heavy mount for strength and, therefore, from the point of view of mobility alone could be regarded as none else than poor design.

Since the gun recoil is not along the axis of the bore a reaction on the projectile normal to the bore is introduced. This reaction reaches a maximum closely at the maximum elevation. It possibly introduces unequal wear on the rifling in the gun tube itself. This reaction further introduces a slight spring during the powder period on the elevating arc and pinion.

APPROXIMATE THEORY OF RECOIL, NEGLECTING NORMAL REACTION OF PROJECTILE ON BORE. Even for a very close approximation the reaction of the projectile normal to the bore during the powder period has a very small effect on the recoil, though it is of importance in estimating the maximum elevating arc reaction during the powder period. If, then we let

P_b = total powder reaction on base of projectile, in lbs.

B = hydraulic braking of recoil cylinders parallel to inclined plane in lbs.

R = total friction of the recoil in lbs.

w_r and m_r = weight and mass of recoiling parts (in lbs)

$\emptyset$ = the angle of elevation of the axle of the bore

θ = the inclination of the inclined plane.

E = displacement of free recoil during powder period (in ft.)

T = total time of powder period (in sec.)

V_f = velocity of free recoil (in ft/sec)

K = the total resistance to recoil, in lbs.

b = length of recoil, in ft.

Then considering the recoiling parts during the powder period, we have,

$P_b\cos(\emptyset+\theta)-(B+R+W_r\sin\theta)= m_r \dfrac{dV}{dt}$ and since

$K=B+R+W_r\sin\theta$

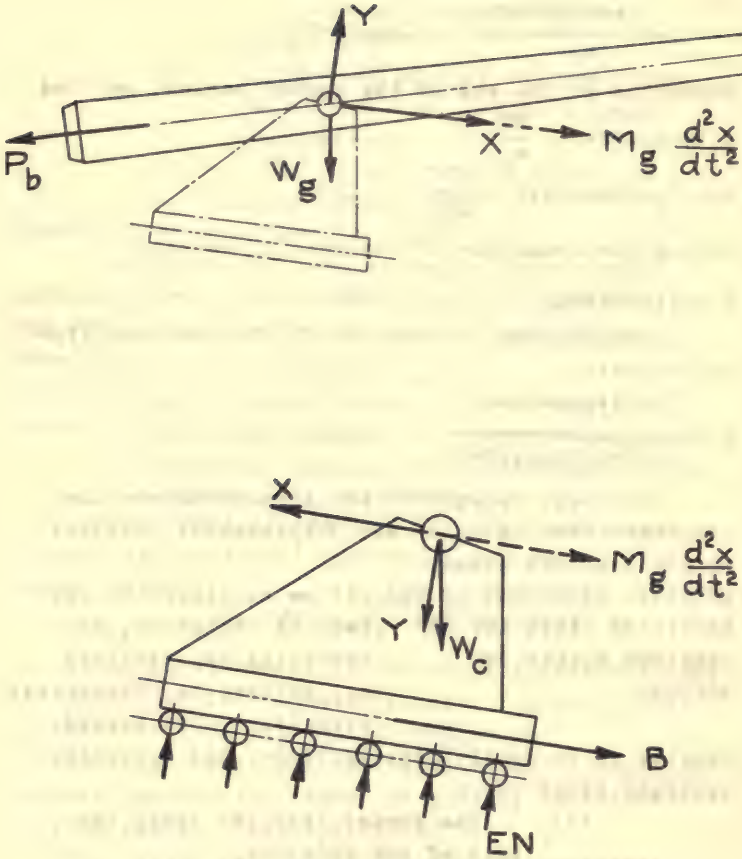


Fig.1

then $\int \frac{P_b \cos(\emptyset+\theta)dt}{m_r} - \frac{KT}{m_r} = V$

but $$\int \frac{P_b \cos(\emptyset+\theta)dt}{m_r} = V_f \cos(\emptyset+\theta)$$

therefore at the end of the powder period, we find

$$V_r = V_f \cos(\emptyset+\theta) - \frac{KT}{m_r} \qquad (1)$$

and $$X_r = E \cos(\emptyset+\theta) - \frac{KT^2}{2m_r} \qquad (2)$$

During the remainder of the recoil, we have

$$\frac{1}{2} m_r V_r^2 = K(b - X_r) \qquad (3)$$

Substituting (1) and (2) in (3) and simplifying we have

$$K = \frac{\frac{1}{2} m_r V_f^2 \cos^2(\emptyset+\theta)}{b-(E-V_f T)\cos(\emptyset+\theta)} \qquad (4)$$

Obviously $V_f \cos(\emptyset+\theta)$ and $E \cos(\emptyset+\theta)$ are the component free velocity and displacement parallel to the inclined plane.

EXTERNAL REACTIONS ON THE RECOILING PARTS AND TOP CARRIAGE ROLLER REACTIONS. If we consider the system, of the gun w_g and recoiling top carriage w_c, we have by D'Alemberts' principle, considering inertia as an equilibriating force, the following external reactions:-

(1) The powder reaction along the axis of the bore —P_b

(2) The inertia force of the recoiling mass, opposite to the motion during the acceleration, and in the direction of the motion during the retardation and parallel to the inclined plane—

$$m_r \frac{d^2x}{dt^2}$$

(3) Weight of the total recoiling parts ---W_r

(4) The normal reaction of the rollers E N

(5) The braking pull exerted along the axis of the hydraulic brake cylinder ---B

(6) The total friction along the roller track ---R

These forces are shown in fig.(1)

Resolving (1), into a couple and a single parallel force through the center of gravity of the recoiling parts and combining with (2) we have, (1) and (2) equivalent to,

A powder pressure couple ---------$P_b d$

where d = the perpendicular distance between the center of gravity of the recoiling parts and the axis of the bore.

A component parallel to the inclined plane through the center of gravity of the recoiling parts -------

$$P_b \cos(\varnothing+\theta) m_r \frac{dV}{dt} = B+R+W_r \sin\varnothing = K$$

and a component normal to the inclined plane through the center of gravity of the recoiling parts --- $P_b \sin(\varnothing+\theta)$ Thus (1) and (2) reduce to

A couple $P_b d$ and the parallel and normal components through the center of gravity of the recoiling parts, K and $P_b \sin(\varnothing+\theta)$

To reduce the couple $P_b d$ and the consequent stresses, the center of gravity of the recoiling parts should be located at the axis of the bore, or slightly below to ensure a positive jump. Since the center of gravity of the gun is at the axis of the bore, the top carriage center of gravity should also be located at the axis of the bore. This is impractical, but if the top carriage is made light as compared with the gun, its effect in lowering the center of gravity of

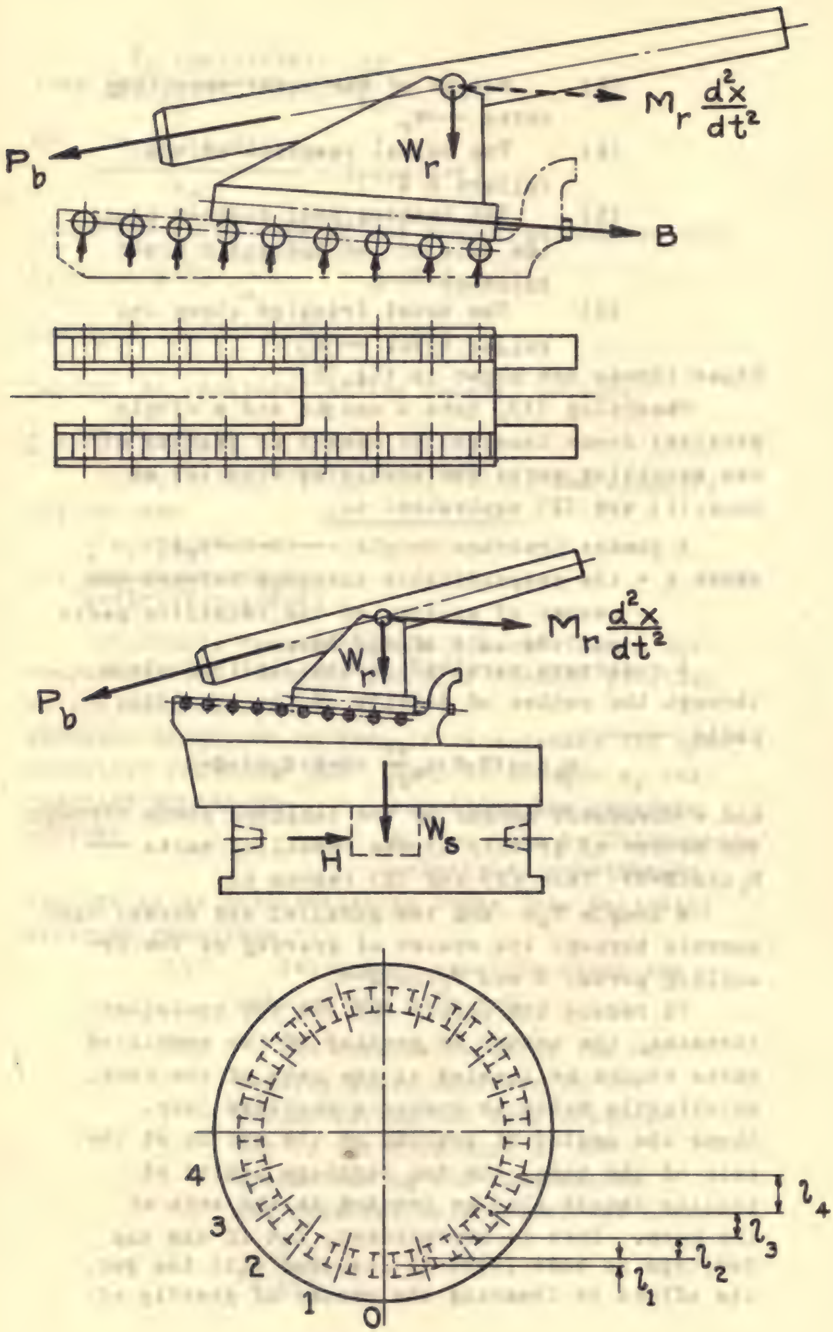


Fig. 2

the total recoiling parts is small.

To compute the roller reactions on the inclined plane, we proceed as follows:

Taking moments about the front roller reaction "O", we have $Kh_r+P_b d+P_b l_r \sin(\phi+\theta)+W_r(l_r \cos\theta-h_r \sin\theta)-Be=N_1 l_1+N_2 l_1$ ----- $N_n l_n$ (5)

where h_r and l_r are the coordinates normal and along the inclined plane of the center of gravity of the recoiling parts with respect to the front roller "O"

e = the moment arm of B with respect to "O"

$N_n l_n$ = the moment of the n <u>th</u> roller reaction about "O"

When the top carriage is light as compared with the gun, the center of gravity may be assumed approximately at the trunnions and therefore $P_b d=0$

Hence (5) reduces to, $Kh_t+P_b l_t \sin(\phi+\theta)+W_r(l_t \cos\theta-h_t \sin\theta)-Be=N_1 l_1+N_2 l_2$ ----- $N_n l_n$ (6)

where h_t and l_t are the coordinates of the trunnion with respect to the front roller "O". Further, we have, $P_b \sin(\phi+\theta)+W_r \cos\theta=N_0+N_1+N_2$----$N_n$ (7)

If we assume the roller base is rigid, we have

$N_1=k(l_1+c)$ $N_2=k(l_2+c)$ -----$N_n=k(l_n+c)$

Therefore if, $\Sigma M_0=N_1 l_1=N_2 l_2$ ----- $N_n l_n$

$\Sigma N=N_0+N_1+N_2$ -------- N_n

we will have, $M_0=k(l_1^2+l_2^2+l_3^2$ ------$l_n^2)$ $kc(l_1+l_2+l_3$----$l_n)$ (8)

$\Sigma N=k(l_1+l_2$ -----$l_n)+(n+1)kc$ (9)

From which we determine k and c

EXTERNAL REACTIONS ON THE MOUNT AND TRAVERSING ROLLER REACTIONS. If we consider the system consisting of the gun, and top carriage, that is the recoiling parts, together with the bottom carriage which rests on a circular base plate supported by traversing rollers,

we may eliminate the mutual reaction between the recoiling parts and bottom carriage since it has no effect on the equilibrium of the system. Further by the use of D'Alembert's principle we may again regard the inertia resistance of the recoiling parts as an equilibriating force.

We have therefore as before,

(1) The powder pressure couple $P_b d$

(2) The total resistance to recoil through the center of gravity of the recoiling parts and in the direction of the recoil ----K

(3) The weight of the system ---W_s

(4) The pintle reaction balancing the horizontal component of (2)

(5) The traversing roller reactions.

Let W_s = weight of system

l_s = moment arm of W_s in battery about rear traversing roller

l_s' = moment arm of W_s at recoil X or b from battery

W_{bc} = weight of bottom carriage

l_{bc} = moment arm of W_{bc} about rear traversing roller

W_r = weight of recoiling parts

l_r' = moment arm of W_r in battery about rear traversing roller

b = length of recoil

The moment of the weight of the system changes during the recoil. If we take moments about the rear traversing roller, we have for the weight during the recoil $W_r(l_r'-X\cos\theta)+W_{bc}l_{bc}=W_sl_s'$ hence $W_sl_s' = W_sl_s - W_rX\cos\theta$ and when X = the length of recoil b, we have $W_sl_s' = W_sl_s-W_rb\cos\phi$ Further if, h_r' and l_r' are the vertical and horizontal battery coordinates of the center of gravity of the recoiling parts with origin at the rear traversing roller then the out of battery coordinates become $(h_r'+b\sin\theta)$ and $(l_r'-b\cos\theta)$ respectively. We have

for the moments about O, <u>in battery</u> $W_s l_s - P_b d - K h_r'$

$\cos\theta - K l_r' \sin\theta + P_b \sin(\emptyset+\theta)(l_r' \cos\theta - h_r'$

$\sin\theta) + 2N_1 l_1 + 2N_2 l_2$ ---$N_n l_n$ (11)

and in the out of battery position

$W_s l_s - W_r b \cos\theta - K(h_r' + b\sin\theta)\cos\theta - K(k_r' - b\cos\theta)\sin\theta$

$+ 2N_1 l_1 + 2N_2 l_2$ ----- $N_n l_n$ (12)

If we assume the center of gravity of the recoiling parts at the trunnions, then $P_b d$ disappears, and $h_r' = h_t'$ and $l_r' = l_t'$ As before $N_0 = kc$, $N_1 = k(l_1 + c)$ -----$N_n = k(l_n + c)$ hence

$2N_1 l_1 + 2N_2 l_2$ -----$N_n l_n = k(2 l_1^2 + 2 l_2^2$ ---$l_n^2)$ (13)

We also note that $P_b \sin(\emptyset+\theta)\cos\theta - K\sin\theta + W_s = \Sigma N$ (14)

where $\Sigma N = k(2 l_1 + 2 l_2$ -----$(2l_{n-1} + l_n) + 2kcn$

From equation (13) and (14) we may solve for k, and c and thus compute the roller reaction N_0, N_1 -----N_n

INTERNAL REACTIONS TRUNNION REACTIONS. With gun lift mounts the trunnions are a part of the gun itself and are located at the center of gravity of the gun. Neglecting the normal reaction of the projectile, and taking moments about the center of gravity of the gun, that is about the trunnions, we have, $E\,j = 0$, (j=moment arm of E about the trunnions), therefore the elevating arc reaction $E = 0$. If X_t and Y_t are the components of the trunnion reaction, parallel and normal to the inclined plane, respectively,

$\overline{W}_g$ = the weight of the gun alone. We have, considering the gun alone, fig.(

$$\left.\begin{aligned} 2X_t &= P_b\cos(\emptyset+\theta) - W_g\sin\theta - m_g\frac{dV}{dt} \\ 2Y_t &= P_b\sin(\emptyset+\theta) + W_g\cos\theta \end{aligned}\right\} \quad (15)$$

but $P_b \cos(\emptyset+\theta) - K = m_r \dfrac{dV}{dt}$

hence $m_g \dfrac{dV}{dt} = \dfrac{m_g}{m_r} P_b \cos (\emptyset + \theta) - K \dfrac{m_g}{m_r}$

Substituting in (15), we have

$$\left.\begin{aligned} 2X_t &= P_b \cos (\emptyset+\theta)(1-\frac{m_g}{m_r})+K\frac{m_g}{m_r} - W_g \sin \theta \\ 2Y_t &= P_b \sin(\emptyset+\theta)+W_g \cos \theta \end{aligned}\right\} \quad (16)$$

which gives us the components of the trunnion reaction. The resultant trunnion reaction, becomes,

$$S_t = \sqrt{X_t^2+Y_t^2} \qquad (17)$$

The elevating arc reaction is zero, except during the first part of the powder pressure period.

To compute this "whipping action" during the powder pressure period, we must plot the moment of the normal reaction of the projectile about the trunnions as the projectile moves along the bore.

The normal reaction of the projectile, equals,

$$N_p = m \frac{dV}{dt} \sin (\emptyset+\theta) \qquad (18)$$

The weight component normal to the bore being neglected since we will assume a fairly large breech preponderence, but

$$\frac{dV}{dt} = \frac{P_b \cos (\emptyset+\theta) - K}{m_r} = \frac{P_b \cos(\emptyset+\theta)}{m_r}$$

If U = the travel up the bore

U_t = the distance from the center of the projectile in its initial position to the center of the trunnions.

Then, the elevating arc reaction becomes,

$$E = \frac{N_p (U-U_t)}{j}$$

$$= \frac{mP_b(U-U_t)\sin 2(\emptyset+\theta)}{2m_r j} \qquad (19)$$

From a plot, the maximum moment was found to occur, when the shot reaches the muzzle, and we then have for the maximum elevating arc reaction,

$$E = \frac{mP_{ob}(U_o-U_t)\sin 2(\emptyset+\theta)}{2m_r j} \qquad (20)$$

where

$P_{ob} = \frac{27}{4} C^2 \frac{U}{(C+U)^3} P_{b_{max}}$ = (pressure on breech when shot leaves muzzle)

$C = U(\frac{27}{16}\frac{P_{max}}{p_e} - 1) \pm \sqrt{(1-\frac{27}{16}\frac{P_{max}}{p_e})^2-1}$ = (twice abscissa of max. pressure)

$P_e = \frac{wv_o^2}{2gU}$ ($p_{b_{max}}$=total max. powder force on breech)

v_o = muzzle velocity; P_e= mean powder reaction on breech

U = travel up bore in feet

REACTIONS ON TOP CARRIAGE. Neglecting the elevating arc reaction during the powder period, the reactions on the top carriage reduce to the following:-

(1) The trunnion reactions divided into X_t and Y_t and equal and opposite to the component reactions exerted on the gun.

(2) The weight of the top carriage acting through its center of gravity --- W_c

(3) The braking pull reaction --- B

(4) The roller reactions of the inclined plane.

Assuming the center of gravity of the top carriage at the trunnions for convenience, we have

$$2X_t - W_c \sin\theta - B = m_c \frac{dV}{dt} \qquad (21)$$

$$2Y_t + W_c \cos\theta = \Sigma N \qquad (22)$$

and taking moments about the front roller reaction, we have $\Sigma EM_0 = 2X_t h_t - 2Y_t l_t + W_c \cos\theta\, l_t - W_c \sin\theta . h_t$

$$- Be - m_c \frac{dV}{dt} h_t \qquad (23)$$

where $m_c \dfrac{dV}{dt} = \dfrac{[P_b \cos(\emptyset+\theta) - K]}{m_r} m_c$

h_t and l_t are the coordinates of the trunnion with respect to the front roller "O", and normal and parallel to the inclined plane.

θ = the perpendicular distance from the front roller to the line of action of B

If, as is usually the case, the center of gravity of the top carriage is not located at the trunnions, we have equation (21) and (22) the same, but equation (23) modified to:-

$$M_0 = 2X_t h_t + 2Y_t l_t + W_c \cos\theta\, l_c - W_c \sin\theta\, h_c - [\frac{P_b \cos(\emptyset+\theta) K}{m_r}]$$

$$m_c h_c - Be \qquad (24)$$

where l_c and h_c are the coordinates of the center of gravity of the top carriage parallel and normal to the inclined plane and with origin at the front roller. As before, the moment of the roller reactions $\Sigma M_0 = N_1 l_1 + N_2 l_2 + N_3 l_3$ ---- $N_n l_n$

$$\Sigma N = N_0 + N_1 + N_2 + N_3 \text{ ---- } N_n$$

therefore $\Sigma M_0 = k(l_1^2 + l_2^2 + l_3^2 \text{ -----} l_n^2) + kc(l_1 + l_2 + l_3 \text{ ---} l_n)$

$$\Sigma N = kl_1 + kl_2 \text{ -----} kl_n + (n+1)kc$$

and $N_0 = kc$, $N_1 = k(l_1 + c)$, $N_2 = k(l_2 + c)$-----$N_n = k(l_n + c)$

that is solving for k and c we determine N_1 N_2 -----N_n knowing the total normal.

Substituting in Eq.(24),

$$2X_t = P_b \cos(\emptyset + \theta)(1 - \frac{m_g}{m_r}) + K \frac{m_g}{m_r} - W_g \sin\theta$$

$2Y_t = P_b \sin(\emptyset+\theta) + W_g \cos\theta$ and noting that, $m_g l_t + m_c l_c = m_r l_r$

$m_g h_t + m_c h_c = m_r h_r$ we have, $P_b[(h_t - h_r)\cos(\emptyset+\theta) + (l_t - l_r)\sin(\emptyset+\theta)] + P_b \sin(\emptyset+\theta) l_r + K h_r - W_r h_r \sin\theta + W_r l_r \cos\theta - Be = \Sigma M_o$ (25)

Now $(h_t - h_r)\cos(\emptyset+\theta) + (l_t - l_r)\sin(\emptyset+\theta)$ is evidently equal to the perpendicular distance between the center of gravity of the total recoiling parts and the axis of the bore. Hence (25) reduces to

$$P_b d + P_b \sin(\emptyset+\theta) l_r + K h_r - W_r h_r \sin\theta + W_r l_r \cos\theta - Be = \Sigma M_o \qquad (26)$$

where $d = (h_t - h_r)\cos(\emptyset+\theta) + (l_t - l_r)\sin(\emptyset+\theta)$

This is evidently the same as equation (5) obtained in the consideration of external force on the recoiling parts.

REACTIONS ON BOTTOM CARRIAGE. The reactions on the bottom carriage consist of the following:-

(1) The braking pull exerted along the axis of the hydraulic recoil cylinder.

(2) The roller reactions normal to the inclined plane.

(3) The horizontal reaction exerted by the pintle bearing.

(4) The supporting reactions exerted by the traversing rollers in a vertical direction.

Evidently (1) and (2) is the reaction of the top carriage on the bottom carriage, which is divided into the components (1) and (2).

Thus in battery, the moments of (1) and (2) about "O" the point of contact of the front roller reaction of the inclined plane reduce to $\Sigma M_o + Be$ but $\Sigma M_o + Be = P_b d + P_b \sin(\emptyset+\theta) l_r + K h_r - W_r h_r \sin\theta + W_r l_r \cos\theta$ where l_r and h_r are the coordinates of the center of gravity along and normal to the plane of the re-

coiling parts with respect to the front roller.

Therefore during the powder pressure period the reaction of the top carriage on the bottom carriage is equivalent to,

(1) A powder pressure couple "$P_b d$"

(2) A component of the powder force normal to the inclined plane and through the center of gravity of the recoiling parts "$P_b \sin(\phi+\theta)$"

(3) The total resistance to recoil parallel to the inclined plane, and through the center of gravity of the recoiling parts "K"

(4) The total weight of the recoiling parts through the center of gravity of the recoiling parts "W_r"

During the pure recoil or subsequent retardation, we have, $\Sigma M_o + Be = Kh_r - W_r h_r \sin\theta + W_r l_r \cos\theta$ and therefore the reaction of the top carriage on the bottom carriage, is equivalent to

(1) The total resistance to recoil parallel to the inclined plane and through the center of gravity of the recoiling parts ---K.

(2) The total weight of the recoiling parts.

To compute the horizontal pintle reaction, we have $H = K\cos\phi - P_b \sin(\phi+\theta)\sin\phi$ the total normal reaction on the traversing rollers, become $\Sigma N = P_b \sin(\phi+\theta)\cos\theta - K\sin\theta + W_r + W_{bc}$

where W_{bc} = weight of bottom carriage

If further l_r' = moment arm of W_r in battery about rear traversing roller

x = recoil displacement from battery

l_{bc} = moment arm of W_{bc} about rear traversing roller

W_s = weight of entire system above traversing rollers

l_s = moment arm of W_s about rear traversing roller

Then, for the moment of the weights about the rear traversing roller, we have,

$$W_r(l_r' - x \cos \emptyset) + W_{bc} l_{bc} = W_s l_s - W_r x \cos \emptyset$$

Therefore, for the moments about the rear traversing roller, we have

$$\Sigma M_o' = W_s l_s - W_r x \cos \emptyset - P_b d + P_b \sin(\emptyset+\theta)[(l_r' - x \cos\theta)\cos\theta - (h_r' + x \sin\theta)\sin\theta] - K[(h_r' + x \sin\theta)\cos\theta - (l_r' - x \cos\theta)\sin\theta]$$

When P_b is a maximum x is negligible; therefore for the maximum roller reaction, we have

$$\Sigma M_o' = W_s l_s - P_b d + P_b \sin(\emptyset+\theta)[l_r' \cos\theta - h_r' \sin\theta] - K[h_r' \cos\theta - l_r' \sin\theta]$$

EXACT THEORY OF RECOIL CONSIDERING NORMAL REACTION TO BORE OF PROJECTILE.

Due to the normal reaction of the powder charge and projectile during the travel up the bore, the recoil is more or less effected, depending of course on the weight of the shell and powder charge as compared with the weight of the recoiling parts. Let

P_b = powder reaction on breech of gun

P_p = powder reaction on base of projectile

P_e = mean powder reaction in bore of bin

N_1 = normal reaction of projectile to axis of bore

N_2 = normal reaction of powder charge to axis of bore

$N = N_1 + N_2$ = the total normal reaction of powder charge and projectile to axis of bore.

$B + R$ = total braking resisting recoil parallel to inclined plane.

w and m = weight and mass of projectile

w_r and m_r = weight of mass of recoiling mass

$\bar{w}$ and $\bar{m}$ = weight and mass of powder charge

$\emptyset$ = the angle of elevation of the axis of the bore

θ = the angle of inclination of the inclined plane

x' and y' = coordinates along and normal to the axis of the bore.

u = travel of the projectile along the bore or relative displacement along the axis of the bore

x = the projection or component of the absolute displacement of the projectile parallel to the inclined plane

Considering the motion of the projectile, we have

$$P_p = m \frac{d^2x'}{dt^2} + mg \sin \emptyset \quad (1)$$

$$N_1 = m \frac{d^2x}{dt^2} \sin (\emptyset+\theta) + mg \cos \emptyset \quad (2)$$

where

$$\frac{d^2x'}{dt^2} = \frac{d^2u}{dt^2} - \frac{d^2x}{dt^2} \cos (\emptyset+\theta) \quad (3)$$

for the motion of the powder charge,

$$P_b - P_p = \frac{\bar{m}}{2}\left[\frac{d^2u}{dt^2} - 2 \frac{d^2x}{dt^2} \cos (\emptyset+\theta)\right] + \bar{m}g \sin\emptyset$$

$$= \frac{\bar{m}}{2}\left[\frac{d^2x'}{dt^2} - \frac{d^2x}{dt^2} \cos (\emptyset+\theta)\right] + \bar{m}g \sin \emptyset \quad (4)$$

$$N_2 = \bar{m} \frac{d^2x}{dt^2} \sin(\emptyset+\theta) + \bar{m}g \cos \emptyset \quad (5)$$

where

$$\left[\frac{\frac{d^2u}{dt^2} - 2 \frac{d^2x}{dt^2} \cos(\emptyset+\theta)}{2}\right]^2 - \left[\frac{d^2x}{dt^2} \sin(\emptyset+\theta)\right]^2 \quad (6)$$

Is the resultant acceleration of the center of gravity of the powder charge, and for the motion of the recoiling parts,

$$P_b \cos(\emptyset+\theta) - N \sin(\emptyset+\theta) - m_r g \sin \theta - (B+R) = m_r \frac{d^2x}{dt^2} \quad (7)$$

where $N = N_1 + N_2$

Combining the above equations, we have

$$(m+\frac{\bar{m}}{2})\frac{d^2x'}{dt^2}\cos(\emptyset+\theta)-\frac{\bar{m}}{2}\frac{d^2x}{dt^2}\cos^2(\emptyset+\theta)+(m+\bar{m})g\sin\emptyset\cos$$

$$(\emptyset+\theta)-(m+\bar{m})\frac{d^2x}{dt^2}\sin^2(\emptyset+\theta)-(m+\bar{m})g\cos\emptyset\sin(\emptyset+\theta)-m_rg\sin\theta$$

$$-(B+R)=m_r\frac{d^2x}{dt^2}$$

Expanding and simplifying, we obtain

$$(m+\frac{\bar{m}}{2})\frac{d^2x'}{dt^2}\cos(\emptyset+\theta)-\frac{\bar{m}}{2}\frac{d^2x}{dt^2}-(m+\frac{\bar{m}}{2})\frac{d^2x}{dt^2}\sin^2(\emptyset+\theta)$$

$$-(m+\bar{m}+m_r)g\sin\theta-(B+R)=m_r\frac{d^2x}{dt^2}$$

that is

$$(m+\frac{\bar{m}}{2})[\frac{d^2x'}{dt^2}\cos(\emptyset+\theta)-\frac{d^2x}{dt^2}\sin^2(\emptyset+\theta)]-(m_r+\frac{\bar{m}}{2})\frac{d^2x}{dt^2}$$

$$=B+R+(m+\bar{m}+m_r)g\sin\theta \qquad (8)$$

It is to be noted that

$[\frac{d^2x'}{dt^2}\cos(\emptyset+\theta)-\frac{d^2x}{dt^2}\sin^2(\emptyset+\theta)]$ is the projection or component of the resuotant acceleration of the projectile parallel to the inclined plane, and $B+R+(m+m_r+\bar{m})g\sin\theta$ is the total external force parallel to the plane. Neglecting gravity and with free recoil($B+R=0$), that is no extraneous force acts, hence we have

$$(m+\frac{\bar{m}}{2})[\frac{d^2x'}{dt^2}\cos(\emptyset+\theta)-\frac{d^2x}{dt^2}\sin^2(\emptyset+\theta)]-(m_r+\frac{\bar{m}}{2})\frac{d^2x}{dt^2} \qquad (9)$$

In terms of the relative acceleration $\frac{d^2u}{dt^2}$

since $\frac{d^2x'}{dt^2}=\frac{d^2u}{dt^2}-\frac{d^2x}{dt^2}\cos(\emptyset+\theta)$,

$$[(m+\frac{\bar{m}}{2})\frac{d^2u}{dt^2}\cos(\emptyset+\theta)+\frac{d^2x}{dt^2}]=(m+\frac{\bar{m}}{2})\frac{d^2x}{dt^2}$$

hence

$$(m+\frac{\bar{m}}{2})\frac{d^2u}{dt^2}\cos(\emptyset+\theta)=(m+\bar{m}+m_r)\frac{d^2x}{dt^2} \qquad (10)$$

and by integration

$$(m+\frac{\bar{m}}{2})\frac{d^2u}{dt^2}\cos(\emptyset+\theta)=(m+\bar{m}+m_r)\frac{dx}{dt} \qquad (11)$$

$$(m+\frac{\bar{m}}{2})u\ \cos\ (\emptyset+\theta)=(m+\bar{m}+m_r)x \qquad (12)$$

Hence (10, (11), and (12) gives us the free acceleration, velocity and displacement up the inclined plane with respect to the corresponding function up the bore of the gun.

Again considering equation (8) and substituting

$$\frac{d^2x'}{dt^2} = \frac{d^2u}{dt^2} - \frac{d^2x}{dt^2}\cos\ (\emptyset+\theta)$$

we have

$$(m+\frac{\bar{m}}{2})\frac{d^2u}{dt^2}\cos(\emptyset+\theta)-(B+R+[m+\bar{m}+m_r]g\ \sin\theta)-(m+\bar{m}+m_r)\frac{d^2x}{dt^2} = 0$$

hence

$$\frac{d^2x}{dt^2} = (\frac{m+\frac{\bar{m}}{2}}{m+\bar{m}+m_r})\frac{d^2u}{dt^2}\cos\ (\emptyset+\theta) - [\frac{B+R(m+\bar{m}+m_r)g\ \sin\theta}{m+\bar{m}+m_r}] \qquad (13)$$

Integrating,

$$\frac{dx}{dt} = (\frac{m+\frac{\bar{m}}{2}}{m+\bar{m}+m_r})\frac{du}{dt}\cos(\emptyset+\theta)-[\frac{B+R+(m+\bar{m}+m_r)g\ \sin\ \theta}{m+\bar{m}+m_r}]t \qquad (14)$$

and

$$x = (\frac{m+\frac{\bar{m}}{2}}{m+\bar{m}+m_r})u\ \cos(\emptyset+\theta)- \frac{1}{2}[\frac{B+R+(m+\bar{m}+m_r)g\sin\ \theta}{m+\bar{m}+m_r}]\ t^2 \qquad (15)$$

which are the general equations of constrained recoil during the travel of the projectile up the bore.

Neglecting m and $\bar{m}$ as small compared with m_r and if we let

$$V_f = \frac{m+\frac{m}{2}}{m+\bar{m}+m_r}\ (\frac{du}{dt})\ \text{and}\ E = (\frac{m+\frac{\bar{m}}{2}}{m+\bar{m}+m_r})u$$

then

$$\frac{dx}{dt} = V_f\ \cos(\emptyset+\alpha)-(\frac{B+R+m_rg\ \sin\ \theta}{m_r})t \qquad (14')$$

$$x = E\ \cos\ (\emptyset+\alpha)\ -\ \frac{1}{2}\ (\frac{B+R+m_rg\sin\theta}{m_r})\ t^2 \qquad (15')$$

which are sufficiently approximate for ordinary

calculations.

NUMERICAL COMPUTATION.

10" Gun Barbette

K = braking force

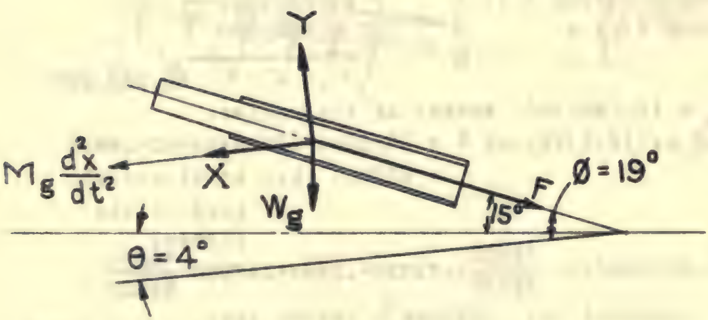
$$K = \frac{m_r\ V_f\ \cos\ (\emptyset+\theta)^2}{2\ b+T\ V_f\cos(\emptyset+\theta)-E\ \cos(\emptyset+\theta)} \qquad (\emptyset+\theta) = 19°$$

$$K = \frac{89.820(29.76\times0.9455)^2}{2(\frac{50}{12}+0.0446\times29.76\times0.9455-0.9455)32.2} = 247000\ \text{lbs.}$$

$$K = \frac{89.820(29.76\times0.9976)^2}{32.2\times2(\frac{50}{12}+.0446\times29.76\times0.9976-0.9976)} = 274000\ \text{lbs.}$$

W_r = 89820 lbs.
V_f = 29.76 ft/sec.
b = 50 in.
T = 0.0446 sec.
E = 1 ft.
Zero elevation $\emptyset$ = 0°
θ = 4°

TRUNNION REACTIONS.



$Y = F \sin(\varnothing+\theta)+W_g \cos\theta$

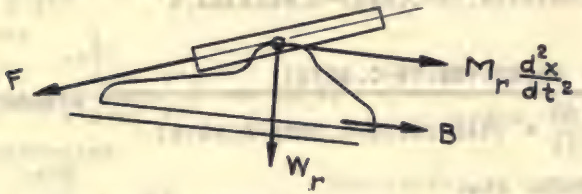
$X = F \cos(\varnothing+\theta(1-\frac{M_g}{M_r})+K\frac{M_g}{m_r} - W_g \sin\theta$

$K = (W_r \sin\theta+B) = 247000$ lbs.

$F = P_m \times A = 32000\times78.54 = 2513000$ lbs.

$W_g = 76830$ lbs.

As a check, we may consider the forces external to the system above the rollers.



$F \cos(\varnothing+\theta)h_t+F \sin(\varnothing+\theta)L_t+W_r\cos\theta L_t-W_r\sin h_t-[F \cos(\varnothing+\theta)-K]h_t-Be = \Sigma M_o$

2376000 × 31 =	73,656,000	
2513000 × .3256 × 15 =	12,273,500	
89820 × .9976 × 15 =	1,344,000	
		87,273,500
89820 × .0698 × 31 =	194,000	
2376000−247000 × 31 =	66,000,000	
240700 × 12 =	2,889,000	
		69,083,000

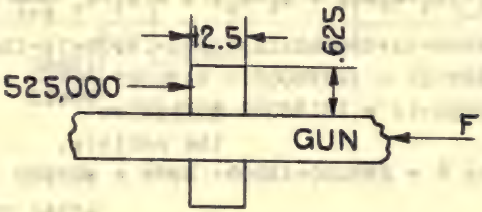
$\Sigma M_o = 18,190,500$ moment of the rollers

$\Sigma N = F \sin(\varnothing+\theta)+W_r\cos\theta = 2513000\times.3256+89820\times.9976$
$= 907600$ lbs. total normal load on the rollers.

$X = 2376000(1-\frac{76830}{89820})-76830\times.0698+247000\frac{76830}{89820}$

$343570-5.363+211284 = 549500$ lbs.

Y = 818000+77000 = 895000 lbs.

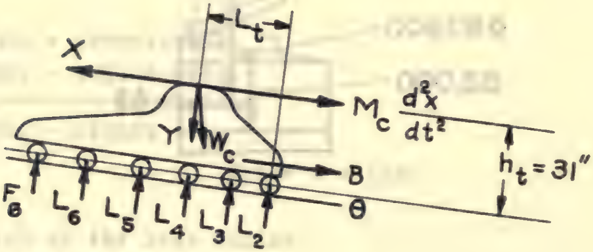


Sectional Modulus=195

Force on the trunnions = $\sqrt{X^2+Y^2} = 10^5\sqrt{30.2+80.1}$
$= 10^6\sqrt{1.103}$
= 1.050000 lbs.

$$S = \frac{525000\times.3.375}{195} = 9080 \text{ lbs/sq.in. fibre stress}$$

ROLLER REACTION.



about front roller, $\frac{F\cos\emptyset - K}{m_r} m_c$

$B = K - W_r \sin\theta = B = 240700$

$\Sigma M_{(o)} = Xh_t + YL_t + W_c \cos\theta\ L_t - W_c \sin\theta\ h_t - m_c \frac{d^2x}{dt^2} h_t - Be$

$= 549500 \times 31 + 895000 \times 15 + 13000 \times .9976 \times 15 - 13000 \times .0698 \times 31 - (2376000 - 247000) \frac{13000}{89820} \times 31 - 240700 \times 12 = 18185000$ moment on the rollers.

$\Sigma N = V + W_c \cos\theta = 895000 + 13000 \times .9976 = 907960$ lbs. total normal load on rollers.

<u>REACTION ON THE TRAVERSING ROLLERS.</u>

$V = F \sin(\phi - \theta) + W_t W_s - K \sin\theta$

$X = (F \cos\phi - K)\cos\theta + F \sin(\phi + \theta)$

W_r = wt. of recoiling part 89000

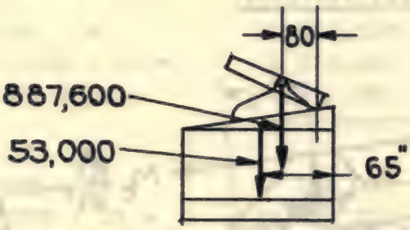
W_s = wt. of the rest 53000

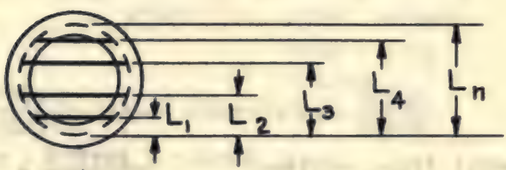
$V = 2513000 \times .2588 + 89000 + 53000(-247000).0698$

$= 650000 + 89000 + 53000 - 17200 = 774,800$

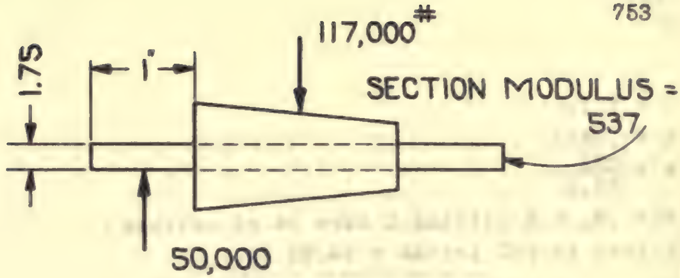
$X = (2513000 \times 9455 - 247000).9976 + 2513000 \times .2588$

$2124000 + 650000 = 2774000$





20 rollers



$M_{(o)} = 722000 \times 80 + 53000 \times 65 = 61450000$ inch/lbs.
moment on the rollers.

$M = K(2\, l_1^2 + 2\, l_2^2 \text{ ----- } l_n^2) + Kc(2\, l_1 + 2 l_2 \text{ ----- } 2\, l_{n-1} + l_n)$

$V = \Sigma N = K(2\, l_1 + 2\, l_2 \text{ ----- } 2\, l_{n-1} + l_n) + 2Kc$

$$6145000 = K(2\times 2.3^2 + 2\times 9.4^2 + 2\times 20.2^2 + 2\times 34.4^2 + 2\times 49.5^2 + 2\times 64.9^2 + 2\times 78.8^2 + 2\times 89.6^2 + 2\times 96.4^2 + 99^2) + Kc$$
$$(2\times 2.3 + 2\times 9.4 + 2\times 20.2 + 2\times 34.4 + 2\times 49.3 + 2\times 78.8 + 2\times 89.6 + 2\times 96.4 + 99)$$

hence $6145000 = 73600K + 990Kc$

$774800 = 990K + 20Kc$

$61430000 = 73600K + 990Kc$

$34000000 = 49000K + 990Kc$

$27450000 = 24600K$

Hence $K = 1180$

Reaction on the last roller
$99 \times 1180 = 117000$ lbs.

Force due to rifling and its effect on the traversing chain.

$F_r t = Iw$

$$F = \frac{Iw}{rt} = \frac{MK^2 w}{rt}$$

$w = 2\ N$

$v = 5$ in.

$t = .0162$

$m = \frac{606}{32.2}$

$K = .8_r = 4$ rifling 1 turn in c5 caliber

1 turn in 250 inches = 20.83 ft.

$$W = 2\pi \frac{25.50}{20.83} = 770 \text{ radius}$$

$$F = \frac{606 \times 4^2 \times 770 \times 2}{32.2 \times 5 \times .0162 \times 12 \times 2} = \frac{606 \times 16 \times 770}{32.2 \times 5 \times .0162 \times 12} = \frac{7465920}{312984}$$

$= 238500$ lbs.

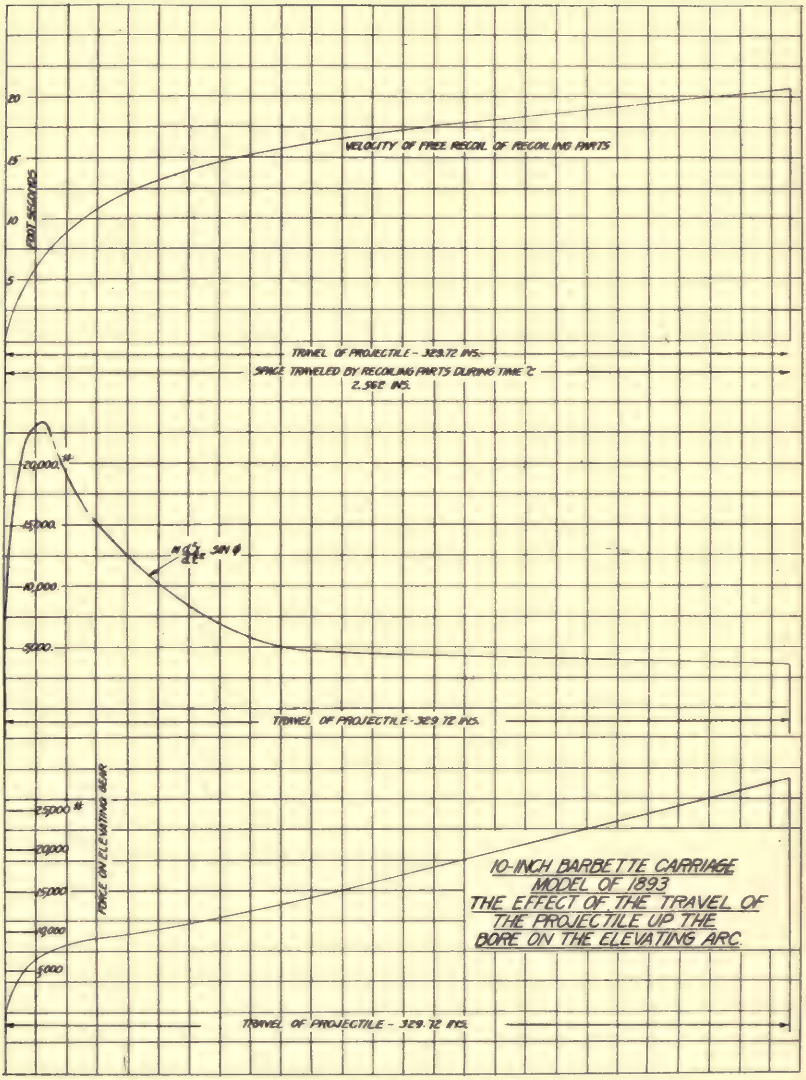
Torque $= 238500 \times 4 = 95400$ ft.lbs.

$95400 \times \sin 15° = 95400 \times .2588 = 24700$ ft.lbs.

" $TR \sin \emptyset = 24700$

$$T_p = \frac{5}{49} \times 24700 = -\ 2500 \text{ lbs.}$$

Tension on the chain at pinion $\frac{2500}{5} = 500$ lbs.


20
15
10
5
FOOT SECONDS
VELOCITY OF FREE RECOIL OF RECOILING PARTS
TRAVEL OF PROJECTILE - 329.72 INS.
SPACE TRAVELED BY RECOILING PARTS DURING TIME τ
2.562 INS.
20,000 #
15,000
10,000
5000
TRAVEL OF PROJECTILE - 329.72 INS.
25,000 #
20,000
15,000
10,000
5000
FORCE ON ELEVATING GEAR
10-INCH BARBETTE CARRIAGE
MODEL OF 1893
THE EFFECT OF THE TRAVEL OF
THE PROJECTILE UP THE
BORE ON THE ELEVATING ARC.
TRAVEL OF PROJECTILE - 329.72 INS.

C H A P T E R XII.

DOUBLE RECOIL SYSTEM.

OBJECT In order to reduce the reaction of recoil on a carriage to a moderate value when the caliber is large a long recoil is necessary. A long recoil requires long guides and in addition is usually prohibitive due to breech clearance necessary to avoid a great loss in stability due to the overhanging of the recoiling weights at low elevation when the gun is out of battery, etc. A long recoil may be avoided by the use of a double recoil system and the stability of a railway or a caterpillar carriage at the same time increased. This latter factor is the real distinctive value of a double recoil system over a corresponding single recoil system.

It is important to note that a caterpillar or railway car braked with a single gun recoil system is essentially a double recoil system, the ground or rail offering a tangential reaction which corresponds to the reaction of the lower recoil system.

Obviously when a top carriage moves up an inclined plane under the recoil reaction of the gun and the resistance of the lower recoil system or when with a single recoil system the caterpillar or railway car runs back on the ground or rail under the recoil reaction of the gun and the resistance of the ground or rail, the recoil reaction of the gun becomes different and the throttling grooves must therefore be necessarily different, then with a single recoil system when a constant recoil reaction is imposed between the gun and top carriage.

CLASSIFICATION. In the design of a double recoil system it is desirable in order to simplify calculation and secure uniformity of stresses throughout the recoil to have both the upper and lower recoil reactions constant throughout recoil. However, in ordnance design it has been customary to mount single recoil mounts, gun and top carriage together on caterpillars, etc., and for augmenting the stability to allow the top carriage to recoil as well up an incline plane, the inclination of the plane being sufficient to bring the system into battery after the recoil. The recoil reaction of the upper system can therefore, with a double recoil no longer be constant since the recoil reaction is the sum of the air reaction, a function of the relative displacement between the gun and top carriage, and the throttling reaction which is a function of the relative velocity. Therefore, with a constant braking on the lower recoil system, to ascertain the displacement of the top carriage up the incline plane, it would be necessary to carry on a somewhat elaborate point by point integration for the various dynamical equations and displacements at each point of the recoil.

Hence in the following discussion we will consider the dynamical relations:-

(1) With a constant resistance for both upper and lower recoil systems.

(2) With a given upper recoil system and a constant resistance for the lower recoil system.

APPROXIMATE THEORY FOR (1). Reactions and velocity for double recoil systems: Let

P = resistance of gun recoil system

W_r or w_r = wt. of recoiling parts (upper)

REACTIONS ON DOUBLE RECOIL SYSTEM

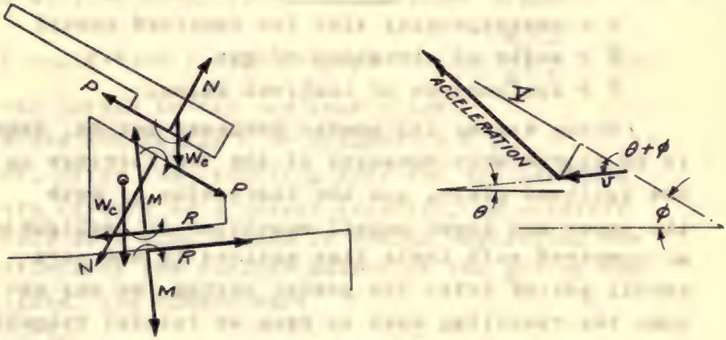


Fig. 1

W_c or w_c = wt. of top carriage and cradle (lower)

V = initial velocity

Z = displacement of gun on carriage, i. e. = relative displacement

N = upper normal reaction between recoiling parts and top carriage

M = lower normal reaction between top carriage and inclined plane.

X = total run up on inclined plane.

V or v = velocity of combined recoil

t = corresponding time for combined recoil

ϕ = angle of elevation of gun

θ = inclination of inclined plane.

Since during the powder pressure period, there is no appreciable movement of the top carriage up the inclined plane, and the timeaction of both the upper and lower recoil reactions is negligible as compared with their time actions in the pure recoil period after the powder period, we may assume the recoiling mass to have an initial velocity V at the beginning of the recoil, where

$$V = 0.9 \left(\frac{wv_o + \bar{w}\ 4700}{w_r}\right)$$

where w = weight of projectile

$\bar{w}$ = weight of charge

v_o = muzzle velocity

and 0.9 is a constant to allow for the effect of the recoil reaction on the recoiling mass during the powder pressure period. Consider now fig.(1)

The retardation of the recoiling parts is the vector difference of the velocities at the end and beginning of time t divided by "t", that is

$\bar{a} = \dfrac{\bar{v} - V}{t}$ hence assuming axes parallel and normal to the guides of the upper recoiling parts, we have the following equations of motion for the recoiling parts,

$$P-W_r\sin\emptyset = \frac{\bar{W}_r}{g}\left(\frac{V-v\cos[\emptyset+\theta]}{t}\right) \qquad (1)$$

and

$$N-W_r\cos\emptyset = \frac{\bar{W}_r}{g}\,\frac{v\ \sin(\theta+\emptyset)}{t} \qquad (2)$$

Since there is no roation the couple between the recoiling parts and top carriage need not to be considered.

Next considering the motions of the carriage above, we have, along the inclined plane:

$$P\ \cos(\emptyset+\theta)-N\ \sin(\emptyset+\theta)-W_c\sin\ \theta - R = \frac{W_c}{g}\,\frac{v}{t} \qquad (3)$$

and normal to the inclined plane,

$$N\ \cos(\emptyset+\theta)+W_c\cos\ \theta\ +P\ \sin(\emptyset+\theta)-M = 0 \qquad (4)$$

If, after the recoiling mass and top carriage are brought to a common velocity, we consider both as a single mass in motion neglecting the effect of the further motion of the gun on its slide, the common mass $\frac{W_r+W_c}{g}$ brought to rest by a constant force R.

Hence the retardation after time t, becomes,

$a_r = \frac{R_g}{W_r+W_c}$ and the interval of common retardation, becomes, $t_r = \frac{W_r+W_c}{R_g}$ and the corresponding displacement $\frac{1}{2}\,a_r t_r^2 = \frac{W_r+W_c}{2R_g}\,v^2$. Therefore, the total displacement (since the top carriage is uniformly accelerated to a velocity v at time t)becomes,

$X = \frac{v}{2}\ t + \frac{W_r+W_c}{2R_g}\,v^2$ Since the relative displacement equals the absolute displacement of the gun parallel to the guides minus the displacement of the top carriages parallel to the guides, we have

$$Z = \frac{V+v\ \cos(\emptyset+\theta)}{2}\ t - \frac{v\ \cos\ (\emptyset+\theta)}{2}\ t \quad \text{hence } Z = \frac{V}{2}\ t$$

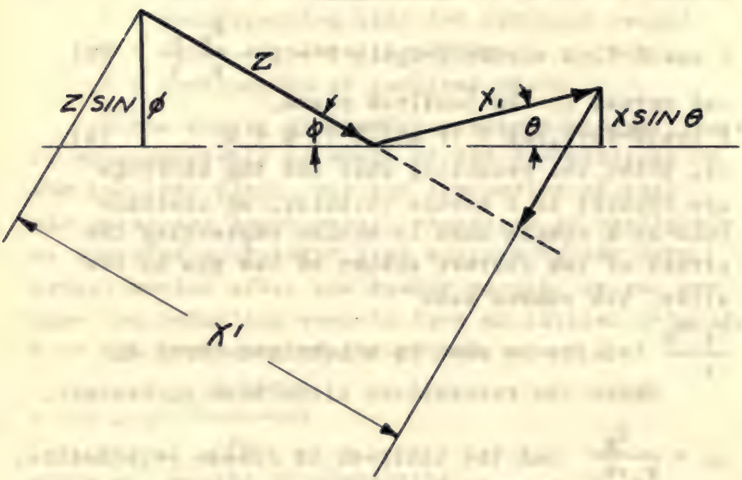


Fig. 2

ENERGY EQUATION FOR DOUBLE RECOIL. Let x' and y' = the coordinates parallel and normal to the gun axis.

x and y = the coordinates parallel and normal to the top carriage inclined plane.

$x_1 = \frac{v}{2} t$ where x_1 = the displacement of the top carriage up the inclined plane at the instant when the recoiling mass and top carriage move at common velocity v,

hence $x_1 = \frac{v}{2} t$. Then for the recoiling parts, we have, $(P-W_r \sin\emptyset)x' = \frac{1}{2} [V^2-v^2\cos^2(\emptyset+\theta)]$ (1') (In direction of upper guides) and

$(N-W_r\cos)x_1\sin(\emptyset+\theta) = \frac{1}{2}m_r v^2\sin^2(\emptyset+\theta)$ (2') (at right angles to upper guides), and for the top carriage alone, we have

$[P \cos(\emptyset+\theta)-N \sin(\emptyset+\theta)-W_c\sin \theta-R]x_1 = \frac{1}{2} M_c v^2$ (3') (Top carriage up plane)

Subtracting (3') from (1'), we have

$$P[x'-x_1\cos(\emptyset+\theta)]-W_r\sin\emptyset x'+N \sin(\emptyset+\theta)x_1+W_c\sin \theta.x_1 +Rx= \frac{1}{2}m_r[V^2-v^2\cos^2 (\emptyset+\theta)]- \frac{1}{2} m_c v^2$$

and substituting (2) in (4), we have

$$P[x'-x_1\cos(\emptyset+\theta)-W_r\sin\emptyset.x'+ \frac{1}{2}m_r v^2\sin^2(\emptyset+\theta) +W_r x_1\cos\emptyset\sin(\emptyset+\theta)- \frac{1}{2} m_r[V^2-v^2\cos^2(\emptyset+\theta)]+ \frac{1}{2} m_c v^2 + w_c \sin\theta.x+R x_1 = 0$$

Now the relative displacement between the gun and top carriage becomes, $Z = x'-x_1 \cos (\emptyset+\theta)$. Hence the above expression reduces to

$$PZ-W_r[x' \sin\emptyset-x_1\cos\emptyset\sin(\emptyset+\theta)]+ \frac{1}{2}m_r v^2+ \frac{1}{2}m_c v^2+W_c \sin\theta.x+R x_1 = \frac{1}{2} m_r v^2 \qquad (7)$$

Now $W_r[x'\sin\emptyset-x_1\cos\emptyset\sin(\emptyset+\theta)]$ is evidently the work done by gravity on the recoiling parts and $W_c \sin \theta.x$ is the same for the top carriage. In terms of the relative displacement Z, the work done by gravity on the recoiling parts may be obtained by consideration of fig.(2)

From fig.(2), we have, $x'=Z+x_1\cos(\emptyset+\theta)$

and the work done by gravity on the recoiling parts becomes, $W_r(Z\sin\phi - x_1\sin\theta)$

$$= W_r[x' - x_1\cos(\phi+\theta)\sin\phi - x_1\sin\theta]$$
$$= W_r(x'\sin\phi - x_1\sin\phi\cos\phi\cos\theta + x_1\sin\phi\sin\theta - x_1\sin\theta)$$
$$= W_r[x'\sin\phi - x_1\sin\phi\cos\phi\cos\theta + x\sin\theta(\sin^2\phi - 1)]$$
$$= W_r(x'\sin\phi - x_1\sin\phi\cos\theta - x_1\sin\theta\cos^2\phi)$$
$$= W_r[x'\sin\phi - x_1\sin(\phi+\theta)\cos\phi]$$

Hence equation (7) reduces to,

$$PZ - W_r(Z\sin\phi - x_1\sin\theta) + W_r x_1\sin\theta + \frac{1}{2}(m_r+m_c)v^2 + Rx_1 = \frac{1}{2}m_r v^2 \qquad (8)$$

where $x_1 = \frac{v}{2}t$. Further since $R(x-x_1) = \frac{1}{2}(m_r+m_c)v^2$, equation (8) reduces to

$$PZ - W_r(Z\sin\phi - x\sin\theta) + W_c\, x\sin\theta + Rx = \frac{1}{2}m_r v^2 \qquad (9)$$

Equation (8) is almost obvious from the theory of energy, since the total initial energy $\frac{1}{2}m_r v^2$ plus the work done by gravity $W_r(Z\sin\phi - x_1\sin\theta) - W_c x_1\sin\theta$ equals the final Kinetic energy of the system $\frac{1}{2}(m_r+m_c)v^2$ plus the work done in the upper and lower recoil brakes $PZ + Rx_1$ to the combined recoil.

Equation (9) is also self evident since the final Kinetic energy of the system equals zero after the system has recoiled the total displacement x up the inclined plane.

RECAPITULATION OF APPROXIMATE FORMULAE FOR DOUBLE RECOIL BRAKES. When the resistance to recoil is assumed constant for both upper and lower recoil systems we may with a very close approximation obtain the principle reaction, by the previous derived formulae. These formulae are recapitulated in the following group for convenience in calculation. Then if,

w = weight of projectile
$\bar{w}$ = weight of charge
v_o = muzzle velocity

W_r or

w_r = weight or recoiling parts

w_o = weight of top carriage

Z = displacement of gun on carriage

X = total run up on inclined plane

N = upper normal reaction between parts.

Ø = angle of elevation of gun

θ = inclination of inclined plane

V = initial velocity recoiling parts

v = velocity of combined recoil

t = corresponding time for combined recoil

x_1= run up on inclined plane to combined recoil

We have

$$V = 0.9 \left(\frac{wv_o+\bar{w}4700}{w_r}\right) \qquad (1')$$

$$P-w_r\sin\phi = \frac{w_r}{g}\left[\frac{V-v\cos(\phi+\theta)}{t}\right] \qquad (2')$$

$$N - w_r \cos \phi = \frac{w_r}{g} \frac{v \sin(\phi+\theta)}{t} \qquad (3')$$

$$P \cos(\phi+\theta)-N\sin(\phi+\theta)-w_r\sin \theta-R = \frac{w_c}{g}\frac{v}{t} \qquad (4')$$

$$x = \frac{v}{2} t + \frac{w_r+w_c}{2R_g} v^2 \qquad (5')$$

$$Z = \frac{V}{2} t \qquad (6')$$

Usually Z and x are given. Hence, the unknowns are V, P, t, v, N and R; therefore a complete solution is possible.

A final check may be made by substitution in the energy equation:

$$PZ -w_r(Z \sin\phi-x \sin \theta)+w_cx \sin \theta + \tfrac{1}{2}(m_r+m_c)v^2+Rx_1 = \tfrac{1}{2}m_rV^2 \qquad (7')$$

where $x_1 = \frac{v}{2} t$ or in the form

$$PZ-w_r(Z\sin\phi-x \sin \theta)+w_cx \sin \theta +Rx = \tfrac{1}{2}m_rV^2$$

In a preliminary layout for a double recoil system, the limitations are usually the length of upper recoil, that is the total relative dis-

placement between the gun and top carriage, and the total run up the inclined plane. A direct solution of the various reactions in terms of these given quantities is especially useful.

$a = w_r \sin\emptyset$

$h = \cos(\emptyset+\theta)$

$b = \frac{W_r V}{g}$

$l = \sin(\emptyset+\theta)$

$$c = \frac{w_r \cos(\emptyset+\theta)}{g}$$

$n = w_r \sin\theta$

$d = w_r \cos\emptyset$

$$g = \frac{w_c}{g}$$

$$f = \frac{w_r \sin(\emptyset+\theta)}{g}$$

$$p = \frac{w_r + w_c}{2g}$$

(6) $t = \frac{2Z}{V}$ same as (5) gives t directly

(7) $p = a + \frac{b}{t} - c\frac{v}{t}$ f(p.V) same as (1)

(8) $N = d + f\frac{v}{t}$ f(N.V) same as (2)

(9) $hp - lN - N - R - g\frac{v}{t}$ f(p.N.R.V.) same as (3)

(10) $X = \frac{vt}{2} + p\frac{v^2}{R}$ f(R.V) same as (4)

<u>Elimination N</u>

(11) $hp - ld - \frac{lf}{t}v - n - \frac{g}{t}v = R$ (8) in (9)

f(P.R.V)

$$(12)\qquad \frac{pV^2}{x-\frac{t}{2}v} = R \qquad \text{from 10} \qquad f(R.V.)$$

Elimination R

$$(13)\qquad P = \frac{ld+n}{h} + \frac{lf+g}{ht}v + \frac{pV^2}{xh-\frac{ht}{2}v} \qquad (12) \text{ in } (11) \qquad f(P.V)$$

Elimination of P

$$(14)\qquad a+\frac{b}{t}-\frac{cv}{t} = \frac{ld+n}{h} + \frac{ld+g}{ht}v + \frac{pV^2}{xh-\frac{ht}{2}v} \qquad (7)-(13)$$

$$(15)\qquad ahx - \frac{aht}{2}v + \frac{bhx}{t} - \frac{bhV}{2} - \frac{chx}{t}V + \frac{ch}{2}V^2 - x(dl+n) + \frac{t(dl+n)V}{2} - \frac{x}{t}(fl+g)V + \frac{fl+g}{2}V^2 - pV^2 = 0$$

$$\frac{ch}{2} - p + \frac{fl+g}{2}V^2 - \left[\frac{aht}{2} + \frac{bh}{2} + \frac{chx}{t} - \frac{t(ld+n)}{2} + \frac{x(lf+g)}{t}\right]V + ahx + \frac{bhx}{t} - x(ld+n) = 0$$

This equation may be put in the form $a'V^2+b'V+c'=0$

$$a' = \frac{w_r\cos^2(\emptyset+\theta)}{2g} - \frac{w_r+W_c}{2g} + \frac{w_r\sin^2(\emptyset+\theta)}{2g} + \frac{W_c}{2g}$$

or $a' = \frac{w_r}{2g} - \frac{w_r+W_c}{2g} + \frac{W_c}{2g}$ hence $a' = 0$

Solution of b'

$$+\frac{aht}{2} = w_r \sin\emptyset \cos(\emptyset+\theta)\frac{Z}{V} = \frac{w_r Z}{V}\sin\emptyset\cos(\emptyset+\theta)$$

$$+\frac{bh}{2}-\frac{w_rV}{2g}\cos(\emptyset+\theta)=\frac{w_rV}{2g}\cos(\emptyset+\theta)$$

$$+\frac{chx}{s}=\frac{w_r\cos(\emptyset+\theta)xV}{2gZ}=\frac{xVw_r}{2gZ}\cos^2(\emptyset+\theta)$$

$$-\frac{dlt}{2}=-2Z\frac{w_r\cos\emptyset\sin(\emptyset+\theta)}{2V}=\frac{2Zw_r}{V}\cos\emptyset\sin(\emptyset+\theta)$$

$$-\frac{nt}{2}=-\frac{W_c\sin\theta Z}{V}=-\frac{W_cZ}{V}\sin\theta$$

$$+\frac{xg}{t}=\frac{xVW_c}{2gZ}$$

hence $\dfrac{xVw_r}{2gZ}-\dfrac{Zw_r}{V}\sin\theta+\dfrac{w_rV}{2Wg}\cos(\emptyset+\theta)-\dfrac{W_cZ}{V}\sin\theta+\dfrac{xVw_c}{2gZ}=b'$

simplifying

$$\frac{xV}{2gZ}(w_r+W_c)-\frac{Z}{V}\sin\theta(w_r+W_c)+\frac{w_rV}{2g}\cos(\emptyset+\theta)=b'$$

$$b'=(W_r+W_c)\underbrace{(\frac{xV}{2gZ})(\frac{xV}{2gZ}-\frac{Z}{V}\sin\theta)}_{C'}+\frac{W_rV}{2g}\cos(\emptyset+\theta)$$

$$+\ abx\ +\ xw_r\sin\emptyset\cos(\emptyset+\theta)$$

$$+\frac{bhx}{t}=x\frac{w_rV^2}{2gZ}\cos(\emptyset+\theta)$$

$$-\ dlx=-\ x\ w_r\cos\ \emptyset\sin(\emptyset+\theta)$$

$$-\ xn=-\ x\ W_c\sin\ \theta$$

$$xw_r[\sin\emptyset\cos(\emptyset+\theta)-\cos\emptyset\sin(\emptyset+\theta)]+\frac{xw_rV^2}{2gZ}\cos(\emptyset+\theta)-xW_c\sin\theta$$

$$-\ xw_r[\sin(\emptyset+\theta)\cos\emptyset-\cos(\emptyset+\theta)\sin\emptyset]$$

$$-\ xw_r\sin\ \theta+\frac{xw_rV^2}{2gZ}\cos(\emptyset+\theta)-\ x\ W_c\sin\ \theta$$

$$C' = \frac{xw_rV^2}{2gZ}\cos(\emptyset+\theta) - x\sin\theta(w_r+W_c)$$

$$v = \frac{c'}{b'} = \frac{\frac{xw_rV^2}{2gZ}\cos(\emptyset+\theta) - x\sin(w_r+W_c)}{(w_r+W_c)(\frac{xV}{2gZ} - \frac{2}{V}\sin\theta) + \frac{w_rV}{2g}\cos(\emptyset+\theta)}$$

As an example of the solution of these equations and a calculation of the prime reactions, the 240 m/m Schneider Howitzer was taken with the top carriage moving up a plane inclined at 6° with the horizontal and with 40" upper recoil and a total of 30" recoil up the inclined plane for the lower recoil.

Muzzle velocity	V_n	1700ft/sec.
Travel up the plane...............	x	30in.
Length of Recoil..................	L	40 in.
Angle of elevation................	Ø	20°
Angle of plane	θ	6°
Weight of carriage................	W_c	11,500 lbs.
Weight of gun.....................	w_r	15,800 lbs.
Weight of the charge..............	W	35 lbs.
Weight of projectile	w	356 lbs.
Relative displacement	Z=L-	40 in.

$$V = 0.9\left(\frac{wV_m+4700\ w}{w_r}\right) = 0.9\left(\frac{356+1700+4700+35}{15800}\right)$$

$$= 0.9\ \frac{6070+1640}{158} = .9\ \frac{7710}{158} = 44\ \text{ft/sec.}$$

$\sin(\emptyset+\theta) = \sin 26° = .4384$

$\cos(\emptyset+\theta) = \cos 26° = .981$

$\sin\theta = \sin 6° = .1045$

$\sin\emptyset = \sin 20° = .342$

$\cos\emptyset = \cos 20° = .9397$

$$b' = 27300\left(\frac{30\times44}{64.4\times40} \quad \frac{3.33}{44}\ .1045\right)+\frac{15800\times44}{64.4}\times .891$$

$$= 27300(.5124 - .0081)+9620$$

$$= 13770+9620$$

$$b' = 23,390$$

$$c' = \frac{30\times15800\times1936}{64.4\times40}\ .891 - 2.5\times27300\times .1045$$

$$= 317406 - 7132$$

$$c' = 310,274$$

$$23390\ V = 310274$$

$$V = \frac{310274}{23390} = 13.2652$$

$$t = \frac{2Z}{V} = \frac{2\times3.33}{44} = .1515 \qquad \text{from (5)}$$

$$2.5 = \frac{13.2652}{2}\times .1515 + \frac{27300}{64.4R}\ \frac{2}{13.2652} \qquad \text{from (4)}$$

$$2.5 - 1.0048 = \frac{74459.408}{R}$$

$$R = \frac{74459.408}{1.4952} = 49800$$

$$N = 15800\times .891 + 490\times\frac{13.2652\times .4384}{.1515} \qquad \text{from (2)}$$

$$= 14080 + 18810$$

$$N = 32890$$

$$P = 15800\times .342 + 490\ \frac{44-13.2652\times .891}{.1515} \qquad \text{from (1)}$$

$$5404+490\times212.$$

$$P = 109,500$$

$$t = 0.1515 \text{ ft/sec.}$$

$$V = 13.2652$$

$$R = 49,800$$

$$N = 33,000 \qquad P = 109,500$$

As a final check on the calculations, the values obtained were substituted in the energy equation. The slight discrepancy between the two sides of the equation is due to numerical approximation.

$$PZ+\frac{V^2}{2}(M_r+M_c)+x\ .W_c\sin\theta+x'R=\frac{1}{2}M_rV+w_r(Z\sin\phi-x'\sin\theta)$$

$$x'=\frac{V}{2}t=1.0048$$

365000+74,5000+1200+50,000=490,700
475,000+16,400 = 491,400
dev. 1.42%

EXACT THEORY FOR CONSTANT RESISTANCE ON BOTH UPPER AND LOWER RECOIL SYSTEMS.

Let x' and y' = the coordinates along and normal to the axis of the bore (upper recoil coordinates).

x and y = the coordinates along and normal to the inclined plane (lower recoil coordinates)

m_r and w_r = mass and weight of recoiling parts

m_c and w_c = mass and weight of top carriage plus cradle

v = velocity of any instant along inclined plane.

v' = absolute velocity of recoiling mass along axis of bore.

t = time from beginning of recoil

ϕ = angle of elevation of gun

θ = angle of plane

E = free recoil displacement for upper recoiling parts during powder period

T = total time of powder pressure period

p = resistance of gun recoil system

N = upper normal reaction between recoiling parts and top carriage

R = lower recoil resistance parallel to inclined plane

n = the coefficient of sliding friction

p_b = total powder pressure on breech at instant t.

Hence t' = time of common recoil

v_1 = common recoil velocity for both recoiling parts.

x_1' = absolute displacement in the direction of the bore to where the recoiling masses move with common velocity

x_1 = corresponding displacement up inclined plane at common velocity

Z = total relative displacement between upper and lower recoiling mass.

V_f = free velocity of recoil (See "Dynamics of Recoil")

R_b' = counter recoil buffer resistance for upper recoil system.

Considering now, the motion of the upper recoiling parts, we have

$$P_b - P + w_r \sin\emptyset = m_r \frac{dv'}{dt} \qquad (1)$$

$$N = w_r \cos\emptyset = m_r \frac{dv}{dt} \sin(\emptyset+\theta) \cdot \qquad (2)$$

From (1), we have

$$\int_0^t \frac{p_b dt}{m_r} - \frac{(p-w_r\sin\theta)}{m_r} t = v'$$

that is,

$$V_f - \frac{(p-w_r\sin\emptyset)}{m_r} t = v' \qquad (3)$$

now, when $t = T_1$ $V_f = V_{f'}$ hence at any time after T, we have

$$V_{f'} - \frac{(p-w_r\sin\emptyset)}{m_r} t = v' \qquad (3')$$

Integrating again for the upper recoil absolute displacement, we have

$$\int_0^t V_f dt - \frac{(p-w_r \sin\emptyset)}{2m_r} t^2 = x'$$

but

$$\int_0^t V_f dt = \int_0^T V_f dt + \int_T^t V_{f'} dt$$

Hence

$$x' = E + V_{f'}(t-T) - \frac{(p-w_r \sin\emptyset)}{2m_r} t^2 \qquad (4)$$

which gives the absolute displacement of the upper recoiling parts along the axis of the bore.

Considering now, the motion of the lower recoiling parts, we have,

$$p \cos(\emptyset+\theta) - N \sin(\emptyset+\theta) - w_c \sin\theta - R = m_c \frac{dv}{dt} \qquad (5)$$

Substituting N from equation (2) into (5) and simplifying, we have

$$p \cos(\emptyset+\theta) - w_r \sin(\emptyset+\theta)\cos\emptyset - w_c \sin\theta - R = [m_c - m_r \sin^2(\emptyset+\theta)\frac{dv}{dt}] \qquad (6)$$

$$\text{Hence } v = \left[\frac{p \cos(\emptyset+\theta) - w_r \sin(\emptyset+\theta)\cos\emptyset - w_c \sin\theta - R}{m_c - m_r \sin^2(\emptyset+\theta)}\right] t \qquad (7)$$

and the corresponding displacement up the plane, becomes

$$x = \left[\frac{p\cos(\emptyset+\theta) - w_r \sin(\emptyset+\theta)\cos\emptyset - w_c \sin\emptyset - R}{2[m_c - m_r \sin^2(\emptyset+\theta)]}\right] t^2 \qquad (8)$$

The relative velocity between the upper and lower recoiling parts, become $v_r = v' - v \cos(\emptyset+\theta)$ (9)
and the corresponding relative displacement
$x_r = x' - x \cos(\emptyset+\theta)$ (10)
When the upper and lower recoiling mass move together with a common velocity, $v_r = 0$, hence $v' = v \cos(\emptyset+\theta)$ hence we obtain the time t' for the

common velocity, from

$$V_{f'} = \left(\frac{p-w_r\sin\emptyset}{m_r}\right) t' \left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{m_c-m_r\sin^2(\emptyset+\theta)}\right]$$

$$\cos(\emptyset+\theta)t'$$

simplified, we have

$$t' = \frac{V_{f'}}{\dfrac{\left(\dfrac{p-w_r\sin\emptyset}{m_r}\right)+\left[\dfrac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset\theta)\cos\emptyset-w_c\sin\theta-R}{m_c-m_r\sin^2(\emptyset+\theta)}\right]}{\cos(\emptyset+\theta)}} \qquad (12)$$

As a check, the time t' for attaining the common velocity of the upper and lower recoil masses, we may equate the components of the absolute velocities of the upper and lower recoiling mass parallel to the inclined plane.

Considering the motion of the recoiling parts parallel to the inclined plane, we have

$$v = V_{f'}\cos(\emptyset+\theta) - \frac{p}{m_r}\cos(\emptyset+\theta)t' + \frac{N}{m_r}\sin(\emptyset+\theta)t'$$

$$- \frac{w_r\sin\theta}{m_r}t'$$

since now the reaction N has a component $N\sin(\emptyset+\theta)$ reacting on the upper recoiling parts parallel to the inclined plane.

Let $Nt' = w_r\cos\emptyset\ t' = m_r\sin(\emptyset+\theta)\ v$ hence

$$V_{f'}\cos(\emptyset+\theta)-\frac{p}{m_r}\cos(\emptyset+\theta)t'+\frac{w_r}{m_r}\sin(\emptyset+\theta)\cos\emptyset t'+\sin^2$$

$$(\emptyset+\theta)\ v - \frac{w_r}{m_r}\sin\theta = v$$

$$v\cos^2(\emptyset+\theta) = V_{f'}(\emptyset+\theta)-\frac{p}{m_r}\cos(\emptyset+\theta)\ t' + \frac{w_r}{m_r}[\sin(\emptyset+\theta)$$

$$\cos\emptyset-\sin\theta]\ t$$

Let $\sin(\emptyset+\theta)\cos\emptyset-\sin\theta=(\sin\emptyset\cos\emptyset+\cos\theta)\cos\emptyset-\sin\theta$

$$= \sin\phi\cos\phi\cos\theta + \cos^2\phi\sin\theta - \sin\theta$$
$$= \sin\phi\cos\phi\cos\theta + \sin\theta\,(\cos^2\phi - 1)$$
$$= \sin\phi\cos\phi\cos\theta - \sin\theta\sin^2\phi$$
$$= \sin\phi\cos(\phi+\theta)$$

hence

$$v\cos(\phi+\theta) = V_{f'} - \left(\frac{p - w_r\sin\phi}{m_r}\right)t'$$

Substituting for v and reducing, we have, as before

$$t' = \frac{V_{f'}}{\dfrac{p - w_r\sin\phi}{m_r} + \left[\dfrac{p\cos(\phi+\theta) - w_r\sin(\phi+\theta) - w_c\sin\theta - R}{m_c - m_r\sin^2(\phi+\theta)}\right]\cos(\phi+\theta)}$$

Knowing the value of t' and substituting in equations (4) and (8), we obtain the total relative displacement between the upper and lower recoiling parts, that is $Z = x_1' - x_1\cos(\phi+\theta)$ (13)
where t' is used in the values of x' and x respectively.

The total energy of the system where the two masses arrive at the common velocity v_1, becomes

$\frac{1}{2}(m_r+R_c)v_1^2 + \int_0^Z p_a dZ$ where $\int_0^Z p_a dZ$ is the potential energy in the recuperator. Let R_b' = the buffer resistance during counter recoil for the upper recoil system, then

$\int_0^Z R_b'\, dZ$ = the work done by the buffer in the upper recoil system.

If now we assume that the counter recoil of the upper system is completed during the recoil of the lower system, we have

$$\frac{1}{2}(m_r+m_c)v_1^2 + \int_0^Z p\,dZ = R(X-x_1) + \int_0^Z R_b'\,dZ$$

$$= w_r(X-x_1)\sin\phi + w_r Z\sin\phi + w_c(X-x_1)\sin\theta \qquad (14)$$

A physical meaning and relationship of the reactions in this equation, may be had by a consideration of the component dynamical equations for the parts of the system.

During this second period of the recoil, we have for the lower recoiling mass, that $[R+w_c\sin\theta+N\sin(\emptyset+\theta)-(p_a-R_b')\cos(\emptyset+\theta)]dx = -m_c dV$ and for the upper recoiling parts, along and normal to the bore $[(p_a-R_b')-w_r\sin\emptyset)d(x\cos(\emptyset+\theta)+Z] = -m_r v_x' dv_x'$ and $(w_r\cos\emptyset-N)d[x\sin(\emptyset+\theta)]=-m_r v_y' dv_y'$ and the above equations and integrating the sum, we have

$$\int_{x_1}^{X} R\,dx + \int_{x_1}^{X} w_r\sin\theta\,dx + \int_{Z}^{o} (p_a-R_b')dZ + w_r\int_{x_1}^{X}[\sin(\emptyset+\theta)\cos\emptyset - \cos(\emptyset+\theta)\sin\emptyset]\,dx$$

$$-\int_{Z}^{o} w_r\sin\emptyset\,dZ = (m_r+m_c)\frac{v_1^2}{2}\ \text{(since } v_x'^2 + v_y'^2 = v_1^2 \text{ for initial valve)}$$

Simplifying we obtain equation (14)

In general the potential energy of the recuperator is partially divided in overcoming the work of the upper recoil buffer $\int_o^Z R_b'\,dZ$ and in augmenting the run up the inclined plane over that if there were no recuperator present. Hence in general

$$\int_o^Z p_a dZ > \int_o^Z R_b'\,dZ \text{ and } \int_o^Z p_a\,dZ - \int_o^Z R_b'\,dZ$$

is the additional energy over that Kinetic energy at common velocity which augments the recoil up the inclined plane.

We may assume with small error, however, that $\int_o^Z p_a\,dZ = \int_o^Z R_b'\,dZ$ or that $\int_o^Z p_a\,dZ - \int_o^Z R_b'\,dZ$ is negligible. This does not imply that $p_a-R_b'=0$. Since $(p_a-R_b')(X-x_1)\cos(\emptyset+\theta)$ (roughly) is the agent by which the upper recoil energy is dissipated.

When the lower recoil is comparatively short

and the resistance of the lower recoil system R is large, we have often a condition, where counter recoil in the upper recoil system becomes impossible and we even have an over run of the upper recoil system.

Thus assuming during the second part of double recoil, that the upper and lower recoil mass move as if one, we have the retardation

$$-\frac{dv}{df} = \frac{R+(w_r+w_c)\sin\theta}{m_r+m_c}$$ and for the upper recoiling mass

$$p_a - w_r \sin \emptyset = m_r\left(-\frac{dv}{dt}\right)\cos(\emptyset+\theta)$$ hence

$$p_a - w_r\sin\emptyset = \frac{m_r}{m_r+m_c}[R+(w_r+w_c)\sin\theta]\cos(\emptyset+\theta)$$

Now if $p_a > w_r \sin \emptyset + \frac{m_r}{m_r+m_c}[R+(w_r+w_c)\sin\emptyset]\cos(\emptyset+\theta)$

Counter recoil of the upper recoil system is possible during the second period of the lower recoil system. If however,

$$p_a < w_r\sin\emptyset + \frac{m_r}{m_r+m_c}[R+(w_r+w_c)\sin\theta]\cos(\emptyset+\theta)$$

We have a tendency of over recoil of the upper recoil system hence counter recoil of the upper recoil system is impossible. For this case the energy equation reduces with exactress to

$$\tfrac{1}{2}(m_r+m_c)v^2 = R(X-x_1)+(w_r+w_c)(X-x_1)\sin\theta \qquad (15)$$

The velocity curve during the second period may be obtained with sufficient exactness by assuming the two masses to recoil together, then

$$R+(w_r+w_c)\sin\theta = -(m_r+m_c)v\,\frac{dv}{dx}$$

and $\int_{x_1}^{x} [R+(w_r+w_c)\sin\theta]dx = \int_{v}^{v_1} (m_r+m_c)v \ dv$

$$[R+(w_r+w_c)\sin\theta](x-x_1) = \left(\frac{m_r+m_c}{2}\right)(v_1^2-v^2)$$

hence

$$v = \sqrt{v_1^2 - \frac{2[R+(w_r+w_c)\sin\theta](x-x_1)}{m_r+m_c}}$$

RECAPITULATION OF FORMULAE FOR CONSTANT RESISTANCE TO RECOIL BOTH UPPER AND LOWER.

From approximate solution with limited upper and lower recoil, calculate P and R. Then <u>during the powder period</u>, we have

$$v' = V_f - \left(\frac{p-w_r\sin\theta}{m_r}\right) t$$

$$v = \left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{m_c-m_r\sin^2(\emptyset+\theta)}\right] t$$

and the relative velocity becomes $v_r=v'-v\cos(\emptyset+\theta)$

Further

$$x' = \int_0^t V_f\,dt - \left(\frac{p-w_r\sin\emptyset}{2m_r}\right)t^2$$

$$x = \left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{2[m_c-m_r\sin^2(\emptyset+\theta)]}\right] t^2$$

and the corresponding relative displacement, becomes $x_r=x' - x\cos(\emptyset+\theta)$. <u>After the powder period during the remainder of the first period of recoil,</u> we have

$$v'=V_f,\left(\frac{p-w_r\sin\emptyset}{m_r}\right)t$$

$$v=\left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{m_c-m_r\sin(\emptyset+\theta)}\right] t$$

and for the relative velocity $v_r=v'-v\cos(\emptyset+\theta)$

Further

$$x'=E+V_f,(t-T) - \left(\frac{p-w_r\sin\emptyset}{2m_r}\right)t^2$$

$$x = \left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset - w_c\sin\theta - R}{2m_c - m_r\sin^2(\emptyset+\theta)}\right]t^2$$

and the corresponding relative displacement becomes $x_r = x' - x\cos(\emptyset+\theta)$. The time for the common velocity becomes,

$$t' = \frac{V_{f'}}{\left(\frac{p-w_r\sin\emptyset}{m_r}\right)+\left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{m_c-m_r\sin^2(\emptyset+\theta)}\right]\cos(\emptyset+\theta)}$$

and the common velocity becomes

$$v_1 = \left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{m_c-m_r\sin^2(\emptyset+\theta)}\right]t'$$

$$x_1' = E + V_{f'}(t'-T) - \left(\frac{p-w_r\sin\emptyset}{2m_r}\right)t'^2$$

$$x_1 = \left[\frac{p\cos(\emptyset+\theta)-w_r\sin(\emptyset+\theta)\cos\emptyset-w_c\sin\theta-R}{2[m_c-m_r\sin^2(\emptyset+\theta)]}\right]t^2$$

and the total relative displacement for the upper recoil system becomes, $Z = x_1' - x_1\cos(\emptyset+\theta)$

During the second period of the recoil, we have

$$v = \frac{v_1^2 - 2(R+w_r+w_c)\sin\theta(x-x_1)}{m_r+m_c}$$

the upper recoiling mass being assumed locked with the lower recoiling parts.

CALCULATION OF THROTTLING GROOVES BOTH UPPER AND LOWER RECOIL.

As a first approximation, it will be assumed that the total friction is mainly guide friction and proportional to the normal reaction between the upper and lower recoiling parts. Then $R_g = nN$ where $n = 0.2$ to 0.3

and $N = w_r\cos\emptyset + m_r\frac{v_1}{t'}\sin(\emptyset+\theta)$ where v_1 and t' have

been already determined. Considering the upper recoiling parts, we have $P=p_a+p_n+R_g=p_a+p_n+nN$ hence $p_n=p-p_a-nN$. Further if the ratio of the final to the initial air pressure = m, we have

$$\frac{p_{af}}{p_{ai}} = m$$

and if A = effective area of upper recuperator and b = Z = recoil displacement on top carriage, then for the initial volume we have

$$V_i = A_a b \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}$$

where k = 1 to 1.41 assume 1.3

Hence $p_a = p_{ai}\left(\frac{V_i}{V_i - A_a x_r}\right)$

x_r = being the relative displacement. Therefore knowing v_r and x_r and the total pull p, we have

$$p_n = p - p_{ai}\left(\frac{V_i}{V_i - A_a}\right)^{k-nN} \quad \text{and} \quad W_{x'} = \frac{KA_n v_r}{13.2\sqrt{\frac{p_n}{A_h}}}$$

where A_h = the hydraulic piston area and k the reciprocal of the throttling constant.

Lower throttling grooves

Knowing R from previous data, we have

$$w_x = \frac{K\ A\ v}{13.2\sqrt{\frac{R}{A}}}$$

where v is the recoil velocity up plane.

EQUIVALENT MASS OF ROTATING PARTS WITH A DOUBLE RECOIL. When a double recoil system, consisting of two separate recoil systems is used, mounted on a railway car or caterpillar, it is customary to consider the car or caterpillar

sufficiently braked to allow no recoil. In fact a salient feature of the design is to make "R" small enough so that the rail or ground friction, induced by proper braking, is sufficient to balance R.

Due to the complication of a double recoil, as well as the impossibility with very large mounts of taking up the recoil energy even with a double recoil system without an excessive recoil displacement it has been the custom to use a single recoil and allow the railway car or caterpillar to run back a limited distance dependent on the magnitude of the braking. The recoil of the car on very large railway mounts may be considerable. This greatly reduces the stresses on low elevation as well as augments the stability. In fact with such mounts stability is of no longer a consideration.

When a single recoil system is used but the car or caterpillar recoils in addition, we obviously have a double recoil system and all the prevous dynamical equations together with the method of computing the throttling on the upper recoil or now the recoil systems holds the same. The lower recoil resistance R is now the tangential reaction exerted at the base of the car wheels or at contact of ground and caterpillar track. In the acceleration of a railway train, railway engineers customarily allow for the rotational inertia of the car wheels by increasing the translatory mass from 8 to 10 percent. Due to the limitations at times of car or caterpillar recoil and the great variation of the magnitude of the rotational inertia as compared with standard railway practice it is important to calculate the exact effect of the rotational inertia in terms of an equivalent addition to the translatory mass.

Consider a railway car or truck with "n" pairs of axles. Let

w_c and m_c = weight and mass of car not in-

cluding wheels.

w_w and m_w = weight and mass of a pair of wheels.

$I = m_w K^2$ = moment of inertia of a pair of wheels about the center line of the axle.

d = tread diam. of a car wheel

k = radius of gyration of a pair of car wheels

N_w = normal reaction at base of a pair of car wheelx

N_s = normal reaction of brake shoe on wheel per pair of wheels

f_w = coefficient of rail friction

f_s = coefficient of brake shoe friction

R_w = tangential force exerted by rail on base of car wheel

p = recoil reaction

N = normal reaction between recoiling parts and car

Ø = angle of elevation

Now independent of rotation or any other motion, the translatory motion of the center of gravity of a system depends only on the external forces applied. Hence

$$p \cos Ø - N \sin Ø - \Sigma R_w = (m_c + \Sigma m_w) \frac{dv}{dt}$$

Considering the motion of a single car wheel, we have for rotations about the center of gravity of a pair of wheels

$$(R_w - N_s f_s) \frac{d}{2} = \frac{2m_w k^2}{d} \frac{dv}{dt}$$

$$R_w = N_s f_s + \frac{4m_w k^2}{d^2} \frac{dv}{dt}$$

hence $p\cos Ø - N\sin Ø - \Sigma N_s f_s = (m_c + \Sigma m_w + \Sigma \frac{4m_w k^2}{d}) \frac{dv}{dt}$ therefore the translatory mass is increased by the term

$\Sigma \frac{4m_w k^2}{d}$ which is the equivalent translatory

mass of rotational inertia. The mass of the lower recoil system therefore, becomes

$m_c + \Sigma m_w(1+\frac{4k^2}{d})$ and this value is to be substituted in the previous dynamic equations. The equivalent resistance for R is now the summation of the brake shoe friction,- that is $R = \Sigma N_s f_s$ and this value is to be used in place of R in the previous dynamic equations.

It is important to note, however, that the actual tangential force exerted at the base of the car wheels is not $\Sigma N_s f_s$

but $\Sigma R_w = \Sigma N_s f_s + \Sigma \frac{4m_w k^2}{d^2} \frac{dv}{dt}$

<u>Consider a caterpillar track and connector mechanism:</u>

Let R_t = total tangential track reaction between track and ground (in lbs)

R_w = total tangential roller reaction on track (in lbs)

r_w = radius of roller wheel (in ft)

r_c = radius of sprocket (in ft)

r_1 = radius of sprocket gear (in ft)

r_2 = radius of brake drum gear (in ft)

r_3 = radius of drum of brake clutch (in ft)

R_1 = tangential reaction between sprocket gear and drum gear (in ft)

T_D = torque exerted on brake drum (lb.ft)

E_o = mechanical efficiency of sprocket mechanism

E_1 = mechanical efficiency of transmission between sprocket gear and drum gear.

E_2 = mechanical efficiency of brake drum and mechanism

E_w = mechanical efficiency of roller trucks.

A_1 = resultant normal bearing reaction of sprocket shaft (lbs)

A_2 = resultant normal bearing reaction of

brake drum shaft (lbs)

f_1 and f_2 = corresponding coefficients of friction

m_c = total mass of caterpillar excluding recoiling parts

$m_w k_w^2$ = moment of inertia of roller wheel (ft.lbs)

$m_s k_s^2$ = moment of inertia of sprocket wheel

$m_{gs} k_{ds}^2$ = moment of inertia of sprocket gear

$m_{gd} k_{ds}^2$ = moment of inertia of drum gear

$m_d k_d^2$ = moment of inertia of drum.

d = the increment change in the radius to account for friction between gear teeth

Considering the motion of the caterpillar, we have

$$p \cos \varnothing - N \sin \varnothing - R_t = m_c \frac{dv}{dt} \qquad (1)$$

The tension in the caterpillar track at the sprocket becomes, $T = R_t - \Sigma R_w$

$$= R_t - \frac{m_w k_w^2}{E_w r_w^2} \frac{dv}{dt} \qquad (2)$$

and its moment about the sprocket axis, (assuming the upper track tension as nil) becomes,

$$\left(R_t - \frac{m_w k_w^2}{r_w^2} \frac{dv}{dt}\right) r_o$$

Considering the angular motion of the sprocket shaft we have

$$\left(R_t - \frac{m_w k_w^2}{E_w r_w^2} \frac{dv}{dt}\right) r_o - R_1 (r_1 + d) - A_1 f_1 r' = \left(\frac{m_s k_s^2 + m_{gs} K_{gs}^2}{r_o}\right) \frac{dv}{dt} \qquad (3)$$

Further considering the angular motion of the drum shaft, we have

$$R_1 (r_2 - d) - A_2 f_2 r'' - T_D = \left(\frac{m_{gd} k_{gd}^2 + m_d k_d^2}{(r_2)(r_o)}\right) r_1 \frac{dv}{dt} \qquad (4)$$

hence

$$R_1 (r_1 + d) = \frac{r_1 + d}{r_2 - d} \left[\frac{T_D}{E_2} + \left(\frac{m_{gd} k_{gd}^2 + m_d k_d^2}{E_2 r_o r_2}\right) r_1 \frac{dv}{dt}\right] \qquad (5)$$

Where E_2 takes care of the friction less in the drum of gear bearing.

The friction loss between the drum gear and sprocket gear may be considered, by letting

$$\frac{r_1+d}{r_2-d} = \frac{1}{E}\,\frac{r_1}{r_2}$$

hence

$$R_1(r_1+d) = \frac{T_D}{E_1E_2}\,\frac{r_1}{r_2} + \left(\frac{m_{gd}k_{gd}^2+m_dk_d^2}{E_1E_2r_or_2^2}\right)r_1^2\,\frac{dv}{dt} \qquad (6)$$

Substituting in (3), we have

$$\left(R_t - \frac{m_wk_w^2}{E_wr_w^2}\,\frac{dv}{dt}\right)r_o = \frac{T_o}{E_oE_1E_2}\,\frac{r_1}{r_2} + \left(\frac{m_{gd}k_{gd}^2+m_dk_d^2}{E_oE_1E_2r_or_2^2}\right)r_1^2\,\frac{dv}{dt}$$

$$+\left(\frac{m_sk_s^2+m_{gs}k_{gs}^2}{E_or_o}\right)\frac{dv}{dt} \qquad (7)$$

where E_o takes care of the friction loss in the sprocket bearing. Therefore, the track reaction becomes,

$$R_t = \left[\frac{m_wk_w^2}{E_wr_w^2} + \left(\frac{m_{gd}k_{gd}^2+m_dk_d^2}{E_oE_1E_2r_or_2^2}\right)r_1^2 + \left(\frac{m_sk_s^2+m_{gs}k_{gs}^2}{E_or_o^2}\right)\right]\frac{dv}{dt} +$$

$$\frac{T_D}{E_oE_1E_2}\,\frac{r_1}{r_2r_o} \qquad (8)$$

and substituting in the equation of translatory motion (Eq.1) we have

$$p\cos\emptyset - N\sin\emptyset - \frac{T_D}{E_oE_1E_2}\,\frac{r_1}{r_2r_o} = \left[m_c + \frac{m_wk_w^2}{E_wr_w^2} +\right.$$

$$\left.\left(\frac{m_{gd}k_{gd}^2+m_dk_d^2}{E_oE_1E_2r_o^2r_2^2}\right)r_1^2 + \left(\frac{m_sk_s^2+m_{gs}k_{gs}^2}{-E_or_o^2}\right)\right]\frac{dv}{dt} \qquad (9)$$

Evidently $\dfrac{T_D}{E_oE_1E_2r_2r_o}$ is the brake torque referred to a reaction at the base of the track, considering the mechanical efficiency of the gearing. The translatory mass is augmented

due to the rotational inertia of the rotating parts by the term

$$\frac{m_w k_w^2}{E_w r_w^2} + \frac{m_{gd} k_{gd}^2 + m_d k_d^2}{E_o E_1 E_2 r_o^2 r_2^2} r_1^2 + \frac{m_s k_s^2 + m_{gs} k_{gs}^2}{E_o r_o^2}$$

which is the equivalent mass of the rotating elements.

It is to be noted that the mechanical efficiency enters in the rotational inertia since the bearing reactions depend upon the external reactions, and the moments of them in turn depend upon the rotational as well as translatory inertia. The effect of the translatory inertia on the rotating element in modifying the bearing reactions will be neglected, being small.

Hence R in the double recoil equations is now the braking torque referred as a tangential force at the track base, that is,

$$R = \frac{T_D}{E_o E_1 E_2} \frac{r_1}{r_2 r_o}$$

The actual track reaction is R_t given by equation (8). As a check on equation (9) we may note that from the energy equation, we have

$$p \cos\emptyset - N \sin\emptyset \, dx = \frac{T_D \theta'}{E_o E_1 E_2} + d\left(\frac{\frac{1}{2} m_d k_d + \frac{1}{2} m_{mg} k_{gd}^2}{E_1 E_2}\right) w'^2 + d\left(\tfrac{1}{2} m_c v^2\right)$$

$$+ d\left(\frac{\frac{1}{2} m_s k_s^2 + \frac{1}{2} m_{gs} k_{gs}^2}{E_o}\right) w^2 + \left(\frac{\frac{1}{2} m_w k_w^2 w_w^2}{E_w}\right)$$

the reaction R_t doing no work. Further, we have

$$d\theta' = \frac{r_1}{r_2 r_o} dx \qquad w' = \frac{r_1}{r_2} \frac{v}{r_o}$$

$$w = \frac{v}{r_o} \qquad w_w = \frac{v}{r_w}$$

hence substituting these values, we find

$$(p\cos\theta - N\sin\theta - \frac{T\ r_1}{E_0 E_1 E_2 r_2 r_0})dx = \frac{1}{2}[m_0\ (\frac{m_d k_d^2 + m_{gd} k_{gd}^2}{E_1 E_2 r_2^2 r_0^2}) r_1^2$$

$$+ (\frac{m_s k_s + m_{gs} k_{gs}^2}{E_0 r_0^2}) + \frac{m_w k_w^2}{E_w r_w^2}] v^2$$

therefore the equivalent translatory mass, to account for the rotational inertia becomes,

$$(\frac{m_d k_d^2 + m_{gd} k_{gd}^2}{E_1 E_2 r_2^2 r_0^2}) r_1^2 + \frac{m_s k_s + m_{gs} k_{gs}^2}{E_0 r_0^2} + \frac{m_w k_w^2}{E_w r_w^2}$$

When the caterpillar track is heavy or there is a long space between the driving sprocket and the front idler sprocket, its inertia effect must be considered. Therefore, let

r_0 = radius of drive gear sprocket (in ft)

r_1 = radius of front idler sprocket (in ft)

$m_i k_i^2$ = moment of inertia of idler sprocket (units lb.ft)

m_t = mass of caterpillar track <u>per unit length</u>

l = length of upper span of caterpillar track (in ft)

$(\pi r_0 m_t) r_0^2$ = moment of inertia for that part of track in contact with driving sprocket (units in lb. ft)

$(\pi r_i m_t) r_i^2$ = moment of inertia for that part of track in contact with front idler sprocket (units lb. ft.)

T_0 = tension at section at point of contact of lower track and drive sprocket wheel (in lbs)

T_3 = tension at point of contact of lower track and front idler sprocket (in lbs)

T_2 = tension at point of contact of upper track and idler sprocket

T_1 = tension at point of contact of upper track and drive sprocket

From kinematics we must have the relative velocity of the track with respect to the frame

equal to v and the corresponding acceleration $\frac{dv}{dt}$ where v is the translatory velocity of the caterpiller at instant with respect to the ground.

Considering the lower track since at any instant it must be at rest, we have for the difference of the tensions at its extremities,

$$T_O - T_3 = R_t - \frac{\Sigma m_w k_w^2}{E_w r_w^2} \frac{dv}{dt} \qquad (10)$$

where the second member is the reaction on the track due to the tangential reaction of the ground and the reaction of the truck rollers.

Considering the angular motion of the drive sprocket shaft, we have

$$(T_O - T_1) r_r - R_1 (r_1 + d) - A_1 f_1 r' = \left(\frac{m_s k_s^2 + m_{gs} k_{gs}^2}{r_o} + \pi\ r_O^2 m_t\right) \frac{dv}{dt} \qquad (11)$$

and for the upper track, we have

$$T_1 - T_2 = 2\ m_t l\ \frac{dv}{dt} \qquad (12)$$

$$(T_2 - T_3) r_1 = \left(\frac{m_i k_i^2}{r_i} + \pi\ r_i^2 m_t\right) \frac{dv}{dt} + A_i f_i r_i''' \qquad (13)$$

From equations (4),(5) and (6) in the preceding discussion, combining with (11), we have,

$$T_O - T_1 = \left[\left(\frac{m_{gd} k_{gd}^2 + m_d k_d^2}{E_O E_1 E_2 r_O^2 r_2^2}\right) r_1^2 + \left(\frac{m_s k_s^2 + m_{gs} k_{gs}^2}{E_O r_O^2} + \pi\ r_O m_t\right)\right] \frac{dv}{dt}$$

$$+ \frac{T_D}{E_O E_1 E_2} \frac{r_1}{r_2 r_O} \qquad (14)$$

Further equation (13) may be simplified by considering the mechanical efficiency of the front idler sprocket mechanism E_i, that is

$$(T_2 - T_3) r_1 = \left(\frac{m_i k_i^2}{r_1} + \pi\, r_i^2\, m_t\right)\frac{dv}{dt} \quad E_i \qquad (15)$$

where E takes care of the bearing friction $A_i f_i r_i$''' and the loss due to the bending of the track at the sprocket.

Now if we combine (10), (12) and (15) with (14), we have

$$R_t = \frac{\Sigma m_w k_w^2}{E_w r_w^2} + m_t\left(2 - 1 + \frac{\pi r_i}{E_i} + \frac{\pi r_o}{E_o}\right) + \frac{m_i k_i^2}{r_i^2}\, E_i + \left(\frac{m_{gd} k_{gd}^2 + m_d k_d^2}{E_o E_1 E_2 r_o^2 r_2^2}\right) r_1^2$$
$$+ \left(\frac{m_s k_s^2 + m_{gs} k_{gs}^2}{E_o r_o^2}\right)\frac{dv}{dt} + \frac{T_D}{E_o E_1 E_2} \frac{r_1}{r_2 r_o} \qquad (16)$$

but $m_t\left(2\ 1 + \frac{\pi r_i}{E_i} + \frac{\pi r_o}{E_o}\right) = M_t$ approx. (total mass of track)

$$\therefore \quad R_t = M_e \frac{dv}{dt} + \frac{T_D}{E_o E_1 E_2} \frac{r_1}{r_2 r_o}$$

Substituting in the equation of translatory motion we have,

$$p\cos - N\sin\emptyset - \frac{T_D}{E_o E_1 E_2} \frac{r_1}{r_2 r_o} = (M_c + M_e)\frac{dv}{dt} \qquad (17)$$

Where M_e given in equation (16) is the equivalent mass that must be added to the translatory mass.

The equivalent inertia may be taken into consideration more simply by the following approximate method.

The primary rotational system, consists of the track, the drive sprocket and front idler, together with the truck rollers.

The reaction of the ground tangentially to the track, = R_t and the truck roller reaction =

$$\Sigma \frac{m_w k_w^2}{r_w^2} \frac{dv}{dt}$$

Hence, for the primary rotational system, we have,

$$(R_t-\Sigma\frac{m_w k_w^2}{r_w^2}\ \frac{dv}{dt})r_o-R_1 r_1=(\frac{m_s k_s^2+m_{gs}k_{gs}^2+m_i k_i^2}{r_o}+2\pi m_t r_o^2$$

$$+2m_t l r_o)\frac{dv}{dt} \qquad (18)$$

but considering the rotational system of the drum and gear, we have

$$R_1 r_2-T_D=(\frac{m_d k_d^2+m_{gd}k_{gd}^2}{r_o r_2})r_1\ \frac{dv}{dt} \qquad (19)$$

Combining (18) and (19) we have

$$R_t=[\frac{\Sigma m_w k_w^2}{r_w^2}+(\frac{m_s k_s^2+m_{gs}k_{gs}^2+m_i k_i^2}{r_o^2})+(\frac{m_d k_d^2+m_{gd}k_{gd}^2}{r_o^2 r_2^2})r_1^2+$$

$$M_t]\frac{dv}{dt}+T_D\frac{r_1}{r_2 r_o} \qquad (20)$$

Hence, we assume the radius of the idler sprocket and driven sprocket the same, namely, r_o

To account for the loss due to friction, let the mechanical efficiency of the greasing referred to the track, be as follows:

E_t = mechanical efficiency of track

E_i = M.E. of front idler

E_o = M.E. of drive sprocket

E_1 = M.E. of gear transmission

E_2 = M.E. of drum shaft

E_w = M.E. of truck rollers

Then equation (20) is modified to:

$$R_t=[\frac{\Sigma m_w k_w^2}{E_w r_w^2}+\frac{m_s k_s^2+m_{gs}k_{gs}^2}{E_o r_o^2}+(\frac{m_d k_d^2+m_{gd}k_{gd}^2}{E_o E_1 E_2 r_o^2 r_2^2})r_1^2+\frac{dv}{E_t}]\frac{dv}{dt}$$

$$+\frac{T_D r_1}{E_o E_1 E_2 r_2 r_o} \qquad (21)$$

that is

$$R_t=M_c\frac{dv}{dt}+\frac{T_D r_1}{E_o E_1 E_2 r_2 r_o}$$

PRIMARY EXTERNAL REACTIONS WITH A DOUBLE RECOIL SYSTEM. With a double recoil system, the

first period when
the top carriage is
accelerated to a
common velocity for both upper and lower recoiling parts and a second period with a retardation for both recoiling masses.

The reactions should be considered during both periods.

<u>External reactions during first period:</u>

By D'Alembert's principle we may regard the inertia force as an equilibrating force, then for the primary external forces of a system consisting of the upper and lower recoiling mass together with caterpillar or railway car.

(1) The inertia resistance of the recoiling mass divided into two components.

(a) The inertia force parallel to axis of the bore through the center of gravity of the upper recoiling parts, p_1' or K_{x_1}

(b) The inertia force normal to the upper guides through the center of gravity of upper recoiling parts, N' or K_{y_1}

(2) The weight of the recoiling mass acting vertically down = W_r

(3) The inertia resistance of the top carriage and cradle acting through the center of gravity of the top carriage and cradle parallel to the inclined plane opposite to the acceleration up the plane = K_x or $m_c \frac{d^2x}{dt^2}$

(4) The total weight of the top carriage and cradle acting vertically

down = w_c

(5) The reaction of the ground on the caterpillar track or the reaction of the rail on the braked wheels of a railway mount using a double recoil, which are divided into the following components:

(a) The tangential reaction of ground or rail.

(b) The normal reaction of ground or rail which is not uniform but distributed so as to produce an upward normal reaction combined with a couple.

When the mount is just stable as with a light caterpillar at zero elevation (5) reduces to a single reaction about which moments are taken and therefore would not be considered for critical stability.

The primary external reactions are shown in fig. (3).

Considering the motion of the upper recoiling parts, we have, during the powder period,

$$P_b - P + W_r \sin \emptyset = m_r \frac{d^2x_1}{dt^2}$$ for the acceleration of the upper recoiling parts parallel to the guides. And

$$N - W_r \cos\emptyset = m_r \frac{d^2y_1}{dt^2} = m_r \frac{dv}{dt} \sin(\emptyset + \theta)$$ for the acceleration of the upper recoiling parts normal to the upper guides.

The external reaction on the recoiling parts when considered with the total mount, becomes, during the powder period,

$$P' = K_{x_1} = P_b - m_r \frac{d^2x_1}{dt^2} = P - W_r \sin\emptyset$$

parallel to the guides in the direction of P_b.

After the powder period during the first period of recoil, we have

$P-W_r \sin\emptyset = -m_r \frac{d^2x_1}{dt^2}$ along the guides and $N-W_r\cos\emptyset = m_r \frac{dv}{dt}\sin(\emptyset+\theta)$ normal to the guides. Therefore the external forces on the recoiling parts during the first period after the powder period, becomes.

$P' = K_{x_1} = -m\frac{d^2x}{dt^2}$ reversed $= m_r\frac{d^2x}{dt^2} = P-W_r\sin\emptyset$ along the bore and

$N' = K_{g_1} = m_r\frac{dv}{dt}\sin(\emptyset+\theta) = N-W_r\cos\emptyset$ normal to the bore.

Hence during the first period of recoil either during or after the powder period, we have

$P' = K_{x_1} = P-W_r\sin\emptyset$ along the bore downward and

$N' = K_{y_1} = m_r\frac{dv}{dt}\sin(\emptyset+\theta)$ normal to the bore dowaward

or $= N-W_r\cos\emptyset$. We have in the above neglected the powder pressure couple,it being at best small, with little or no effect on stability. The inertia force of the top carriage is evidently

$m_c\frac{dv}{dt}$ reversed parallel to the inclined plane. Hence the external forces not including (5) become

(1) $P' = K_{y_1} = P-W_r\sin\emptyset$ along the bore

$N'+K_{y_1} = m_r\frac{dv}{dt}\sin(\emptyset+\theta)$ normal to the bore

(2) W_r = upper recoiling weight, vertically down.

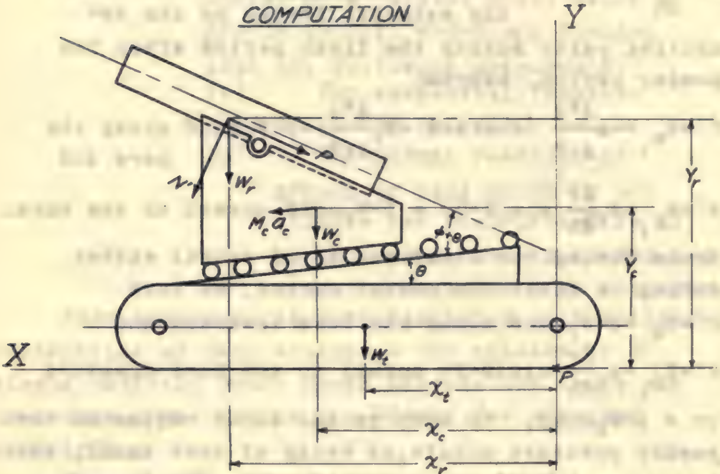
(3) $m_c\frac{dv}{dt}$ = inertia force of lower recoiling parts parallel to inclined plane.

(4) W_c = weight of lower recoiling parts, vertically down.

<u>The external reactions during the second period:</u>

During the second period, $m_c\frac{dv}{dt} = K_{x_1}$ reverses in direction since the lower recoiling mass now becomes retarded. On the assumption that during this period the upper recoiling parts have the same motion as the lower recoiling parts, we have

STABILITY COMPUTATION



a_c = Acceleration of Carriage up plane $= \frac{dv}{dt}$

$N' = M_c a_c \text{ Sine}(\phi+\theta)$

P = Braking Force $= P_1 - W_r \text{ Sine } \phi$

ϕ = Angle of Elevation

θ = Angle of inclined Plane

Moments about P

$$= (N' \text{Cos}\, \phi + P \text{ Sine}\, \phi + W_r) x_r + (M_c a_c \text{ Sine}\, \theta + W_c) x_c + (N' \text{ Sine}\, \phi - P \text{ Cos}\, \phi) Y_r + (M_c a_c \text{ Cos}\, \theta) Y_c + W_t x_t$$

For any other point

$$x'_r = x_r - (S' \text{Cos}\, \phi + S \text{ Cos}\, \theta), \quad Y'_r = Y_r - (S' \text{Sine}\, \phi - S \text{ Sine}\, \theta)$$

$$x'_c = x_c - S \text{ Cos}\, \theta, \quad Y'_c = Y_c + S \text{ Sine}\, \theta$$

$$\left(S' = \int v_{rel}\, \partial t, \quad S = \int v_c\, \partial t\right)$$

Fig. 3

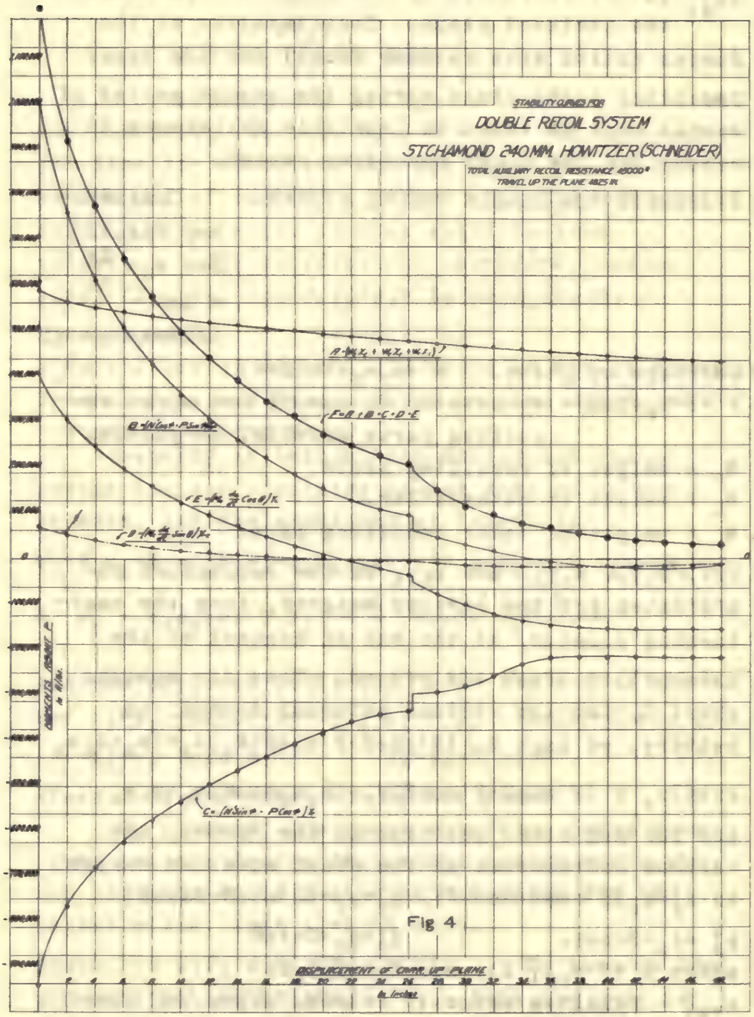

STABILITY CURVES FOR
DOUBLE RECOIL SYSTEM
ST. CHAMOND 240 MM. HOWITZER (SCHNEIDER)
TOTAL AUXILIARY RECOIL RESISTANCE 45000#
TRAVEL UP THE PLANE 48.25 IN.
DISPLACEMENT OF CARR. UP PLANE
In Inches

Fig 4

for the inertia force of the upper recoiling parts:

$m_r\frac{dv}{dt}$ in the direction up the plane and parallel to the inclined plane. Consideration of the forces acting when counter recoil for the upper recoiling parts place during the second period of recoil will be taken in "variable resistance to recoil for the upper recoiling parts".

STABILITY FOR DOUBLE RECOIL SYSTEM. Considering fig.(3) let $a_c = \frac{dv}{dt}$ = acceleration of carriage up plane. $N' = m_r a_c \sin(\phi+\theta)$

$P' = P - W_r\sin\phi$ = resistance to recoil for upper recoiling parts parallel to guides.

W_r = weight of recoiling parts

W_t = weight of caterpillar.

W_c = weight of lower recoiling parts

Let, $x_r y_r$, $x_c y_c$, and $x_t y_t$ be the respective coordinates for the various weights, from the overturning point O, at the end of contact of the caterpillar track and ground. Then for moments about O, for the various external forces, in battery, we have $M_o = (N'\cos\phi + P'\sin\phi + W_r)x_r + (w_c + m_c a_c \sin\theta)x_c + (N'\sin\phi - P'\cos\phi)y_r + (m_c a_c \cos\theta)y_c + w_t x_t$

and for any other position in the recoil, the various coordinates of the above equation change to $x_r' = x_r(S'\cos\phi + S\cos\theta)$, $y_r' = y_r - (S'\sin\phi - S\sin\theta)$

$x_c' = x_c - S\cos\theta$, $y_c' = y_c + S\sin\theta$

where $S' = \int v_{rel}\,dt$, and $S = \int v_c dt$

v_{rel} = relative velocity between upper and lower recoiling parts

v_c = velocity of carriagrs up inclined plane.

Further let $A = W_t x_t + W_r X_r' + W_c X_c'$

$$B = (N'\cos\phi + P'\sin\phi)x_r'$$

$$C = (N'\sin\phi - P'\cos\phi)y_r'$$

$$D = (m_c \frac{dv_c}{dt} \sin \theta) x_c$$

$$E = (m_c \frac{dv_c}{dt} \cos \theta) y_c$$

$$F = A+B+C+D+E$$

For stability, we must have $F = 0$. The critical position fir stability for the first period is at the end of the first period when the two recoiling parts begin to move with the same velocity. The coordinates, therefore, become

$x_r' = x_r - (Z\cos\emptyset + x_1 \cos\theta), y_r' = y_r - (Z\sin\emptyset - x_1 \sin\theta)$

$x_c' = x_c - x_1 \cos\theta, \qquad y_c' = y_c + x_1 \sin\theta$

x_1=displacement up inclined plane at common velocity

Z=total relative displacement between upper and lower recoiling mass.

Assuming as before that during the second period the two recoiling masses move as if one, we have, for the condition of stability

$$M_o = W_t x_t + W_c x_c' + W_r x_r' - m_c a_c \sin\theta\, x_c' - m_c a_c \cos\theta\, y_c' - m_r a_c \sin\theta\, x_r' - m_r a_c \cos\theta\, y_r' = 0 \quad \text{where } a_c = \frac{R}{m_c + m_r}$$

and the critical stability is at the end of recoil and $x_r' = x_r - (Z\cos\emptyset + X\cos\theta)$; $y_r' = y_r - (Z\sin\emptyset - X\sin\theta)$

$x_c' = x_c - X\cos\theta; \qquad y_c' = y_c + X\sin\theta$

where X is the total run up the plane.

If, however, the upper recoiling parts move into battery during the second period while the top carriage still continues moving up the inclined plane, then $x_r' = x_r - X\cos\theta$.

STABILITY WITH A SINGLE RECOIL AND CATERPILLAR BRAKED. We have as before the same inertia and weight moments but in addition rotational inertia couples. Since the effect of a couple is entirely independent of the axis about which moments are taken, we merely have to add in the previous

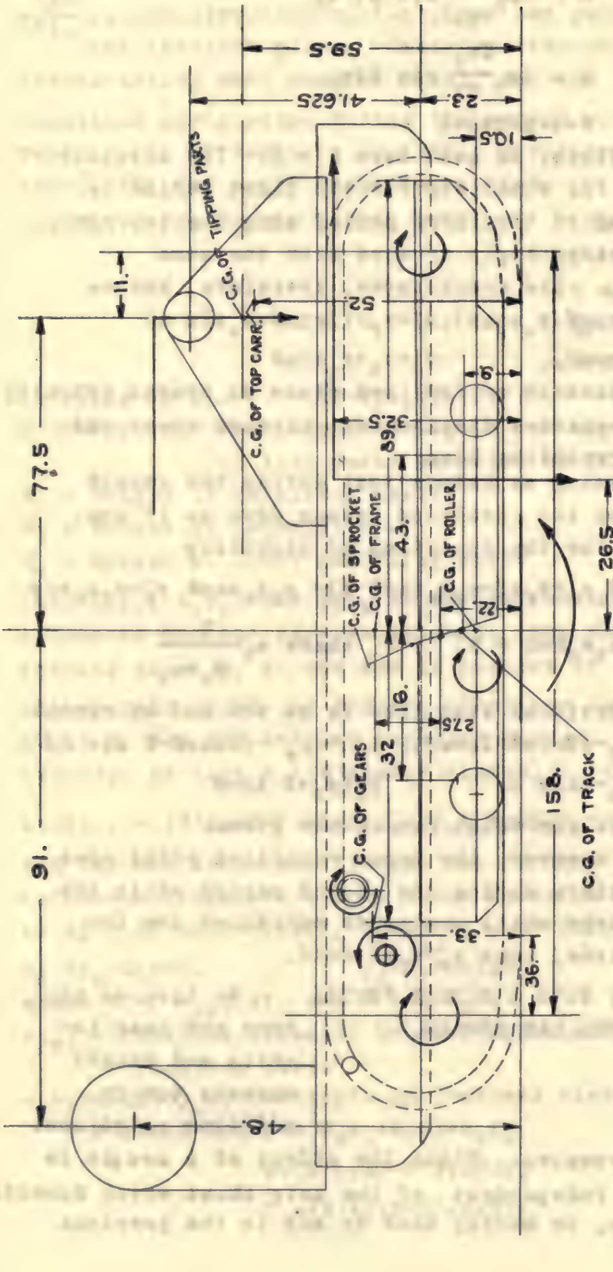

59.5
41.625
23.
10.5
11.
C.G. OF TIPPING PARTS
C.G. OF TOP CARR.
52
77.5
9.
32.5
89.5
43.
C.G. OF SPROCKET
C.G. OF FRAME
C.G. OF ROLLER
26.5
22
16.
27.5
32
C.G. OF GEARS
158.
C.G. OF TRACK
91.
33.
36.
48.

Fig. 5

moment equation the additional rotational inertia terms, taking of course account of the algebraic sign of the inertia couple.

The following inertia couples are introduced with a caterpillar using a simple mechanism as assumed before:

<u>Stabilizing inertia couples:-</u>

$\left(\frac{m_s k_s^2 + m_{gs} k_{gs}^2}{r_o}\right)\frac{dv}{dt}$ = drive sprocket and bear couple

$\frac{m_w k_w^2}{r_w} \frac{dv}{dt}$ = roller truck couples

$\frac{m_i k_i^2}{r_i} \frac{dv}{dt}$ = front idler couple

$m_t[\pi(r_o^2+r_i^2)+2\ l\ r]$ = track inertia moment where $r = \frac{r_o+r_i}{2}$ and l = total span of track

<u>Overturning inertia couple:-</u>

$\left(\frac{m_d k_d^2 + m_{gd} k_{gd}^2}{r_o r_2}\right) r_1 \frac{dv}{dt}$ = drum shaft inertia couple

Therefore the stability equation becomes, F = A+B+C+D+E+G+H+I+J+L and for stability $F \geq 0$. Where during the first period $A = W_r x_t + W_r x_r + W_c X_c'$, $B = (N'\cos\emptyset + P'\sin\emptyset)x_r'$, $C = (N'\sin\emptyset - P'\cos\emptyset)y_r'$, $D = (m_c \frac{dv_c}{dt} \sin\theta)x_c$, $E = (m_c \frac{dv_c}{dt}\cos\theta)y_c$,

$G = \left(\frac{m_s k_s^2 + m_{gs} k_{gs}^2}{r_o}\right)\frac{dv}{dt}$, $H = \Sigma \frac{m_w k_w^2}{r_w} \frac{dv}{dt}$,

$I = \frac{m_i k_i^2}{r_i} \frac{dv}{dt}$, $J = m_t[\pi(r_o^2+r_1^2)+2\ l\ r]$

$K = -\left(\frac{m_d k_d^2 + m_{gd} k_{gd}^2}{r_o r_2}\right) r_1 \frac{dv}{dt}$ where the coordinates refer to point of contact of ground and track at rear end of track. During the second period, the inertia couples become

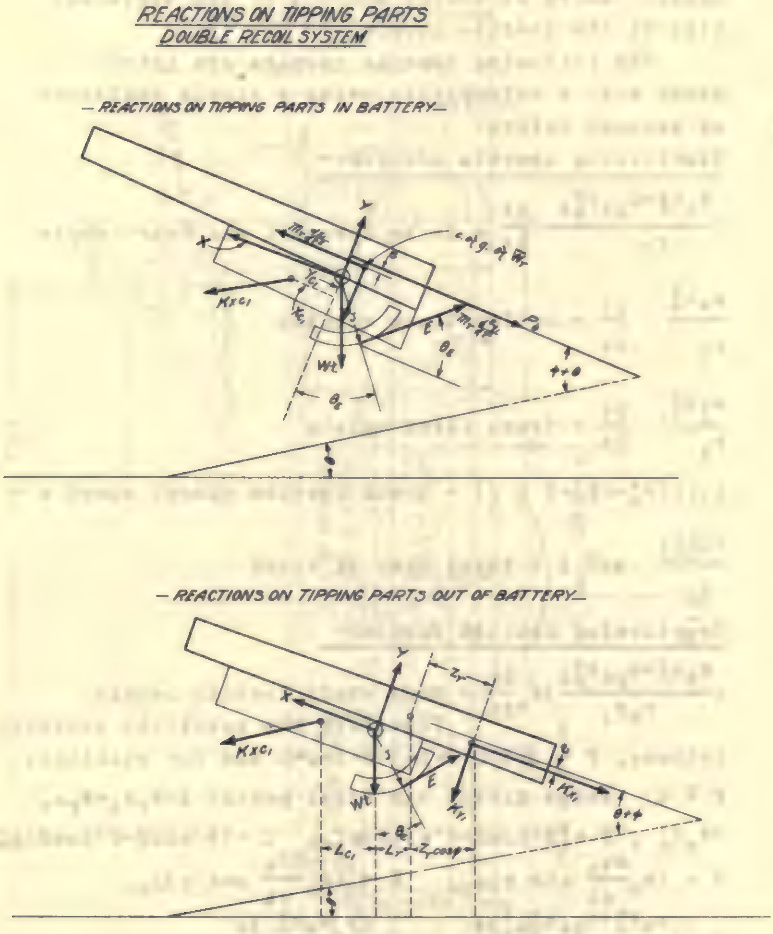

REACTIONS ON TIPPING PARTS
DOUBLE RECOIL SYSTEM
— REACTIONS ON TIPPING PARTS IN BATTERY—
— REACTIONS ON TIPPING PARTS OUT OF BATTERY—

Fig. 6

reversed, therefore, $A-D-E-m_r\frac{dv_c}{dt}\sin\theta \cdot x_r - m_r\frac{dv_c}{dt}\cos\theta\ y_r - G-H-I-J-K \geqq 0$ where $\frac{dv_c}{dt}$ is determined by the relation,

$$\frac{T_D r_1}{E_0 E_1 E_2 r_0 r_2} = (m_r+m_c+m_e)\frac{dv_c}{dt}$$

where m_r=mass of recoiling parts, m_c=mass of caterpillar and mount excluding recoiling parts, m_e= equivalent mass for rotational inertia.

ELEVATING ARC AND TRUNNION REACTION OF THE TIPPING PARTS. In computing the various reactions in a double recoil system, we must consider the inertia effect of the various parts in modifying these reactions over their static values or as would occur with a single recoil.

The primary inertia forces induced by the double recoil are:

<u>For the upper recoiling parts:</u>

(1) The inertia force of the upper recoiling mass divided into components through the center of gravity of the upper recoiling parts, parallel and normal to the axis of the bore, respectively.

<u>For the lower recoiling mass:</u>

(2) The inertia force of the top carriage and cradle acting through the center of gravity of this combined mass, opposite to the acceleration, and parallel to the inclined plane.

The inertia resistance of the top carriage and cradle may be divided into two parallel components through the center of gravity of the cradle and top carriage respectively the

magnitude of the components being proportional to their respective masses.

x_1 and y_1 = coordinates of upper recoiling parts parallel and normal to the axis of bore.

x and y = coordinates of lower recoiling parts parallel and normal to inclined plane.

K_{x_1} = inertia component along bore of upper recoiling mass through its center of gravity (lbs)

K_{y_1} = inertia component normal to bore of upper recoiling mass through its center of gravity (lbs)

K_{xc_1} = inertia force of cradle through its center of gravity and parallel to inclined plane (lbs)

K_{xc_2} = inertia force of top carriage through its center of gravity and parallel to inclined plane (lbs)

W_r = weight of upper recoiling parts (lbs)

x_r and y_r = coordinates from trunnions of center of gravity of upper recoiling parts in battery, parallel and normal to bore (ft)

W_{c_1} = weight of cradle (lbs)

x_{c_1} and y_{c_1} = coordinates from trunnions of cradle parallel and normal to bore (ft)

W_{c_2} = weight of top carriage (lbs)

W_c = total weight of lower recoil parts (lbs)

$W_c = W_{c_1} + W_{c_2}$

W_t = weight of tipping parts

X_t and Y_t = components of trunnion reactions parallel and normal to axis of bore of gun.

E = elevating arc reaction (lbs)

j = elevating arc radius about trunnions or perpendicular distance to line of act on of E (lbs)

B = total braking between upper and lower recoiling parts (lbs)

R_1 = total friction between upper and lower recoiling parts (lbs)

P = total resistance between upper and lower recoiling parts (lbs)

N = total normal reaction between upper and lower recoiling parts (lbs)

Z = relative displacement of upper recoiling parts with respect to lower recoiling parts.

P_b = powder reaction on base of breech (lbs)

e = distance from P to center of gravity of upper recoiling parts (in)

Then during the acceleration for the upper recoiling parts, we have

$P_b = (B+R_1-W_r\sin\emptyset)=m_r\dfrac{d^2x_1}{dt^2}$ along the bore and the external reaction on the upper recoiling parts during the powder acceleration, becomes

$$
\begin{aligned}
K_{x_1} &= P_b-m_r\frac{d^2x_1}{dt^2} \text{ (lbs) along the bore}\\
&= B+R-W_r\sin\emptyset \quad \text{along the bore}\\
&= P-W_r\sin\emptyset \quad \text{along the bore}
\end{aligned}
$$

During the retardation,

$m_r\dfrac{d^2x_1}{dt^2} = -(B+R-W_r\sin\emptyset)$ and the external reaction on the recoiling parts parallel to the bore is the inertia force,

$$
\begin{aligned}
K_{x_1} &= -m_r\frac{d^2x_1}{dt^2} = B+R-W_r\sin\emptyset\\
&= P-W_r\sin\emptyset
\end{aligned}
$$

Hence either during the acceleration or retardation

the external component parallel to the bore on the recoiling parts equals the total resistance to recoil off the upper recoiling parts. The inertia force normal to the bore, becomes,

$K_{y_1} = m_r \frac{d^2 y_1}{dt^2}$ (lbs) Since $\frac{dy_1}{dt} = v \sin(\emptyset+\theta)$, where v is the velocity of the lower recoiling parts up the plane.

$$K_{y_1} = m_r \frac{dv}{dt} \sin(\emptyset+\theta) \text{ (lbs)}$$

$$= N - W_r \cos\emptyset \qquad \text{(lbs)}$$

For the lower recoiling parts, we have

$$K_{xc_1} = -m_{c_1} \frac{dv}{dt} \text{ (lbs) along the inclined plane}$$

and $K_{xc_2} = -m_{c_2} \frac{dv}{dt}$ (lbs) along the inclined plane

Elevating arc and trunnion reactions:

Let us now consider the tipping parts, that is the recoiling parts, together with the cradle.

By the use of D'Alemberts principle the problem in Kinetics is reduced to one of statics, provided, we introduce the proper inertia forces.

Further, the mutual reaction between the upper recoiling parts and the cradle of the lower recoiling parts, becomes, an internal force for the system consisting of the tipping parts.

Therefore, introducing the inertia forces, we have:

For the reactions of the tipping parts in battery:-

Along the bore: fig.(6)

$$P_b - m_r \frac{d^2 x_1}{dt^2} + E\cos\theta_e + W_t \sin\emptyset - K_{xc_1} \cos(\emptyset+\theta) - 2X = 0$$

Normal to the bore:

$$W_t\cos\phi - E\sin\theta - K_{xc'}\sin(\theta+\phi) + K_{y'} - 2Y = 0$$

Moments about the trunnion:

$$P_b e + \left(P_b - m_r\frac{d^2x_t}{dt^2}\right)y_r + K_{y'}x_r + K_{xc'}\cos(\theta+\phi)y_{cj} - K_{xc'}\sin$$

$(\phi+\theta)x_{c'} - Ej = 0$ since in the battery position the center of gravity of the tipping parts is located at the axis of the trunnions. Further,

$P_b - m_r\dfrac{d^2x_t}{dt^2} = B + R - W_r\sin\phi = K_{x'}$ We have, for the elevating arc reaction,

$$E = \frac{P_b e + K_{x'}y_r + K_{y'}x_r + K_{xc'}[y_{c'}\cos(\phi+\theta) - x_{c'}\sin(\phi+\theta)]}{j}$$

and for the components of the trunnion reaction

$$2X = K_{x'} + E\cos\theta_e + W_t\sin\phi - K_{xc'}\cos(\phi+\theta)$$

$$2X = W_t\cos\phi + K_{y'} - E\sin\theta_e - K_{xc'}\sin(\phi+\theta)$$

For the reactions of the tipping parts out of battery:

In any intermediate position, out of battery the entire tipping parts are displaced backwards up the inclined plane but in addition we have a relative displacement between the recoiling parts and the cradle of the top carriage, equal to Z (in).

Therefore, the moment of the tipping parts about the trunnions, become $W_r(l_r + Z\cos\phi) + W_{c'}l_{c'} = M_t$ where l_r and $l_{c'}$ are the horizontal coordinates of the upper recoiling parts and cradle in the battery position. Since center of gravity of the tipping parts are located at the trunnion in the battery position, we have $W_r l_r + W_{c'}l_{c'} = 0$ hence $M_t = W_r Z\cos\phi$. Then, the reactions along the bore

$K_x' + E\cos\theta_e + W_t\sin\phi - K_{xc}'\cos(\phi+\theta) - 2X = 0$

Normal to the bore:

$W_t\cos\phi + K_y' - E\sin\theta_e - K_{xc}'\sin(\phi+\theta) - 2Y = 0$

Moments about the trunnion:

$K_x'y_r + K_y'x_r + K_{xc}'\cos(\theta+\phi)y_c' - K_{xc}'\sin(\theta+\phi)x_c' + W_r x_r \cos\phi - Ej = 0$

Hence, we have for the elevating arc reaction for a relative displacement Z out of battery

$$E = \frac{K_x'y_r + K_y'(x_r+Z) + K_{xc}'[y_c'\cos(\theta+\phi) - x_c'\sin(\theta+\phi) + W_r Z_r\cos\phi]}{j}$$

and for the components of the trunnion reactions,

$2X = K_x' + E\cos\theta_e + W_t\sin\phi - K_{xc}'\cos(\theta+\phi)$

$2Y = W_t\cos\phi + K_y' - E\sin\theta_e - K_{xc}'\sin(\theta+\phi)$

REACTION BETWEEN UPPER AND LOWER RECOILING PARTS.

In the calculation of guide and clip reactions and the bending stresses in the cradle it is necessary to know the nature of the reaction between the upper and lower recoiling parts as well as its distribution.

The reaction between the two recoiling masses, consists of:

(1) The resultant braking reaction acting parallel to the guides and through the controid of the various pulls.

(2) The guide friction acting along the guides.

(3) The normal clip reactions, which may be divided into:

(a) a normal component perpendicular to the axis of the

REACTION BETWEEN UPPER AND RECOILING PARTS
DOUBLE RECOIL SYSTEM

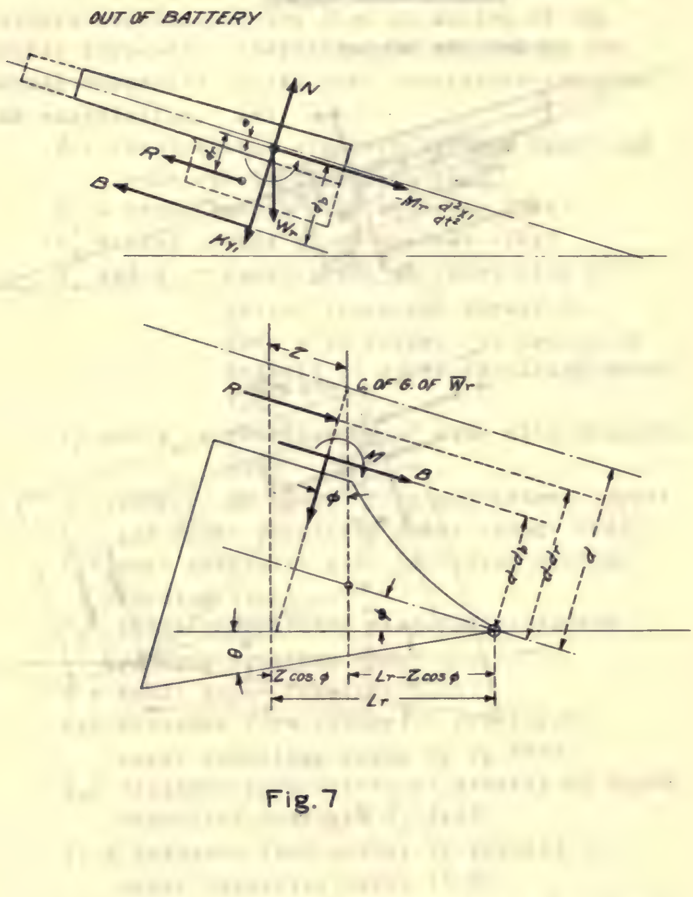


Fig. 7

REACTION BETWEEN UPPER AND LOWER RECOILING PARTS:
DOUBLE RECOIL SYSTEM

IN BATTERY POSITION

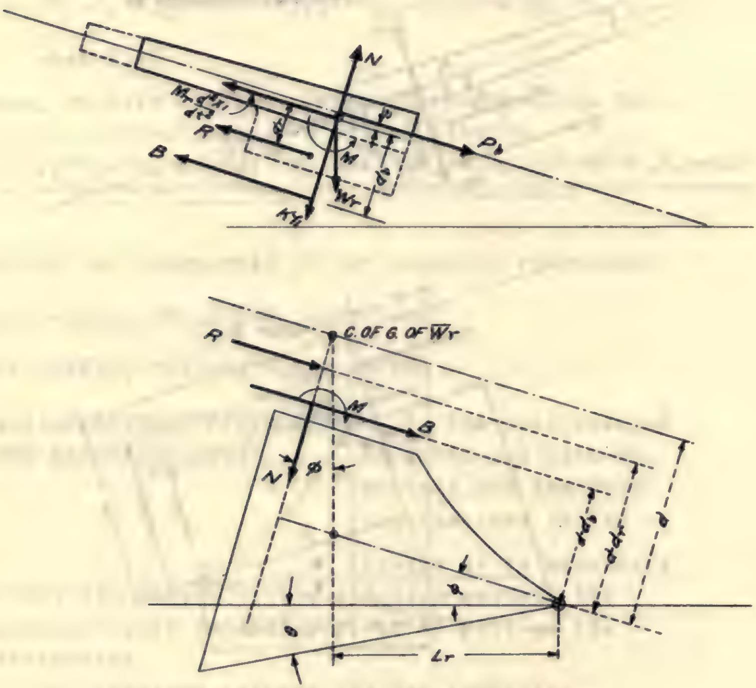


Fig. 8

bore.

(b) a couple between the two parts.

The magnitude of the couple depends upon the assumed position of the line of action of the normal component; therefore, we may assume the normal component in its most convenient position for calculation. Let

N = total normal reaction between upper and lower recoiling parts (lbs)

N_1 = front normal clip reaction (lbs)

N_2 = rear normal clip reaction (lbs)

x_1 and y_1 = coordinates of front clip reaction along and normal to bore with respect to center of gravity of upper recoiling parts (in)

x_2 and y_2 = coordinates of rear clip reaction (in)

M = couple or moment reaction between upper and lower recoiling parts (inch- lbs)

P_h=total hydraulic pull including packing friction (lbs)

P_a= total recuperator reaction including packing friction (lbs)

R = total guide friction (lbs)

d_h= distance from center of gravity of upper recoiling parts to P_h (in)

d_a= distance from center of gravity of upper recoiling parts to B_a (in)

d_r = distance from center of gravity of upper recoiling parts to R

n = coefficient of guide friction (0.15 approx.)

$B = \Sigma P_h + \Sigma P_a$=Total braking (lbs)

$R = n(N_1+N_2)$=guide friction (lbs)

l_v = horizontal distance from rear roller contact of top carriage and inclined

plane to line of action of W_r

d = distance from A, normal to line through center of gravity of upper recoiling parts and parallel to bore

Then $B\,d_b=\Sigma P_h d_h+\Sigma P_a d_a$. Considering the reactions on the recoiling mass in battery, we have,

$$P_b-m_r\frac{d^2x_1}{dt^2}=B+R-W_r\sin\emptyset,\text{ along the bore}$$

$$N=K_{y'}+W_r\cos\emptyset,\text{ normal to the bore}$$

$$M=P_be+Bd_b+R\,d_r,\text{moments about center of gravity.}$$

Taking moments about A fig.(7) at the rear roller contact of top carriage and inclined plane, we have, for the moment of the reaction exerted by the upper recoiling parts, on the lower

$$M_a=B(d-d_b)+R(d-d_r)+M-N\left(\frac{l_r}{\cos\emptyset}+d\tan\emptyset\right)$$

Substituting for M, its value $M=Bd_b+Rd_r+P_be$ and for N, its value $N=W_r\cos\emptyset+K_{y'}$ we have

$$M_a=P_be+(B+R-W_r\sin\emptyset-K_{y'}\tan\emptyset)d$$
$$-\left(W_r+\frac{K_{y'}}{\cos\emptyset}\right)l_r$$

Hence, the reaction of the upper recoiling part on the lower, during the powder period, is equivalent to:

(1) A couple P_be

(2) A force through the center of gravity of the recoiling parts parallel to the bore: $B+R-W_r\sin\emptyset-K_{y'}\tan\emptyset$

(3) A vertical force through the center of gravity of the recoiling parts,

$$W_r+\frac{K_{y'}}{\cos\emptyset}$$

After the powder period, the reactions on the recoiling parts, become

$$-m_r \frac{d^2 x_1}{dt^2} = B+R-W_r \sin\emptyset, \text{ along the bore}$$

$N = K_{y'} + W_r \cos\emptyset$, normal to the bore

$M = Bd_b + Rd_r$, moments about the center of gravity

Taking moments about A, fig.(8) at the rear roller contact of top carriage and inclined plane, we have, for the moment of the reaction exerted by the upper recoiling parts, on the lower, the recoiling parts having a relative displacement Z.

$$M_A = B(d-d_b) + R(d-d_r) + M - N\left(\frac{l_r - Z\cos\emptyset}{\cos\emptyset} + d\tan\emptyset\right)$$

Substituting, as before, for M and N, we have

$$M_A = (B+R-W_r\sin\emptyset - K_{y'}\tan\emptyset)d - \left(W_r + \frac{K_{y'}}{\cos\emptyset}\right)(l - Z\cos\emptyset)$$

Hence, the reaction of the upper recoiling parts on the lower, after the powder period, that is during the retardation of the upper recoiling parts, is equivalent to:

(1) A force through the center of gravity of the recoiling parts parallel to the bore

$$B+R-W_r\sin\emptyset - K_{y'}\tan\emptyset$$

(2) A vertical force through the center of gravity of the recoiling parts:

$$W_r + \frac{K_{y'}}{\cos\emptyset}$$

The mutual reaction between the upper and lower recoiling parts, can be determined immediately as follows:

(1) B+R along the bore

(2) N normal to the bore

(3) M a couple between the parts

Now, $N = W_r\cos\emptyset + K_{y'}$ (algebraically)

$$\bar{N} = \overline{W_r} - \overline{W_r\sin\emptyset} + \overline{\frac{K_{y'}}{\cos\emptyset}} - \overline{K_{y'}\tan\emptyset} \quad \text{(vectorially)}$$

and through the center of gravity of the recoiling parts. Further B and R may be resolved into a vector

$\overline{B+R}$ parallel to B and R through the center of gravity of the recoiling parts and a couple Bd_b+Rd_r. Hence combining B+R and M, we have

$\overline{B+R}+\overline{Bd_b}+\overline{Rd_r}+\bar{M}=\overline{B+R}$ (through the center of gravity of the recoiling parts, parallel to the bore)

since $\bar{M}=-\overline{Bd_b}-\overline{Rd_r}$

Combining, the parallel components through the center of gravity of the recoiling parts, we have

$B+R-W_r\sin\emptyset-K_{y'}\tan\emptyset$, along the bore

$W_r+\dfrac{K_{y'}}{\cos\emptyset}$ vertically

$-(bd_b+Rd_r)$ a couple between the parts

<u>Guide and clip reactions:</u>

For the front clip reaction, we have

$$N_1=N_{x_2}-\frac{M}{l}$$

$$=(W_r\cos\emptyset+K_{y'})x_2-\left(\frac{Bd_b+Rd_r}{l}\right)$$ (lbs) acting upward on recoiling parts, and

for the rear clip reaction, $N_2=Nx_1+\frac{M}{l}$

$$=(W_r\cos\emptyset+K_{y'})x_1+\left(\frac{Bd_b+Rd_r}{l}\right)$$ (lbs) acting upward on recoiling parts.

VARIABLE BRAKING ON UPPER RECOIL BRAKE DOUBLE RECOIL SYSTEM.

Usually we have a given upper recoil system with a constant braking for the lower recoil system, since given single reçoil mounts are converted into a double recoil system by allowing the top carriage to slide along an inclined plane. Further, in the

design of a double recoil system, since at high elevations of the gun, the component of the reaction of the upper recoiling parts along the plane is small, the movement up the plane therefore, becomes relatively small. Hence in a design layout the throttling grooves of the upper recoil system may be calculated on the basis of a single recoil at maximum elevation.

We have therefore a very important class of double recoil systems, where the upper recoil throttling is based on a single or static recoil and the lower recoil braking is designed for an approximate constant resistance at minimum elevation.

The recoil braking of the upper recoil system consists of the sum of the following components:

(1) The recuperator reaction, which is a function of the relative displacement between the gun and top carriage.

(2) The throttling reaction, which is proportional to the square of the relative velocity at a given relative displacement.

(3) The guide and packing frictions, which depend upon the normal reaction between the parts, etc., but can be assumed approximately constant.

The lower recoil braking will be assumed constant at minimum elevation.

Considering fig. (9)

Let

P_b = powder reaction (lbs)

B = total braking (lbs)

P_f = total friction (assumed constant) (lbs)

P_h = hydraulic braking (lbs)

$p = Pa + Ph + P_f$

P_{hs}= hydraulic pull from static force diagram
P_a= recuperator reaction (lbs)
N=normal reaction between upper and lower recoiling parts (lbs)
R = brake reaction of lower recoil system (lbs)
Ø= angle of elevation
θ= angle of inclination of inclined plane
W_r= weight of upper recoiling parts (lbs)
W_c= weight of lower recoiling parts (lbs)
V_f= free velocity of recoil (ft/sec)
V_r=absolute velocity of recoiling parts parallel to axis of gun (ft/sec)
V_c= velocity of lower recoiling parts along the inclined plane (ft/sec)
Z = relative displacement between upper and lower recoiling parts (ft)
X_c = displacement of top carriage up inclined plane (ft)
X_r = absolute displacement of gun parallel to axis of bore (ft)
B'= counter recoil buffer reaction

During the powder pressure period:

On the upper recoiling parts, we have

$$P_b - p + W_r \sin Ø = m_r \frac{d^2x_r}{dt^2} \quad \text{(1) along the bore}$$

$$N - W_r \cos Ø = m_r \frac{d^2y_r}{dt^2} \quad \text{(2) normal to the bore.}$$

On the lower recoiling parts, we have

$$P\cos(Ø+\theta) - N\sin(Ø+\theta) - R - W_c \sin\theta = m_c \frac{d^2x_c}{dt^2} \quad \text{(3) up the}$$

inclined plane. We have further, the following kinematical relations:-

$$v_r = \frac{dx_r}{dt} = v_{rel} + v_c \cos(Ø+\theta) \qquad X_r = X_{rel} + X_c$$

$$\frac{dy_r}{d_t} = v_c \sin(Ø+\theta) \qquad : \qquad Y_r = y_c \sin(Ø+\theta)$$

hence $\dfrac{d^2x_r}{dt^2} = \dfrac{dv_{rel}}{dt} + \dfrac{dv}{dt}\cos(\emptyset+\theta)$

$\dfrac{d^2y_r}{dt^2} = \dfrac{dv_c}{dt}\sin(\emptyset+\theta)$

Now between any two instants t_{n-1} and t_n we have, from eq.(1)

$$\int_{t_{n-1}}^{t_n} \frac{P_b\,dt}{m_r} - \left(\frac{p-W_r\sin\emptyset}{m_r}\right)(t_n-t_{n-1}) = V_r^n - V_r^{n-1}$$

and $V_{f_n} - V_{f_{n-1}} \left(\dfrac{p-W_r\sin\emptyset}{m_r}\right)\Delta t_n = V_r^n - V_r^{n-1}$

therefore $V_r^n = V_r^{n-1} + (V_{f_n} - V_{f_{n-1}}) - \left(\dfrac{p-W_r\sin\emptyset}{m_r}\right)\Delta t_n$

which gives a "point by point" method for determining the absolute velocity of the gun parallel to the bore.

Now if we substitute for the normal reaction N in (3) its value

$$N = W_r\cos\emptyset + m_r\frac{dv_c}{dt}\sin(\emptyset+\theta)$$

we have

$$P\cos(\emptyset+\theta) - W_r\sin(\emptyset+\theta)\cos\emptyset - m_r\frac{dv_c}{dt}\sin(\emptyset+\theta) - W_r\sin\theta - R = m_c\frac{dv_c}{dt}$$

hence

$$\frac{dv_c}{dt} = \frac{p\cos(\emptyset+\theta) - W_c\sin\theta - W_r\sin(\emptyset+\theta)\cos\emptyset - R}{m_c + m_r\sin^2(\emptyset+\theta)}$$

and between instants t_{n-1} and t_n, we have

$$V_c^n = V_c^{n-1} + \frac{[p\cos(\emptyset+\theta) - W_c\sin\theta - W_r\sin(\emptyset+\theta)\cos\emptyset - R]}{m_c + m_r\sin^2(\emptyset+\theta)}\Delta t_n$$

The total braking P, becomes $P = P_a + P_b + P_f$ (lbs)

In the static or single recoil, the top car-

riage stationary, we have $P_{hs}=C_o \dfrac{v_s^2}{w_{xn}^2}$

Now for the same relative displacement between the upper and lower recoiling parts for the double recoil, the throttling area is the same, namely w_{xn}, then

$$P_h=C_o \frac{v_{rel}^2}{w_{xn}^2} \quad \text{hence } P_h=P_{hs} \frac{v_{rel}^2}{v_s^2}$$

Therefore, from a static force diagram, knowing the relative displacement = static recoil displacement, we may determine P_{hs} and v_s^2. If V_{rel} has been determined for the point, the hydraulic braking is readily determined from the above equation.

The recuperator reaction is determined from the static force diagram when the relative displacement is known. When the upper recoiling parts begin to counter recoil relatively to the lower recoiling parts, we have

$$P=P_a-B' \frac{v_{rel}^2}{v_s^2} -P_f$$

Procedure for recoil calculations

We must first construct a static force and velocity diagram for the upper recoil system as would occur if the mount had a single recoil, the top carriage being fixed. Let

v_o = muzzle velocity (ft/sec.)

u= travel up bore (ft)

w = weight of projectile (lbs)

$\bar{w}$ = weight of charge (lbs)

P_m=max. total powder reaction (lbs)

then average pressure on breech $P_e = \dfrac{wv_o^2}{2gu}$ (lbs)

Pressure on breech when shot leaves muzzle —

$$P_{ob} = \frac{27}{4} b^2 \frac{u}{(b+u)^3} 1.12 P_m \quad \text{(lbs)}$$

where $b=\left(\frac{27}{16}\frac{P_m}{P_e}-1\right)\pm\sqrt{\left(1-\frac{27}{16}\frac{P_m}{P_e}\right)^2-1}$ (ft)

Time of travel to bore $t_o = \frac{3}{2} u_o$ (sec)

Max. free velocity of recoil $V_f = \frac{wv_o+4700\bar{w}}{w_r}$ (ft/sec)

Free velocity of recoil when shot leaves muzzle —

$$V_{fo}=\frac{(w+0.5\bar{w})v_o}{w_r} \quad \text{(ft/sec)}$$

Time during expansion of powder gases —

$$t_{1o}=\frac{2(V_f-V_o)}{P_{ob}}\frac{w_r}{g} \quad \text{(sec)}$$

Total powder period $T = t_{1o}+t_o$ (sec)

Free displacement of recoil during travel up bore—

$$X_{fo}=\frac{w+0.5\bar{w}}{w_r}u \quad \text{(ft)}$$

Free displacement during expansion of gases —

$$X_{f'o} = \frac{P_{ob}}{w_r} g \frac{(T-t_o)^2}{3} + V_{fo}(T-t_o) \quad \text{(ft)}$$

Total displacement of free recoil during powder period— $E=X_{fo}+X_{f'o}$

Three points are sufficient to establish approximate the velocity curve during the powder period. They may be taken at times t_o, t_m and T respectively.

The total resistance to recoil for constant resistance to recoil,

$$K_o=\frac{\frac{1}{2}m_rV_f^2}{b-E+V_fT}$$

for variable resistance to recoil

$$K_o = \frac{m_rV_f^2+m(b-E)^2}{2\left[b-E+V_fT-\frac{m}{2}\frac{T^2}{m_r}(b-E)\right]}$$

At t_o when the shot leaves the muzzle —

$$V_o = V_{fo} - \frac{K_o t_o}{m_r} \quad \text{(ft/sec)}$$

$$X_o = X_{fo} - \frac{K_o t_o^2}{2m_r} \quad \text{(ft)}$$

At t_m when we have max. restrained recoil velocity,

$$V_m = V_{fm} - \frac{K_o t_o}{m_r} \quad \text{(ft/sec)}$$

$$X_m = X_{fm} - \frac{K_o t_o^2}{2m_r} \quad \text{(ft)}$$

where $V_{fm} = V_{fo} + P_{ob}(t_m - t_o)\left[1 - \frac{P_{ob}(t_m - t_o)}{4m_r(V_f - V_o)}\right]$ (ft/sec)

$$X_{fm} = X_{fo} + \left[V_{fo} + \frac{P_{ob}}{m_r}(t_m - t_o) - \frac{(t_m - t_o)^2}{6m_r(V_v - V_o)}\right. \quad \text{(ft)}$$

$$t_m = T - \frac{K(T - t_o)}{P_{ob}} \quad \text{(sec)}$$

At time T, the end of the powder period —

$$V_r = V_f - \frac{K_o T}{m_r} \quad \text{(ft/sec)}$$

$$E_r = X_f - \frac{K_o T^2}{2m_r} \quad \text{(ft)}$$

After the powder period, during the retardation, we have for constant resistance to recoil,

$$V_x = \sqrt{\frac{2K_o}{m_r}(b - X)} \quad \text{(ft/sec)}$$

for variable resistance to recoil,

$$V_x = \sqrt{\frac{\left[(K_o - \frac{m}{2}(b + X - 2E_r)(b - X)\right]}{m_r}} \quad \text{(ft/sec)}$$

where b = the total length of static recoil (ft)

$m = C_s \frac{W_r}{h}$; C_s=0.85 approx.; h= perpendicular distance from spade to line of action of K.

Construction of static force diagram:

We have, for a constant resistance throughout recoil, $K=P_{hs}+P_a+P_f-W_r\sin\emptyset$ hence $P_{hs}+P_a+P_f=K+W_r\sin\emptyset$ (a constant)

For variable recoil, in battery $K=K_o$, out of battery $K=k$

where $k = K_o-m(b-E_r)$ and $K = K_o$ during the powder period.

$=K_o-m(x-E_r)=k+m(b-x)$

hence $P_{hs}+P_a+P_f=K_o+W_r\sin\emptyset-m(X-E_r)$

where $m = C_s \frac{W_r}{h}$

Value of components P_f, P_a and P_h. For a first approximation, the friction component becomes, $P_f=0.2W_r\cos\emptyset+p$ (estimated packing friction) and will be assumed constant. The recuperator reaction becomes,

$$P_a=P_{ai}+\frac{P_{af}-P_{ai}}{b}X \text{ for springs}$$

where P_{ai}=total initial spring reaction

P_{af}=total final spring reaction

$$P_a=P_{ai}\left(\frac{V_o}{V_o-A_vX}\right)^k \quad \text{where } k=1.1 \text{ to } 1.3$$

V_o=initial volume (cu.ft)

$$V_o=A_vb\left(\frac{m_o^{\frac{1}{k}}}{m_o^{\frac{1}{k}}-1}\right)=\frac{P_{ai}}{P_{ai}}\left(\frac{m_o^{\frac{1}{k}}}{m_o^{\frac{1}{k}}-1}\right)$$

A_v = effective area of recuperator piston (sq.in)

P_{ai}= initial air pressure (lbs/sq.in)

m_o = ratio of compression (from 1.5 to 2)

The hydraulic throttling reaction, becomes for constant recoil, $P_{hs}=(K+W_r\sin\emptyset)-P_a-P_f$

for variable recoil $P_{hs}=K_o+W_r\sin\emptyset-m(X-E_r)-P_a-P_f$ where the value of P_a corresponds to the displacement X.

Construction of static counter recoil diagram:

The counter recoil may be divided into and acceleration period, controlled or regulated by a throttling resistance through a constant orifice, and the retardation period where the recoiling mass is brought to rest into battery by a constant resistance to recoil, with a varying buffer throttling. If

P_a=the recuperator reaction

P_f=total friction of counter recoil assumed the same as for recoil and constant.

B_s'=static buffer reaction

l_o=length of constant orifice period (ft)

l_b=length of variable orifice period (ft)

Then during the acceleration,

$$P_a-P_f-W_r\sin\emptyset-B_s'=m_r v\,\frac{dv}{dx} \text{ where } B_s'=c_o'\,\frac{v_s^2}{w_o^2} \;(w_o=\text{ a constant})$$

and during the retardation

$$B_s' + W_r\sin\emptyset + P_f - P_a = -\,m_r\,v\,\frac{dv}{dx} \text{ where } B_s'=\frac{c_o' v_s^2}{w_x^2}$$

Now $\frac{c_o'}{w_o^2}$ may be determined by assuming a max. counter recoil velocity = 3.5 ft/sec. at max. velocity, we have,

$$P_a-P_f-W_r\sin\emptyset-\left(\frac{c_o'}{w_o^2}\right)v_{ms}^2 = 0 \quad \text{and assuming } v_{ms}^2,$$

we readily determine $\frac{c_o'}{w_o^2} = G$

The velocity and force curve during the first period may therefore be constructed as follows:

(1) Plot the recuperator reaction against counter recoil displacement, that is,

$$P_a = P_{ai}\left[\frac{V_o}{V_o - A_v(b-X)}\right]^k$$ where $k = 1.1$ to 1.3
b = length of recoil

(2) Assume $P_f = (0.2W_r\cos\emptyset)$ (estimated packing friction) Constant for the counter recoil.

(3) Divide the acceleration period into "n" intervals and take the mean air pressure for this interval. Then, knowing the velocity at the beginning of the interval, we can compute the velocity at the end of the interval by the formula, -

$$\log\left(A - \frac{c_o'v_2^2}{w_o^2}\right) = \log\left(A - \frac{c_o'v_1^2}{w_o^2}\right) - \frac{2C(x_2 - x_1)}{2.3m_r w_o^2}$$

where $A = P_a - P_f - W_r\sin\emptyset$

$\frac{c_o'}{w_o^2}$ = G and determined as outline above.

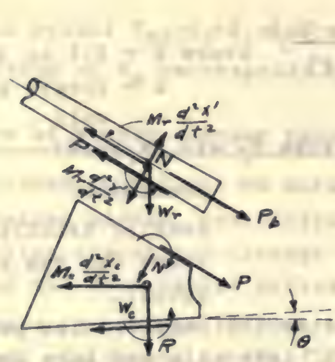
(4) Next construct from the velocity curve a static buffer against counter recoil displacement, that is

$$B_s' = \left(\frac{c_o'}{w_o^2}\right) v_n^2$$

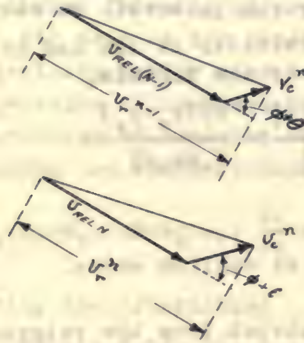
The velocity and force curve during the retardation period of counter recoil may be constructed, as follows:-

(1) The total resistance to counter recoil being assumed constant during this period, we have

$B_s' + W_r\sin\emptyset + P_f - P_a = K_v$ whence, $v = \sqrt{\frac{2K_v(b-x)}{m_r}}$



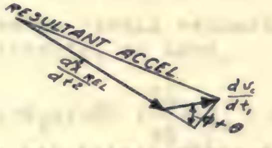
VELOCITY DIAGRAM FOR UPPER RECOILING PARTS: RECOIL



AT INTERVAL (N-1)

AT INTERVAL N

ACCELERATION DIAGRAM FOR UPPER RECOILING PARTS: RECOIL



$$\frac{d^2X'}{dt^2} = \frac{d^2X_{REL}}{dt^2} + \frac{dv_c}{dt}\cos(\phi+\theta)$$

$$\frac{d^2Y'}{dt^2} = \frac{dv_c}{dt}\sin(\phi+\theta)$$

Fig. 9

where $K_v = \dfrac{\frac{1}{2} m_r v_m^2}{l_b}$ and v_m is determined from the previous point by point method to the end of the displacement l_o. Then, the velocity and buffer force against recoil displacement is determined, since

$$B_s' = \frac{\frac{1}{2} m_r v_m^2}{l_b} + P_{ai}\left[\frac{V_o}{V_o - A_v(b-X)}\right]^k - P_f - W_r \sin\emptyset$$

and $v = \sqrt{\dfrac{2K_v(b-X)}{m_r}}$ where $K_v = \dfrac{\frac{1}{2} m_r v_m^2}{l_b}$

Dynamical equations of double recoil for point by point method of procedure for construction of reaction and velocity plots:

Let $\emptyset$ = min. angle of elevation of gun

P = total pull between upward and lower recoiling parts (lbs)

P_{hs}= static hydraulic pull (lbs)

P_a= recuperator reaction (lbs)

P_f= total friction assumed constant (lbs)

V_f=free velocity of recoil(ft/sec)

V_r=velocity of upper recoiling parts parallel to upper guides (ft/sec)

V_{rel}= relative velocity between upper and lower recoiling parts (ft/sec)

V_o= velocity of lower recoiling parts up plane (ft/sec)

X = displacement of top carriage up inclined plane (ft)

B_s' = static counter recoil buffer reaction (lbs)

R = lower recoil reaction parallel to inclined plane (lbs)

Then, during the powder pressure period,

$$P = P_{hs}\left(\frac{v_{rel}}{v_s}\right)^2 + P_a + P_f \qquad (1)$$

$$V_r^n = V_r^{n-1} \quad (V_f^n - V_f^{n-1}) - \frac{P - W_r \sin\emptyset}{m_r} \Delta t \qquad (2)$$

$$V_c^n = V_c^{n-1} + \frac{[P\cos(\emptyset+\theta) - W_c \sin\theta - R - W_r \cos\emptyset \sin(\emptyset+\theta)]\Delta t}{m_c + m_r \sin^2(\emptyset+\theta)} \qquad (3)$$

$$V_{rel} = V_r - V_c \cos(\emptyset+\theta) \qquad (4)$$

$$X_{rel}^n = \frac{X_{rel}^{n-1} + V_{rel}^n + V_{rel}^{n-1}}{2} \Delta t \qquad (5)$$

$$X_c^n = X_c^{n-1} + \frac{V_c^n + V_c^{n-1}}{2} \Delta t \qquad (6)$$

After the powder period,

$$V_r^n = V_r^{n-1} - \frac{P - W_r \sin\emptyset}{m_r} \Delta t \qquad (2')$$

After gun begins relative counter recoil,

$$P = P_a - B'\left(\frac{v_{rel}^2}{v_s^2}\right) - P_f \qquad (1')$$

In determining $P_{hs} v_s$ and P_a the relative displacement must be equal to the static displacement of the recoil, that is $X_{rel} = X_s$, from which we determine P_{hs} v_s and P_a.

240 M/M HOWITZER, GAS-ELECTRIC TYPE, DOUBLE RECOIL.

24° Elevation, R = 45,000 lbs.

Points	Intervals sec	Total Time Seconds	Total Braking Force	VELOCITY: Gun Parallel to Gun Slides	VELOCITY: Carriage 7° Plane	VELOCITY: Relative	DISPLACEMENT: Relative	DISPLACEMENT: Up Plane ft.
				During Powder Pressure Period				
1	.004		151700	15.93	.703	15.332	.0307	.0014
2	.006		142400	32.849	1.648	31.438	.1710	.0084
3	.01		140000	41.431	3.310	38.593	.5217	.0332
4	.012		138300	41.351	5.105	36.971	.9751	.0837
				After Powder Pressure Period				
5	.016		128900	37.351	7.225	31.161	1.5201	.1824
6	.02		117900	32.801	9.405	25.051	2.082	.3487
7	.02		108000	28.65	11.195	19.061	2.523	.5548
8	.02		96000	24.99	12.51	14.271	2.856	.7919
9	.02		86500	21.721	13.45	10.191	3.101	1.051
10	.02		77100	18.835	14.03	6.805	3.271	1.326
11	.02		67700	16.33	14.23	4.12	3.3804	1.6091
12	.02		61000	14.10	14.175	1.95	3.441	1.893
13	.02		56600	12.052	13.941	.092	3.461	2.174
14	.002		54900	11.854	13.911	.076	3.46	2.202
				Gun beginning to C'Recoil				
15	.01		46800	11.03	13.601	.64	3.457	2.339
16	.01		45430	10.233	13.263	1.137	3.449	2.474
17	.01		42300	9.501	12.86	1.519	3.436	2.605
18	.01		38790	8.841	12.397	1.779	3.419	2.731
19	.01		35680	8.243	11.868	1.937	3.40	2.852
20	.01		33550	7.689	11.297	1.981	3.38	2.968
21	.01		33070	7.144	10.717	2.056	3.36	3.078
22	.01		31600	6.63	10.107	2.03	3.34	3.182

240 M/M HOWITZER, GAS-ELECTRIC TYPE, DOUBLE RECOIL.

				(continued)				
23	.01		31740	6.113	9.5	2.037	3.32	3.28
24	.01		31500	5.601	8.889	2.019	3.3	3.372
25	.012		31680	4.982	8.159	2.018	3.276	3.474
26	.014		31350	4.269	7.299	1.981	3.248	3.582
27	.016		31590	3.446	6.323	1.974	3.216	3.690
28	.018		31360	2.530	5.215	1.94	3.181	3.794
29	.02		31490	1.506	3.99	1.914	3.142	3.886
30	.02		31520	.481	2.765	1.889	3.104	3.954
31	.01		31470	.031	2.153	1.878	3.085	3.979
32	.02		31350	1.05	1.063	1.962	3.047	4.011
33	.016		30000	1.82	.036	1.85	3.017	4.02

240 M/M HOWITZER, TRACTOR MOUNT, DOUBLE RECOIL.

0° Elevation, R = 80000

Points	Intervals Sec	Total Time Sec	Total Braking Force	VELOCITY Gun	Carriage 6 Plane		DISPLACEMENT Relative	Up Plane Ft
				During Powder Pressure Period				
1	.004		152000	15.760	.753	15.011	.0300	.0015
2	.004		144600	28.578	1.425	27.160	.1143	.0059
3	.004		143900	35.505	2.087	33.428	.2355	.0129
4	.004		140300	39.258	2.713	36.558	.3755	.0225
5	.004		137600	41.235	3.306	37.943	.5245	.0345
6	.004		134600	42.035	3.869	38.185	.6768	.0489
7	.008		133500	41.355	4.984	36.400	.9751	.0834
				After Powder Pressure Period				
8	.02		125800	36.225	7.303	28.965	1.6288	.2072

240 M/M HOWITZER, TRACTOR MOUNT, DOUBLE RECOIL.

(Continued)

9	.02		110800	31.705	8.808	22.945	2.1479	.3683
10	.02		100500	27.605	9.753	17.905	2.5565	.5539
11	.04		91300	20.145	10.621	9.575	2.1063	.9613
12	.02		74400	17.105	10.138	7.005	3.2722	1.1689
13	.02		69000	14.290	9.360	4.980	3.3920	1.3640
14	.02		64600	11.650	8.340	3.360	3.4750	1.5410
15	.02		61500	9.140	7.150	2.040	3.529	1.696
16	.02		58300	6.760	5.790	1.010	3.560	1.825
17	.023		56700	4.100	4.120	.0	3.577	1.939
				Gun beginning to C'Recoil				
18	.004		47700	3.710	3.730	.0	.1592	1.955
19	.004		47700	3.320	3.340	.0	.1592	1.969
20	.004		47700	2.930	2.950	-.005	.1592	1.982
21	.004		47700	2.540	2.560	-.010	.1592	1.993
22	.004		47700	2.150	2.170	-.010	.1592	2.003
23	.004		47700	1.760	1.780	-.010	.1592	2.010
24	.004		47700	1.370	1.390	-.010	.1592	2.016
25	.004		47700	.980	1.00	.015	.1592	2.021
26	.004		47700	.590	.610	.016	.1593	2.021
27	.004		47700	.200	.220	.019	.1592	2.026

UP Plane (in)	Kx_t	Ky_t	K_x	$P_h\left(\frac{V^2_{rel}}{V^2_s}\right)$
0	145000	45000	62000	118000
2	124000	32000	46000	96000
4	112000	26000	38000	82000
6	103500	21000	30000	70000
8	95000	17000	24000	58000
10	89000	13000	18000	49000
12	82000	10000	14000	40000
14	76000	7200	11000	32000
16	70000	5000	7700	24000
18	64200	2100	4000	18000
20	60000	1000	1000	13000
22	55000	- 2000	- 1500	8000
24	52000	- 3000	- 2500	5000
26	50000	- 6000	- 8500	2000

Gun beginning to Counter Recoil

	K_{x_1}	K_{y_1}	K_x	$B'(\frac{V_r^2}{V_s^2}-1)$
28	41000	- 8000	- 11000	0
30	38000	-10000	-14000	1370
36	27000	-14000	-21000	12600
42	25000	-16000	-22000	12700
46	24800	-16000	-22000	12800
48	24700	-15000	-21800	13000

Acceleration up plane.

In battery 60° Elevation.

$v_1 = .102 \quad v_2 = .207 \quad v_3 = .292 \quad v_4 = .345$

$a_1 = .102 \quad a_2 = .105 \quad a_3 = .085 \quad a_4 = .053$

$$\text{average} = .086 \qquad \text{acc.} = \frac{.086}{.002} = 43 \text{ 1/sec.}^2$$

Out of battery 60° Elevation

$v_1 = 6.550 \quad v_2 = 6.550 \quad v_3 = 6.350 \quad v_4 = 6.150$

$a_1 = .050 \quad a_2 = .150 \quad a_3 = .020 \text{av.} = .133$

$$\text{acc.} = \frac{.133}{.010} = 13.3 \text{ 1/sec.}^2$$

(Reversed)

In battery 30° Elevation

$v_1 = .318 \quad v_2 = .475 \quad v_3 = .600 \quad v_4 = .720$

$a_1 = .318 \quad a_2 = .157 \quad a_3 = .125 \quad a_4 = .120$

$$\text{average} = .180 \qquad \text{acc.} = \frac{180}{.002} = 90 \text{ 1/sec.}^2$$

Out of battery 30° Elevation

$v_1 = 8.630 \quad v_2 = 8.323 \quad v_3 = 7.970 \quad v_4 = 7.600$

$a_1 = .307 \quad a_2 = .353 \quad a_3 = .370$

$$\text{average} = .343 \qquad \text{acc.} = \frac{.343}{.010} = 34.\text{sec.}^2$$

(Reversed)

CALCULATION OF STABILITY
DOUBLE RECOIL (240 MM. TRACTOR)

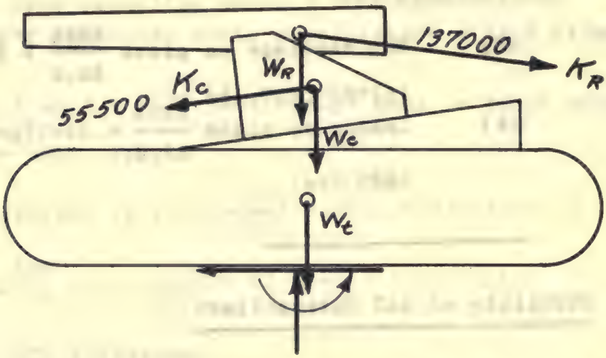


Fig. 10

Out of battery 30° Elevation

(1) Recoiling parts along bore

50000-15790×.5=42100.lbs.

(2) Recoiling parts up plane acceleration = 34. ft/sec.²

$$\frac{15790}{32.2} \times 34 = 16700 \text{ lbs. normal comp. } 16700 \times .91355 = 15200. \text{ lbs.}$$

(3) Top carriage up plane $\frac{5231}{32.2} \times 34 =$ 162×34=5510.lbs.

(4) Cradle up plane $\frac{5513}{32.2} \times 34 = 171 \times 34 =$ 5820 lbs.

Stability of 240 Caterpillar.

Moments taken at 0° Elevation, Howitzer out of battery, about a point under of rear track sprocket.

(1)	Weight of recoiling parts	15790×59.		931,610
	Weight of cradle	5231	115.	601,565
	Weight of top carriage	5513	80	441,040
	Weight of bottom carriage	5250	45.	236,250
	Weight of tractor	55000	128.	7,040,000
	Inertia of recoiling parts	58000	93	5,394,000
	Inertia of recoiling parts	6700	59	395,300
	Inertia of cradle	10820	86	930,520
	Inertia of top carriage	11580	71	822,180
				1,703,785

Inertia forces 60° Elevation in battery

(1) Along bore = 159000-15790 cos 30°(Recoiling parts) 15000-13765=145325 lbs.

(2) Up plane = acceleration = 43.ft/sec.²

$\frac{15790}{32.2} \times 43 = 21070$ lbs. normal comp. = $21070 \times .91355$

= 19250 lbs.

(3) Top carriage up plane $\frac{5231}{32.2} \times 123. = 162. \times 43. = 6966.$ lbs.

(4) Cradle up plane $\frac{5513}{32.2} \times 12.3 = 171 \times 43. = 7350$ lbs.

Out of battery 60° Elevation

(1) Along bore recoiling parts = 70000−13675=56325

(2) Up plane recoiling parts acceleration = 13.3 ft/sec.*

$\frac{15790}{32.2} \times 13.3 = 6517.$lbs. normal comp. = $6517 \times .91355$

= 5950. lbs.

(3) Top carriage up plane $\frac{5231}{32.2} \times 13.3 = 162 \times 13.3 =$

2155. lbs.

(4) Cradle up plane $\frac{5513}{32.2} \times 13.3 = 171 \times 2274.$lbs.

In battery 30° Elevation.

(1) Recoiling parts along bore 147000−15790×.5=139100 lbs.

(2) Recoiling parts up plane. Acceleration 90 ft/sec.

$\frac{15790}{32.2} \times 90 = 44200$ lbs. Normal comp. $44200 \times .91355$

= 40300 lbs.

(3) T.C. up plane $\frac{5231}{32.2} \times 90 = 162. \times 90 = 14600.$ lbs.

(4) Cradle up plane $\frac{5513}{32.2} \times 90 = 171 \times 90 = 15400$ lbs.

(2) About center line rearmost roller.(110in.from trunnions)

Weight of recoiling parts	15790.×	41.	647000.
Weight of cradle	5231.	97.	507000.
Weight of top carriage	5513.	62.	342000.
Weight of bottom carriage	5250.	27.	142000.
Weight of tractor	55000.	110.	6050000.

Inertia of recoiling parts	58000.	93.	-5394000.
Inertia of recoiling parts	6700.	59.	- 395000.
Inertia of cradle	10820.	86.	- 930000.
Inertia of top carriage	11580.	71.	- 822000.
			+ 147000

Direct Pads on Rollers.

0° In battery

Weight of recoiling parts	15700×.99452	+ 15703.
Weight of cradle	5231×.99452	+ 5202.
Weight of top carriage	5513×.99452	+ 5483.
Inertia of recoiling parts	17500×.99452	+ 17404.
Inertia of recoiling parts	140000×.10453	+ 14634.
		58426.

$$\text{Hydraulic resistance}\left\{\begin{array}{l}29967-28433+140000\times.99453\\17500\times.10452-26534.\times.10452\end{array}\right\}=$$

76200.

0° Out of battery.

Weight -	26534×.99452	+ 26389.
Inertia of recoiling parts	6700.×.99452	- 6663.
Inertia of recoiling parts	58000×.10453	+ 6063.
		25789.

$$\text{Hydraulic resistance}\left\{\begin{array}{l}+22400 \quad 58000\times.99453\\+6700\times.10452-26534.\times.10452\end{array}\right\}=$$

78330

60° In battery,

Weight 15790+5231+5513 .99452 + 26389.

Inertia of recoiling

parts (145324 ×.91355) + 132762.

Inertia of recoiling

parts (19250.×.40674) - 7838.

166981

Hydraulic resistance $\left\{\begin{array}{l}6966+7350+19250\times.91355 \\ 145325\times.40674+26534\times.10453\end{array}\right\}=$

24400 lbs.

60° Out of battery

Weight (15790+5231+5513).99452 + 26389

Inertia of recoiling parts(56325×

.91355)+ 51456

Inertia of recoiling

parts (5950.×.40674) - 2420.

75425

Hydraulic resistance $\left\{\begin{array}{l}2274+2155+56375\times.40674 \\ 5950\times.91355-26534\times.10453\end{array}\right\}=$

32800

0° In. 80000 Out =

30° In. Out =

60° In. Out =

Weight of bottom carriage 5250 lbs.

At 30° Elevation. Hydraulic resistance. In battery.

139000×.80902-40300×.58779-14600-15400-26594

×.10453

114000-23700-14600-15400-2780=57520

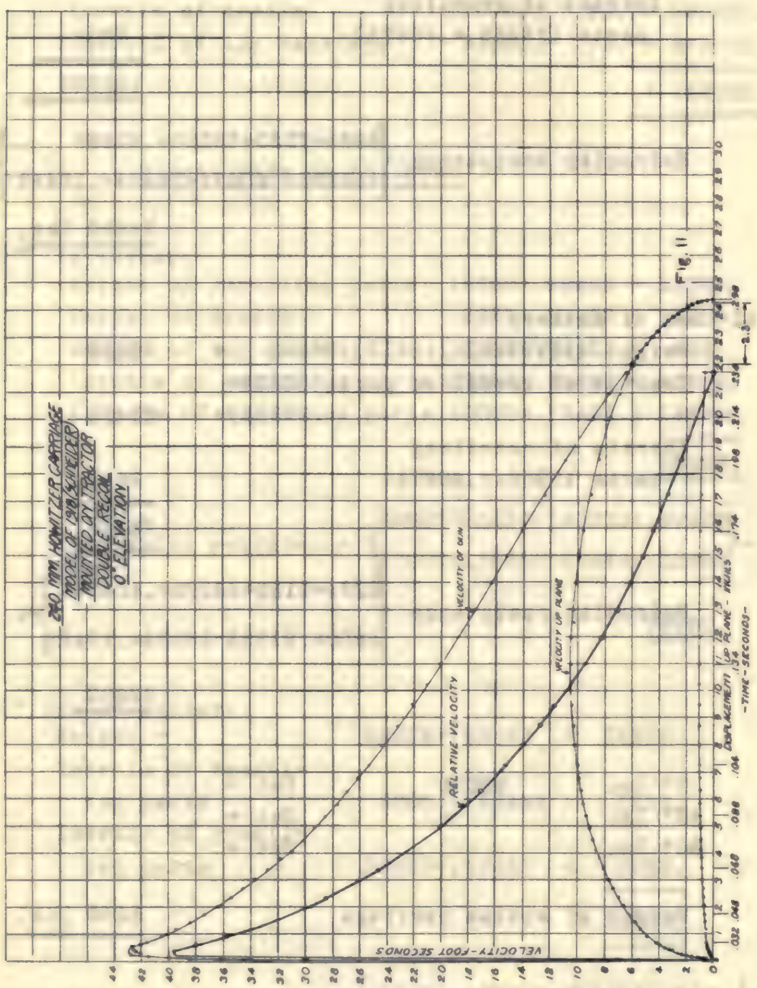

230 MM. HOWITZER CARRIAGE
MODEL OF 1918 (SCHNEIDER)
MOUNTED ON TRACTOR
DOUBLE RECOIL
0° ELEVATION
VELOCITY OF GUN
VELOCITY UP PLANE
RELATIVE VELOCITY
VELOCITY - FOOT SECONDS
DISPLACEMENT UP PLANE - INCHS
-TIME - SECONDS-

Fig. 11

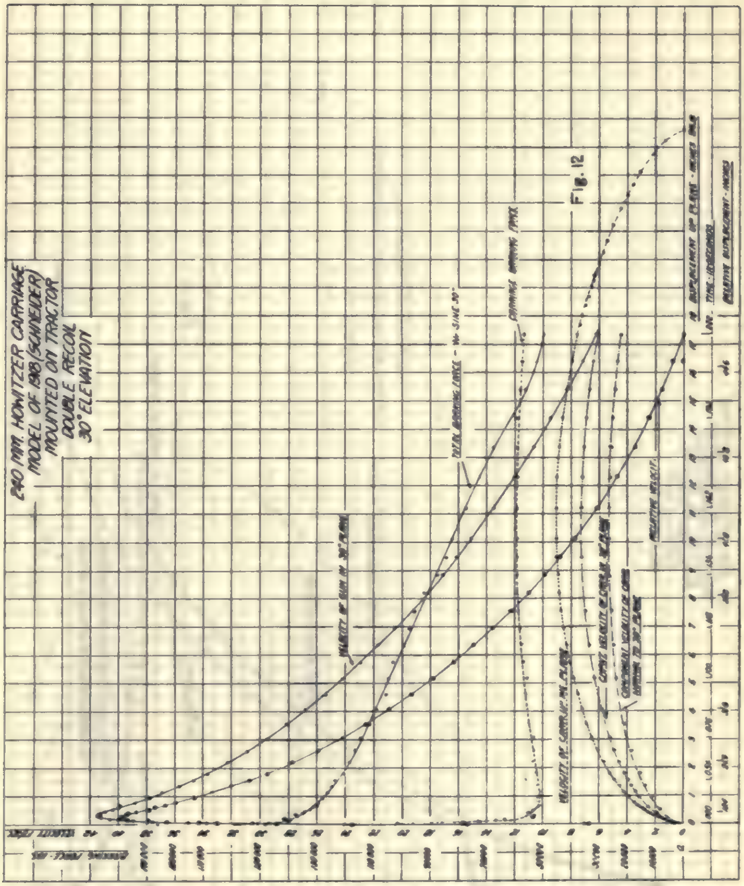
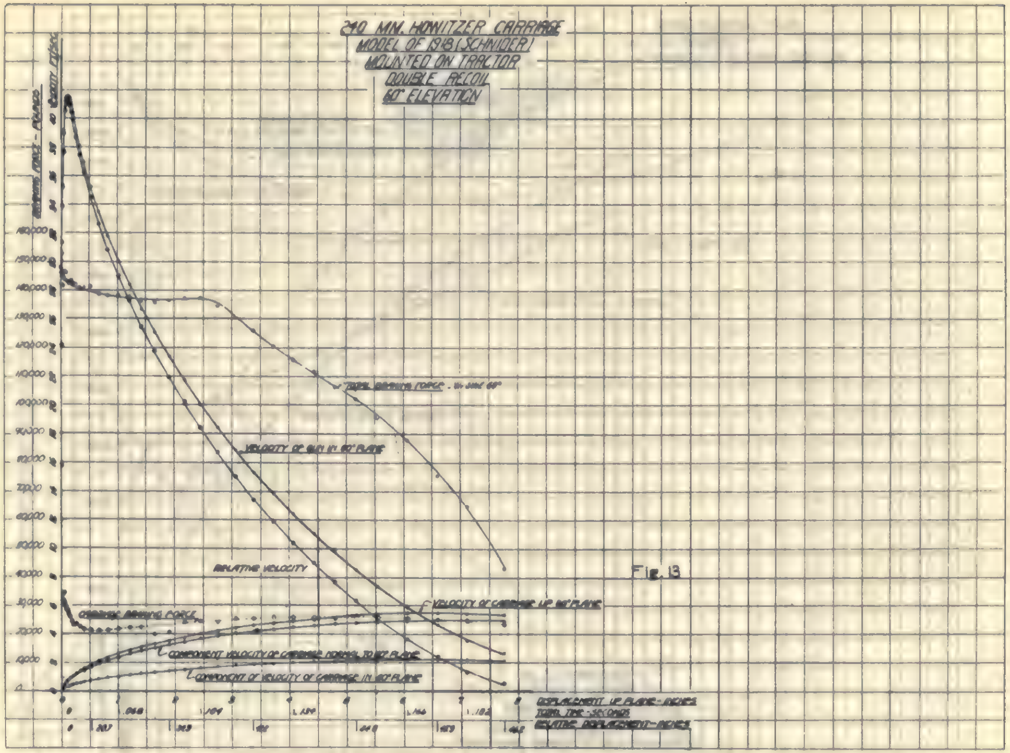

240 MM. HOWITZER CARRIAGE
MODEL OF 1918 (SCHNEIDER)
MOUNTED ON TRACTOR
DOUBLE RECOIL
30° ELEVATION
VELOCITY OF GUN IN 30° PLANE
TOTAL BRAKING FORCE - W SINE 30°
CARRIAGE BRAKING FORCE
RELATIVE VELOCITY

Fig. 12


240 MM. HOWITZER CARRIAGE
MODEL OF 1918 (SCHNIDER)
MOUNTED ON TRACTOR
DOUBLE RECOIL
60° ELEVATION
VELOCITY OF GUN IN 60° PLANE
RELATIVE VELOCITY
CARRIAGE BRAKING FORCE
VELOCITY OF CARRIAGE UP 60° PLANE
COMPONENT VELOCITY OF CARRIAGE NORMAL TO 60° PLANE
COMPONENT OF VELOCITY OF CARRIAGE IN 60° PLANE
DISPLACEMENT UP PLANE—INCHES
TOTAL TIME—SECONDS
RELATIVE DISPLACEMENT—INCHES
Fig. 13

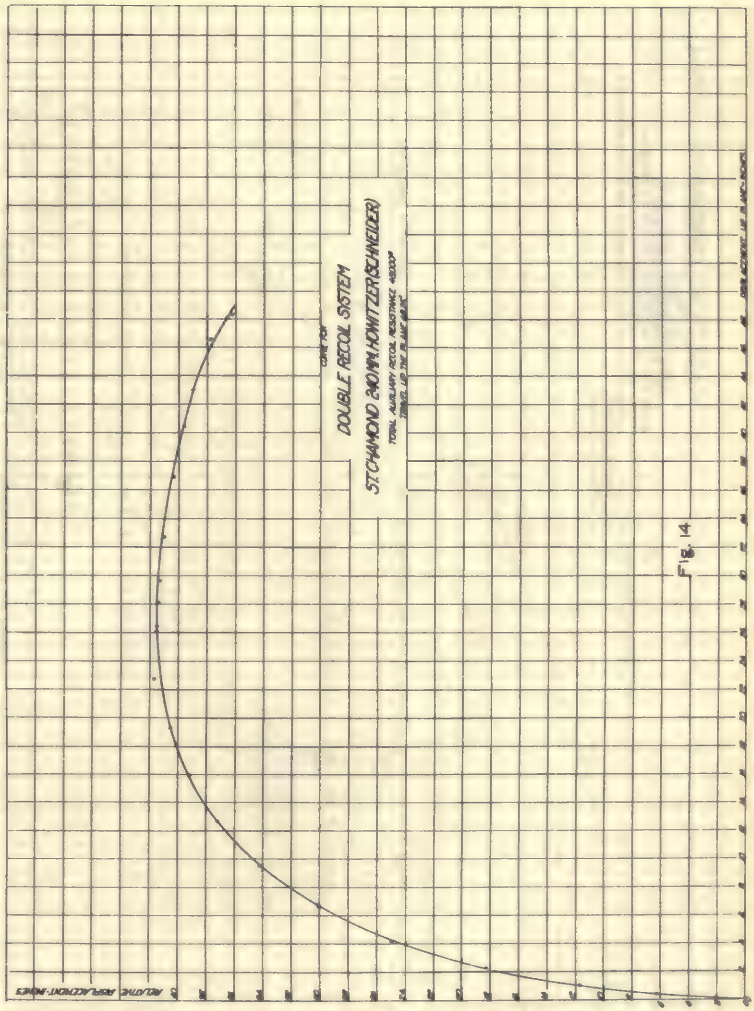

DOUBLE RECOIL SYSTEM
ST. CHAMOND 240 MM. HOWITZER (SCHNEIDER)
RELATIVE DISPLACEMENT-INCHES
DISPLACEMENT UP PLANE-INCHES

Fig. 14

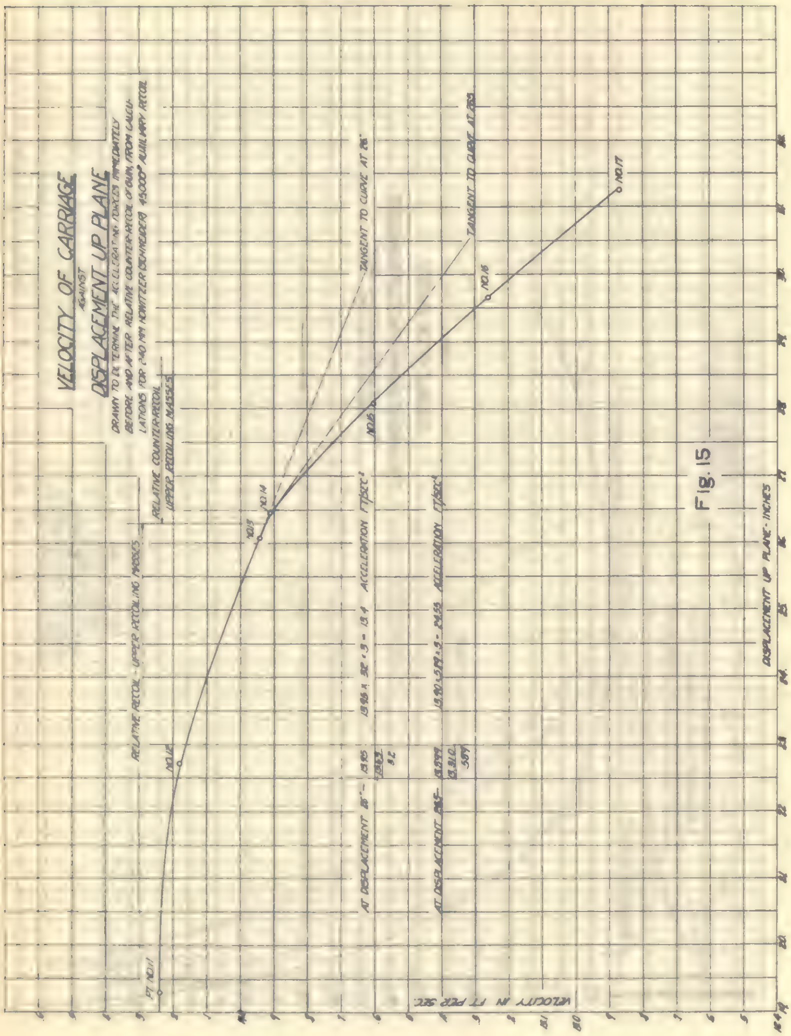

VELOCITY OF CARRIAGE
AGAINST
DISPLACEMENT UP PLANE
DRAWN TO DETERMINE THE ACCELERATING FORCES IMMEDIATELY BEFORE AND AFTER RELATIVE COUNTER-RECOIL OF GUN, FROM CALCULATIONS FOR 240 MM HOWITZER (SCHNEIDER) 45000# AUXILIARY RECOIL
RELATIVE RECOIL - UPPER RECOILING MASSES
RELATIVE COUNTER-RECOIL UPPER RECOILING MASSES
PT. NO.11
NO.12
NO.13
NO.14
NO.15
NO.16
NO.17
TANGENT TO CURVE AT 26"
TANGENT TO CURVE AT 28.5
AT DISPLACEMENT 26" - 13.95
13.63
3.2
13.95 x 32 ÷ 3 = 13.4 ACCELERATION FT/SEC²
AT DISPLACEMENT 28.5 - 13.999
13.910
5.89
13.90 x 5.89 ÷ 3 = 24.55 ACCELERATION FT/SEC²
VELOCITY IN FT PER SEC.
DISPLACEMENT UP PLANE - INCHES
19
20
21
22
23
24
25
26
27
28
29
30
31
32
13.1
13.0
12.4

Fig. 15

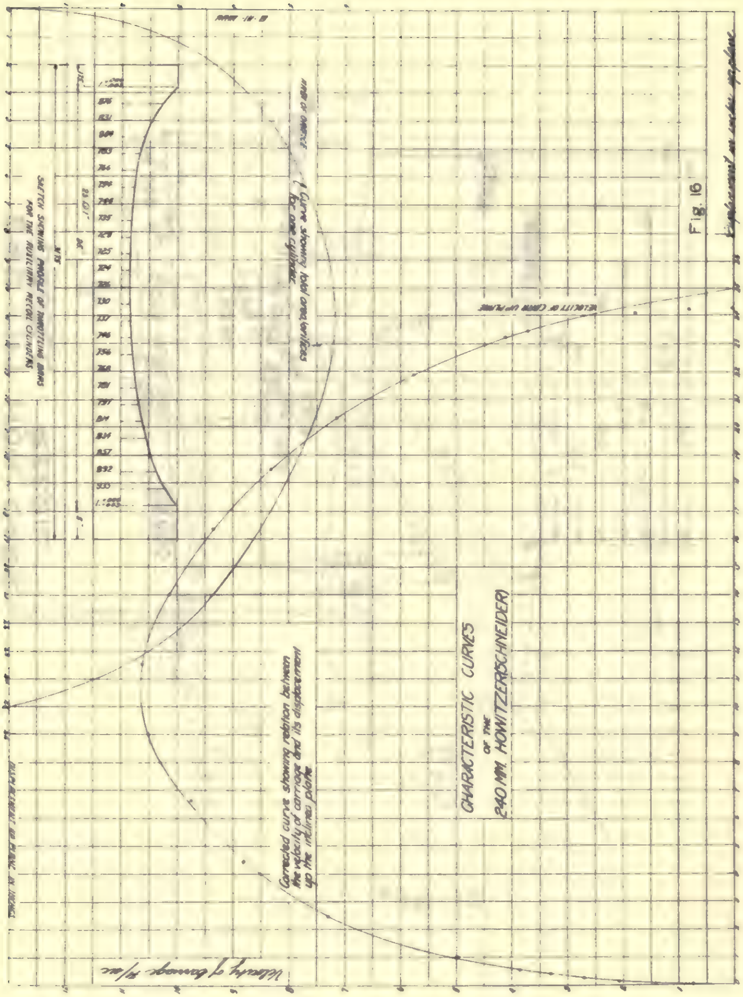

CHARACTERISTIC CURVES
OF THE
(240 MM. HOWITZER SCHNEIDER)
Corrected curve showing relation between the velocity of carriage and its displacement up the inclined plane
Curve showing total area orifices for one cylinder
SKETCH SHOWING PROFILE OF THROTTLING BARS FOR THE AUXILIARY RECOIL CYLINDERS
VELOCITY OF CARR UP PLANE
Velocity of carriage ft/sec

Fig. 16

240 MM HOW. CARR.
DOUBLE RECOIL

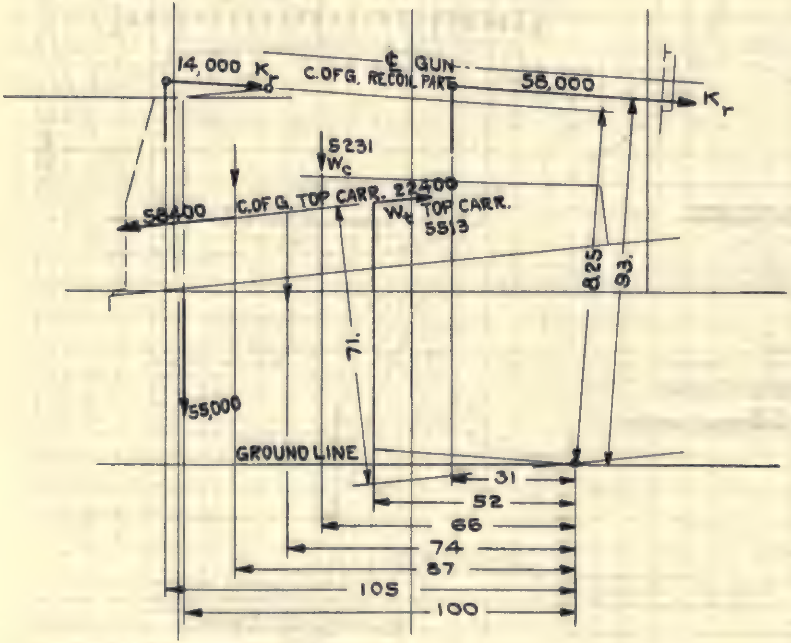


Fig. 17

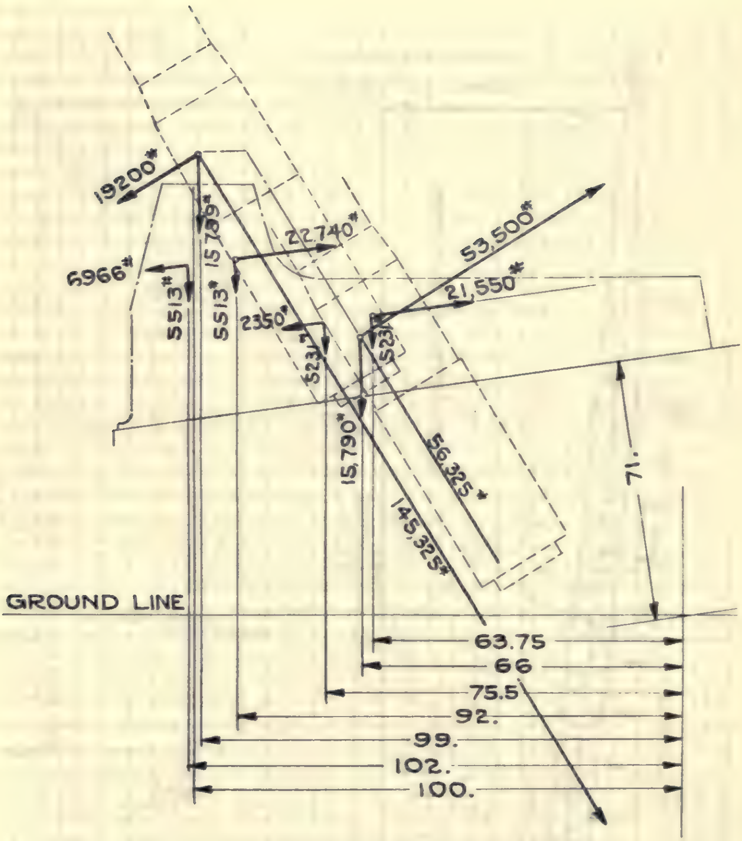

19200#
15789#
22740#
53,500#
21,550#
6966#
5513#
5513#
2350#
5231
5231
15,790#
56,325#
145,325#
71.
GROUND LINE
63.75
66
75.5
92.
99.
102.
100.

Fig. 18

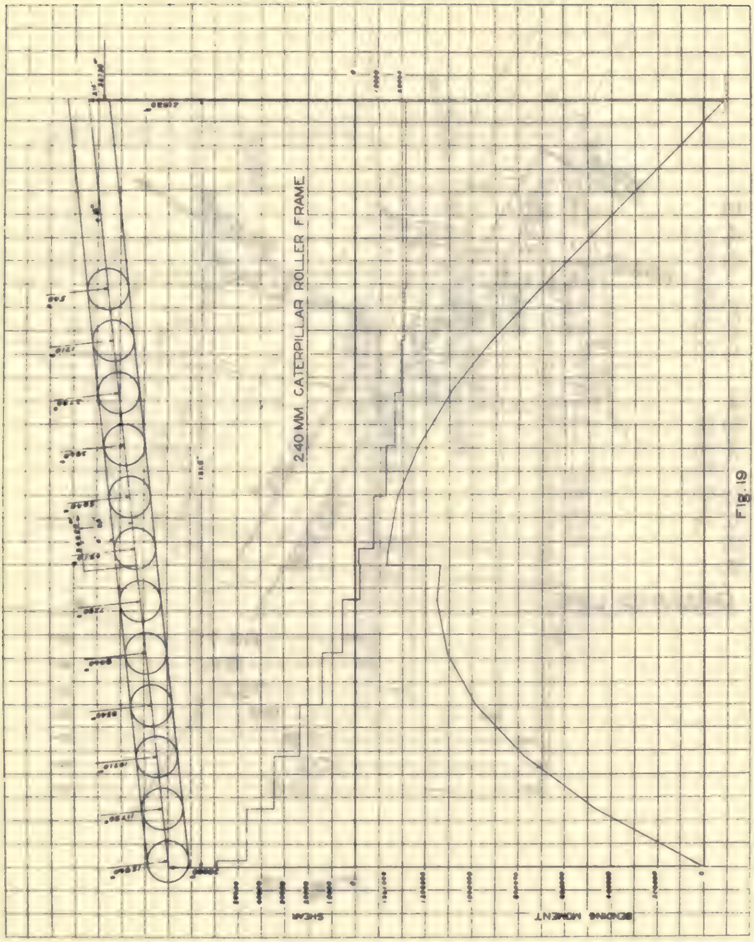
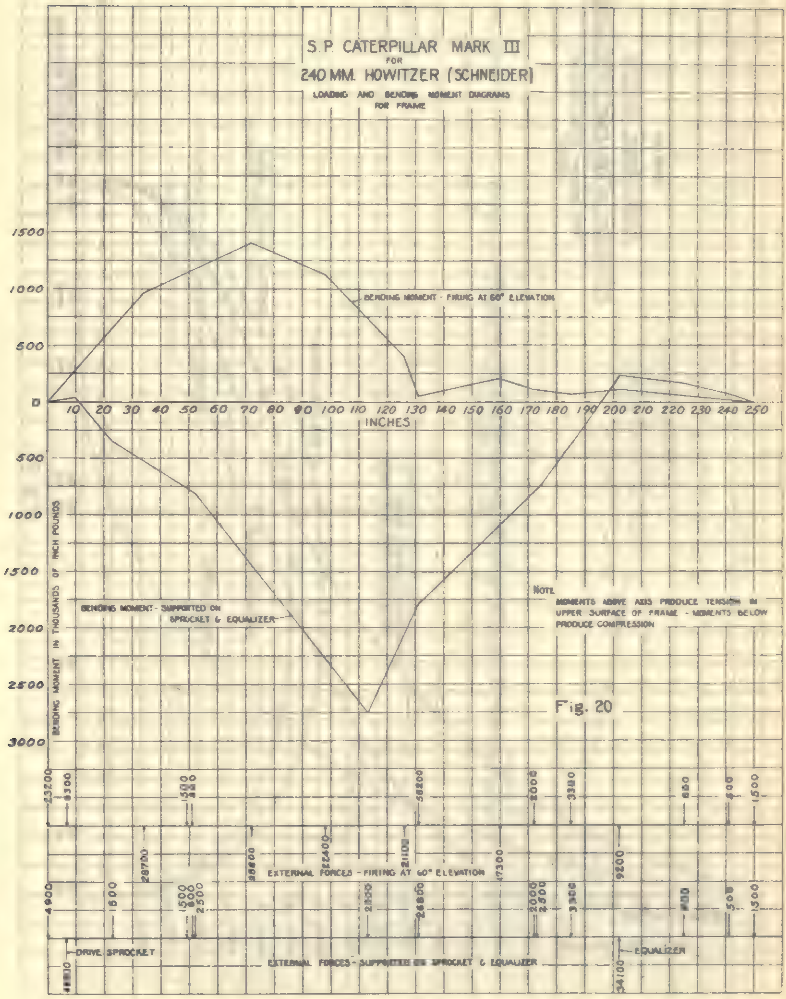


Fig. 19



S.P. CATERPILLAR MARK III
FOR
240 MM. HOWITZER (SCHNEIDER)
LOADING AND BENDING MOMENT DIAGRAMS
FOR FRAME
BENDING MOMENT - FIRING AT 60° ELEVATION
BENDING MOMENT - SUPPORTED ON SPROCKET & EQUALIZER
INCHES
BENDING MOMENT IN THOUSANDS OF INCH POUNDS
NOTE
MOMENTS ABOVE AXIS PRODUCE TENSION IN UPPER SURFACE OF FRAME - MOMENTS BELOW PRODUCE COMPRESSION
EXTERNAL FORCES - FIRING AT 60° ELEVATION
EXTERNAL FORCES - SUPPORTED ON SPROCKET & EQUALIZER
DRIVE SPROCKET
EQUALIZER

Fig. 20

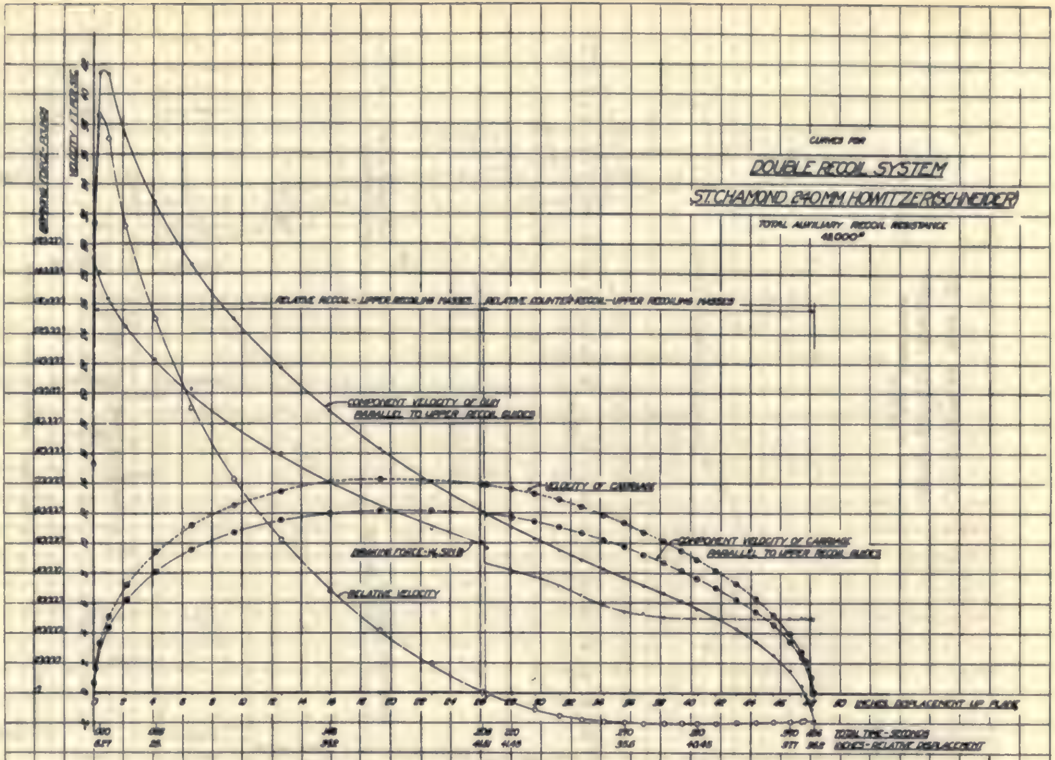

CURVES FOR
DOUBLE RECOIL SYSTEM
ST. CHAMOND 240 MM HOWITZER (SCHNEIDER)
TOTAL AUXILIARY RECOIL RESISTANCE
RELATIVE RECOIL - UPPER RECOILING MASSES
RELATIVE COUNTER-RECOIL - UPPER RECOILING MASSES
COMPONENT VELOCITY OF GUN PARALLEL TO UPPER RECOIL GUIDES
VELOCITY OF CARRIAGE
COMPONENT VELOCITY OF CARRIAGE PARALLEL TO UPPER RECOIL GUIDES
BRAKING FORCE
RELATIVE VELOCITY
VELOCITY - FT. PER SEC.
INCHES DISPLACEMENT UP PLANE
TOTAL TIME - SECONDS
INCHES - RELATIVE DISPLACEMENT

Fig. 21

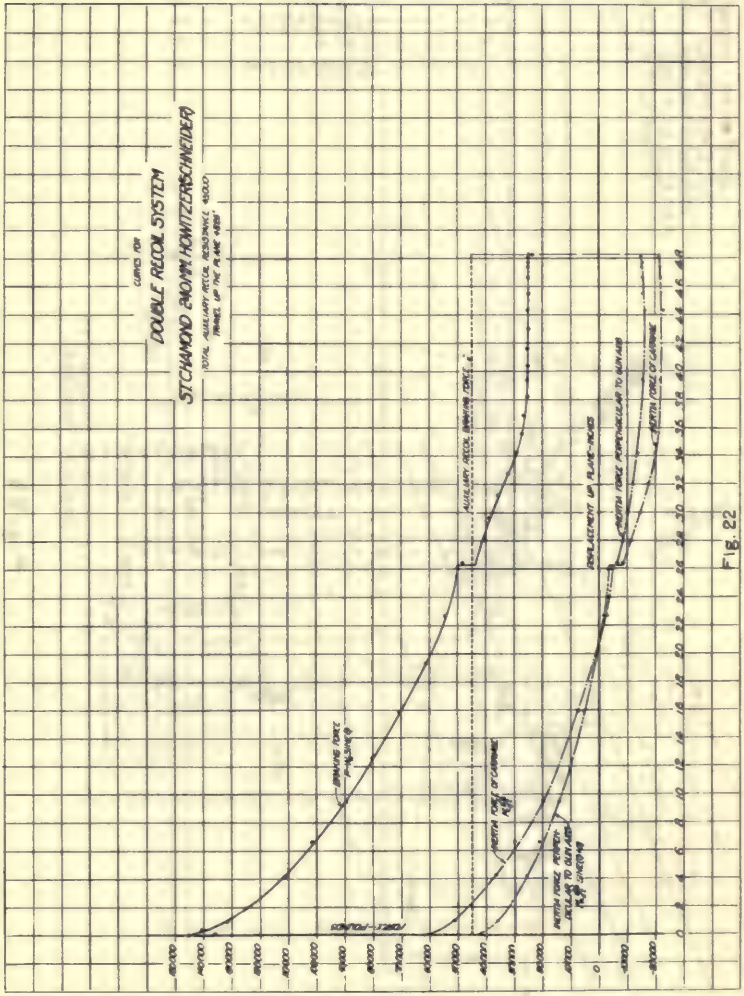

CURVES FOR
DOUBLE RECOIL SYSTEM
ST. CHAMOND 240MM. HOWITZER (SCHNEIDER)
AUXILIARY RECOIL BRAKING FORCE
BRAKING FORCE
INERTIA FORCE OF CARRIAGE
INERTIA FORCE PERPENDICULAR TO GUN AXIS
DISPLACEMENT UP PLANE—INCHES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48

Fig. 22

240 MM HOWITZER
ST. CHAMOND TRACTOR
45,000# AUXILIARY RECOIL
24° ELEVATION 7° PLANE
STABILITY DIAGRAMS

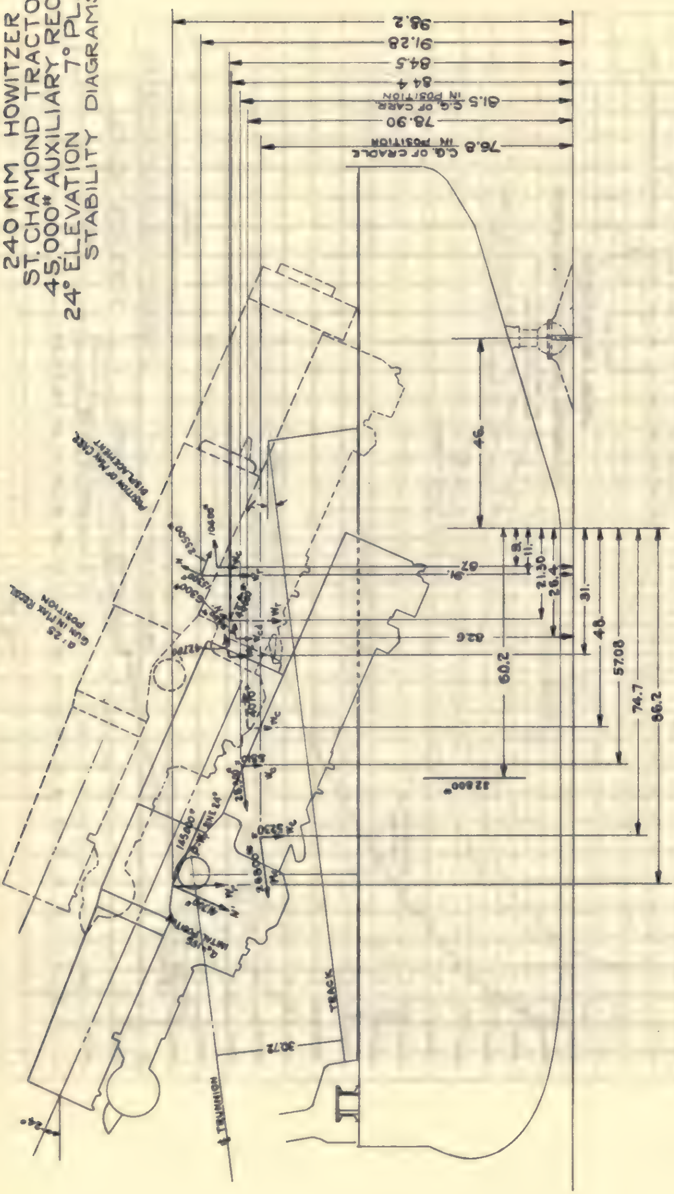


Fig. 23

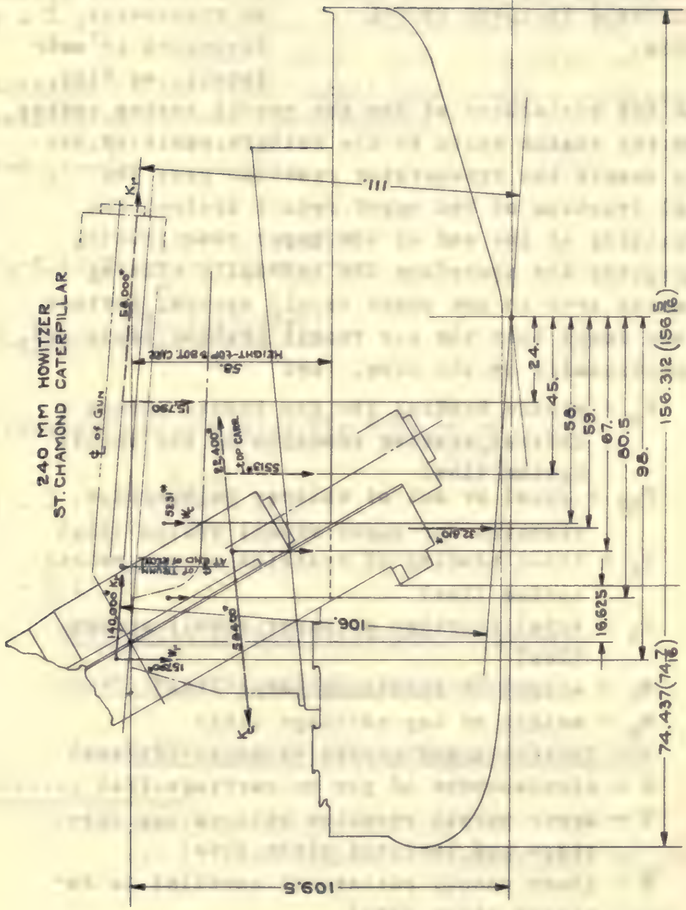

240 MM HOWITZER
ST. CHAMOND CATERPILLAR
℄ of Gun
HEIGHT-TOP BOT. CARR
TOP CARR
AT END of RECOIL
109.5
111.
106.
58.
24.
45.
56.
59.
67.
80.5
98.
16.625
156.312 (156 5/16)
74.437 (74 7/16)

Fig. 24

Hydraulic resistance out of battery

$42100\times.80902+15200\times.58779+5510+5820-2780$

$34100+\ 8940\ \ 5510+\ \ 5820-2780=51590.$ lbs.

THEORY FOR VARIABLE RESISTANCE IN UPPER RECOIL AND CONSTANT RESISTANCE IN LOWER RECOIL SYSTEM.

From the point by point method as previously discussed in some detail, we find, that the resistance of the gun recoil system varies from its static value in the battery position, to very nearly the recuperator reaction plus the total friction of the upper recoil system, the throttling at the end of the upper recoil being negligible and therefore the hydraulic braking becoming zero in the upper recoil system; further it was found that the gun recoil braking falls off proportionally on the time. Let

P_s = static braking for gun recoil system = initial braking reaction on gun recoil system (lbs)

P_{af} = final or out of battery recuperator reaction for upper recoil system (lbs)

P_f = final braking of reaction on gun recoil system (lbs)

R_t = total friction of upper recoil system (lbs)

W_r = weight of recoiling parts (lbs)

W_c = weight of top carriage (lbs)

V = initial upper recoil velocity (ft/sec)

Z = displacement of gun on carriage (ft)

N = upper normal reaction between top carriage and inclined plane (lbs)

R = lower recoil resistance parallel to inclined plane (lbs)

X = total run upon inclined plane (ft)

v_t = velocity of combined recoil Rel. vels.=0

t' = prime P_o common recoil

The mean braking zone for the upper recoiling parts, becomes,

$$P_o = \frac{P_s+P_{af}+R_t}{2}$$

Further the distance run up the inclined plane during the time t_1' was found to be approximately $X = \frac{3}{4} v_1 t'$. The approximate equations for the double recoil, with a variable resistance in the upper recoil system and a constant resistance for the lower, become,

$$V=0.g\left(\frac{W\ V_o\ +\ \text{to}\ 4700}{W_r}\right) \qquad (1)$$

$$P_o-W_r\sin\emptyset= \frac{W_r}{g}\left[\frac{V-v_1\cos(\emptyset+\theta)}{t}\right] \qquad (2)$$

$$N-W_r\cos\emptyset = \frac{W_r}{g}\,\frac{v_1\sin(\emptyset+\theta)}{t} \qquad (3)$$

$$P.\cos(\emptyset+\theta)-N\sin(\emptyset+\theta)-W_c\sin\theta-R= \frac{W_c}{g}\,\frac{v_1}{t'} \qquad (4)$$

$$X = \frac{3}{4} v_1 t' + \frac{W_r+W_c}{2Rg} v_1^2 \qquad (5)$$

$$Z = \frac{V}{2} t'$$

DERIVATION OF THE DYNAMICAL EQUATIONS

POINT BY POINT METHOD COMPUTATION:

Total pull between upper and lower recoiling parts:

This reaction is composed of:-

(1) the hydraulic braking pull $=P_{hs}\left(\frac{v_{rel}}{v_s}\right)^2$ (lbs)

(2) the recuperator reaction at the relative displacement under consideration -- P_a (lbs)

(3) the friction between the recoiling parts -- P_f (lbs)

Hence $P=P_{bs}\dfrac{v_{rel}^2}{v_s^2}+P_a+P_f$ (lbs)

REACTIONS ON THE UPPER RECOILING PARTS:

If P_b= the powder reaction, then for the gun along its axis, we have,

$$P_b-m_r\frac{d^2x'}{dt^2}-P+W_r\sin\emptyset=0 \qquad (1)$$

and normal to its axis

$$N-m_r\frac{d^2y'}{dt^2}-W_r\cos\emptyset=0 \qquad (2)$$

Integrating equation (1), we have,

$$\int_{t_{n-1}}^{t_m}\frac{P_b dt}{m_r}-\left(\frac{P-W_r\sin\emptyset}{m_r}\right)\Delta t=\Delta v_r$$

$$V_f^n-V_f^{n-1}-\left(\frac{P-W_r\sin\emptyset}{m_r}\right)\Delta t=v_r^n-v_r^{n-1}$$

hence

$$V_r^n=V_r^{n-1}+(V_f^n-V_f^{n-1})-\left(\frac{P-W_r\sin\emptyset)}{m_r}\right)\Delta t \quad (1a)$$

From a somewhat different point of view, we have from (1)

$$P_b-m_r\frac{d^2x'}{dt^2}-P+W_r\sin\emptyset=0 \quad \text{since } \frac{d^2x'}{dt^2}=\frac{d^2x_{rel}}{dt^2}$$

$$+\frac{dv_c}{dt}\cos(\emptyset+\theta) \quad \text{See acceleration diagram,}$$

then $P_b-m_r\left[\dfrac{d^2x_{rel}}{dt^2}+\dfrac{dv_c}{dt}\cos(\emptyset+\theta)\right]-P+W_r\sin\emptyset+\theta$

Integrating, we have

$$\int\frac{P_b dt}{m_r}-\left(\frac{P-W_r\sin\emptyset}{m_r}\right)t=V_{rel}+v_c\cos(\emptyset+\theta)=v_r \text{ hence, as}$$

before,

$$V^n=V^{n-1}+(V^n-V^{n-1})-\left(\frac{P-W_r\sin\emptyset}{m_r}\right)\Delta t$$

From the vector disgram of acceleration,

$$\frac{d^2v'}{dt^2} = \frac{dv_c}{dt}\sin(\emptyset+\theta) \quad \text{hence equation (2) becomes,}$$

$$N - m_r\frac{dv_c}{dt}\sin(\emptyset+\theta) - W_r\cos\emptyset = 0 \qquad (2a)$$

REACTIONS ON THE LOWER RECOILING PARTS

These reactions are N and P reversed (the mutual couple having no effect on the translation) of the upper recoiling parts, the braking reaction R of the lower recoil brake and the weight and kinetic reactions of the top carriage.

The normal reaction and couple exerted by the plane has no effect on the motion of the system, then, along the inclined plane,

$$P\cos(\emptyset+\theta) - N\sin(\emptyset+\theta) - W_c\sin\theta - R - m_o\frac{dv_c}{dt} = 0$$

Substituting $N = m_r\dfrac{dv_c}{dt}\sin(\emptyset+\theta) + W_r\cos\emptyset$

we have

$$P\cos(\emptyset+\theta) - m_r\frac{dv_c}{dt}\sin^2(\emptyset+\theta) - W_c\sin\theta - W_r\cos\emptyset\sin(\emptyset+\theta)$$

$$-R - m_c\frac{dv_c}{dt} = 0, \text{ combining terms and simplifying, we have}$$

$$[m_r\sin^2(\emptyset+\theta)+m_c]\frac{dv_c}{dt} = P\cos(\emptyset+\theta) - W_c\sin\emptyset - W_r\cos\emptyset+\theta - R$$

hence

$$\frac{dv_c}{dt} = \frac{P\cos(\emptyset+\theta) - W_c\sin\theta - W_r\sin(\emptyset+\theta)\cos\emptyset - R}{m_r\sin^2(\emptyset+\theta)+m_c}$$

and between any two intervals,

$$v_c^n = v_c^{n-1} + \left[\frac{P\cos(\emptyset+\theta) - W_c\sin\theta - W_r\cos\emptyset\sin(\emptyset+\theta) - R}{m_r\sin^2(\emptyset+\theta) + m_c}\right]\Delta t$$

GEOMETRICAL RELATIONS.

To compute P it is necessary to compute the relative velocity and displacement respectively for any given interval in the recoil. Obviously from a velocity diagram

$v_{rel}=v_r-v_c\cos(\emptyset+\theta)$ and the relative displacement

$v^n_{rel}-X^{n-1}_{rel} + \dfrac{v^n_{rel}+v^{n-1}_{rel}}{2}\,\Delta t$ and the displacement

up the inclined plane $x^n_c = x^{n-1}_c + \dfrac{v^n_c+v^{n-1}_c}{2}\,\Delta t$

METHOD OF COMPUTATION.

Knowing v^{n-1}_c , v^{n-1}_r and v^{n-1}_{rel} at the beginning of the interval, we have,

$$P=P_{hs}\left(\frac{v^{n-1}_{rel}}{v_s}\right)^2 +P_a+P_f \text{ at relative displacement } x^{n-1}_{rel}$$

then $$v^n_c=v^{n-1}_c +\left[\frac{P\cos(\emptyset+\theta)-W_c\sin\theta-R-W_r\cos\emptyset\sin(\emptyset+\theta)}{m_c+m_r\sin^2(\emptyset+\theta)}\right]\Delta t$$

and
$$v^n_r=v^{n-1}_r +(V^n_f-V^{n-1}_f)-\left(\frac{P-W_r\sin\emptyset}{m_r}\right)\Delta t$$

From these values, we have $v^n_{rel}=v^n_r-v^n_c\cos(\emptyset+\theta)$ and therefore

$$x^n_{rel} = x^{n-1}_{rel} + \frac{v^n_{rel}+v^{n-1}_{rel}}{2}\,\Delta t$$

and
$$x^n_c=x^{n-1}_c + \frac{v_c+v^{n-1}_c}{2}\,\Delta t$$

After the powder period, obviously the expression for v_r reduces to,

$$v^n_r=v^{n-1}_r - \left(\frac{P-W_r\sin\emptyset}{m_r}\right)\Delta t$$

RELATIVE COUNTER RECOIL OF THE UPPER RECOILING PARTS:

In the expression for P, the hydraulic reaction and friction reverses. If B' is the c'recoil buffer force in the upper recoil system at a given relative displacement, then

$P_h = -B'\left(\frac{v_{rel}^2}{v_s^2}\right)$; $P_f = -P_f$ assuming friction the same.

hence,

$P = P_a - B'\left(\frac{v_{rel}^2}{v_s^2}\right) - P_f$ (lbs) The remaining expressions are the same as before.

This method of computation is sufficiently accurate and was followed in the recoil calculations illustrated.

APPROXIMATE CALCULATIONS FOR STABILITY WITH A DOUBLE RECOIL.

Reactions and velocity for double recoil system:

- P = resistance of gun recoil system
- W_r = weight of recoiling parts (upper)
- W_c = weight of top carriage and cradle (lower)
- V = initial velocity
- z = displacement of gun on carriage
- R = reaction of lower recoil system
- N = upper normal reaction between recoiling parts and top carriage
- M = lower normal reaction between top carriage and tractor.
- X = total run up on inclined plane.
- v = velocity of combined recoil
- t = corresponding time.
- $\emptyset$ = angle of elevation of gun
- θ = inclination of plane.

Values assumed for computation of recoil:

P =

R =

$\theta = 6°$

W_r = 15,790 lbs.

W_c = 11,570 lbs.

$\emptyset$ =

General equations for double recoil:

$$P - W_r \sin\emptyset = \frac{W_r}{g}\left[\frac{V - v\cos(\emptyset+\theta)}{t}\right] \quad (1)$$

$$N-W_r\cos\emptyset = \frac{W_r}{g}\,\frac{v\sin(\theta+\emptyset)}{t} \qquad (2)$$

$$P\cos(\emptyset+\theta)-N\sin(\emptyset+\theta)-W_c\sin\theta-R = \frac{W_c}{g}\,\frac{v}{t} \qquad (3)$$

$$[N\cos(\emptyset+\theta)+W_c\cos\theta+P\sin(\emptyset+\theta)-M = 0] \qquad (4)$$

$$X = \frac{v}{2}\,t + \frac{W_r+W_c}{2Rg}\,v^2 \qquad (5)$$

$$Pz + \frac{1}{2}\left(\frac{W_r+W_c}{g}\right)v^2 + R\,\frac{v}{2}\,t = \frac{1}{2}\,m_rV^2 \qquad (6)$$

$$V = 0.9\left(\frac{wxv_m+4700\bar{w}}{W_r}\right) \qquad (7)$$

$$z = \frac{v}{2}\,t \qquad (8)$$

Energy equation:

$PX_o = \frac{1}{2}m_r[v^2-v^2\cos^2(\emptyset+\theta)]$ Indication of P

$Nx\sin(\emptyset+\theta) = \frac{1}{2}\,m_rv^2(\emptyset+\theta)$

$[P\cos(\emptyset+\theta)-N\sin(\emptyset+\theta)-R]x = \frac{1}{2}M_cv^2$

$O[X_c-X\cos(\emptyset+\theta)] + \frac{1}{2}\,m_rv^2\sin^2(\emptyset+\theta)+R\,x = \frac{1}{2}m_r$

$[V^2-v^2\cos^2(\emptyset+\theta)] - \frac{1}{2}\,m_cv^2$

hence $Pz + \frac{1}{2}m_rv^2 + \frac{1}{2}M_cv^2 + \frac{Rvt}{2} - \frac{1}{2}\,m_rV^2$

Further

$$V = 0.9\left(\frac{wxv_m+4700\times\bar{w}}{W_r}\right)$$

where w = weight of shot.

$\bar{w}$ = weight of powder

W_r = weight of recoiling parts

v_m = muzzle velocity of shot

240 M/M DOUBLE RECOIL MOUNTED ON MARK III MI CATERPILLAR.

APPROXIMATE DOUBLE RECOIL CALCULATIONS FOR 240 M/M SCHNEIDER HOWITZER AT 0° ELEVATION OF HOWITZER.

Given: W_r = 15790 lbs. — weight of recoiling parts

W_c = 11570 lbs.— weight of sliding carriage.

$\theta = 6°$ — angle of inclined plane.

$V = 45$ ft/m — max. velocity of upper recoiling parts at beginning of recoil.

$R = 80000$ lbs. — resistance to recoil on lower recoil system.

<u>From static force,</u>

Diagram 240 M/M Howitzer,

$P_s = 155000$ max.pull

$R_t + P_{af} = 60{,}000$ maximum recuperator reaction plus friction at end of recoil.

<u>Approximate Calculations,</u>

$$P_o = \frac{P_s + P_{af} + R_t}{2} \quad \text{whence } P_s = 155{,}000 \text{ (lbs)}$$

$$P_{af} + R_t = 60{,}000 \text{ (lbs)}$$

$$215{,}000$$

hence $P_c = 107{,}500$ lbs.mean reaction

$$X = \frac{3}{4}(10.4)(0.190) + \frac{1}{2}\left(\frac{27360}{32.2}\right)\frac{\overline{10.4}^2}{80{,}000}$$

$$= 1.480 + 0.58 = \underline{2.06 \text{ ft.}}$$

$Z = 22.5 \times 0.158 = 3.56$ ft. $= \underline{42.7 \text{ in.}}$

<u>Check on Z by energy method:</u>

$$107{,}500\,Z = \frac{1}{2}\left(\frac{27360}{32.2}\right)\overline{10.4}^2 = \frac{80{,}000 \times 10.4}{2} \times 0.105$$

$$= \frac{1}{2}\,\frac{15790}{32.2}\,\overline{45}^2$$

$107{,}500Z + 46{,}000 + 68{,}200 = 497{,}000$

$Z = 3.56$ ft. <u>Check</u>

$R = 80{,}000$ lbs. For horizontal recoil, $-\emptyset = 0$

$\theta = 6°$ $\sin\theta = 0.1045$ $\cos\theta = 0.9945$

$$107{,}500 = \frac{15790}{32.2}\left(\frac{45 - 0.9945\,v}{t}\right) \qquad (1)$$

$$N-15790 - \frac{15790}{32.2} \times 0.1045 \frac{v}{t} \qquad (2)$$

$$0.9945\times107,500-N\times0.1045-11570\times0.1045-80,000 = \frac{11570}{32.2}\ \frac{v}{t}$$

$$(3)$$

$$107,000-0.1045N-1210-80,000=359 \frac{v}{t}$$

$$\frac{v}{t} = \frac{25,790-0.1045N}{359} = \frac{N-15,790}{51.2} \text{ hence } 7.11N=136,290$$

$$\underline{N = 19,170 \text{ lbs.}}$$

$$t = \frac{51.2\ v}{N-15790} = \frac{490(45-0.9945v)}{107,500} = \frac{51.2\ v}{3,380} = 0.21 -.00453v$$

$51.2v=692-15.3v$

$66.5v=692$ hence $\underline{v = 10.4 \text{ ft/sec.}}$

$$t = \frac{51.2\times10.f}{3380} = \underline{.1575 \text{ sec.}}$$

Total time = T+t=.032.158=.190 sec.

240 M/M DOUBLE RECOIL MOUNTED ON MARK IV MI CATERPILLAR.

APPROXIMATE CALCULATIONS FOR 240 M/M GAS-ELECTRIC DOUBLE RECOIL SYSTEM AT 24° ELEVATION OF HOWITZER.

Given: W_r = 15790 lbs. — weight of recoiling parts.

W_c = 11570 lbs.— weight of sliding carriage.

$\theta = 7°$ — angle of inclined plane.

V = 45 ft/m. — max. velocity of upper recoiling parts at beginning of recoil.

R = 45,000 lbs. — resistance to recoil on lower recoil system.

From static force,

Diagram 240 M/M Howitzer

P_s = 155,000 max.pull

$R_t + P_{af}$ = 60,000 maximum recuperator reaction plus friction at end of recoil.

Approximate calculations,

$$P_o = \frac{P_s+P_{af}+R}{2} \text{ whence } P_s = 155,000 \text{ lbs.}$$

$$P_{aft}R = \frac{60,000}{215,000}$$

107,500 lbs.

R = 45,000 lbs., Ø = 24° Elevation of gun, θ= 7°—angle of inclined plane

Ø+θ= 31° sin(Ø+θ)=.5150 cos(Ø+θ)=.3572

sin θ = .1219 $W_r$sinØ=15790×.515=6140 lbs.

$$107,500=6140=490\left(\frac{45-0.8572v}{t}\right) \quad (1)$$

$$N-15,790\times.8572=490\times\frac{.5150\ v}{t} \quad (2)$$

$$107,500\times.8572-N.515-11,570\times.1219-45,000-359\ \frac{v}{t} \quad (3)$$

$$359\ \frac{v}{t} = 93,200-0.515N-1410-45,000$$

$$\frac{v}{t} = \frac{46,800-0.515\ N}{359} = \frac{N-13520}{252}$$

46,800-0.515 N=1.425N-19250

N = 34,000 lbs.

$$t = \frac{252\ v}{N-13,520} - \frac{490(45-0.8572\ v)}{107,500-6140}$$

$$\frac{252\ v}{20480} = \frac{22080-420v}{101,360}\ ;\ 252v = 4460-85v$$

337v= 4480 hence v = 13.23 ft/sec.

$$t = \frac{252\times13.23}{20,480} = 0.163\ \text{sec.}$$

Total time T+t=0.163+.032=0.195 sec.

$$x = \frac{3}{4}(13.23\times0.195) + \frac{27360}{2\times32.2}\ \frac{\overline{13.23}^2}{45000}$$

x = 1.955+1.65=3.59 ft. = 43 (in)

$$Z = \frac{v}{2}\ t = \frac{45}{2}\ \times0.163= 3.67\ \text{ft.}=44\ \text{in. check.}$$

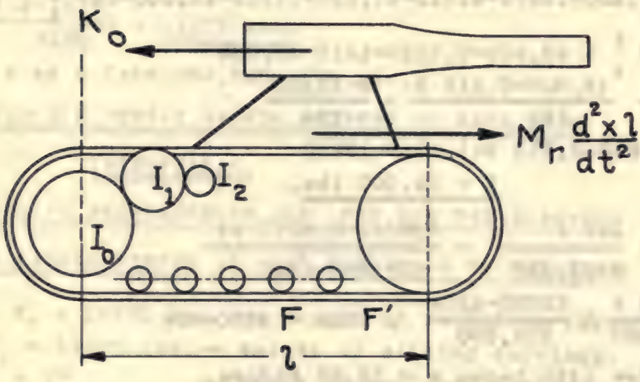
$$107,500\ Z + \frac{1}{2}\left(\frac{27360}{32.2}\right)\overline{13.23}^2 + \frac{45000}{2}\times 73.23\times 0.163$$

$$= \frac{1}{2}\ \frac{15,790}{32.2}\overline{45}^2$$

74,500+48,500=497,000

$Z = 3.48$ ft.

The discrepancy between this value of Z of the above is due to the fact that work done by gravity is omitted in the energy equations.



Theory of stability not braked.

By D'Alembert's principle, we have

$$K_D - F - M_e \frac{d^2x}{dt^2} = 0 \qquad (1)$$

$$K_o = P_s - m_r \frac{d_x^2 l}{dt^2} \qquad (2)$$

where K_D = dynamic inertia resistance of recoiling parts = 0.9 K (assembled approximate)

F = tractive force reaction

r = radius of traction rim

$$(F-\Sigma F)+r-Rr_o=I_o\frac{d^2\theta}{dt^2}+I_\phi\frac{d^2\theta}{dt^2}+WmlR(\frac{d^2\theta}{dt^2}R) \qquad (1)$$

$$(F-\Sigma F')r-Rr_o=(I_o-I_o'=I_t=2mlR^2)\frac{d^2\theta}{dt^2} \qquad (2)$$

$$\Sigma_1 F'r'=I_o''\frac{\frac{d^2\theta}{4t^2}}{r''} \qquad (3)$$

$$(R-R')r_1=I_1\frac{r_o}{r_1}\frac{d^2\theta}{dt^2} \qquad (4)$$

$$(R''-R')-r_2=I_2\frac{r_o}{r_1r_2}\frac{d^2\theta}{dt^2} \qquad (5)$$

$$I_t=2\pi r\ mr^2$$

<u>Hence we have the following equations:</u>

$$K_D-F=M\frac{d^2x}{dt^2} \qquad (1)$$

$$(F-\Sigma F')\ r-Rr_o=[I_o-I_o+3mr^2(\pi r+1)]\frac{\frac{d^2x}{dt^2}}{r} \qquad (2)$$

$$\Sigma F'r''=I_o''\frac{d^2x}{dt^2} \qquad (3)$$

$$(R-R')r_1=I_1\frac{r_o}{r_1}\frac{d^2\theta}{dt^2} \qquad (4)$$

$$(R''-R')r_2=I_2\frac{r_o}{r_1r_2}\frac{d^2\theta}{dt^2} \qquad (5)$$

The reaction of the truck rollers on track,

$$F'=\frac{10.1\frac{d^2x}{dt^2}}{\frac{1}{4}}=40.4\frac{d^2x}{dt^2}$$

The reaction of the clutch shaft pinion,

$$R_1(\tfrac{1}{4})=\frac{1.75}{1.43}\frac{24}{5.2}\frac{20.35}{6.00}\frac{d^2x}{dt^2}=19.1\frac{d^2x}{dt^2}\quad \text{hence } R_1=\frac{d^2x}{dt^2}$$

The reaction of the drive gear pinion,

$$R(0.2165)-R_1(0.846)=\frac{2.27}{1.43}\frac{24}{5.2}\frac{d^2x}{dt^2}=7.32\frac{d^2x}{dt^2}$$

$$\therefore R(0.2165)-(76.4\times0.846)\frac{d^2x}{dt^2}=7.32\ \frac{d^2x}{dt^2}$$

$$R(0.2165)-(64.6+7.32)\frac{d^2x}{dt^2}$$

$$\therefore R = \frac{71.92}{0.2165} = 332.0\ \frac{d^2x}{dt^2}$$

$$R = 331.0\ \frac{d^2x}{dt^2} = 331.\frac{d^2x}{dt^2}$$

$$F'=40.4\ \frac{d^2x}{dt^2}$$

and $I_0+2mr^2(\pi r+1)\ \dfrac{\frac{d^2x}{dt^2}}{r}\ +I_0'\ \dfrac{\frac{d^2x}{dt^2}}{r'}$

$I_0=35.54 \qquad m= \dfrac{150}{32.2} = 4.65$

$I_0'=18.5 \qquad r = 1.43$ ft. $\quad l = 158$ in. or 13.2 ft.

hence

$$\frac{35.54+9.30\times\overline{1.43}^2\ (\pi 1.43+13.2)}{1.43}\ \frac{d^2x}{dt^2}\quad \frac{18.5}{1.25}\ \frac{d^2x}{dt^2}$$

$$= 274.8\ \frac{d^2x}{dt^2}$$

$$(F-40.4\ \frac{d^2x}{dt^2})1.43-331\ \frac{d^2x}{dt^2} = 274.8\ \frac{d^2x}{dt^2}$$

$$\text{hence } F = \frac{663.4}{1.43}\ \frac{d^2x}{dt^2} = 463\ \frac{d^2x}{dt^2}$$

$$\frac{21.259}{32.2} + \frac{15000}{32.2} - 292 = 838$$

$$50000-463\ \frac{d^2x}{dt^2}=838\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{50000}{1301} = 38.4\ \text{ft/sec.}^2$$

$$\tfrac{1}{2}MV_x^2+\tfrac{1}{2}I_0(\frac{dw_0}{dt})^2\ +\tfrac{1}{2}I_1(\frac{d0_1}{dt})^2\ +\tfrac{1}{2}I_2(\frac{d0}{dt^2})^2\ + ---$$

$$(\tfrac{1}{2}M+\tfrac{1}{2}I_0k_0^2+\tfrac{1}{2}I_1k_1^2+\tfrac{1}{2}I_1k_1^2\ ---)V_r^2$$

$$K=(M+I_0k_0^2+I_rk_1^2\ --)(v\frac{dv}{dx})$$

Check on equivalent mass of rotating parts:

Kinetic energy of rotating parts in terms of translatory mass,

$$\frac{1}{2}\ 28+2.27\left(\frac{24}{3.2}\right)^2+1.75\left(\frac{24}{3.2}\ \frac{20.333}{6}\right)^2\left(\frac{v}{1.43}\right)^2$$

$$+\frac{1}{2}\ 10.1\left(\frac{v}{0.3}\right)^2+\frac{1}{2}\ 18.5\left(\frac{v}{1.25}\right)^2+\frac{1}{2}\ 9.3(\pi 1.43+13.2\ v^2$$

$$\frac{1}{2}\ \frac{28+48.6+430}{2.05}=123.2$$

$$\frac{1}{2}\ 10.1\ \frac{1}{\left(\frac{1}{4}\right)}=20.2$$

$$\frac{1}{2}\ \frac{18.5}{(1.25)^2}=5.61$$

$$\frac{1}{2}0.3(\pi 1.43+13.2)=\frac{82.2}{231.21}$$

$$\frac{1}{2}\ M_{rot_1}$$

$$M_{rot}=463.0$$

CALCULATION OF STABILITY.

Evaluation of inertia couples:

Track rollers $20.2\ \frac{d^2x}{dt^2}=653$

Track inertia sprockets $274.8\ \frac{d^2x}{dt^2}=8900$

Intermediate gear $7.32\ \frac{d^2x}{dt^2}=237$

Clutch $19.1\ \frac{d^2x}{dt^2}=616$

Resultant couple effect:

653	10,169	
8900	237	
616	9,932	ft.lbs. stabilizing moment, due to
	119,000	ft.lbs inertia couples of
10169		wheels:

K_d=0.9K=50000lbs. Overturning moment.

50000×72=3,600,000(overturning moment)lbs.

Stabilizing moment:	6248×69.5=	2,480,000	3,520,000
	9396×111=	1,040,000	119,000
		3,520,000	3,639,000

$$M_c\frac{d^2x}{dt^2} = \frac{36248}{32.2} \times 32.3 = 36400.$$

36400×32.5=1,185,000

3,639,000
1,185,000

4,824,000

Dynamic

Overturning moment 3,600,000 lbs.
Stabilizing moment 4,824,000 lbs.

Static

Overturning moment 4,100,000
Stabilizing moment 3,520,000

$$50000-15000-464\frac{d^2x}{dt^2} = 1270\frac{d^2x}{dt^2}$$

$$35000=1734\frac{d^2x}{dt^2} \quad \text{hence } \frac{d^2x}{dt^2} = 20.2 \text{ ft/sec.}$$

$$Ft = M V \qquad t = \frac{292\times 38.61}{50000} = 0.226 \text{ sec.}$$

$$S_1 = \frac{20.2\times .226}{2} = 2.28 \text{ ft.}$$

$$S_2 = \frac{M}{2R}V^2 \; \frac{1734}{30000} \times \overline{3.9}^2 = 0.87 \text{ ft.}$$

$$S = S_1 + S_2 = 2.28$$

.87

3.15

CHAPTER XIII

MISCELLANEOUS PROBLEMS AND TYPES OF CARRIAGES.

GENERAL DYNAMIC EQUATION OF RECOIL DURING POWDER PRESSURE PERIOD. The following theory is perfectly general and specially applicable for types of mounts that do not recoil along the axis of the bore. Let

m = mass of projectile

$\bar{m}$ = mass of the powder charge

v = absolute velocity of the shot up the bore ft/sec.

v_x = component of v parallel to recoil path ft/sec.

v_y = component of v normal to recoil path ft/sec.

v_{rel} = relative velocity of the shot in the bore

m_r = mass of the recoiling parts

P = mean powder force

$\emptyset$ = angle between axis of bore and path of recoil

N_s = normal reaction between projectile

R = total resistance of the recoil system (lbs)

u = travel up the bore (ft)

X = retarded recoil displacement (ft)

X_f = free recoil displacement (ft)

B = angle between absolute velocity of projectile v and path of recoil.

Assume half the charge to move with the projectile and half with the gun.

The reaction between the gun and projectile, becomes $P \cos \emptyset - N_s \sin\emptyset$ along the bore

$P\sin\emptyset - N_s\cos\emptyset$ normal to the bore.

The equation of motion of the recoiling parts, becomes, along the recoil path,

$P\cos\emptyset - N_s\sin\emptyset - R = (m_r + 0.5\,\bar{m})\dfrac{dV}{dt}$ Integrating and dividing by m_r,

we have $\left(\dfrac{P\cos\emptyset - N_s\sin\emptyset}{m_r + 0.5\bar{m}}\right)dt - \dfrac{Rdt}{m_r + 0.5\bar{m}} = V$ Now from the vector diagram of velocities, we have, adding vectorily,

$\overline{v_{rel}} + V = v$ but since $V = V_f$ approx. that is the retarded velocity of recoil is approximately equal to the free velocity of recoil, we have $\overline{v_{rel}} + V_f = v$ (approx.). Now in the free recoil

$\left(\dfrac{P\cos\emptyset - N_s\sin\emptyset}{m_r + 0.5\,\bar{m}}\right)dt = V_f$ that is the expression $\left(\dfrac{P\cos\emptyset - N_s\sin\emptyset}{m_r + 0.5\bar{m}}\right)dt$ is measured by V_f and which assumes, for given intervals of time P and N_s are not greatly different in the free recoil as compared with the retarded recoil. If R was sufficiently great to prevent an appreciable recoil N_s would disappear but P would not vary even then greatly for given intervals of time between free and stationary recoil. Further N_s is small even in free recoil as compared with P, hence the above expression would be but slightly modified.

Next, considering the motion of the projectile in a direction parallel to the recoil, we have $(P\cos\emptyset - N_s\sin\emptyset)dt = (m + 0.5)v_x$ but since $v_x = v_{rel}\cos\emptyset - V_f$ we have $(P\cos\emptyset - N_s\sin\emptyset)dt = (m + 0.5\bar{m})(v_{rel}\cos\emptyset - V_f)$

Combining with the expression for free recoil of the recoiling parts, $(m_r + 0.5\bar{m})V_f = (m + 0.5\bar{m})(v_{rel}\cos\emptyset - V_f)$

$$\text{Hence, } V_f = \frac{(m + 0.5\bar{m})v_{rel}\cos\emptyset}{m_r + m + \bar{m}}$$

$$X_f = \frac{(m + 0.5\bar{m})u}{m_r + m + \bar{m}}\cos\emptyset$$

Since B equals the angle v makes with the recoil

path, we also have $(P\cos\emptyset - N_s\sin\emptyset)dt = (m+0.5\bar{m})v\cos B$ and therefore $(m_r+0.5\bar{m})V_f = (m+0.5\bar{m})v\cos B$

hence $V_f = \frac{m+0.5\bar{m}}{m_r+0.5\bar{m}} v \cos B$ Now B differs very little from Ø, and assuming B=Ø hardly modifies the recoil effect; further $0.5\,\bar{m}$ is negligible as compared with m_r. Hence

$V_f = \frac{m+0.5\bar{m}}{m_r} v \cos \emptyset$ approx. The dynamic equations of recoil, become therefore

$$V = \int \left(\frac{P\cos\emptyset - N_s\sin\emptyset}{m+0.5\bar{m}}\right)dt \int \frac{R\,dt}{m_r+0.5\bar{m}}$$

$$= \frac{(m+0.5\bar{m})v_{rel}\cos\emptyset}{m_r+m+\bar{m}} - \frac{Rt}{m_r+0.5\bar{m}}$$

$$= \frac{m+0.5\bar{m}}{m_r} v \cos\emptyset - \frac{Rt}{m_r} \quad \text{(approx.)}$$

Integrating again $X = \int v_f dt - \frac{Rt^2}{2m_r} = \frac{(m+0.5\bar{m})u}{m_r+m+\bar{m}} \cos\emptyset - \frac{Rt^2}{2m_r}$

During the after effect period of the powder gases, the reaction of the powder is approximately along the axis of the bore and the procedure of computation has been previously discussed in detail.

The effect of the reaction N_s is to deviate the motion of the projectile, causing the projectile to leave the muzzle at an angle somewhat greater than the angle Ø.

To compute this angle, we have, $v \sin B = v_{rel}\sin\emptyset$

hence $\sin B = \frac{v_{rel}}{v} \sin\emptyset = \left(\frac{m_r+m+\bar{m}}{m+0.5\bar{m}}\right)\left(\frac{m+0.5\bar{m}}{m_r+0.5\bar{m}}\right)\frac{\cos B}{\cos\emptyset}$

$\tan B = \frac{m_r+m+\bar{m}}{m_r+0.5\bar{m}} \tan\emptyset$ and $B = \tan^{-1}\left(\frac{m_r+m+\bar{m}}{m_r+0.5\bar{m}}\right)\tan\emptyset$

The increase in the apparent angle of elevation becomes B−Ø and is usually small and may be neglected in recoil problems.

On the other hand, to compute N_s is important since it causes an additional load on the elevating mechanism during the travel of the shot up the bore

$$N_s = (m+0.5\bar{m})\frac{dV}{dt}\sin\emptyset = \frac{(m+0.5\bar{m})^2}{m_r}\sin\emptyset\cos\emptyset\frac{dv}{dt}$$

$$N_s = \frac{(m+0.5\bar{m})^2}{2m_r}\sin 2\emptyset\frac{dv}{dt} \quad \text{since } P = (m+0.5\bar{m})\frac{dv}{dt} \text{ (approx)}$$

we have $N_s = \dfrac{m+0.5\bar{m}}{2m_r}\sin 2\emptyset P$ in other words the normal reaction of the projectile when a gun recoils at an angle $\emptyset$ with the axis of the bore, is always proportional to the powder reaction which varies from point to point along the bore. Though the max. reaction occurs practically at the beginning of recoil, the moment is usually found greatest when the shot reaches the muzzle of the gun.

REACTIONS AND GENERAL EQUATIONS IN A RECOILING MOUNT. Consider the recoiling parts to be constrained in movement always parallel to the axis of the bore, the constraints being offered by suitable guides or a gun sleeve fixed to the cradle. We will assume rotation possible about the axis of the trunnions. Let

P_b = the powder reaction on the breech (lbs)
Q_1 and Q_2 = the front and rear clip reactions (lbs)
tan u = f = the coefficient of guide friction
M_r and W_r = mass and weight of the recoiling parts (lbs)
B = total braking force (lbs)
X and Y = the components of the trunnion reaction parallel and normal to the axis of the bore (lbs)
E = elevating gear reaction (lbs)
j = distance from trunnion to line of action of E (ft)
M_c and W_e = mass and weight of the cradle (lbs)
θ_e = angle between E and axis of bore
v_r = relative velocity of recoiling parts in cradle (ft/sec)

w = angular velocity of tipping parts about the trunnion (rad/sec)

I_r = moment of inertia of recoiling parts about the center of gravity of the recoiling parts.

I_{tr} = moment of inertia of recoiling parts about trunnion axis

I_{tc} = moment of inertia of cradle about the trunnions.

x_o and y_o = battery coordinates of the center of gravity of the recoiling parts with respect to the trunnion.

x_c and y_c = coordinates of the center of gravity of the cradle with respect to the trunnion.

d_{tb} = distance from trunnion to line of action of B.

$T = \sqrt{X^2+Y^2}$ = total trunnion reaction.

r' = radius of trunnion bearing

n_1 = friction angle in the trunnion bearing

$x_1 x_2 y_1$ and y_2 = coordinates of the front and rear clip reactions with respect to the trunnions.

REACTIONS ON THE RECOILING PARTS.

The reactions on the recoiling parts, consist of the reactions of the cradle $Q_1 Q_2$ and B, the reaction of the powder P_b and the various inertia forces as shown in the diagram.

Referring to fig.(1) and considering the motion of the recoiling parts assuming by D'Alemberts principle, kinetic equilibrium, we have

(1) Along the axis of the gun

$$P_b - B - (Q_1+Q_2)\sin u + W_r \sin\emptyset - m_r w^2 (x_o - x) - m_r y_o \frac{dw}{dt} - m_r \frac{dv_r}{dt} = 0$$

(2) Normal to the bore

$$(Q_2 - Q_1)\cos u - W_r \cos\emptyset + m_r w^2 y_o - m_r (x_o - x)\frac{dw}{dt} + 2 m_r w v_r = 0$$

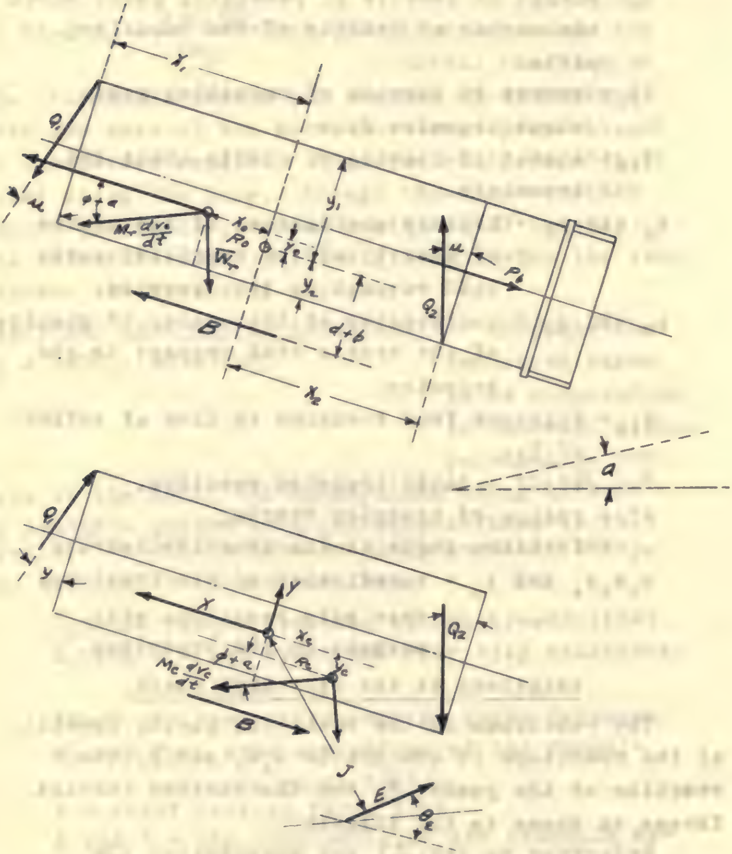


Fig. 1

(3) Moments about the trunnion,

$$P(e+s)+Bd_{tb}-m_r\frac{dv_r}{dt}s-I_t\frac{dw}{dt}+2m_rwv_r(x_o-x)-W_r\cos\emptyset(x_o-x)$$

$$+W_r\sin\emptyset y_o-Q_1\cos u.x_1-Q_2\cos u.x_2-Q_1\sin u.y+Q_2\sin u.y_2=0$$

where $I_t=I_r+m_rr_o^2=I_r+m_r[(x_o-x)^2+y_o^2$ and thus a variable with the recoil.

REACTIONS ON THE CRADLE.

Referring to the cradle, we have the reactions Q_1 Q_2 and B reversed, of the recoiling parts on the cradle, the trunnion reaction divided into components X and Y, the elevating gear reaction E and the various inertia forces as shown in fig.(1) passing through the center of gravity of the cradle, together with the inertia couple $I_o\frac{dw}{dt}$.

Referring to fig. (1) we have,

(1') Along the direction of the bore or guides

$$B+(Q_1+Q_2)\sin u+W_c\sin\emptyset+m_cw^2x_c+m_cy_c\frac{dw}{dt}+E\cos\theta_e-X=0$$

(2') Normal to the guides,

$$-(Q_2-Q_1)\cos u-W_c\cos\emptyset+m_cx_c\frac{dw}{dt}-m_cw^2y_c+E\sin\theta+Y=0$$

(3') For moments about the trunnion axis,

$$Q_1\cos u.x_1+Q_2\cos ux_2+Q_1\sin uy_1-Q_2\sin uy_2+W_c\cos\emptyset.x_c$$

$$-W_e\sin\emptyset.y_c-Bd_{tb}-Ej-I_{tc}\frac{dw}{dt}-Tr'\sin u_1=0$$

where $I_{tc}=I_c+m_cr_c^2=I_c+m_c(x_c^2+y_c^2)$

EXTERNAL REACTIONS ON THE TIPPING PARTS.

Assuming the tipping parts to be balanced about the trunnions, which is customary in order that the tipping parts may be rapidly elevated, we have $W_rx_c-W_cx_c=0$ and $m_rx_o-m_cx_c=0$

$W_ry_o-W_cy_c=0$ $m_ry_o-m_cy_c=0$

and for the total weight of the tipping parts W_t

$W_t=W_r+W_c$ and $M_t=m_r+m_c$ If now, we combine,

(1) and (1'), (2) and (2'), (3) and (3') and noting the above relations, we will have for the kinetic equilibrium of the tipping parts,

(1") along the bore

$$P_b + W_t \sin\phi + m_r w^2 x - m_r \frac{dv_r}{dt} + E\cos\theta_e - X = 0$$

(2") normal to the bore

$$-W_t \cos\phi - m_r x \frac{dw}{dt} + 2m_r w v_r + E\sin\theta_e + Y = 0$$

(3") moments about the trunnion

$$P_b(e+s) - m_r \frac{dv_r}{dt} s - I_t \frac{dw}{dt} - I_{tc}\frac{dw}{dt} + 2m_r w v_r (x_o - x) + W_r x \cos\phi$$

$$-Ej - Tr'\sin u_t = 0$$

Therefore, we have for the retardation, exerted by the top carriage on the tipping parts,

<u>For the trunnion reactions,</u>

$$X = P_b + W_t \sin\phi + m_r\left(w^2 x - \frac{dv_r}{dt}\right) + E\cos\theta_e$$

$$Y = W_t \cos\phi - E\sin\theta_e - m_r\left(x\frac{dw}{dt} + 2w\, v_r\right)$$

<u>For the elevating gear reaction,</u>

$$E = \frac{P_b(e+s) - (I_t + I_{tc})\frac{dw}{dt} + W_r x\cos\phi - Tr'\sin u_1 + m_r\left[2wv_r(x_c - x) - \frac{dv_r}{dt}.s\right]}{j}$$

APPLICATIONS OF THE PRECEEDING FORMULAE.

When the brake cylinders recoil with the gun as in the slide or sleigh containing the recoil cylinders and rigidly attached to the gun used with the Schneider material, the center of gravity of the recoiling parts falls considerably below the axis of the bore. To offset the effect of the large powder pressure couple and reduce the reaction on the elevating arc, we may employ a counterweight at the top of the gun to raise the

center of gravity nearer the axis of the bore as was done on the 155 m/m Schneider Howitzer or we may introduce a friction disk at the elevating pinion, this allowing rotation of the tipping parts about the trunnion.

In other types of mounts, a spring buffer may be introduced in the elevating gear thus reducing the elevating gear to a small finite value, and the moment effect of the powder pressure couple being distributed over a longer period.

If now we neglect, w^2x and $2w\ v_r$ as small and during the powder pressure period x being small we may neglect also, $x\frac{dw}{dt}$ and $W_r x \cos\phi$. The reactions on the tipping parts, become

$$X=P_b-m_r\frac{dv_r}{dt}+W_t\sin\phi+E\cos\theta_e$$

$$Y=W_t\cos\phi-E\sin\theta_e$$

and

$$E=\frac{P_b(e+s)-Tr'\sin u_1-(I_t+I_{tc})\frac{dw}{dt}-m_r\frac{dv_r}{dt}s}{j}$$

$$\text{Now } P_b-m_r\frac{dv_r}{dt}=B+(Q_1+Q_2)\sin u+m_r y_c\frac{dw}{dt}-W_r\sin\phi$$

$$=K+m_r y_o\frac{dw}{dt}$$

where K= the total resistance to recoil during the recoil neglecting the rotation effect during the powder period.

If y_o is small, that is if the trunnions are approximately on the axis of the bore, we have,

$$P_b=m_r\frac{dv_r}{dt}=K\ \text{(approx.)}$$

Assuming the brake disk on the elevating pinion shaft to offer a given torque, we may readily compute E. In other words, the pinion bearing is designed for a given reaction. This reaction should be comparable with the reaction required in the out of battery position of the recoiling parts.

That is $E = c\left(\frac{Ks+W_r b \cos\emptyset}{j}\right)$ where c = 2 to 3 depending upon max. allowable angular displacement of tipping parts, where b = length of recoil, K = resistance to recoil, s = distance from K to trunnions. The trunnion reactions become simply, $X = K+W_t \sin\emptyset + E \cos\theta_e$, $Y = W_t \cos\emptyset - E\,\theta_e$

Thus the trunnion reactions are fairly independent of the rotation about the trunnions, being primarily dependent only on the elevating gear reaction, the total resistance to recoil and the weight of the tipping parts. The additional forces induced by rotation about the trunnion can be treated as secondary forces.

The total trunnion reaction becomes, $T=\sqrt{X^2+Y^2}$ (lbs)

To determine the angular acceleration with a given elevating gear reaction E. We have, approx.

$P_b e+Ks-Tr'\sin u_1-Ej=(I_t+I_{tc})\frac{dw}{dt}$ hence

$$\frac{dw}{dt} = \frac{P_b e+Ks-Tr'\sin u_1-Ej}{I_t+I_{tc}}$$

Assuming $Tr'\sin u_1$ Ks and Ej as constant,

then,

$$w=\int^t \frac{P_b e dt}{I_t+I_{tc}} - \left(\frac{Tr'\sin u_1+Ej-Ks}{I_t+I_{tc}}\right)t$$

since obviously $Tr'\sin u_1+Ej$ must be greater than Ks. Further since

$$\int_0^t P_b dt = (m+0.5\,\bar{m})v$$

where m = mass of projectile

$\bar{m}$ = mass of powder charge

v = velocity of projectile in bore (ft/sec)

we have

$$w = \frac{(m+0.5\bar{m})ve}{I_t+I_{tc}} - \left(\frac{Tr'\sin u_1+Ej-Ks}{I_t+I_{tc}}\right)t$$

$$\theta = \frac{(m+0.5\bar{m})ve}{I_t+I_{tc}} - \left(\frac{Tr'\sin u_1+Ej-Ks}{2(I_t+I_{tc})}\right)t^2$$

where u = travel up the bore, (ft) To allow for the reaction effect of the powder gases, we will assume the free angular displacement at the end of the powder period, given by

$\theta_f = \dfrac{(m+2\bar{m})ve}{I_t+I_{tc}}$ Hence the angular velocity and displacement at the end of the powder period become

$$w_1 = \frac{(mv+4700\bar{m})}{I_t+I_{tc}}e - \left(\frac{Tr'\sin u_1+Ej-Ks}{I_t+I_{tc}}\right)t_1$$

$$\theta_1 = \frac{(m+2\bar{m})ue}{I_t+I_{tc}} - \left(\frac{Tr'\sin u_1+Ej-Ks}{I_t+I_{tc}}\right)\frac{t_1^2}{2}$$

where t_1 = total powder period (sec)

v = muzzle velocity of projectile (ft/sec)

u = travel up the bore (ft)

The remaining angular displacement is that due to a constant torque $(Tr'\sin u_1+Ej-Ks)$ acting on a rotating mass with an initial angular velocity w_1. Hence $(Tr'\sin u_1+Ej-Ks)(\theta_t-\theta_1) = \frac{1}{2}(I_t+I_{tc})\,w_1^2$

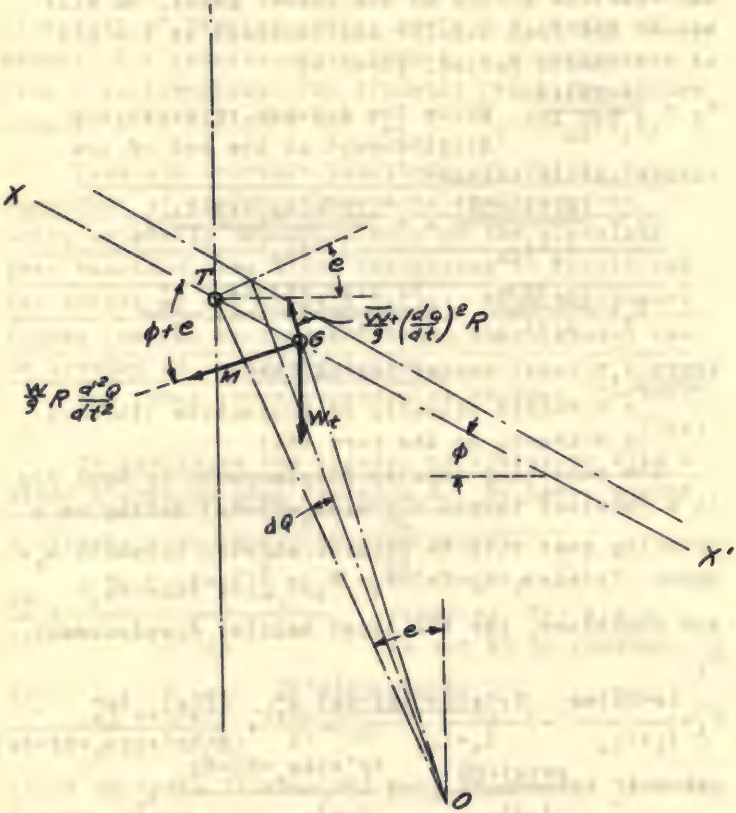
and therefore, for the total angular displacement θ_t

$$\theta_t = \frac{(m+2\bar{m})ue}{I_t+I_{tc}} - \left(\frac{Tr'\sin u_1+Ej-Ks}{I_t+I_{tc}}\right)\frac{t_1^2}{2} + \frac{(I_t+I_{tc})w_1^2}{2(Tr'\sin u_1+Ej-Ks)}$$

where $w_1 = \left(\dfrac{mv+4700\bar{m}}{I_t+I_{tc}}\right)e - \left(\dfrac{Tr'\sin u_1+Ej-Ks}{I_t+I_{tc}}\right)t_1$

and t_1 is computed by the methods of interior ballistics and $T=\sqrt{X^2+Y^2}$ using a suitable value of E, we may compute from the above expressions the total angular twist.

GENERAL EQUATIONS:- ROTATION OF THE TRUNNIONS ABOUT A FIXED AXIS OR A TRANSLATION OF THE TRUNNIONS. With rotation of the trunnions about an axis, the elevating gear reaction is usually reversed and the magnitude of the reversed action on the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e
T
$\frac{W_t}{g}\left(\frac{dQ}{dt}\right)^2 R$
$\phi+e$
G
$\frac{W}{g} R \frac{d^2Q}{dt^2}$
M
W_t
ϕ
dQ
X'
e
O

Fig. 2

elevating gear depends upon the product of the angular acceleration about this axis and the total moment of inertia about the trunnions of the tipping parts. Thus, in the rolling of a ship or in the jump of a field carriage where the angular acceleration upon the mount may be considerable, and with heavy tipping parts, a large reversed reaction is exerted on the elevating gear, which in turn modified the trunnion reactions. This same phenomena occurs in a double recoil system, or in a railway mount where the mount below the recoiling parts is accelerated up in an inclined plane or along the rails.

Let us first consider, the angular motion induced in the tipping parts when the elevating gear reaction is nil.

Assuming the trunnions to rotate about an axis O, fig.(2) and the axis of the bore and center of gravity of the recoiling parts to be along the trunnion axis, then,

The Kinetic reactions on the tipping parts, become

(1) The trunnion reaction X and Y which impress the angular acceleration on the tipping parts. Due to the friction of the trunnions $T = \sqrt{X^2+Y^2}$ has a moment about the trunnion axis: $T\,r'\sin u$.

(2) The tangential component of the inertia force of the tipping parts $= \frac{W_t}{g} R \frac{d^2\theta}{dt^2}$ and its moment about the trunnion axis becomes,

$$\frac{W_t}{g} R \frac{d^2\theta}{dt^2}(Tm) .$$

Let $Tm = \frac{W_r x}{W_t} \sin(\emptyset+e)$ hence $\frac{W_t}{g} R' \frac{d^2Q}{dt^2}(Tm) = \frac{W_r}{g} R' \times \frac{d^2Q}{dt^2} \sin(\bar{\emptyset}+e)$

(3) The centrifugal component of the inertia force of the tipping parts = $\frac{W_t}{g} R \left(\frac{dQ}{dt}\right)^2$ and its moment about the trunnion axis becomes

$$\frac{W_t}{g} R'\left(\frac{dQ}{dt}\right)^2 mG = \frac{W_r}{g} R' X\left(\frac{dQ}{dt}\right)^2 \cos(\emptyset + e)$$

(4) The rotational inertia couple of the tipping parts

$=(I_{tr}+I_{tc}) \frac{dw}{dt}$ where w = the angular velocity about the trunnions, I_{tr} = moment of inertia of the recoiling parts about the trunnions

I_{tc} = moment of inertia of the cradle about the trunnions

(5) The weight of the tipping parts, its moment being $W_t(TG)\cos\emptyset = W_t \left(\frac{W_r x}{W_t}\right)\cos\emptyset = W_r x \cos\emptyset$

(6) The complementary centrifugal inertia force due to the relative motion of the recoiling parts= $2m_r w \frac{dx}{dt}$ where x = the relative displacement of the recoiling parts. Its moment about the trunnion becomes,

$2m_r(x_o - x) w \frac{dx}{dt}$

(7) The powder reaction, and the relative inertia resistance due to the relative acceleration of the recoiling parts. We are not concerned with these reactions, since their moment effect is nil, it being assumed that their line

of action passes through the trunnion.

We have, therefore for the moment equation about the trunnion, considering the kinetic equilibrium of the various inertia forces,

$$Tr'\sin a_1+(I_{tr}+I_{tc})\frac{dw}{dt} - 2m_r w\frac{dx}{dt} - W_r X\cos\emptyset$$

$$-\frac{W_r}{g}R'X\frac{d^2Q}{dt^2}\sin(\emptyset+e)+\frac{W_r}{g}R'X\left(\frac{dQ}{dt}\right)^2\cos(\emptyset+e)=0$$

If we assume R large, for an elementary displacement, R dQ may be considered rectangular, hence the term

$\frac{W_r}{g}R'X\left(\frac{dQ}{dt}\right)^2\cos(\emptyset+e)$ may be omitted.

Further R = R' approx, R being the distance from axis O to the trunnion.

In experiments, conducted by the French at "Sevran-Livry" the term $2m_r w\frac{dx}{dt}$ was found to be negligible. Hence the equation of angular motion about the trunnion axis without an elevating gear interposed becomes,

$$T\,r\sin a_1+(I_{tr}+I_{tc})\frac{dw}{dt} - W_r X\cos\emptyset - \frac{W_r}{g}RX\frac{d^2Q}{dt^2}\sin(\emptyset+e)=0$$

since $Tr\sin a_1$ and $W_r X\cos\emptyset$ are small for a large angular acceleration, we have, approximately,

$$(I_{tr}+I_{tc})\frac{dw}{dt} - \frac{W_r}{g}RX\frac{d^2Q}{dt^2}\sin(\emptyset+e)=0$$

From this equation we observe that immediately upon the recoiling parts becoming out of battery, when the acceleration of the top carriage is backwards, as would occur in the jump of a field carriage, the upward rolling of a ship or in the recoil of the top carriage in a double recoil system or railway mount, we have an angular acceleration tending to cause a reversal or stress in the elevating gear.

ANGULAR ACCELERATION OF THE TIPPING PARTS.

Invariable Elevating Gear Reaction Introduced.

In this case, the angular acceleration of the tipping parts, is the same as the angular acceleration of the system about the fixed or instantaneous axis O. To impress this angular acceleration on the tipping parts, as would occur in the jump of a field carriage, or in the upward rolling of a ship, the elevating gear reaction is lessened or completely reversed when the trunnions are located along the bore. Considering fig.(3)

Let P_b = the powder reaction on the breech (lbs)

Q_1 and Q_2 = the front and rear clip reactions (lbs)

$\tan u = f$ = coefficient of guide friction

m_r and w_r = mass and weight of recoiling parts (lbs)

B = total braking force (lbs)

X and Y = components of the trunnion reaction parallel and normal to the bore (lbs)

E = elevating gear reaction (lbs)

j = distance from trunnion axis to line of action of E (ft)

θ_e = angle between E and the axis of the bore

V_r = relative velocity of recoiling parts in cradle (ft/sec)

$\frac{dQ}{dt}$ = angular velocity impressed on tipping parts (rad/sec)

I_r = moment of inertia of recoiling parts about center of gravity of recoiling parts

I_{tr} moment of inertia of recoiling parts about trunnion axis.

I_{tc} = moment of inertia of the cradle about the trunnion axis.

x_o and y_o = battery coordinates of the center of gravity of the cradle with respect to the

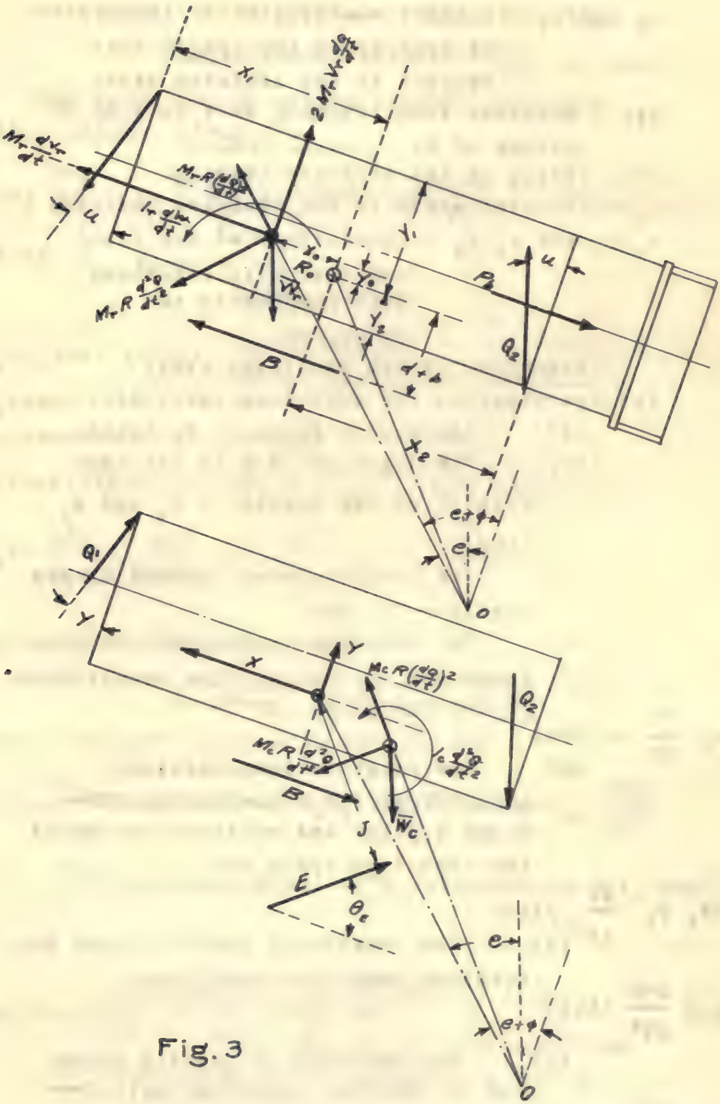

X_1
$2M_r V_r \frac{d\theta}{dt}$
$M_r \frac{dv_r}{dt}$
$M_r R\left(\frac{d\theta}{dt}\right)^2$
$I_r \frac{d\omega}{dt}$
u
X_0
R_0
Y_0
Y_1
$M_r R \frac{d^2\theta}{dt^2}$
$\overline{W}_r$
Y_2
P_2
u
Q_2
B
$d+b$
X_2
$e+\phi$
e
O
Q_1
Y
X
Y
$M_c R\left(\frac{d\theta}{dt}\right)^2$
Q_2
$M_c R \frac{d^2\theta}{dt^2}$
$I_c \frac{d^2\theta}{dt^2}$
B
$\overline{W}_c$
J
E
θ_E
e
$e+\phi$
O

Fig. 3

trunnion axis,

x_c and y_1 = battery coordinates of the center of gravity of the cradle with respect to the trunnion axis.

d_{tb} = distance from trunnion axis to line of action of B.

r' = radius of the trunnion bearing.

u_1 = friction angle in the trunnion bearing.

x_1 y_1 and x_2 y_2 = coordinates of the front and rear clip reactions with respect to the trunnions.

REACTIONS ON THE RECOILING PARTS.

The reactions on the recoiling parts are:

(1) The powder force -- P_b (lbs)

(2) The reactions due to the constraint of the cradle -- Q_1 and Q_2 (lbs)

(3) The braking force exerted by the cradle -- B (lbs)

(4) The relative tangential inertia force due to the relative acceleration of the recoiling parts --

$m_r \dfrac{dv_r}{dt}$ --(lbs)

(5) The relative complementary centrifugal force due to the combined angular and relative motion of the recoiling parts ---

$2m_r v_r \dfrac{dQ}{dt}$ (lbs)

(6) The tangential inertia force due rotation about the axis O ---

$m_r R \dfrac{d^2Q}{dt^2}$ (lbs)

(7) The centrifugal inertia force due to rotation about the axis O---

$m_r R(\dfrac{dQ}{dt})^2$ (lbs)

(8) The weight of the recoiling parts --- W_r (lbs)

(9) The angular couple resisting angular acceleration $I_r \frac{d^2Q}{dt^2}$ (ft.lbs)

The equations of motion for the recoiling parts, become, along x x' --

$$P_b - m_r \frac{dv_r}{dt} - m_r R \frac{d^2Q}{dt^2} \cos(e+\emptyset) - m_r R \left(\frac{dQ}{dt}\right) \sin(e+\emptyset) + W_r \sin\emptyset$$

$$- B - (Q_1 + Q_2) \text{ since } u = 0 \qquad (1)$$

along y y'

$$(Q_2 - Q_1)\cos u + 2m_r v_r \frac{dQ}{dt} + m_r R \left(\frac{dQ}{dt}\right)^2 \cos(e+\emptyset) - m_r R \frac{d^2Q}{dt} \sin(e+\emptyset)$$

$$-W_r \cos\emptyset = 0 \qquad (2)$$

for moments about the axis O, we have,

$$P_b[R\cos(e+\emptyset) + (s+e)] - \Sigma M_o(Q_1 + Q_2 + B) - m_r \frac{dv_r}{dt}[R\cos(e+\emptyset) + s]$$

$$-m_r R^2 \frac{d^2Q}{dt^2} - I_r \frac{d^2Q}{dt^2} + 2m_r v_r \frac{dQ}{dt}[x_x - x + R\sin(\emptyset+e)]$$

$$-W_r[(x_o - x)\cos\emptyset + R\sin e - y_o \sin\emptyset]$$

Now $m_r R^2 + I_r = I_{or}$ moment of inertia of recoiling parts about axis O.

Hence the above expression reduces to,

$$\left(P_b - m_r \frac{dv_r}{dt}\right)[R\cos(e+\emptyset) + s] + P_b e_b - \Sigma M_o(Q_1 + Q_2 + B) - I_{or} \frac{d^2Q}{dt^2}$$

$$+2m_r v_r \frac{dQ}{dt}[x_o - x + R\sin(\emptyset+e)] - W_r[(x_o - x)\cos\emptyset + R\sin e - y_o \sin\emptyset]$$

$$(3)$$

REACTIONS ON THE CRADLE:

The reactions on the cradle are:

(1) The reactions of the recoiling parts on the cradle $Q_1 Q_2$ and B.

(2) The trunnion reaction $T = \sqrt{X^2 + Y^2}$ and having a moment about its center line Tr' sin u.

(3) The elevating gear reaction E

(4) The tangential inertia force

$$m_cR\frac{d^2Q}{dt^2}$$

(5) The centrifugal inertia $m_cR(\frac{dQ}{dt})^2$

(6) The weight of the cradle W_c

The equations of motion, become

along the x x' axis

$$(Q_1+Q_2)\sin u+B+W_c\sin\emptyset+E\cos\theta_e$$

$$-m_cR\frac{d^2Q}{dt^2}\cos(e+\emptyset)-m_cR(\frac{dQ}{dt})^2\sin(e+\emptyset)-X=0 \quad (1')$$

along the y y' axis

$$Y+E\sin\theta_e-W_c\cos\emptyset-(Q_2-Q_1)\cos u+m_cR(\frac{dQ}{dt})^2\cos(e+\emptyset) \quad (2')$$

for moments about the axis O,

$$\Sigma M_o(Q_1+Q_2+B)-XR\cos(\emptyset+e)+YR\sin(\emptyset+e)-m_cR^2\frac{d^2Q}{dt^2}$$

$$-I_c\frac{d^2Q}{dt^2}+E\cos\theta_e[R\cos(\emptyset+e)-J\cos\theta_e]+E\sin\theta_e[R\sin(e+\emptyset)-J$$

$$\sin\theta_e]-W_c(R\sin e-x_c\cos\emptyset+y_c\sin\emptyset)=0$$

Now, $m_cR^2+I_c=I_{oc}$ the moment of inertia about the axis O of the cradle.

and $E\cos\theta_e[R\cos(\emptyset+e)-J\cos\theta_e]+E\sin\theta_e[R\sin(\emptyset+e)-J\sin\theta_e]$

$$=ER\cos\theta\cos(\emptyset+e)+ER\sin\theta_e\sin(\emptyset+e)-EJ(\cos^2\theta_e+\sin^2\theta_e)$$

$$=ER\cos(\emptyset+e-\theta_e)-EJ=E[R\cos(\emptyset+e-\theta_e)-J]$$

Hence the moment equation of the cradle about O, reduces to $\Sigma M_o(Q_1+Q_2+B)-XR\cos(\emptyset+e)+YR\sin(\emptyset+e)-I_{oc}\frac{d^2Q}{dt^2}$

$$+E[R\cos(\emptyset+e-\theta_e)-J]-W_c(R\sin e-x_c\cos\emptyset+y_c\sin\emptyset)=0 \quad (3')$$

REACTIONS ON THE TIPPING PARTS.

Since the tipping parts are balanced about the

trunnions in the battery position, we have,

$W_r x_o - W_c x_c = 0 \qquad m_r x_o - m_c x_c = 0$

and

$W_r y_o - W_c y_c = 0 \qquad m_r y_c - m_c y_c = 0$

Adding (1) and (1'), (2) and (2') and (3) and (3'), we have

$$P_b - m_r \frac{dv_r}{dt} - (m_r + m_c) R \frac{d^2Q}{dt^2} \cos(\emptyset + e) - (m_r + m_c) R \left(\frac{dQ}{dt}\right)^2 \sin(e + \emptyset) +$$

$$(W_r + W_c) \sin\emptyset + E\cos\theta_e - X = 0 \qquad (1'')$$

$$Y + E\sin\theta_e - (W_r + W_c)\cos\emptyset + 2m_r v_r \frac{dQ}{dt} + (m_r + m_c) R \left(\frac{dQ}{dt}\right)^2 \cos(\emptyset + e)$$

$$-(m_r + m_c) R \frac{d^2Q}{dt^2} \sin(\emptyset + e) = 0 \qquad (2'')$$

$$\left(P_b - m_r \frac{dv_r}{dt}\right)[R\cos(\emptyset + e) + s] + P_b e_b - (I_{or} + I_{oc}) \frac{d^2Q}{dt^2} + 2m_r v_r \frac{dQ}{dt}$$

$$[x_o + x + R(\emptyset + e)] + W_r x\cos\emptyset - (W_r + W_c) R\sin e - XR\cos(\emptyset + e)$$

$$+ YR\sin(\emptyset + e) + E[R\cos(\emptyset + e - \theta_e) - J] = 0 \qquad (3'')$$

Equations (1"),(2"),(3") are the general equations of a recoiling system, where the relative translation is along the axis of the bore and the trunnions have a rotation about some fixed axis O.

These equations may be simplified as follows:

$W_t = W_r + W_c$ and $m_t = m_r + m_c$ where W_t = the total weight of the tipping parts.

m_t = the total mass of the tipping parts

Further $I_{or} = I_{tr} + m_r R^2$

$I_{oc} = I_{tc} + m_c R^2$ and $I_{or} + I_{oc} = I_{tr} + I_{tc} + m_t R^2$

$R\cos(\emptyset + e) + s = R\cos(\emptyset + e)$ approx.

$$X = P_b - m_r \frac{dv_r}{dt} - m_t R\left[\frac{d^2Q}{dt^2}\cos(\emptyset + e) + \left(\frac{dQ}{dt}\right)^2 \sin\emptyset + e\right] + W_t \sin\emptyset + E\cos\theta_e$$

$$Y = W_t\cos\emptyset + m_t R\left[\frac{d^2Q}{dt^2}\sin(\emptyset+e) - \left(\frac{dQ}{dt}\right)^2\cos(\emptyset+e)\right] - 2m_r v_r\frac{dQ}{dt} - E\sin\theta_e$$

and

$$E = \frac{(I_{tr}+I_{tc}+m_tR^2)\frac{d^2Q}{dt^2} - (P_b - m_r\frac{dv_r}{dt})[R\cos(\emptyset+e)+s] - P_b e_b - 2m_r v_r \frac{dQ}{dt} E\sin\theta_e - W_r x\cos\emptyset + W_t R\sin e + XR\cos(\emptyset+e) - Y\sin(\emptyset+e)}{R\cos(\emptyset+e-\theta_e) - J}$$

Substituting the values of X and Y in the equation of E and simplifying, we have,

$$E = \frac{(P_b - m_r\frac{dv_r}{dt})s + P_b e_b + 2m_r v_r\frac{dQ}{dt}(x_o - x) + W_r x\cos\emptyset - (I_{tr}+I_{tc})\frac{d^2Q}{dt^2}}{J} \qquad (4)$$

which is evidently the moment equation of the various kinetic reactions on the tipping parts about the trunnion as an axis. Since the term

$2m_r v_r\frac{dQ}{dt}(x_o - x)$ is always small, the elevating gear reaction, reduces to

$$E = \frac{(P_b - m_r\frac{dv_r}{dt})s + P_b e_b + W_r x\cos\emptyset - (I_{tr}+I_{tc})\frac{d^2Q}{dt}}{J} \qquad (4')$$

where $I_{tr} = I_r + m_r[(x_o - x)^2 + y_o^2]$

I_r = moment of inertia about center of gravity of recoiling parts. Hence I_{tr} is a variable depending upon the displacement in the recoil x, also $I_{tc} = I_c + m_c(x_c^2 + y_c^2)$ a constant

I_c = moment of inertia about center of gravity of the cradle.

The equation (4) or (4') is of special importance in the study of the variation of the elevation gear reaction. The angular acceleration $\frac{d^2Q}{dt^2}$ will be determined in the following discussion

on the jump of a carriage.

In the case when s and $e_b=0$, that is when the center of gravity of the recoiling parts and trunnion axis lie along the axis of the bore, we have

$$E = \frac{-(I_{tr}+I_{tc})\frac{d^2Q}{dt^2}}{J}$$

Thus the elevating gear reaction is reversed and its moment about the trunnion imparts the required angular momentum in the tipping parts. We calculate the value of $(-E)$ we must determine the maximum angular acceleration $\frac{d^2Q}{dt^2}$.

The condition that there will be no reversal of stress on the elevating gear on the jump of a field carriage, is that

$$W_r x \cos\emptyset + (P_b - m_r\frac{dv_r}{dt})s + P_b e_b \geqq (I_{tr}+I_{tc})\frac{d^2Q}{dt^2}$$

Now roughly $P_b - m_r \frac{dv_r}{dt} = K$ the static resistance to recoil, hence for no reversal of stress on the elevating gear,

In the battery position:

$$Ks + P_b e_b \geqq (I_{tr}+I_{tc})\frac{d^2Q}{dt^2}$$

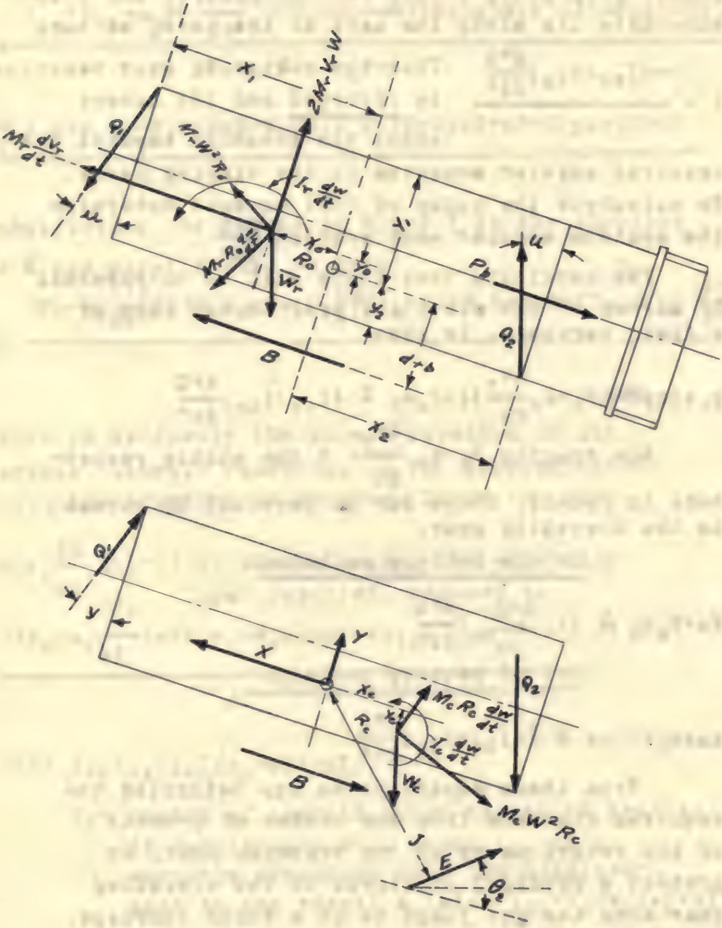
Cut of battery position:

$$Ks + W_r b \cos\emptyset \geqq (I_{tr}+I_{tc})\frac{d^2Q}{dt^2}$$

From these equations we may determine the required distance from the center of gravity of the recoil parts to the trunnion axis, to prevent a reversal of stress on the elevating gear when the gun jumps as in a field carriage.

RECTILINEAR ACCELERATION OF THE TIPPING PARTS

With a double recoil system, or in the case of a railway recoiling along the rails, the trunnions are accelerated to the rear due to the recoil re-



X_1
$2M_r V_r W$
Q_1
$M_r \frac{dV_r}{dt}$
$M_r W^2 R_o$
$I_r \frac{dw}{dt}$
Y_1
u
X_o
R_o
Y_o
$\overline{W_r}$
Y_2
P_b
Q_2
B
$d+b$
X_2
Q'
y
Y
X
X_c
$M_c R_c \frac{dw}{dt}$
R_c
$I_c \frac{dw}{dt}$
W_c
$M_c W^2 R_c$
J
E
θ_2

Fig. 4

action of the gun. Thus the tipping parts are subjected to a rectilinear acceleration to the rear and the elevating gear reactions is increased. Considering fig.(4) we have the various reactions as the recoiling parts and cradle as shown.

REACTIONS ON THE RECOILING PARTS.

The reactions on the recoiling parts, consist,

(1) The powder force ---P_b (lbs)

(2) The reactions due to the constraint of the cradle ----Q_1 and Q_2 (lbs)

(3) The braking force exerted by the cradle -- B (lbs)

(4) The weight of the recoiling parts -- W_r (lbs)

(5) The kinetic reaction of the recoiling parts due to the relative acceleration-- $m_r\frac{dv_r}{dt}$ (lbs)

(6) The kinetic reaction due to the acceleration $(\frac{dv_c}{dt})$ of the constraint of the recoiling parts, i.e. the top carriage and cradle -- $m_r\frac{dv_c}{dt}$ (lbs)

Then, for the kinetic equilibrium of the recoiling parts, along the axis of the bore

$$P_b-m_r\frac{dv_r}{dt} - m_r\frac{dv_c}{dt}\cos(\emptyset+a)+W_r\sin\emptyset-(Q_1+Q_2)\sin u - B=0 \quad (1)$$

along the normal axis to the bore,

$$(Q_2-Q_1)\cos u-W_r\cos\emptyset-m_r\frac{dv_c}{dt}\sin(\emptyset+a)=0 \quad (2)$$

and for moments about the trunnion axis,

$$P_b(y_c+e)+Bd_b-(Qx_1+Q_2x_2)\cos u-(Q_1y_1-Q_2y_2)\sin u$$
$$-m_r[\frac{dv_r}{dt}+\frac{dv_c}{dt}\cos(\emptyset+a)]u_o-W_r\cos\emptyset(x_o-x)$$

$$-m_r\frac{dv_c}{dt}\sin(\emptyset+a)(x_o-x)+W_r\sin\emptyset y_o=0 \qquad (3)$$

For the kinetic equilibrium of the cradle, we have, along the axis of the bore or guides,

$$(Q_1+Q_2)\sin u+E\cos\theta_e+B-m_c\frac{dv_c}{dt}\cos(\emptyset+a)+W_c\sin\emptyset-X=0 \qquad (1')$$

along the normal to the guides or bore,

$$Y+E\sin\theta_2-(Q_2-Q_1)\cos u-W_c\cos\emptyset-m_c\frac{dv_c}{dt}\sin(\emptyset+a)=0 \qquad (2')$$

and for moments about the axis of the trunnions,

$$(Q_1x_1+Q_2x_2)\cos u+(Q_1y_1-Q_2y_2)\sin u-Bd_b+m_c\frac{dv_c}{dt}\cos(\emptyset+a)y_c$$

$$+m_c\frac{dv_c}{dt}x_c\sin(\emptyset+a)+w_c\cos\emptyset x_c-W_c\sin\emptyset y_c-Ej=0 \qquad (3')$$

REACTIONS ON THE TIPPING PARTS.

Since the tipping parts are usually balanced about the trunnions in the battery position, we have,

$$W_rx_o-W_cx_c=0 \qquad m_rx_c-m_cx_c=0$$

and

$$W_ry_o-W_cy_c=0 \qquad m_ry_o-m_cy_c=0$$

Adding equations (1) and (1'), (2) and (2'), (3) and (3'), we have, then, along the axis of the bore,

$$P_b-m_r\frac{dv_r}{dt}-(m_r+m_c)\frac{dv_c}{dt}\cos(\emptyset+a)+(W_r+W_c)\sin\emptyset+E\cos\theta_e-X=0 \qquad (1'')$$

along the normal to the axis of the bore,

$$Y+E\sin\theta_e-(W_r+W_c)\cos\emptyset-(m_r+m_c)\frac{dv_c}{dt}\sin(\emptyset+a)=0 \qquad (2'')$$

and for moments about the trunnion,

$$P_b(y_o+\theta_b)-m_r\frac{dv_r}{dt}y_o\times W_rx\cos\emptyset+m_r\frac{dv_c}{dt}\sin(\emptyset+a).x-Ej=0 \qquad (3'')$$

The elevating gear reaction, becomes,

$$E = \frac{(P_b - m_r\frac{dv_r}{dt})y_o + P_b a_b + W_r x\ \cos\emptyset + m_r\frac{dv_c}{dt}\ \sin(\emptyset+a).x}{J}$$

From equation (1),

$$P_b - m_r[\frac{dv_r}{dt} - \frac{dv_c}{dt}\cos(\emptyset+a)] + W_r\sin\emptyset - (Q_1+Q_2)\sin\ u - B = 0$$

Since the displacement and velocity of the top carriage is small at the beginning of recoil, the relation v_r= the static velocity v_s, that is the velocity of the recoiling parts when the top carriage is assumed stationary.

The braking force B, equals, $B=F_v+\frac{cv_r^2}{w_x^2}$

but since $v_r - v_s$ approx.

$$B=F_v+\frac{cv_r^2}{w_x^2}=F_v+\frac{cv_c^2}{w_x^2}=B_s$$ where B_s= the static recoil braking force.

Hence the kinetic reaction along the bore, becomes,

$$P_b - m_r[\frac{dv_r}{dt}+\frac{dv_c}{dt}\cos(\emptyset+a)] = B_s+(Q_1+Q_2)\sin\ u - W_r\sin\emptyset$$

But for the static resistance to recoil, we have

$$K_x = B_s + (Q_1+Q_2)\sin\ u - W_r\sin\ \emptyset$$

hence

$$P_b - m_r[\frac{dv_x}{dt}+\frac{dv_c}{dt}\ \cos(\emptyset+a)] = K_x$$ Therefore, the elevating gear reaction reduces to,

$$E=\frac{P_b e_b + K_x y_o + W_r x\cos\emptyset + m_r\frac{dv_c}{dt}[x\sin(\emptyset+a)+y_o\cos(\emptyset+a)]}{J}$$

This equation is of special interest since in the battery position, we find,

$$E=\frac{P_b e_b + K_x y_o + m_r\frac{dv_c}{dt}y_o\cos(\emptyset+a)}{J}$$ when the top carriage moves.

$$E=\frac{P_b e_b + K_x y_o}{J}$$ when the top carriage is stationary.

Thus we have only a slightly additional load

brought on the elevating gear $\dfrac{m_r\dfrac{dv_c}{dt}\,y_o\cos(\phi+a)}{J}$ (lbs)

This value however is somewhat compensated by the slightly decreased value of K_x due to the fact that the relative velocity is somewhat less than the static velocity of recoil.

RECAPITULATION.

Reaction of Top Carriage on Tipping Parts:

For the Trunnion Reactions

$$X=P_b-m_r\frac{dv_r}{dt}-(m_r+m_c)\frac{dv_c}{dt}\cos(\phi+a)+(W_r+W_c)\sin\phi+E\cos\theta_e$$

$$Y=(W_r+W_c)\cos\phi+(m_r+m_c)\frac{dv_c}{dt}\sin(\phi+a)-E\sin\theta_e$$

For the Elevating Gear Reaction

$$E=\frac{P_be_b+(P_b-m_r\frac{dv_r}{dt})y_c+W_rx\cos\phi+m_r\frac{dv_c}{dt}.x\sin(\phi+a)}{J}$$

If we define $K_x=B+(Q_1+Q_2)\sin u+W_r\sin\phi$

then

$K_x-P_b=m_r[\dfrac{dv_r}{dt}+\dfrac{dv_c}{dt}\cos(\phi+a)]$ and $W_t=W_r+W_c$ total weight of the tipping parts, $M_t=m_r=m_c$ Total mass of tipping parts.

For the Trunnion Reactions

$$X=K_x=m_c\frac{dv_c}{dt}\cos(\phi+a)+W_t\sin\phi+E\cos\theta_e$$

$$Y=W_t\cos\phi+m_t\frac{dv_c}{dt}\sin(\phi+a)-E\sin\theta_e$$

For the Elevating Gear Reaction:

$$E=\frac{P_be_b+K_xy_o+W_rx\cos\phi+m_r\frac{dv_c}{dt}[x\sin(\phi+a)+y_o\cos(\phi+a)]}{J}$$

ON THE JUMP OF A FIELD CARRIAGE Mounts are frequently designed for stability at a given minimum elevation and yet may be fired at a lower elevation. Con-

sideration, therefore, must be given to the inertia loadings and corresponding reactions induced by the jump of the carriage. In the following discussion it will be assumed the total mount to rotate about its spade point.

By the application of D'Alembert's principle we introduce the various inertia effects as kinetic reactions, the mutual reactions between the parts, of course having no effect on the kinetic equilibrium of the total system, gun cradle and carriage.

From the acceleration diagram we have for the recoiling parts,

(1) The relative acceleration along the axis of the bore ---

$$\frac{dv_r}{dt} \ (\text{ft/sec.}^2)$$

(2) The tangential acceleration of the recoiling parts about the axis O ---

$$R \frac{dw}{dt}$$

(3) The centripetal acceleration of the recoiling parts towards the axis O---

$$w^2 R$$

(4) The acceleration due to the relative motion combined with the rotation of the recoiling parts $2w\ v_r$

The accelerations in the remainder of the mount, the carriage proper, become

(1) The tangential acceleration -- $L_c \dfrac{dw}{dt}$

(2) The centripetal acceleration-- $w^2 L_c$

KINETIC EQUILIBRIUM OF THE SYSTEM.

(Gun and Carriage)

From the principle of D'Alembert, we have the external reactions in equilibrium with the various kinetic reactions induced by the angular rotation of the mount and the relative acceleration of the gun.

The forces and kinetic reactions on the system

gun and carriage are :

(1) The total powder reaction P_b

(2) The weights of the recoiling parts and carriage W_t and W_c (lbs)

(3) The tangential inertia force of the recoiling parts due to the angular acceleration about the spade point O ---

$$M_R R \frac{dw}{dt} \quad \text{(lbs)}$$

(4) The centrifugal inertia force of the recoiling parts due to the angular velocity about the spade point O ---

$$M_R \; R \; w^2 \; \text{(lbs)}$$

(5) The inertia resistance due to the relative acceleration of the recoiling parts

$$m_r \frac{dv_r}{dt} \quad \text{(lbs)}$$

(6) The inertia resistance due to the combined rotation of the recoiling parts

$$2 \; m_r w v_r \; \text{(lbs)}$$

(7) The tangential inertia force of the carriage proper due to the angular acceleration about the spade point O ----

$$m_c L_c \frac{dw}{dt} \quad \text{(lbs)}$$

(8) The centrifugal inertia force due to the angular velocity about the spade point O ---- $m_o \; L_c w^2$ (lbs)

(9) The inertia couple about the center of gravity of the recoiling parts due to the angular acceleration of the system --

$$I_R \frac{dw}{dt} \quad \text{(ft.lbs)}$$

(10) The inertia couple about the center of gravity of the carriage proper due to the angular acceleration of the system --

$I_c \dfrac{dw}{dt}$ (ft.lbs)

For moments about the axis O, we have,

$$P_s(d+e)-m_R \frac{dv_R}{dt} d-m_R R^2 \frac{dw}{dt} +2mg\ w\ v_R$$

$$(x_o-x)-W_R[(x_o-x)\cos\emptyset-d\ \sin\emptyset]$$

$$I_s \frac{dw}{dt} = m_c L_c^{-2} \frac{dw}{dt} -I_c \frac{dw}{dt} -W_c L_c \cos(\theta-B) = 0 \quad (1\ )$$

since $\theta = \frac{\pi}{2} +\emptyset+Q$ whence Q = angle turned in rotating about O, we have $\dfrac{d\theta}{dt} = \dfrac{dQ}{dt} = w$ for the angular velocity

$\dfrac{d^2\theta}{dt^2} = \dfrac{d^2Q}{dt^2} = \dfrac{dw}{dt}$ for the angular acceleration.

Considering now the recoiling parts, above, we have

$$P_b-m_R\frac{dv_R}{dt} -m_R\ d\ \frac{dw}{dt} -m_R w^2(x_o-x)-B-R_t+W_r\sin\emptyset=0.$$

Simplifying we have, $P_b-m_R[\dfrac{dv_R}{dt} + d\ \dfrac{dw}{dt}+w^2(x_o-x)]-B-R_t$

$+W_r\sin\emptyset=0$ (2)

whence $B=F_v+P_h$ -- the total braking reaction (lbs)

F_v = the recuperator reaction (lbs)

P_h = the total hydraulic resistance (lbs)

Now

$$P_h=P_{hs} \frac{v_R^2}{v_s^2} \quad \text{whence}$$

v_s = static recoil velocity (ft/sec)

P_{hs} = corresponding static hydraulic braking reaction (lbs)

We thus see that $P_b - m_R[\frac{dv_R}{dt} + d\frac{dw}{dt} + w^2(x_o - x)] = P_{hs}\frac{v_R^2}{v_s^2} + R_t$

$$F_v - W_R \sin\phi \qquad (3)$$

From equation (1), we have

$$\frac{dw}{dt} = \frac{P_b(d+e) - m_R\frac{dv_R}{dt}d + 2m_R w\, v_R(x_o - x) - W_R[(x_o - x)\cos\phi - d \sin\phi] - W_c L_c \cos(\phi - B)}{m_R R^2 - m_c L_o = I_R + I_c} \qquad (4)$$

If I_s = the moment of inertia of the system about the axis O, then $I_s = m_R R^2 + m_c L_o + I_R + I_c$ <u>a variable</u> since $m_R R^2 = m_R[d^2 + (x_o - x)^2]$ <u>a function of x</u>, hence

$$\frac{dw}{dt} = \frac{P_b(d+e) - m_R\frac{dv_R}{dt}d + 2m_R w\, v_R(x_o - x) - W_R[(x_o - x)\cos\phi - d \sin] - W_c L_c \cos(\theta - B)}{I_s} \qquad (4a)$$

Substituting the value $\frac{dw}{dt}$ in equation (3), we have a dynamical equation in terms of $\frac{dv_R}{dt}$ and w. If now, we construct a table for the various intervals of time, we may compute v_R, $\frac{dv_R}{dt}$, w and $\frac{dw}{dt}$ by the methods of a point by point procedure.

APPROXIMATE CALCULATIONS FOR THE JUMP OF A CARRIAGE

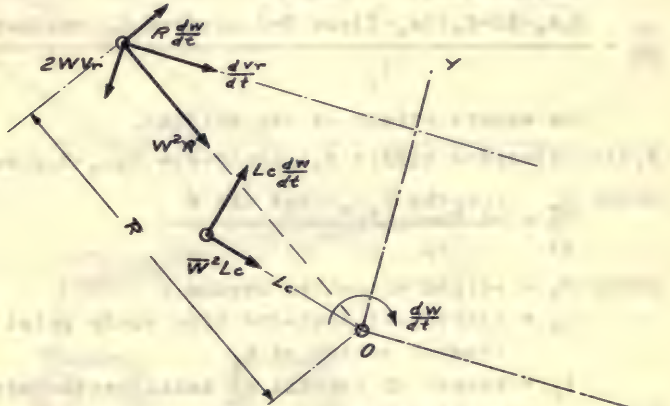
From equation (3) in the previous article, we have

$$P_b - m_R[\frac{dv_R}{dt} + d.\frac{dw}{dt} + w^2(x_o - x)] - P_{hs}\frac{v_R^2}{v_s^2} + F_v + R_t - W_R \sin\phi$$

The terms $m_R\, d\frac{dw}{dt}$ and $m_R\, w^2\, (x_o - x)$ are usually small compared with $m_R\frac{dv_R}{dt}$, hence $v_R = v_s$ approx.

and $P_b - m_R\frac{dv_R}{dt}$ = K the static resistance to recoil (approx)

ACCELERATION COMPONENTS ON JUMP OF FIELD CARRIAGE



INERTIA FORCES ON JUMP OF FIELD CARRIAGE

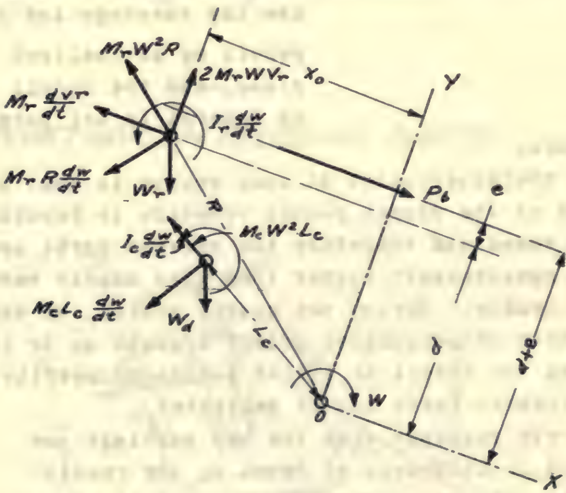


Fig. 5

Substituting in equation (4a) of the previous article, and omitting the term $2\ m_r w\ v_r(x_o - x)$ which is small, we have

$$\frac{dw}{dt} = \frac{P_b e_b + Kd - W_r[(x_o - X)\cos\phi - d\sin\phi] - W_c L_c \cos(\theta - B)}{I_s}$$

The moment effect of the weights,

$W_r[(x_o - x)\cos\phi - d\sin\phi] = W_c L_c \cos(\theta - B) = W_s L_s - W_R x\cos\phi.$

Hence $$\frac{dw}{dt} = \frac{P_b e_b = Kd - W_s L_s + W_R x\cos\phi}{I_s}$$

where W_s = weight of entire system

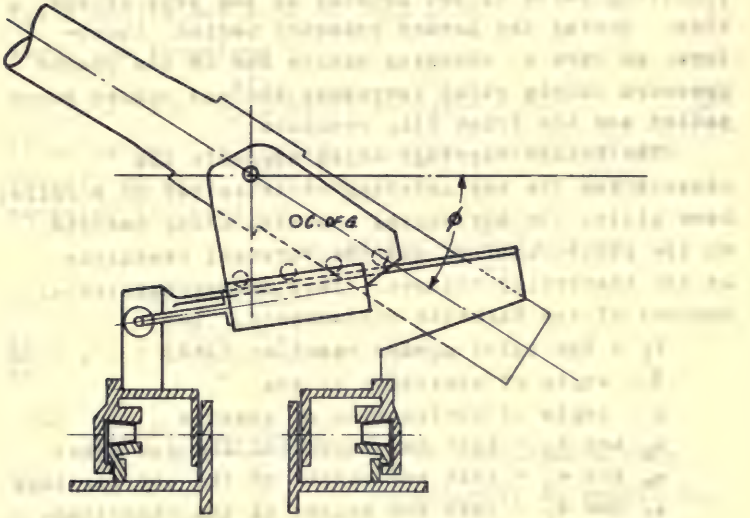
L_s = horizontal distance from spade point to line of action of W_s

I_s = moment of inertia of total system about spade axis

BARBETTE CHASSIS MOUNTS. In this type of mount, the top carriage and gun recoil up an inclined plane, and the recoil in general is not parallel to the bore.

The characteristics of such mounts is that a component of the direct powder reaction is brought upon the mount and therefore the various parts are stressed considerably higher then with mounts recoiling in a cradle. During the powder period, we have an impulsive or percussion effect brought on to the mount, and the effect of finite forces as gravity and the braking force may be neglected.

The gun together with the top carriage are considered in this type of mount as the recoiling parts. The gun has trunnions, and the trunnions are located at the center of gravity of the gun along the axis of the bore. Since there is no regular acceleration in the recoil, the reaction on the elevating gear is practically nil. Due to the weight and position of the center of gravity



REACTIONS ON RECOILING PARTS

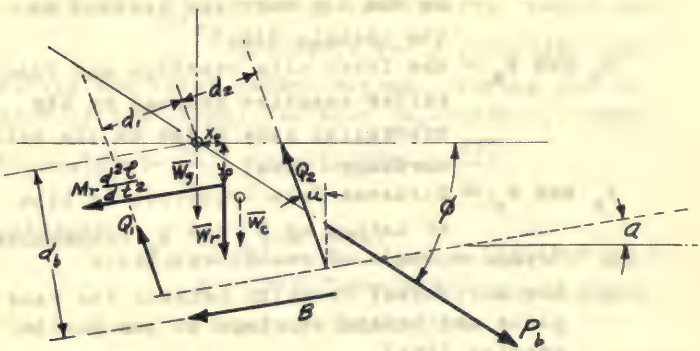


Fig. 6

of the top carriage, the center of gravity of the recoiling parts is not located at the axis of the bore. During the powder pressure period, therefore, we have a whipping action due to the powder pressure couple which increases the end roller reaction and the front clip reaction.

The bottom carriage which supports the chassis for the top carriage is traversed on a roller base plate, the horizontal reaction being carried on the pintle bearing and the vertical reactions by the traversing rollers. This arrangement is typical of any Barbette emplacement. Let

P_b = the total powder reaction (lbs)

$\emptyset$ = angle of elevation of gun

a = angle of inclination of chassis

m_g and w_g = mass and weight of the gun (lbs)

m_c and w_c = mass and weight of the top carriage

m_r and w_r = mass and weight of the recoiling parts

B = the total braking reaction (lbs)

d_b = distance from trunnion to line of action of B (ft)

Q_1 and Q_2 = the front and rear roller reactions on the top carriage exerted by the chassis (lbs)

R_1 and R_2 = the front clip reaction and rear roller reaction exerted by the traversing base plate on the bottom carriage (lbs)

d_1 and d_2 = distance from trunnions to line of action of Q_1 and Q_2 respectively.

n = friction angle of roller reactions.

H = the horizontal reaction between the base plate and bottom carriage at the pintle bearing (lbs)

REACTIONS ON THE RECOILING PARTS

GUN AND TOP CARRIAGE TOGETHER

We have for the motion of the recoiling parts, along the chassis:-

$$P_b\cos(\emptyset+a)-W_r\sin a-B(Q_1+Q_2)\sin u-m_r\frac{d^2l}{dt^2}=0 \qquad (1)$$

normal to the chassis:-

$$P_b\sin(\emptyset+a)+W_r\cos a-(Q_1+Q_2)\cos u=0 \qquad (2)$$

about the trunnions:-

$$Bd_b+Q_2d_2+W_rx_0+m_r\frac{d^2l}{dt^2}[(y_0\cos a)+x_0\sin a]=0 \qquad (3)$$

If we assume the braking constant throughout the recoil, we have, $B+W_r\sin a+(Q_1+Q_2)\sin u = K$ and equation (1) becomes,

$$P_b\cos(\emptyset+a)-m_r\frac{d^2l}{dt^2} - K=0$$

Integrating, we find,

$$\frac{dl}{dt} = v = \int \frac{P_b\cos(\emptyset+a)}{m_r}dt - \frac{K}{m_r}t$$

now

$$\int \frac{P_b dt}{m_r} = \frac{mv+4700\,\bar{m}}{m_r} = V_f$$ or the maximum free velocity of recoil for a recoiling mass m_r, hence

$$\left(\frac{dl}{dt}\right)=v_1=V_f\cos(\emptyset+a)=\frac{Kt}{m_r}$$

Integrating again,

$$l_1 = \int_0^T V_f\cos(\emptyset+a)dt-\frac{KT^2}{2m_r}$$

$= E\cos(\emptyset+a) - \dfrac{KT^2}{2m_r}$ where E is the free recoil displacement for a recoiling mass m_r during the total powder period. During the remainder of the recoil, we have $\frac{1}{2}m_rv_1^2=K(b-l_1)$ hence

$$\frac{1}{2}m_r\left[V_f\cos(\emptyset+a)-\frac{Kt}{m_r}\right]^2=K\left[b-E\cos(\emptyset+a)+\frac{KT^2}{2m_r}\right]$$

Simplifying,

$$K=\frac{m_rV_f^2\cos^2(\emptyset+a)}{2[b-(E-V_fT)\cos(\emptyset+a)]}$$ where E and T are obtained by the methods of Interior Ballistics.

EFFECT OF CHASSIS ROLLER REACTIONS ON THE RECOIL BRAKE.

Assuming only the end roller reactions to come into play, we have, from eq. (1) and (2),

$$\tan u = \frac{K-B-W_r \sin a}{P_b \sin(\phi+a)+W_r \cos a} = f$$

hence $K - B - W_r \sin a = f[P_b \sin(\phi+a)+W_r \cos a]$ where f = coefficient of roller friction. After the powder period, $K-B-W_r \sin a = f\, W_r \cos a$, therefore during the powder period, $B_1 = K-W_r \sin\phi - f[P_b \sin(\phi+a)+W_r \cos a]$ in the recoil, $B = K-W_r \sin\phi - fW_r \cos a$, and the charge of required braking, becomes, $B-B_1 = fP_b \sin(\phi+a)$

REACTIONS ON THE BOTTOM CARRIAGE.

The reactions on the bottom carriage are:-

(1) Q_1 and Q_2 reversed, the roller reactions on the chassis of the top carriage (lbs)

(2) V reversed, the braking reaction (lbs)

(3) The horizontal pintle bearing reaction H.

(4) The weight of the bottom carriage W_{tc}.

(5) The traversing roller and clip reactions R_1 and R_2 (lbs)

Then, resolving forces along and normal to the chassis, we have,

$$(Q_1+Q_2)\sin u + B - H\cos a + (R_2-R_1)\sin a - W_{tc}\sin a = 0 \quad (1')$$

$$(Q_2+Q_1)\cos u + W_{tc}\cos a - H\sin a - (R_2-R_1)\cos a = 0 \quad (2')$$

and for moments about the trunnion,

$$H\, d_h - Bd_b + Q_2 d_2 - Q_1 d_1 - R_2 r_2 - R_1 r_1 - W_{tc} x_o' = 0 \quad (3')$$

where x_o' = the momentum of W_{tc} about the trunnion.

EXTERNAL REACTIONS ON THE SYSTEM CONSISTING OF THE TOTAL MOUNT.

Adding equations (1) and (1'), we have,

$$P_b \cos(\phi+a) - W_r \sin a - m_r \frac{d^2 l}{dt^2} - H\cos a + (R_2-R_1)\sin a - W_{tc} \sin a = 0 \quad (1'')$$

Since $P_b\cos(\emptyset+a)-m_r\frac{d^2l}{dt^2}=K$ and $W_r+W_{tc}=W_s$ the total weight of the mount. Equation (1") reduces to,

$$K-W_s\sin a - H\cos a + (R_2-R_1)\sin a = 0 \qquad (1")$$

Adding (2) and (2'), we have

$$P_b\sin(\emptyset+a)+W_r\cos a+W_{tc}\cos a-H\sin a-(R_2-R_1)\cos a = 0 \qquad (2")$$

Adding (3) and (3'), we have,

$$Hd_h-R_2r_2-R_1r_1+W_rx_o-W_{tc}x_o^1+m_r\frac{d^2l}{dt^2}(y_o\cos a+x_o\sin a)=0 \qquad (3")$$

Eliminating (R_2-R_1) from (1") and (2"), we have

$$K\cos\emptyset-H+P_b\sin a\sin(\emptyset+a)=0 \qquad (a)$$

Eliminating H from (1") and (2"),

$$(R_2-R_1)+K\sin a-P_b\sin(\emptyset+a)\cos a-W_s=0 \qquad (b)$$

and equation (3") reduces to for moments about the trunnion,

$$Hd_h-R_2r_2-R_1r_1+W_r x_o-W_{tc}x_o'+[P_b\cos(\emptyset+a)-K](y_o\cos a+x_o\sin a)=0 \qquad (c)$$

From (a) and (b), we have, $H=K\cos\emptyset+P_b\sin a\sin(\emptyset+a)$

$R_2-R_1=P_b\sin(\emptyset+a)\cos a+W_s-K\sin a$. Substituting the value of H in (c) and combining with (b) we obtain R_2 and R_1 respectively.

PERCUSSION REACTIONS:

The percussion reactions take place during the powder period and are reactions of a magnitude comparable with the powder forces. In an ordinary cradle recoil, the direct effect of the powder reactions are practically eliminated by allowing the gun to recoil along the bore. In mounts of the chassis type, especially when the gun elevates, we have a large component of the powder reaction, which causes the chassis to offer a corresponding reaction.

REACTION ON TOP CARRIAGE

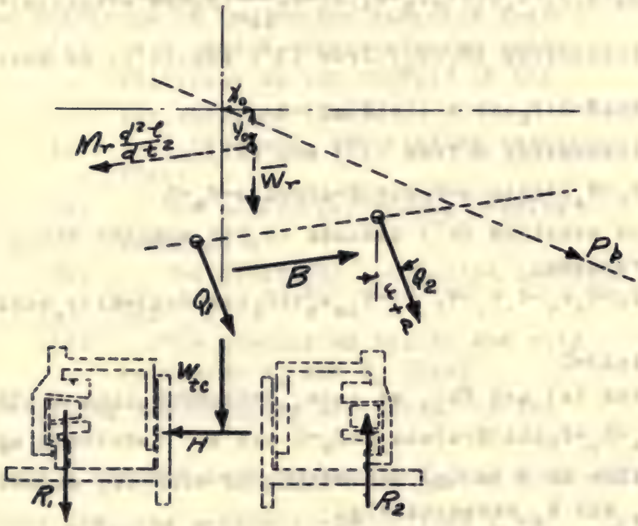
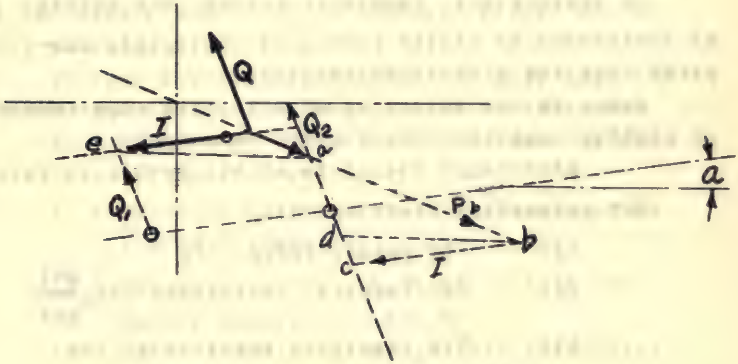


Fig. 7

PERCUSSION REACTION ON RECOILING PARTS



PERCUSSION REACTION ON TOP CARRIAGE

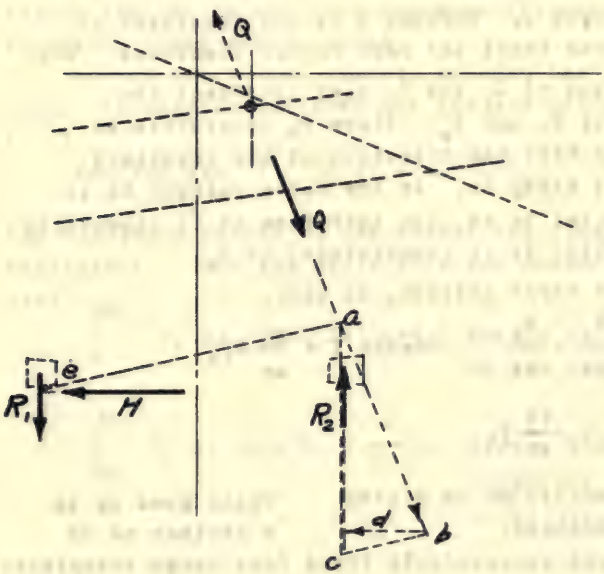


Fig. 8

In dealing with impulsive forces, the effect of continuous or finite forces is negligible compared with the percussion reactions.

Hence in the following we will omit such forces as gravity, and the recoil brake reaction.

PERCUSSION REACTIONS ON THE RECOILING PARTS:

The percussion reactions are,

(1) The powder force --P_b

(2) The inertia resistance $I=m_r\frac{d^2l}{dt^2}$

(3) The resultant reaction of the chassis ---Q

P_b acts along the bore, I acts parallel to the chassis and through the center of gravity of the recoiling parts, while Q balances these reactions at their intersection, as shown in fig.(8).

The force polygon of the percussions is abc, where a b $//P_b$, bc//I and ca//Q. The direction of Q is slightly inclined to the chassis due to the friction angle u. Further Q is the resultant of Q_1 and Q_2 the front and rear roller reactions. Now the resultant of Q_1 and I, must intersect the resultant of P_b and Q_2. Since P_b intersects at Q_2 at a, we have the direction of the resultant of Q, and I along ae. In the force polygon bd is drawn parallel to ae, and therefore cd is proportional to Q_1 while da is proportional to Q_2.

In the force polygon, we have,

$$\frac{P_b}{ab} = \frac{I}{bc} = \frac{Q_1}{cd} = \frac{Q_2}{da} \qquad \text{hence } I = \frac{vc}{ab} P_b,$$

$$Q_1 = \frac{cd}{ab} P_b, \quad Q_2 = \frac{da}{ab} P_b$$

DYNAMICAL RELATIONS ON FIRING FROM AN AEROPLANE. Small guns up to a caliber of 75 m/m have been successfully fired from large aeroplanes.

Larger calibers may be possible by the introduction of the muzzle brake, which thereby reduces the recoil reaction.

In this discussion, however, we will take the simple case of a gun without a muzzle brake. Let

V_o = horizontal velocity of the plane before firing (ft/sec) V_o

V_1 = velocity of the plane immediately after firing (ft/sec) V_1

V_r = velocity of the gun at the end of the powder period (ft/sec) V_r

v = muzzle velocity of projectile (absolute) (ft/sec)

P_b = powder reaction (lbs)

R = recoil reaction (lbs)

m_r and w_r = mass and weight of recoiling parts (lbs)

m_s and w_s = mass and weight of equivalent weight of aeroplane + weight of cradle and mount (lbs)

Assume the gun to be fired horizontally while the aeroplane flies horizontally:

During the powder period, we have the mutual impulsive reaction between the gun and aeroplane = $P_b\,dt$

For the gun, $\int_0^{t_1} P_b\,dt = m_r(V_o - V_r)$ (1)

the impulsive effect of the recoil reaction R being negligible. For the projectile and powder, we have,

$$\int_0^{\Delta t_o} P_b\,dt = (m+0.5\bar{m})(v-V_o) \text{ during the travel up the bore.}$$

$$\int_{t_o}^{\Delta t_1} P_b\,dt = \bar{m}\left[4700-\left(\frac{v-V_o}{2}\right)\right] \text{ during the powder expansion.}$$

- - - - - - - - - - -

$$\int_0^{\Delta t_1} P_b \, dt = \bar{m}(v-V_o)+\bar{m}4700 = mv+\bar{m}4700 \text{ (approx)} \tag{2}$$

Let us now consider the effect of the recoil reaction R on the aeroplane and fixed part of the mount. On firing the aeroplane the aeroplane acts somewhat as an elastic beam, more or less supported by the air reactions at the ends. We may consider, the equivalent mass of the aeroplane and mount attached = 0.7 to 0.8 the actual mass of the plane and mount. We will denote m_s as this equivalent mass.

Then, for the motion of the aeroplane during the recoil period, we have,

$$R = \frac{m_s(V_o-V_1)}{t} \quad \text{(lbs)} \tag{3}$$

and for the motion of the recoiling parts during this same period,

$$R = \frac{m_r(V_1-V_r)}{t} \quad \text{(lbs)} \tag{4}$$

since the recoiling parts must have the same velocity as the plane at the end of recoil.

It is interesting to note the magnitude of the relation of the various velocities for a typical small mount.

V_o = 100 miles/hour = 146.6 ft/sec.

V_o-V_r = 30 ft/sec. roughly; V_r = 116 ft.sec.roughly.

V_1 = between 116 and 146 ft/sec. say 130 ft/sec.

Thus we have a check in the velocity of the plane of several feet per second, the magnitude of which depends of course on the ballistics and relations of the various masses.

Combining the previous equations, we have,

$$m_r(V_o-V_r)=mv+\bar{m}\,4700 \tag{5}$$

$$m_s(V_o-V_1)=m_r(V_1-V_r) \tag{6}$$

- - - - - - - - - -

$$(m_s+m_r)(V_o-V_1)=mv+\bar{m}\,4700 \tag{7}$$

That is, as we should expect from first principles,

the momentum imparted to the aeroplane backwards, equals the momentum imparted to the projectile and powder forwards.

Let us now assume the recoil reaction constant, and let b equal the length of recoil.

Now due to the superior motion of the aeroplane as compared with that of the gun, during the recoil the aeroplane does work on the gun, in bringing the velocity from the smaller value V_r to the larger value V_1 hence

$$R\left(\frac{V_o+V_1}{2}t-b\right)=\frac{m_r(V_1^2-V_r^2)}{2}$$

The energy taken from the aeroplane, becomes

$$R\left(\frac{V_o+V_1}{2}\right)t=\frac{m_s(V_o^2-V_1^2)}{2}$$

hence $-R\,b=\frac{m_r}{2}(V_1^2-V_r^2)-\frac{m_s}{2}(V_o^2-V_1^2)$ therefore, the recoil reaction, becomes

$$R=\frac{1}{b}\left[\left(\frac{m_sV_o^2}{2}+\frac{m_rV_r^2}{2}\right)-\left(\frac{m_s+m_r}{2}\right)V_1^2\right] \quad \text{(lbs)}$$

where $V_r=V_o-\left(\frac{mv+4700\bar{m}}{m_r}\right)$ (ft/sec)

and $V_1=V_o-\left(\frac{mv+4700\bar{m}}{m_s+m_r}\right)$ (ft/sec)

DISAPPEARING AND OTHER TYPES OF CARRIAGES.

TYPES OF DISAPPEARING CARRIAGES. Disappearing gun carriages, as evident by their terminology, are designed, so that in the recoil the gun is brought down below a parapet and disappears from the enemy's view. The gun is loaded in the lower position. By introducing a counterweight, the gun is brought by gravity to the firing position, the gun during the firing period only being above the parapet.

Disappearing gun carriages may be broadly

classified in two general types:-

(1) Revolving or rotating types, where the gun lever rotates about a fixed axis, as in the Moncrieff, Howell and Krupp carriages.

(2) Sliding carriage types, where the Cardon system of linkage is used, the gun lever being constrained to move at two of its points along guides practically at right angles, as in the Buffington Crozier models.

APPROXIMATE THEORY OF THE ROTATING TYPE OF DISAPPEARING CARRIAGE.

The following assumptions are made and the validity of these assumptions will be considered more in detail later:-

(1) The center of gravity of the gun will be assumed at the gun trunnion.

(2) The angular displacement of the gun lever, during the powder period, will be assumed small and will therefore not effect the initial geometrical conditions greatly.

(3) The inertia effect of the elevating rods, will be assumed negligible as compared with that of the gun, lever, gun and counterweight.

(4) The elevating arm, will be assumed approximately parallel to the axis of the gun lever and roughly equal to the upper half of the gun lever.

(5) The angular movement of the gun itself during the powder period will be assumed very small.

From assumptions (3), (4) and (5) we may neglect the reaction of the elevating arm during the powder action period, for the following reasons:

(a) The tangential component of

the elevating arm reaction becomes zero due to assumption (3).

(b) Condition (4) assumes the instantaneous center of the gun practically at infinity. Hence the angular velocity of the gun at the end of the powder period is negligible; the angular acceleration therefore may be assumed zero, and the normal reaction of the elevating arm becomes zero.

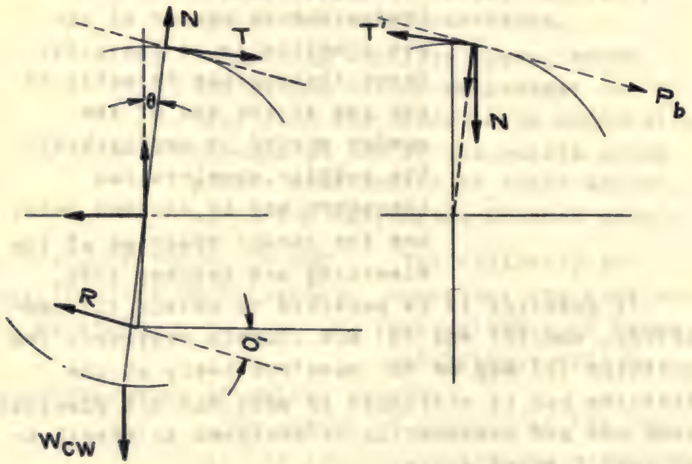
In practice it is possible to obtain (1) completely, and (2) and (3) are closely realized. The condition (4) may be met constructively at one elevation but is difficult to meet for all elevations, since the gun customarily is designed to recoil to the same loading angle.

To reduce the reaction on the elevating arm it is customary to introduce a kick down buffer at the bottom end of the arm, and thus during the powder period a small minor reaction comparable with the buffer resistance is introduced between the elevating arm and gun. This reaction may be neglected as compared with the major reactions of the gun lever.

Therefore, as a first approximation, however, we will neglect the reaction of the elevating arm, and assume the center of gravity of the gun located at the trunnions. Let

W_g = weight of the gun (lbs)
W_r = weight of the gun lever (lbs)
W_{cw} = weight of the counterweight (lbs)
$I_r = W_r k_r^2$ = moment of inertia of gun lever about fixed axis of rotation.
$I_{cw} = W_{cw} k_{cw}^2$ = moment of inertia of counterweight about fixed axis of rotation.

REACTIONS ON THE ROCKER AT GUN
DURING POWDER PERIOD



REACTIONS ON THE ROCKER AT GUN
AFTER POWDER PERIOD

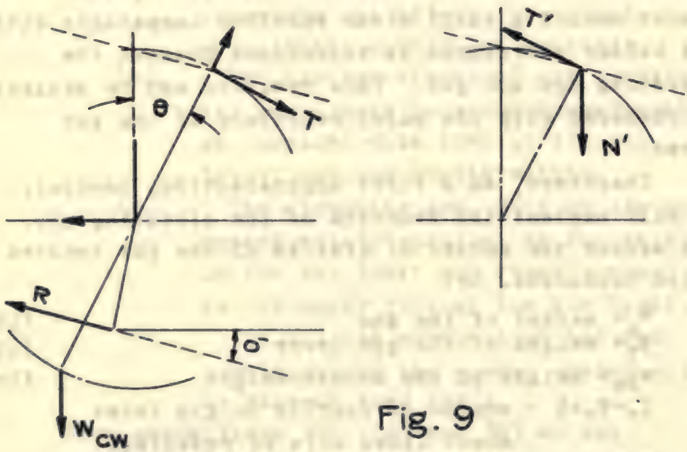


Fig. 9

T and N = tangential and normal trunnion reaction (lbs)

X and Y = horizontal and vertical reactions at axis of rotation of gun lever (lbs)

P = total powder reaction (lbs)

P_m = maximum powder reaction (lbs)

$\emptyset$ = angle of elevation of gun

θ_i = initial angle of gun lever with respect vertical

θ_f = final angle of gun lever with respect vertical

r = radius of upper half of gun lever (ft)

r' = radius to center of gravity of counterweight (ft)

R = reaction of oscillating cylinder brake

d_i = initial angle R makes with the normal to r'

r_o = distance from axis along r' to line of action of R.

m = mass of projectile

$\bar{m}$ = mass of powder charge

v = muzzle velocity (ft/sec)

$T_f V_f$ = total friction torque resisting rotation

From fig.(9). the gun axis makes an agle $\emptyset - \theta_i$ with the tangent of the path in the initial position of the gun.

For the motion of the gun lever, we have for moments about the fixed axis,

$$Tr = (I_r + I_{cw}) \frac{d^2\theta}{dt^2} + R \cos d \cdot r_o + T_f r_f \qquad (1)$$

and for the motion of the gun along the tangent to its initial path,

$$P\cos(\emptyset - \theta_i) - T = \frac{w_g}{g} r \frac{d^2\theta}{dt^2} \qquad (2)$$

If s = the displacement along the arc of the gun trunnion

V = the corresponding tangential velocity of the gun trunnions,

$\frac{ds}{dt} = r\frac{d\theta}{dt}$; $\frac{d^2s}{dt^2} = \frac{dV}{dt} = r\frac{d^2\theta}{dt^2}$ Hence, combining the two equations, we have,

$$P\cos(\emptyset-\theta_i)=\left(\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}\right)\frac{dV}{dt}+R\cos d;\ \frac{r_o}{r}+T_f\ \frac{r_f}{r} \qquad (3)$$

Evidently $\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}$ may be regarded as the so called equivalent translatory mass at the gun trunnions due to the rotational inertia effect of the gun lever and counterweight.

Integrating equation (3), we have,

$$V = \int \frac{P\cos(\emptyset-\theta_i)}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}\,dt - \int \frac{R\cos d\ \frac{r_o}{r}\ dt}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}} - \int \frac{T_f\frac{r_f}{r}\ dt}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}$$

Now both $\emptyset$ and d as well as the friction torque $T_f r_f$ vary during the powder period but as the change is small, we are quite justified in assuming them constant. Further, since, $Pdt=(m+0.5\bar{m})v$ (during the travel of the shot up the bore), we have

$$\frac{P\cos(\emptyset-\theta_i)}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}\,dt = \frac{(m+0.5\bar{m})v}{\left(\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}\right)}\cos(\emptyset-\theta_i)$$

or in terms of the free velocity of recoil,

$$V_f\cos(\emptyset-\theta_i)= \frac{(m+0.5\bar{m})v}{\left(\frac{w_r}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}\right)}\cos\ (\emptyset-\theta_i)$$

where V_f is the equivalent free velocity with a recoiling mass equal to

$$\left(\frac{w_r}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}\right)$$

Integrating again, we have

$$X_f\cos(\emptyset-\theta_i)=\left[\frac{m+0.5\bar{m}}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}\right]x'\ \cos(\emptyset-\theta_i)$$

where x' = the absolute displacement of the projectile up the bore. Now

$x'\cos(\emptyset-\theta_i) = u\cos(\emptyset-\theta_i) - X_f\cos(\emptyset-\theta_i)$ hence, we have

$$X_f\cos(\emptyset-\theta_i) = \left[\frac{(m+0.5\bar{m})u\cos(\emptyset-\theta_i)}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}+m+0.5\bar{m}}\right]$$

now $m+0.5\bar{m}$ is small compared with $\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}$ hence we may assume

$$X_f\cos(\emptyset-\theta_i) = \frac{(m+0.5\bar{m})u\cos(\emptyset-\theta_i)}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}} \quad \text{(ft)}$$

The equations of recoil, become therefore

$$V = V_f\cos(\emptyset-\theta_i) - \frac{(R\cos d\frac{r_o}{r}+T_f\frac{r_f}{r})t}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}$$

and

$$S = X_f\cos(\emptyset-\theta_i) - \frac{(R\cos d\frac{r_o}{r}+T_f\frac{r_f}{r})t^2}{2(\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2})} \quad \text{(approx)}$$

where $V_f = \frac{(m+0.5\bar{m})v}{(\frac{w_r}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2})}$ and $X_f \frac{(m+0.5\bar{m})u}{(\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2})+m+0.5\bar{m}}$

We see the equations of recoil during the powder period are exactly similar to the previous recoil equation, the recoiling mass now including the inertia effect of the rotating elements. Hence the previous interior ballistic formulas are immediately applicable for the computation of the free recoil displacement E and the time of the powder period t_e.

For the maximum velocity of recoil, we have

$$V_{fm} = \frac{mv+4700\bar{m}}{\frac{w_r}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}} \quad \text{(ft/sec)}$$

and the max. velocity of constrained recoil along the path of the gun trunnion, becomes,

$$V_m = V_{fm}\cos(\phi-\theta_i) - \frac{(R\cos d\frac{r_o}{r}+T_f\frac{r_f}{r})t_c}{\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}$$

$$S_m = E\cos(\phi-\theta_i) - \frac{(R\cos d\frac{r_o}{r}+T_f\frac{r_f}{r})t_g^2}{2(\frac{w_g}{g}+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2})}$$

The corresponding maximum angular velocities and angular displacements, become,

$$w_m = (\frac{d\theta}{dt})m = \frac{V_m}{r} \quad \text{and} \quad \theta_m = \frac{S_m}{r}$$

The energy of recoil at the end of the powder period becomes,

$$A_m = \frac{1}{2}(I_r+I_{cw}+\frac{w_g}{g}r^2)w_m^2$$

From the energy equation we may easily consider the remainder of the recoil.

Since the brake and friction resistances are small compared with the powder reaction and the inertia resistance of the rotating parts, we may assume with sufficient accuracy that $V_m = V_{fm}\cos(\phi-\theta_i)$ and $S_m = E\cos(\phi-\theta_i)$ We have, for the recoil energy at any angular displacement θ.

$$\int_{\theta_i}^{\theta_f} (R\cos d.r_o)d\theta + \int_{\theta_i}^{\theta_f} T_f r_f d\theta + W_{cw}r'(\cos\theta_i - \cos\theta)$$

$- W_{gr}\,r(\cos\theta_i - \cos\theta) = A_m - A$ where W_{gr} = weight of gun and that portion of the rocker, not including the counter weight reduced to an equivalent weight at the gun trunnion, that is

$$W_{gr}=W_g+\frac{W_r r_r}{r}$$ r_r = distance from axis to center of gravity of rocker. Since d varies with the angular displacement of the gun lever, from a layout we may readily evaluate the term

$\int_{\theta_i}^{\theta}(R\cos d\ r_o)d\theta$ provided R is assumed constant which is usually the case.

Further since $T_f r_f$ does not vary greatly we may assume it constant. As a close approximation,

$T'_f r'_f = u(W_g+W_r+W_{cw})r'_f$ $\quad u = 0.15$ roughly

$T''_f . r''_f = u\ W_g$ $\quad$ where r'_f = radius of bearing of axis of rotation of rocker

r''_f = radius of trunnion. Further $T_f r_f = T'_f r'_f + T''_f r''_f$

Hence

$$\int_{\theta_i}^{\theta_f} T_f r_f\ d\theta = T_f r_f(\theta_f-\theta_i) \quad \text{(ft/lbs)}$$

now $A = \frac{1}{2}(I_r+I_{cw}+\frac{W_g}{g}r^2)w^2$ hence $\frac{d\theta}{dt} = w = \sqrt{\frac{2A}{I_r+I_{cw}+\frac{W_g}{g}r^2}}$

(rad/sec)

REACTIONS ON THE CORDAN LINKAGE DISAPPEARING CARRIAGE DURING THE POWDER PERIOD.

Reactions on the Gun: The center of gravity of the gun is assumed at the trunnion axis of the gun. The angular acceleration of the gun is assumed small and the reaction of the elevating arm on the gun is considered a secondary force, this being possible by a proper arrangement of the parts or by the introduction of a kick down buffer at the base of the elevating arm.

The primary reactions on the gun consist:

(1) The powder force along the axis of the bore = P_b

REACTIONS ON THE GUN

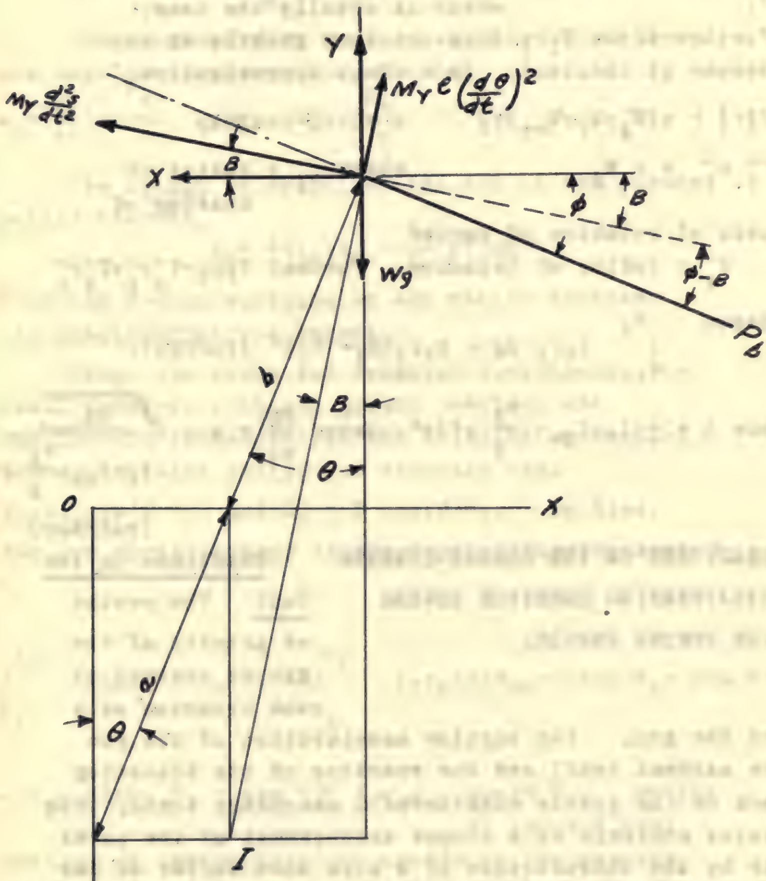


Fig. 10

(2) The trunnion reactions divided into horizontal and vertical components X and Y respectively.

(3) The weight of the gun acting through the trunnion axis = W_g

(4) The tangential inertia force along the path of the movement of the trunnions or normal to a line from instantaneous axis to the trunnion axis =

$$m_g\frac{d^2s}{dt^2}$$

(5) The centrifugal inertia force, normal to the path of the trunnion axis and proportional to the square of the angular velocity =

$$m_g l\left(\frac{d\theta}{dt}\right)^2$$

The secondary reactions on the gun are:

(1) The elevation arm reaction on the gun comparable with the kick down buffer reaction at the base of the elevating arm.

(2) The inertia couple due to the angular acceleration of the gun about the trunnion axis.

In the following analysis, we will neglect the effect of the secondary reactions. The forces on the gun neglecting the secondary forces are shown in fig.(

Since we assume the rotation negligible, we have the equations of motion,

$$P_b\cos\emptyset - m_g\frac{d^2s}{dt^2}\cos B + m_g l\left(\frac{d\theta}{dt}\right)^2\sin B - X = 0$$

$$P_b\sin\emptyset - m_g\frac{d^2s}{dt^2}\sin B - m_g k\left(\frac{d\theta}{dt}\right)^2\cos B + W_g - Y = 0$$

where $\tan B = \frac{b}{a+b}\tan\theta$; $\frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2}$ approx. since l does not change greatly during the powder period.

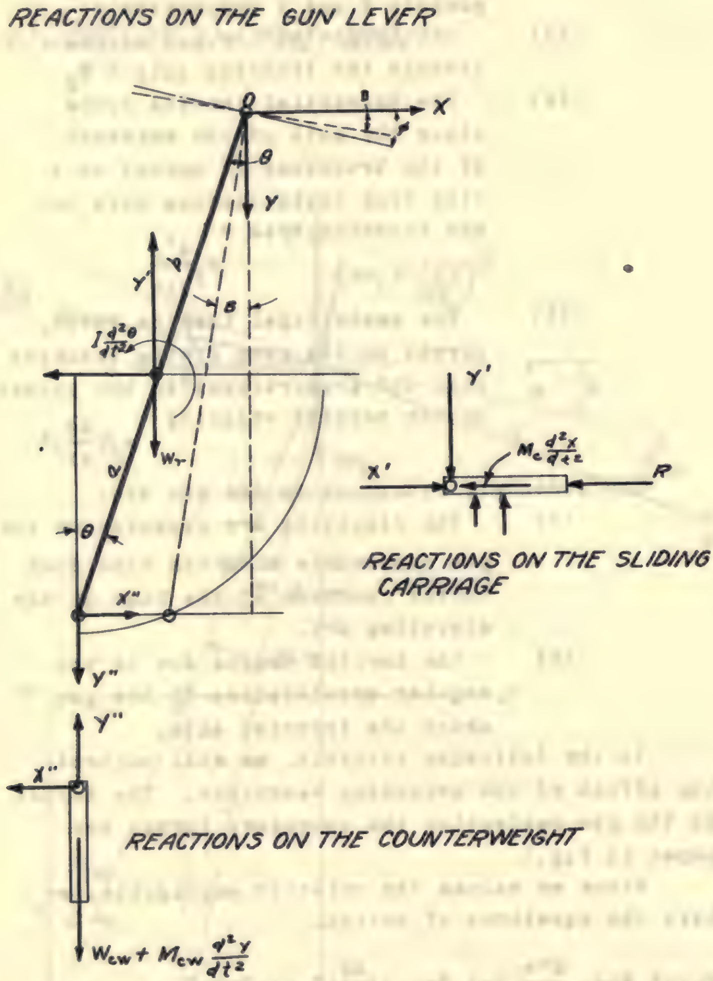

REACTIONS ON THE GUN LEVER
O
X
Y
θ
Y'
X''
Y''
$I\frac{d^2\theta}{dt^2}$
W_T
REACTIONS ON THE SLIDING CARRIAGE
X'
R
$M_c\frac{d^2x}{dt^2}$
REACTIONS ON THE COUNTERWEIGHT
$W_{cw} + M_{cw}\frac{d^2y}{dt^2}$

Fig. 11

$l = \sqrt{(a^2+2ab)\cos^2\theta+b^2}$ Hence, the trunnion reactions become,

$$X=P_b\cos\emptyset-m_g l\frac{d^2\theta}{dt^2}\cos B+m_g l(\frac{d\theta}{dt})^2\sin B$$

$$Y = P_b\sin\emptyset-m_g l\frac{d^2\theta}{dt^2}\sin B-m_g l(\frac{d\theta}{dt})^2\cos B+W_g$$

REACTIONS ON THE ROCKER.

The reactions on the rocker, are:

(1) The reaction of the gun on the rocker divided into components X and Y.

(2) The reaction of the sliding carriage on the rocker at the rocker trunnion, divided into components X' and Y'.

(3) The reaction of the counterweight cross head at the wrist pin of the cross head, divided into components X" and Y".

(4) The weight of the rocker at the center of gravity assumed at the rocker trunnion W_r.

(5) The rotational inertia couple due to the angular acceleration of the rocker = $I_r\frac{d^2\theta}{dt^2}$

(6) The tangential inertia force of the rocker along $OX = m_r\frac{d^2x}{dt^2}$ acting through center of gravity of rocker.

(7) The centrifugal inertia force of the rocker normal to $OX = m_r(IA)(\frac{d\theta}{dt})^2$

The equations of motion of the rocker, become, along OX ----

$$X-X'+X"-m_r\frac{d^2x}{dt^2} = 0$$

along OY ----

$$Y+Y''+W_r-m_r(IA)\left(\frac{d\theta}{dt}\right)^2=0$$

about the instantaneous axis I,

$$X(a+b)\cos\theta+Y\,b\sin\theta-X'a\cos\theta-Y''\,a\sin\theta - m_r\frac{d^2x}{dt^2}\,a\cos\theta - I_r\frac{d^2\theta}{dt^2} = 0$$

REACTIONS ON THE SLIDING CARRIAGE AND COUNTER WEIGHT RESPECTIVELY IN THE DIRECTION OF THEIR MOTIONS.

Considering the sliding carriage, we have,

$X'-R-m_c\frac{d^2x}{dt^2}=0$ Where R is the hydraulic brake reaction on the carriage and for the counterweight,

$$Y''-m_{cw}\frac{d^2y}{dt^2} - w_{cw}=0$$

EQUATION OF MOTION OF THE SYSTEM DURING THE POWDER PRESSURE PERIOD.

Substituting the values of X', Y", X and Y in the moment equation about the instaneous axis of the rocker, we have,

$$P_b[(a+b)\cos\theta\cos\phi+b\sin\theta]-m_g[(a+b)^2\cos^2\theta+b^2\sin^2\theta]\frac{d^2\theta}{dt^2}$$

$$+W_g b\sin\theta-[R+(m_r+m_c)\frac{d^2x}{dt^2}]a\cos\theta\; m_{cw}\frac{d^2y}{dt^2}\,a\sin\theta$$

$$-W_{cw}\,a\sin\theta - I_r\frac{d^2\theta}{dt^2} = 0$$

Now $x = a\sin\theta$

$$\frac{dx}{dt} = a\cos\theta\frac{d\theta}{dt}$$

$$\frac{d^2x}{dt^2} = a\cos\theta\frac{d^2\theta}{dt^2} - a\sin\theta\left(\frac{d\theta}{dt}\right)^2$$

and $y = a \cos \theta$

$$\frac{dy}{dt} = - a \sin \theta \frac{d\theta}{dt}$$

$$\frac{d^2y}{dt^2} = - a \sin \theta \frac{d^2\theta}{dt^2} - a \cos \theta \left(\frac{d\theta}{dt}\right)^2$$

If we assume the positive direction of y upward, then

$$\frac{d^2y}{dt^2} = a \sin \theta \frac{d^2\theta}{dt^2} + a \cos \theta \left(\frac{d\theta}{dt}\right)^2$$

Substituting these values in the above equation we have the general dynamical equation of the disappearing carriage during the powder pressure in terms of a single coordinate variable θ.

The differential equation of motion, becomes,

$$P_b[(a+b)\cos \theta \cos\emptyset + b \sin \theta \sin \emptyset] - m_g[(a+b)^2\cos^2 \theta + b^2 \sin^2 \theta] \frac{d^2\theta}{dt^2} + W_g b \sin \theta - R\, a \cos \theta - (m_r+m_c)a^2\cos^2 \theta \frac{d^2\theta}{dt^2} - m_{cw}a^2 \sin^2 \theta \frac{d^2\theta}{dt^2} + (m_r+m_c)a \sin \theta \cos \theta\left(\frac{d\theta}{dt}\right)^2 - m_{cw}\, a \sin \theta \cos \theta\left(\frac{d\theta}{dt}\right)^2 - W_{cw}\, a \sin \theta - I_r \frac{d^2\theta}{dt^2} = 0$$

Combining terms, we have,

$$P_b[(a+b)\cos \theta \cos\emptyset + b \sin \theta \sin\emptyset] - \left\{m_g [(a+b)^2 \cos^2 \theta + b^2 \sin^2 \theta] + (m_r+m_c)a^2 \cos^2 \theta + m_{cw}a^2 \sin^2 \theta + I_r\right\} \frac{d^2\theta}{dt^2} + [(m_c+m_r)a \sin \theta \cos \theta - m_{cw}\, a \sin \theta \cos \theta]\left(\frac{d\theta}{dt}\right)^2 - R\, a \cos \theta + W_g\, b \sin \theta - W_{cw}a \sin \theta = 0$$

The equation is in the form of $A\frac{d^2\theta}{dt^2} + B\left(\frac{d\theta}{dt}\right)^2 + C = 0$

where

$A = m_g[(a+b)^2\cos^2\theta + b^2\sin^2\theta] + (m_r+m_c)a^2\cos^2\theta + m_{cw}a^2$

$\sin\theta + I_r$

$B = -[(m_c+m_r)a\sin\theta\cos\theta\ m_{cw}a\sin\theta\cos\theta]$

$C = -P_b[(a+b)\cos\theta\cos\phi + b\sin\theta\sin\phi] + Ra\cos\theta - W_g$

$b\sin\theta + W_{cw}\ a\sin\theta$

CALCULATION OF THE RECOIL DURING THE POWDER PRESSURE PERIOD.

The general equation of motion for the system, becomes,

$$P_b[(a+b)\cos\theta\cos\phi + b\sin\phi\sin\phi] - [m_g(a+b)^2\cos^2\theta + b^2\sin^2\theta] + [(m_r+m_c)a^2\cos^2\theta + m_{cw}a^2\sin^2\theta + I_r]$$

$$\frac{d^2\theta}{dt^2} + [(m_c+m_r)a\sin\theta\cos\theta - m_{cw}a\sin\theta\cos\theta]\left(\frac{d\theta}{dt}\right)^2$$

$$-R\,a\cos\theta + W_g b\sin\theta - W_{cw}\,a\sin\theta = 0$$

We may write this, as

$$AP_b - B\frac{d^2\theta}{dt^2} + C\left(\frac{d\theta}{dt}\right)^2 - D = 0$$

where $A = (a+b)\cos\theta\cos\phi + b\sin\theta\sin\phi$

$B = m_g[(a+b)^2\cos^2\theta + b^2\sin\theta]$
$+ [(m_r+m_c)a^2\cos\theta + m_{cw}a^2$
$\sin^2\theta + I_r]$

$C = (m_c+m_r)a\sin\theta\cos\theta - m_{cw}a\sin\theta\cos\theta$

$D = Ra\cos\theta - W_g b\sin\theta + W_{cw}a\sin\theta$

Integrating, we have

$$A\int_0^t P_b dt - B\left(\frac{d\theta}{dt}\right) + C\int_0^t \left(\frac{d\theta}{dt}\right)^2 - Dt = 0$$

now $\int P_b dt = (m+0.5\bar{m})v$ during the travel up the bore

where m = mass of the projectile

$\bar{m}$ = mass of the powder charge

v = velocity of the projectile in the bore.

hence

KINEMATICS OF DISAPPEARING CARRIAGE

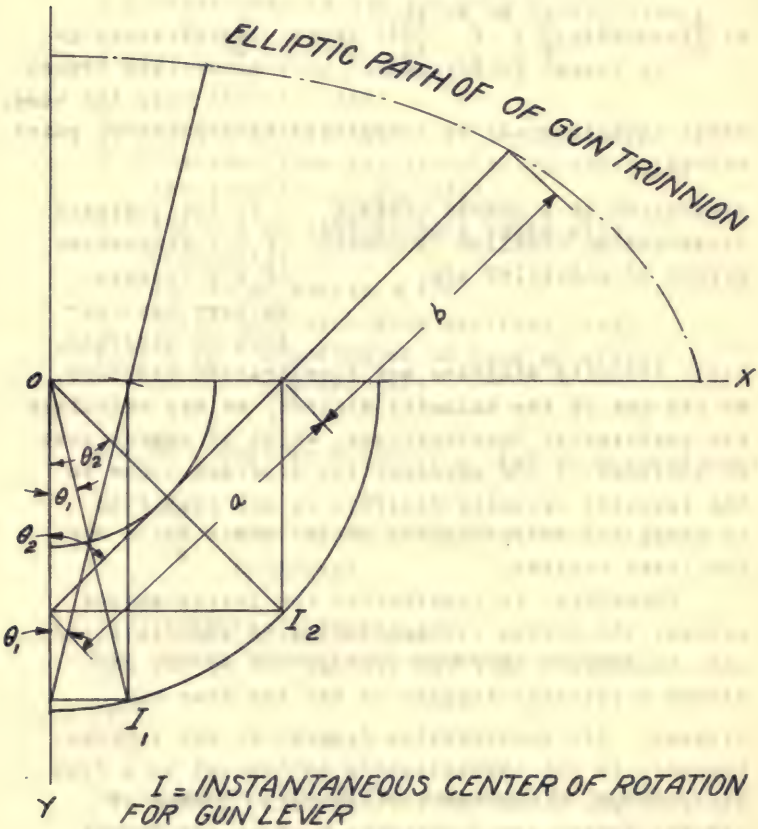


Fig. 12

$$\frac{d\theta}{dt} = \frac{A}{B}(m+0.5\bar{m})v + \frac{C}{B}\int_0^t \left(\frac{d\theta}{dt}\right)^2 dt - \frac{D}{B}t$$ which is the general expression of the angular velocity of the system during the travel up the bore. Integrating again,

$$\theta = \frac{A}{B}(m+0.5\bar{m})u + \frac{C}{B}\int_0^t \int_0^t \left(\frac{d\theta}{dt}\right)^2 dt.dt - \frac{D}{2B}t^2$$ where u= the travel up the bore.

These equations may be integrated by a point by point method.

KINEMATICS OF A CORDAN LINKAGE DISAPPEARING CARRIAGE INCLUDING EFFECT OF ELEVATING ARM.

In the analysis of the kinematics of any linkage, we have two systems of diagrams, viz:- velocity diagrams and acceleration diagrams. By the use of the velocity diagram, we may calculate the centripetal accelerations, which of course must be included in the acceleration diagrams. Due to the required velocity diagram, we are justified in using the instantaneous center about which the gun lever rotates.

Therefore, in considering the instantaneous center, the cordan linkage including the elevating arm, becomes, a four bar linkage and we may construct a velocity diagram as for any four bar linkage. The acceleration diagram of the linkage, however, is not theoretically equivalent to a four bar linkage, since the instantaneous center of the gun lever, has a definite path in the recoil. Hence the distance from the Instantaneous center of the gun trunnion, changes in the recoil and the tangential acceleration becomes,

$$\frac{d(wl)}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt}$$ Since l does not change greatly during the powder period we are justified in omitting

$w \frac{dl}{dt}$ being small then $\frac{d(wl)}{dt} = l\frac{dw}{dt} \quad l\frac{d^2\theta}{dt^2}$

Consider any position of the linkage during the powder period: see fig.(13).
Let

θ = angle made by gun lever or rocker with vertical (rad)

l = distance from instantaneous center to gun trunnion (ft)

B = angle made by l with vertical (rad)

d = distance from gun trunnion to elevating arm trunnion on gun (ft)

Ø = angle d or axis of bore makes with horizontal

C = length of elevating arm

Q = angle made by c with vertical (rad)

x_o and y_o = coordinates of base of elevating arm (ft)

$w = \frac{d\theta}{dt}$ = angular velocity of gun lever (rad/sec)

$w' = \frac{dQ}{dt}$ = angular velocity of elevating arm (rad/sec)

VELOCITY DIAGRAM.

The linear velocity of point O, becomes,

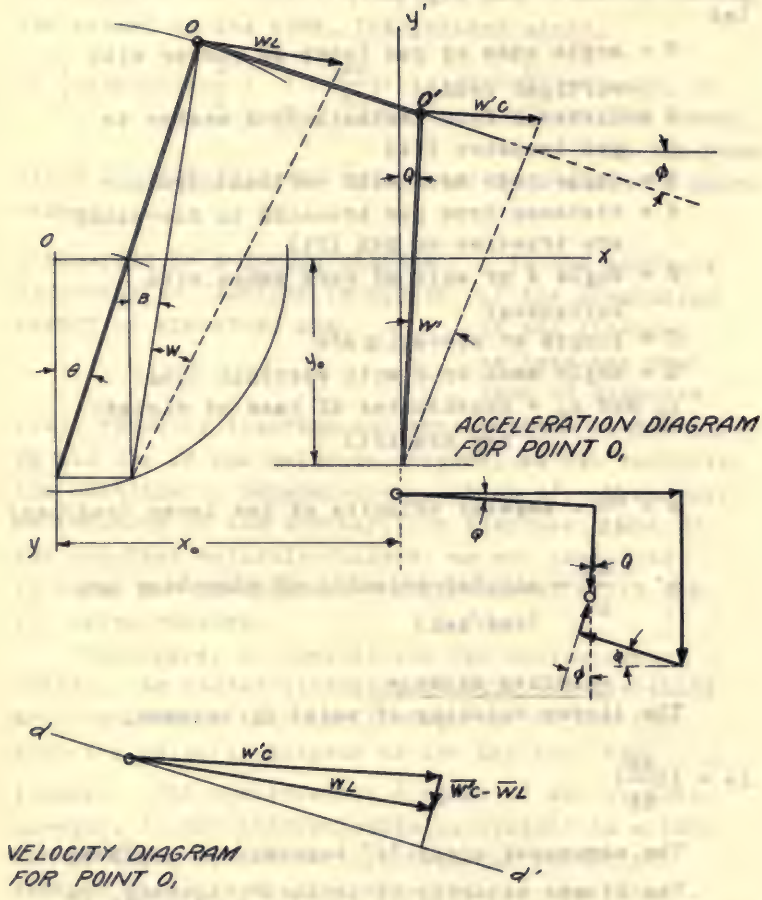
$$lw = l(\frac{d\theta}{dt})$$

The component along "d" becomes, $lw \cos(\phi - B)$

The linear velocity of point Q', becomes, cw'
Its component along d, becomes, $cw' \cos(\phi - Q)$
Hence $lw \cos(\phi - B) = cw' \cos(\phi - Q)$

$$\text{and } w' = \frac{l}{c}\,\frac{\cos(\phi-B)}{\cos(\phi-Q)}\,w \quad \therefore \frac{dQ}{dt} = \frac{l}{c}\,\frac{\cos(\phi-B)}{\cos(\phi-Q)}\,\frac{d\theta}{dt} \qquad (1)$$

The angular rotation about the trunnions, equals,



$$\frac{d\phi}{dt} = \frac{W'c - WL}{OO_1}$$

Fig. 13

$$\frac{d\phi}{dt} = \frac{\overline{Vel.O}-\overline{Vel.O}}{d} = \frac{w'c\ \sin(\phi-Q)-wl\ \sin(\phi-B)}{d}$$

$$= [l\ \frac{\cos(\phi-B)}{\cos(\phi-Q)}\ \sin(\phi-Q)-l\ \sin(\phi-B)]\frac{w}{d}$$

$$= \frac{l}{d}\ [\tan(\phi-Q)\cos(\phi-B)-\sin(\phi-B)]\,(\frac{d\theta}{dt}) \qquad (2)$$

ACCELERATION DIAGRAM:

The acceleration of point O, the center of gravity of the gun, along the x axis,

$$\frac{d^2x}{dt^2}g = l\ \frac{d^2\theta}{dt^2}\cos B - l(\frac{d\theta}{dt})^2 \sin B$$

Along the y axis,

$$\frac{d^2y}{dt^2}g = l\frac{d^2\theta}{dt^2}\sin B + l(\frac{d\theta}{dt})^2\cos B$$

These values do not include the effect of the small change in "l" in the powder period. To include this, we have $x_g=(a+b)\sin\theta$

$$\frac{dx_g}{dt} = (a+b)\cos\theta\ \frac{d\theta}{dt}$$

$$\frac{d^2x_g}{dt^2} = (a+b)\ \cos\theta\ \frac{d^2\theta}{dt^2} - (a+b)\sin\theta(\frac{d\theta}{dt})^2 \qquad (3)$$

and $y_b = -\ b\ \cos\ \theta$

$$\frac{dy_g}{dt} = b\ \sin\theta\ \frac{d\theta}{dt}$$

$$\frac{d^2y_g}{dt^2} = b\ \sin\theta\ \frac{d^2\theta}{dt^2} + b\ \cos\theta\ (\frac{d\theta}{dt})^2 \qquad (4)$$

The acceleration of O', is divided into the following components:

(1) The acceleration of O, divided into two components

$$\frac{d^2x_g}{dt^2}\ \text{and}\ \frac{d^2y_g}{dt^2}$$

(2) The centripetal acceleration of O', about O directed along d towards O =

$$d\left(\frac{d\emptyset}{dt}\right)^2$$

(3) The tangential acceleration of O', about O, normal to d and equal to

$$d.\frac{d^2\emptyset}{dt^2}$$

Since O', is a common point for both the gun and the elevating arm, we have, also, the acceleration of O', divided into,

(1) The tangential acceleration of the gun lever at O

$$= c\,\frac{d^2Q}{dt^2} = c\,\frac{dw'}{dt}$$

(2) The centripetal acceleration of the gun lever at O

$$= c\left(\frac{dQ}{dt}\right)^2 = cw'^2$$

From the acceleration diagram, we have the following vector equation

$$\overline{\frac{d^2x_g}{dt^2}}+\overline{\frac{d^2y_g}{dt^2}}+\overline{d\left(\frac{d\emptyset}{dt}\right)^2}+\overline{d\frac{d^2\emptyset}{dt^2}}=\overline{c\frac{d^2Q}{dt^2}}+\overline{c\left(\frac{dQ}{dt}\right)^2}$$

The two unknowns in the equation, are

$d\frac{d^2\emptyset}{dt^2}$ and $c\,\frac{d^2Q}{dt^2}$ which we will denote by a_d and a_c but we have two coordinate equations along ox and oy and hence a solution is possible.

The solution may either graphical or analytical.

We have, from (3) and (4)

$$\frac{d^2x_g}{dt^2} = (a+b)\left[\cos\theta\,\frac{d^2\theta}{dt^2} - \sin\theta\left(\frac{d\theta}{dt}\right)^2\right]$$

$$\frac{d^2y_g}{dt^2} = (b)[\sin\theta\frac{d^2\theta}{dt^2}+\cos\theta(\frac{d\theta}{dt})^2]$$

$$d(\frac{d\emptyset}{dt})^2=[\tan(\emptyset-Q)\cos(\emptyset-B)-\sin(\emptyset-B)]^2 \frac{l^2(\frac{d\theta}{dt})^2}{d}$$

$$c(\frac{dQ}{dt})^2 = cw'^2 = \frac{l^2}{c}\frac{\cos^2(\emptyset-B)}{\cos^2(\emptyset-Q)}(\frac{d\theta}{dt})^2$$ From the acceleration diagram, we have along the x axis.

$$\frac{d^2y_g}{dt^2} - d(\frac{d\emptyset}{dt})^2\cos\emptyset+a_d\sin\emptyset=a_c\cos Q-c(\frac{dQ}{dt})^2\sin Q$$

$$\frac{d^2y_g}{dt^2} - d(\frac{d\emptyset}{dt})^2\sin\emptyset-a_d\cos\emptyset=a_c\sin Q+c(\frac{dQ}{dt})^2\cos Q$$

then,

$$a_c\sin Q\cos Q=\frac{d^2x_g}{dt^2}\sin Q-d(\frac{d\emptyset}{dt})^2\sin Q\cos\emptyset+a_d\sin Q\sin\emptyset+c(\frac{dQ}{dt})^2\sin^2 Q$$

$$a_c\sin Q\cos Q=\frac{d^2y_g}{dt^2}\cos Q-d(\frac{d\emptyset}{dt})^2\sin\emptyset\cos Q-a_d\cos\emptyset\cos Q-c(\frac{dQ}{dt})^2\cos^2 Q$$

ROTATING TYPE CARRIAGE: REACTIONS ON TRUNNION AND FIXED AXIS OF ROCKER. The maximum reactions on trunnions and main bearing(fixed axis of rocker) are at a maximum value at the maximum powder pressure, and therefore we only need to consider these values in determining the strength of parts. The powder reaction is mainly balanced by the inertia resistance offered by the gun and the revolving parts. The reaction exerted on the rocker at the trunnions, is that needed to overcome the angular inertia of the rocker and counterweight which in turn must be equal to the powder reaction increases the inertia resistance offered by the gun. Therefore the heavier the gun as compared with the re-

volving parts, the smaller the effect of the powder reaction.

The reaction of the main bearing is considerably augmented over that of the trunnion reactions due to the tangential inertia forces of the counterweight. The development of the Cordan linkage in which the rocker bearing is allowed to slide back on a top carriage has been largely to decrease the reaction at the main bearing when fixed as in the revolving type.

At the maximum powder force, the recoil velocity of the gun is small and therefore the centrifugal force of the gun may be neglected. The tangential component of the trunnion reaction, becomes.

$$T=P_{max}\cos(\emptyset-\theta_i)+W_g\sin\theta_i-m_g\frac{d^2s}{dt^2} \quad \text{where} \quad \frac{d^2s}{dt^2}=\frac{P\cos(\emptyset-\theta_i)}{m_g+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}$$

and for the normal component, $N=P_{max}\sin(\emptyset-\theta_i)+W_g\cos\theta_i$

Therefore, we have

$$T=P_{max}\cos(\emptyset-\theta_i)\left(1-\frac{m_g}{m_g+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}\right)+W_g\sin\theta_i$$

$$N=P_{max}\sin(\emptyset-\theta_i)+W_g\cos\theta_i$$

and for the resultant trunnion reaction $S_t=\sqrt{N^2+T^2}$

The maximum bending moment in the rocker or gun lever occurs at a section adjacent to the center bearing of the rocker. This bending moment is due to the moment of the reaction of the gun at the trunnion minus the inertia moment of that part of the rocker above the section, which is practically one-half the mass of the rocker or gun lever.

The moment of the inertia resistance of the rocker, becomes,

$$\frac{1}{2}I_r\frac{d^2\theta}{dt^2}=\frac{1}{2}I_r\frac{\frac{d^2s}{dt^2}}{r}=\frac{1}{2}\frac{I_r}{r^2}r\frac{d^2s}{dt^2} \quad \text{where } \frac{I_r}{r^2} \text{ is the}$$

equivalent mass of the gun lever referred to the

trunnions. The maximum bending moment at center section of the rocker or gun lever, becomes,

$M_o=(T - \frac{1}{2}\frac{I_r}{r^2}\frac{d^2s}{dt^2})r$ or in terms of the maximum powder force

$$M_o=[P_{max}\cos(\emptyset-\theta_i)(1 - \frac{m_g-\frac{1}{2}\frac{I_r}{r^2}}{m_g+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}})+W_g\sin\theta_i]r$$

In addition the section is subjected to a compression, $C_o=N+\frac{1}{2}W_r\cos\theta_i=P_{max}\sin(\emptyset-\theta_i)+(W_g+\frac{1}{2}W_r)\cos\theta_i$ We will now consider the reaction at the fixed axis of the rocker or gun lever. Since the tangential inertia effect of the rocker practically balances, we will consider the reaction on the main center bearing as due only to the reaction of the gun at the trunnions and the inertia of the counterweight. The tangential inertia resistance of the counterweight, is

$F_{cw}=\Sigma mq.\frac{d^2\theta}{dt^2}$ where q is the distance to any mass particle of the counterweight measured from the axis of rotation of the gun lever. If r_{cw} = the distance to the center of gravity of the counterweight, then $\Sigma mq=M_{cw}r_{cw}$ hence

$$F_{cw}=M_{cw}\ r_{cw}\ \frac{d^2\theta}{dt^2}=P_{max}\cos(\emptyset-\theta_i)\frac{r_{cw}}{r}\frac{M_{cw}}{\Sigma M}$$

It is to be noted that the point of application of F_{cw} is not at the center of gravity of the counterweight, but rather at the center of percussion of the counterweight with respect to the axis of rotation of the gun lever. If k is the distance from the axis of rotation to the center of percussion, $F_{cw}k=\Sigma Mq^2\frac{d^2\theta}{dt^2}=M_{cw}\ Z^2\frac{d^2\theta}{dt^2}$ where Z = the radius of gyration of the counterweight with respect to the fixed axis, $Z=\sqrt{\frac{I_{cw}}{M_{cw}}}$ therefore $k=\frac{Z^2}{r_{cw}}$ Resolving the resultant reactions at the fixed

axis of the gun lever into components normal and along the axis of the gun lever, we have, neglecting the centrifugal forces as small,

$X=T+F_{cw}+(W_{cw}+W_r)\sin\theta_i$

$Y=N+(W_{cw}+W_r)\cos\theta_i$ or substituting values for N, T, and F_{cw},

$$X=P_{max}\cos(\emptyset-\theta_i)\left(1+\frac{M_{cw}\frac{r_{cw}}{r}\cdot M_g}{M_g+\frac{I_r}{r^2}+\frac{I_{cw}}{r^2}}\right)+(W_g+W_{cw}+W_r)\sin\theta_i$$

$$Y=P_{mas}\sin(\emptyset-\theta_i)+(W_g+W_{cw}+W_r)\cos\theta_i$$

and for the resultant we have, $S_o\sqrt{X^2\ Y^2}$

From these equations, it is easy to see, that the reaction at the fixed axis is increased over that at the gun trunnions by the tangential inertia of the counterweight $M_{cw}\dfrac{r_{cw}}{r}\cdot\dfrac{P_{max}\cos(\emptyset-\theta_i)}{\Sigma M}$

With a heavy counterweight, this term is larger and the bearing load at the fixed axis becomes very great with large guns. To reduce this reaction and consequent weight of members, etc., the Cordan linkage disappearing carriage developed by Buffington.Crozier and the Krupp linkage have been used for the larger gun mounts. Subtracting, we have,

$$\frac{d^2x_g}{dt^2}\sin Q-\frac{d^2y_2}{dt^2}\cos Q+d\left(\frac{d\emptyset}{dt}\right)^2\sin(\emptyset-Q)+a_d\cos(\emptyset-Q)+c\left(\frac{dQ}{dt}\right)^2=0$$

Substituting the values of $\dfrac{d^2x_g}{dt^2}$ and $\dfrac{d^2y_g}{dt^2}$, we have

$$(a+b)\left[\cos\theta\frac{d^2\theta}{dt^2}-\sin\theta\left(\frac{d\theta}{dt}\right)^2\right]\sin Q-b\left[\sin\theta\frac{d^2\theta}{dt^2}+\cos\left(\frac{d\theta}{dt}\right)^2\right]$$

$$\cos Q+d\left(\frac{d\emptyset}{dt}\right)^2\sin(\emptyset-Q)+a_d\cos(\emptyset-Q)+c\left(\frac{dQ}{dt}\right)^2=0$$

Expanding and simplifying, we find

$$a[\cos\theta \frac{d^2\theta}{dt^2} - \sin\theta (\frac{d\theta}{dt})^2]\sin Q - b[\sin(\theta-Q)\frac{d^2\theta}{dt^2} + \cos(\theta-Q)$$

$$(\frac{d\theta}{dt})^2] + d(\frac{d\emptyset}{dt})^2 \sin(\emptyset-Q) + a_d \cos(\emptyset-Q) + c(\frac{dQ}{dt})^2 = 0$$

Now $(\frac{d\emptyset}{dt})^2 = \frac{l^2}{d^2}[\tan(\emptyset-Q)\cos(\emptyset-B) - \sin(\emptyset-B)]^2 (\frac{d\theta}{dt})^2$

and $(\frac{dQ}{dt})^2 = \frac{l^2}{c^2} \frac{\cos^2(\emptyset-B)}{\cos^2(\emptyset-Q)} (\frac{d\theta}{dt})^2$

hence

$$a_d = \frac{b[\sin(\theta-Q)\frac{d^2\theta}{dt^2} + \cos(\theta-Q)(\frac{d\theta}{dt})^2] - a[\cos\theta \frac{d^2\theta}{dt^2} - \sin}{\cos(\emptyset-Q)}$$

$$\frac{\theta(\frac{d\theta}{dt})^2]\sin Q - \left\{\frac{l^2}{d}[\tan(\emptyset-Q)\cos(\emptyset-B) - \sin(\emptyset-B)]^2\right.}{}$$

$$\frac{\left.\sin(\emptyset-Q) + \frac{l^2}{c} \frac{\cos^2(\emptyset-B)}{\cos^2(\emptyset-B)}\right\} (\frac{d\theta}{dt})^2}{}$$

Combining the acceleration and velocity terms, we have

$$a_d = \frac{[b\sin(\theta-Q) - a\cos\theta\sin Q]}{\cos(\emptyset-Q)} \frac{d^2\theta}{dt^2} + \frac{\{a\sin\theta\sin Q + b}{\cos(\emptyset-Q)}$$

$$\frac{\cos(\theta-Q) - \frac{l^2}{d^2}[\tan(\emptyset-Q)\cos(\emptyset-B) - \sin(\emptyset-B)]^2 \sin(\emptyset-Q)\}}{}$$

$$(\frac{d\theta}{dt})^2 + \frac{l^2}{c} \frac{\cos^2(\emptyset-B)}{\cos^2(\emptyset-B)} (\frac{d\theta}{dt})^2$$

Therefore the angular acceleration of the gun, becomes,

$$\frac{d^2\emptyset}{dt^2} = \frac{a_d}{d} = \frac{A_d \frac{d^2\theta}{dt^2} + B_d(\frac{d\theta}{dt})^2}{d\cos(\emptyset - Q)} \quad (\text{rad/sec.}^2)$$

where $A_d = b\sin(\theta - Q) - a\cos\theta\sin Q$

$$B_d = a\sin\theta\sin Q + b\cos(\theta - Q) - \frac{l^2}{d}[\tan(\emptyset - Q)$$

$$\cos(\emptyset - B) - \sin(\emptyset - B)]^2\sin(\emptyset - Q) + \frac{l^2}{c}\,\frac{\cos^2(\emptyset - B)}{\cos^2(\emptyset - Q)}$$

For the acceleration $a_c = c\,\dfrac{d^2Q}{dt^2}$ we eliminate a_d in the equations:-

$$-a_d\sin\emptyset\cos\emptyset = \frac{d^2x_g}{dt^2}\cos\emptyset - D\left(\frac{d\emptyset}{dt}\right)^2\cos^2\emptyset + c\left(\frac{dQ}{dt}\right)^2\sin Q\cos\emptyset$$
$$-a_c\cos Q\cos\emptyset$$

$$+a_d\sin\emptyset\cos\emptyset = \frac{d^2y_g}{dt^2}\sin\emptyset - D\left(\frac{d\emptyset}{dt}\right)^2\sin^2\emptyset - c\left(\frac{dQ}{dt}\right)^2\cos Q\sin\emptyset$$
$$-a_c\sin Q\sin\emptyset$$

Adding, we have

$$\frac{d^2x_g}{dt^2}\cos\emptyset + \frac{d^2y_g}{dt^2}\sin\emptyset - d\left(\frac{d\emptyset}{dt}\right)^2\ \emptyset - c\left(\frac{dQ}{dt}\right)^2\sin(\emptyset - Q) - a_c\cos(\emptyset - Q) = 0$$

hence

$$a_c = \frac{\dfrac{d^2x_g}{dt^2}\cos + \dfrac{d^2y_g}{dt^2}\sin\emptyset - d\left(\dfrac{d\emptyset}{dt}\right)^2 - c\left(\dfrac{dQ}{dt}\right)\ \ in(\emptyset - Q)}{\cos(\emptyset - Q)}$$

Substituting for $\dfrac{d^2x_g}{dt^2}, \dfrac{d^2y_g}{dt^2}, \left(\dfrac{d\emptyset}{dt}\right)^2$ and $\left(\dfrac{dQ}{dt}\right)^2$ we obtain

$$a_c = \frac{(a+b)\left[\cos\theta\dfrac{d^2\theta}{dt^2} - \sin\theta\left(\dfrac{d\theta}{dt}\right)^2\right]\cos\emptyset + b\left[\sin\theta\,\dfrac{d^2\theta}{dt^2} + \cos\theta\left(\dfrac{d\theta}{dt}\right)^2\right]}{\cos(\emptyset - Q)}$$

$$\underline{\sin\emptyset} - \frac{\dfrac{1}{d}[\tan(\emptyset - Q)\cos(\emptyset - B) - \sin(\emptyset - B)]^2\left(\dfrac{d\theta}{dt}\right)^2 + \dfrac{l^2}{c}}{\cos(\emptyset - Q)}$$

$$\underline{\frac{\cos^2(\emptyset - B)}{\cos^2(\emptyset - Q)}\quad\left(\frac{d\theta}{dt}\right)^2}$$

Combining, we have,

$$a_c = \frac{a\cos\theta\cos\phi + b\cos(\theta-\phi)}{\cos(\phi-Q)}\,\frac{d^2\theta}{dt^2} - \frac{\left\{b\sin(\theta-\phi) + a\cos\phi\sin\theta + \frac{l^2}{d}\left[\tan(\phi-Q)\cos(\phi-B) - \sin(\phi-B)\right]^2 + \frac{l^2}{c}\,\frac{\cos^2(\phi-B)}{\cos^2(\phi-Q)}\right\}}{\cos(\phi-Q)}\left(\frac{d\theta}{dt}\right)^2$$

Therefore since $a_c = c\,\frac{d^2Q}{dt^2}$, the angular acceleration of the gun lever, becomes,

$$\frac{d^2Q}{dt^2} = \frac{A_c\frac{d^2\theta}{dt^2} + B_e\left(\frac{d\theta}{dt}\right)^2}{c\cos(\phi-Q)} \quad \text{rad.sec.}^2$$

where $A_c = a\cos\theta\cos\phi + b\cos(\theta-\phi)$

$$B_c = -\left\{b\sin(\theta-\phi) + a\cos\phi\sin\theta + \frac{l^2}{d}\left[\tan(\phi-B) - \sin(\phi-B)\right]^2 + \frac{l^2}{c}\,\frac{\cos^2(\phi-B)}{\cos^2(\phi-Q)}\right\}$$

RECAPITULATION OF VELOCITIES AND ACCELERATIONS IN A CORDAN LINKAGE DISAPPEARING GUN CARRIAGE:

Let $a+b$ = total length of gun lever

a = distance from cross head to top carriage trunnion

b = distance from top carriage trunnion to gun trunnion

d = distance from gun trunnion to elevating arm trunnion measured along gun.

c = length of elevating arm.

θ = angle gun lever makes with vertical

ϕ = angle turned by gun

Q = angle elevating arm makes with vertical

$\frac{d\theta}{dt}$ = angular velocity of gun lever

$\frac{d\phi}{dt}$ = angular velocity of gun

$\frac{dQ}{dt}$ = angular velocity of elevating arm

$\frac{d^2\theta}{dt^2}$ = angular acceleration of gun lever

$\frac{d^2\phi}{dt^2}$ = angular acceleration of gun

$\frac{d^2Q}{dt^2}$ = angular acceleration of elevating arm

$\frac{dx_g}{dt}$ = horizontal linear velocity of gun at trunnions

$\frac{dy_g}{dt}$ = vertical linear velocity of gun at trunnions

$\frac{d^2x_g}{dt^2}$ = horizontal acceleration of gun at trunnions

$\frac{d^2y_g}{dt^2}$ = vertical acceleration of gun at trunnions

$\frac{dx}{dt}$ = velocity of top carriage

$\frac{d^2x}{dt^2}$ = acceleration of top carriage

$\frac{dy}{dt}$ = velocity of counterweight and crosshead

$\frac{d^2y}{dt^2}$ = acceleration of counterweight and crosshead

Then, in terms of the angular velocity and acceleration of the gun lever,

(a) The velocity and acceleration of top carriage, are

$$\frac{dx}{dt} = a \cos\theta \frac{d\theta}{dt} \qquad \text{(ft/sec)}$$

$$\frac{d^2x}{dt^2} = a \cos\theta \frac{d^2\theta}{dt^2} - a \sin\theta\left(\frac{d\theta}{dt}\right)^2 \qquad \text{(ft/sec}^2\text{)}$$

(b) The velocity and acceleration of the top carriage, are

$$\frac{dy}{dt} = - a \sin\theta \frac{d\theta}{dt} \quad \text{(ft/sec)}$$

$$\frac{d^2y}{dt^2} = - a \sin\theta \frac{d^2\theta}{dt^2} - a \cos\theta \left(\frac{d\theta}{dt}\right)^2 \quad \text{(ft/sec}^2\text{)}$$

(c) The velocity and acceleration of the gun, are

$$\frac{dx_g}{dt} = (a+b)\cos\theta \frac{d\theta}{dt} \quad \text{(ft/sec)}$$

$$\frac{d^2x_g}{dt^2} = (a+b)\cos\theta \frac{d^2\theta}{dt^2} - (a+b)\sin\theta \left(\frac{d\theta}{dt}\right)^2 \quad \text{(ft/sec}^2\text{)}$$

$$\frac{dy_g}{dt} = b \sin\theta \frac{d\theta}{dt} \quad \text{(ft/sec)}$$

$$\frac{d^2y_g}{dt^2} = b \sin\theta \frac{d^2\theta}{dt^2} + b \cos\theta \left(\frac{d\theta}{dt}\right)^2 \quad \text{(ft/sec}^2\text{)}$$

$$\frac{d\emptyset}{dt} = \frac{1}{d}[\tan(\emptyset-Q)\cos(\emptyset-B) - \sin(\emptyset-B)]\frac{d\theta}{dt} \quad \text{(rad/sec)}$$

$$\frac{d^2\emptyset}{dt^2} = \frac{A_d\frac{d^2\theta}{dt^2} + B_d\frac{d\theta}{dt}}{d\cos(\emptyset-Q)} \quad \text{(rad/sec}^2\text{)}$$

where $A_d = b \sin(\theta-Q) - a \cos\theta \sin Q$

$$B_d = a \sin\theta \sin Q + b \cos(\theta-Q) - \frac{1^2}{d}[\tan(\emptyset-Q)$$

$$\cos(\emptyset-B) - \sin(\emptyset-B)]^2 \sin(\emptyset-Q) + \frac{1^2}{c} \frac{\cos^2(\emptyset-B)}{\cos^2(\emptyset-Q)}$$

(d) The velocity and acceleration of the elevating arm, are

$$\frac{dQ}{dt} = \frac{1}{c} \frac{\cos(\emptyset-B)}{\cos(\emptyset-Q)} \left(\frac{d\theta}{dt}\right) \quad \text{(rad/sec)}$$

$$\frac{d^2Q}{dt^2} = \frac{A_c\frac{d^2\theta}{dt^2} + B_c\left(\frac{d\theta}{dt}\right)^2}{c\cos(\emptyset-Q)}$$

where $A_c = a \cos\theta \cos\emptyset + b \cos(\theta-\emptyset)$

$$B_c = -\left\{ b\sin(\theta-\phi)+a\cos\phi\sin\theta + \frac{l^2}{c}\left[\tan(\phi-Q)\right.\right.$$

$$\left.\left.\cos(\phi-B)-\sin(\phi-B)\right]^2 + \frac{l^2}{c}\;\frac{\cos^2(\phi-B)}{\cos^2(\phi-Q)}\right\}$$

<u>Coordinates of the system:</u>

Displacement of top carriage = x

Displacement of counterweight = y

Distance from instantaneous center of gun lever to gun trunnion:

$$l = \sqrt{(a+b)^2\cos^2\theta+b^2\sin^2\theta}$$

COORDINATES OF THE CORDAN LINKAGE DISAPPEARING CARRIAGE.

In estimating the work done by the various weights and resistances during the retardation period of the recoil it is necessary to compute the various displacements of the parts of the system in terms of the independent coordinate of the system.

From the diagram, to determine ϕ and Q in terms of the angle θ made by the gun lever with the vertical, we have

$$\left.\begin{aligned} x_o &= (a+b)\sin\theta+d\cos\phi-c\sin Q \\ y_o &= -b\cos\theta+d\sin\phi+c\cos Q \end{aligned}\right\}$$

which may be written,

$$\left.\begin{aligned} d\cos\phi &= x_o-(a+b)\sin\theta+c\sin Q \\ d\sin\phi &= y_o+b\cos\theta-c\cos Q \end{aligned}\right\}$$

Squaring and adding, we have $d^2=[x_o-(a+b)\sin\theta]^2$

$+2[x_o-(a+b)\sin\theta]c\sin Q+(y_o+b\cos\theta)^2+2(y_o+b\cos\theta)$

$c\cos Q+c^2$

This equation may be put in the form,

$$[x_o-(a+b)\sin\theta]\sin Q+(y_o+b\cos\theta)\cos Q = \frac{d^2-c^2}{2}$$
$$+\frac{[x_o-(a+b)\sin\theta)^2+(y_o+b\cos\theta)^2}{2}$$

hence $m\sin(A+Q) = \frac{1}{2}\left\{d^2-c^2+[x_o-(a+b)\sin\theta]^2+(y_o+b\cos\theta)^2\right\}$

where $m = \sqrt{[x_o-(a+b)\sin\theta]^2+(y_o+b\cos\theta)^2}$

$$A = \tan^{-1}\left[\frac{x_o-(a+b)\sin\theta}{y_o+b\cos\theta}\right]$$

From this equation we may solve for Q in terms of θ, and substituting in either equation below,

$$\cos\phi = \frac{1}{d}[x_o-(a+b)\sin\theta+c\sin Q]$$

$$\sin\phi = \frac{1}{d}[y_o+b\cos\theta - c\cos Q]$$

we may then calculate the value of ϕ in terms of the independent variable. Further if,

Displacement of top carriage = x

Displacement of counterweight = y

The distance from instantaneous center of gun lever to the gun trunnion, is

$$l = \sqrt{(a+b)^2\cos^2\theta+b^2\sin^2\theta}$$

REACTIONS ON THE PARTS OF CORDAN LINKAGE.

Considering the reactions on the gun, it will be assumed that the center of gravity is located at the gun trunnion. The gun is subjected to a translatory acceleration divided into horizontal and vertical components as well as an angular acceleration due to the reaction of the elevating arm. Let

(1) P_b = the powder pressure along the axis of the bore

(2) X and Y = the horizontal and vertical reactions at the gun trunnions.

(3) W_g = the weight of the gun acting through the gun trunnion

(4) $m_g \frac{d^2x}{dt^2}$ and $m_g \frac{d^2y}{dt^2}$ = the inertia components along the horizontal and vertical axis

(5) $I_g \frac{d^2Q}{dt^2}$ = the inertia angular resistance

(6) X''' and Y''' = the horizontal and vertical components exerted by the elevating arm on the gun

Then for the motion of the gun, we have

$$P_b \cos \emptyset - X - m_g \frac{d^2x_g}{dt^2} - X''' = 0$$

$$P_b \sin\emptyset - Y - m_g \frac{d^2y_g}{dt^2} - Y''' = 0$$

and

$$Y''' d \cos\emptyset - X''' d \sin\emptyset - I_g \frac{d^2\emptyset}{dt^2} = 0$$

For the elevating arm, we have $(X''' \cos Q + Y''' \sin Q)c - I_c \frac{d^2Q}{dt^2} = 0$

where I_c = the moment of inertia about its fixed axis.

Combining with the moment equation of the gun, we have

$$X''' = \frac{I_c \frac{d^2Q}{dt^2} d \cos \emptyset - I_g \frac{d^2\emptyset}{dt^2} c \sin Q}{cd \cos(\emptyset - Q)} \quad \text{(lbs)}$$

$$Y''' = \frac{I_c \frac{d^2Q}{dt^2} d \sin\emptyset + I_g \frac{d^2\emptyset}{dt^2} c \cos Q}{cd \cos(\emptyset - Q)} \quad \text{(lbs)}$$

Next, to obtain the reactions X and Y we must consider the dynamical equations of the gun lever. By taking moments about the instantaneous center of the gun

lever, we eliminate the unknown normal reactions of the constraints of the carriage and counter-weight.

Then for moments about the instantaneous center of the gun lever, we have

$$X(a+b)\cos\theta+Y\,b\sin\theta-X'a\cos\theta-Y''a\sin\theta-m_R\frac{d^2x}{dt^2}a\cos\theta-I_R\frac{d^2\theta}{dt^2}=0 \quad \text{where } X'=R+m_c\frac{d^2x}{dt^2}.$$

R = the hydraulic brake reaction on the carriage "m_c".

$$Y''=m_{cw}\frac{d^2y}{dt^2}+w_{cw}$$

m_{cw} and w_{cw} = mass at weight of the counterweight. Combining, we have the dynamical equation of the motion of the disappearing gun carriage during the powder pressure period, as follows:

$$\left[P_b\cos\emptyset-m_g\frac{d^2x_g}{dt^2}-\frac{I_c\frac{d^2Q}{dt^2}d\cos\emptyset-I_g\frac{d^2\emptyset}{dt^2}c\sin Q}{cd\cos(\emptyset-Q)}\right](a+b)\cos\theta$$

$$+\left[P_b\sin\emptyset-m_g\frac{d^2y_g}{dt^2}-\frac{I_c\frac{d^2Q}{dt^2}d\sin\emptyset+I_g\frac{d^2\emptyset}{dt^2}c\cos Q}{cd\cos(\emptyset-Q)}\right]b\sin\theta$$

$$-\left(R+m_c\frac{d^2x}{dt^2}\right)a\cos\theta-\left(m_{cw}\frac{d^2y}{dt^2}+w_{cw}\right)a\sin\theta$$

$$-m_R\frac{d^2x}{dt^2}a\cos\theta-I_R\frac{d^2\theta}{dt^2}=0.$$

For a solution of this equation we must substitute for the various accelerations their value in terms of a function of the acceleration $\frac{d^2\theta}{dt^2}$. The hydraulic brake resistance R may readily be obtained by considering the energy equation

of the linkage to its recoiled position.

If A_g = kinetic energy of gun at end of powder period (ft/lbs)

A_c = Kinetic energy of top carriage at end of powder period (ft/lbs)

A_e = kinetic energy of elevating gun at end of powder period (ft/lbs)

A_R = kinetic energy of gun lever at end of powder period (ft/lbs)

A_w = kinetic energy of counterweight at end of powder period

Then for the kinetic energy of the gun, we have, if l = the distance to gun trunnion from the instantaneous center of gun movement, and k radius of gyration about center of gravity or trunnions of the gun.

$$A_g = \frac{1}{2} m_g l^2 \left\{ 1 + \frac{k^2}{d^2} \left[\tan(\emptyset - Q)\cos(\emptyset - B) - \sin(\emptyset - B)\right]^2 \right\} \left(\frac{d\theta}{dt}\right)^2$$

(ft/lbs)

For the kinetic energy of the elevating arm.

$$A_e = \frac{1}{2} I_e \frac{l^2}{c^2} \frac{\cos^2(\emptyset - B)}{\cos^2(\emptyset - Q)} \left(\frac{d\theta}{dt}\right)^2$$

For the kinetic energy of the gun lever, if k_R =

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* NOTE: If the path of the sliding carriage has an inclination to the horizontal equal to angle d, then for the equation of the gun lever, we have

$$X(a+b)\cos\Theta + Y(b\sin\Theta - a\cos\Theta\tan d) - \left(R + m_c\frac{d^2x}{dt^2}\right)$$

$$\frac{a\cos\Theta}{\cos\alpha} - Y'' a\sin\Theta - m_R\frac{d^2x}{dt^2}\frac{a\cos\Theta}{\cos\alpha} - I_R\frac{d^2\Theta}{dt^2} = 0$$

Substituting the values of X and Y as in the previous equations, we have the general dynamical equation of motion.

radius of gyration about the center of gravity of the gun lever, we have

$$A_R = \frac{1}{2} m_R(a^2\cos^2\theta+k_R^2)(\frac{d\theta}{dt})^2$$

If the top carriage and sides are inclined plane making angle α with the horizontal,

$$A_R = \frac{1}{2}m_R[a^2\cos^2 \theta(1+\tan^2 \alpha)+k_R^2](\frac{d\theta}{dt})^2$$

For the kinetic energy of the top carriage

$A_C=\frac{1}{2}m_C a^2\cos^2 \theta(\frac{d\theta}{dt})^2$ for horizontal plane and

$A_C = \frac{1}{2} m_C a^2\cos^2 \theta(1+\tan^2 \alpha)(\frac{d\theta}{dt})^2$ for inclined plane

For the kinetic energy of the counterweight and cross head,

$$A_W = \frac{1}{2} m_W a^2 \sin^2 \theta(\frac{d\theta}{dt})^2$$

When the sliding carriage rides an inclined plane the kinetic energy of the counterweight, becomes,

$$A_W = \frac{1}{2}m_W a^2(\sin \theta+\cos \theta \tan \alpha)^2 (\frac{d\theta}{dt})^2$$

From the principle of energy,

$$A_g+A_e+A_R+A_C+A_W = \bar{W}_B+\bar{W}_{CW}-\bar{W}_g-\bar{W}_e-\bar{W}_R-\bar{W}_C$$

where $\bar{W}_B$ = work resisted by the recoil brake

$\bar{W}_{CW}$ = work resisted by the weight of the counterweight

$\bar{W}_g$ = work done by the weight of the gun

$\bar{W}_e$ = work done by the weight of the elevating arm

$\bar{W}_R$ = work done by the weight of the gun lever

$\bar{W}_C$ = work done by the weight of the sliding carriage

During the powder period, the sliding carriage moves a distance E and the gun lever angle increases from θ_0 to θ_1. The length of recoil = L and the recoiled position of the gun lever makes an angle θ with the vertical.

Work resisted by the recoil brake = $\bar{W}_B$. If R= the brake resistance, then for the work of the recoil brake during the retardation period, we have

$\bar{W}_B = R(L-E)$ (ft/lbs) where obviously $L-E = a(\sin\theta_2 - \sin\theta_1)$ and with an inclined plane sliding carriage,

$$L-E = \frac{a}{\cos\alpha}(\sin\theta_2 - \sin\theta_1)$$

Work resisted by the weight of the counterweight = $\bar{W}_{cw}$

$\bar{W}_{cw} = w_w\, y_w$ $\qquad w_w$ = weight of counterweight

where $y_w = a(\cos\theta_1 - \cos\theta_2)$ and with an inclined plane sliding carriage $y_w = a(\cos\theta_1 - \cos\theta_2) + L\sin\alpha$

Work due to the weight of the gun = $\bar{W}_g$

$\bar{W}_g = w_g y_g$ where $y_g = (a+b)(\cos\theta_1 - \cos\theta_2)$ and likewise with an inclined plane sliding carriage $y_g = (a+b)(\cos\theta_1 - \cos\theta_2)$

Work due to the weight of the sliding carriage and gun lever = $\bar{W}_r + \bar{W}_c$. Assuming the center of gravity of the gun lever at the gun lever trunnion, the center of gravity of the gun lever has the same displacement as the sliding carriage. Hence $\bar{W}_r + \bar{W}_c = (w_r + w_c)y_c$ where $y_c = (L-E)\sin\alpha = \frac{a}{\cos\alpha}(\sin\theta_2 - \sin\theta_1)$. Hence when the plane is horizontal no work is done by the weights of the gun lever or sliding carriage.

Work due to the weight of the elevating arm = $\bar{W}_e$

$\bar{W}_e = w_e y_e$ $\qquad$ where $y_e = d_e(\cos Q_1 - \cos Q_2)$

$\qquad d_e$ = distance to center of gravity from fixed axis of elevating arm.

$$Q_1 = \sin^{-1}\left\{\left[\frac{\left\{d^2 - c^2 + [x_0 - (a+b)\sin\theta_1]^2 + (y_0 + b\cos\theta_1)^2\right\}}{2\sqrt{[x_0 - (a+b)\sin\theta_1]^2 + (y_0 + b\cos\theta_1)^2}}\right]\right\}$$

$$- \tan^{-1} \left[\frac{x_0-(a+b)\sin\theta_1}{y_0+b\cos\theta_1}\right]$$

$$Q_2 = \sin^{-1}\left\{\left[\frac{\left\{d^2-c^2+[x_0-(a+b)\sin\theta_2]^2+(y_0+b\cos\theta_2)^2\right\}}{2\sqrt{[x_0-(a+b)\sin\theta_2]^2+(y_0+b\cos\theta_2)^2}}\right]\right\}$$

$$- \tan^{-1}\left[\frac{x_0-(a+b)\sin\theta_2}{y_0+b\cos\theta_2}\right]$$

EQUIVALENT MASS OF CORDAN LINKAGE. During the powder period, it is convenient to express the dynamical equation of recoil in terms of the external moments or forces and the equivalent mass of the system times the acceleration of the coordinate considered. The equivalent mass and corresponding reactions may be referred as a function of the angle made by the gun lever with the vertical or as a function of the displacement of the sliding carriage.

(1) Equivalent mass referred to angle "θ" of gun lever with vertical:-

From the dynamical equation of recoil for the Cordan linkage previously derived, we have, for moments about the instantaneous center of the gun lever, $P_b[a\cos\theta\cos(\theta-\emptyset)]-Ra\cos\theta-\bar{W}_{cw}$

$$a\sin\theta = m_g\left[(a+b)\cos\theta\frac{d^2x_g}{dt^2}+b\sin\theta\frac{d^2y_g}{dt^2}\right]+m_c a$$

$$\cos\theta\frac{d^2x}{dt^2}+m_{cw}\,a\sin\theta\frac{d^2y}{dt^2}+I_r\frac{d^2\theta}{dt^2}+I_c\frac{[a\cos\theta\cos\emptyset+b}{c\cos(\emptyset-Q)}$$

$$\frac{\cos(\theta-\emptyset)]}{}\frac{d^2Q}{dt^2}+I_g\frac{[b\sin(\theta-Q)-a\sin Q-\cos\theta]}{d\cos(\emptyset-Q)}\frac{d^2\emptyset}{dt^2}$$

Neglecting the centrifugal components of the accelerations, as small,

$$\frac{d^2x}{dt^2} = a\cos\theta\frac{d^2\theta}{dt^2}\;;\qquad \frac{d^2x_g}{dt^2}=(a+b)\cos\theta\frac{d^2\theta}{dt^2}$$

$$\frac{d^2y}{dt^2} = a \sin\theta \frac{d^2\theta}{dt^2}; \quad \frac{d^2y_g}{dt^2} = b \sin\theta \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\emptyset}{dt^2} = \frac{1}{d\cos(\emptyset-Q)} [b \sin(\theta-Q) - a \sin Q \cos\theta] \frac{d^2\theta}{dt^2}$$

$$\frac{d^2Q}{dt^2} = \frac{1}{c\cos(\emptyset-Q)} [a \cos\theta \cos\emptyset + b \cos(\theta-\emptyset)] \frac{d^2\theta}{dt^2}$$

Substituting, we have $P_b[a \cos\theta\cos\emptyset + b\cos(\theta-\emptyset)] -$

$$Ra \cos\theta - \overline{W}_{cw} a \sin\theta =$$

$$\left\{ m_g[(a^2+2ab)\cos^2\theta + b^2] + m_c a^2\cos^2\theta + m_{cw} a^2 \sin^2\theta + I_r + I_c \frac{[a\cos\theta\cos\emptyset + b\cos(\theta-\emptyset)]^2}{c^2\cos^2(\emptyset-Q)} + I_g \frac{[b\sin(\theta-Q) - a\sin Q\cos\theta]^2}{d^2\cos^2(\emptyset-Q)} \right\} \frac{d^2\theta}{dt^2}$$

Thus the equation is in the form of $AP_b - BR - C\overline{W}_{cw} = D\frac{d^2\theta}{dt^2}$

where $A = a\cos\theta\cos\emptyset + b\cos(\theta-\emptyset)$

$B = a\cos\theta$

$C = a\sin\theta$ and for the equivalent mass "D"

$$D = m_g[(a^2+2ab)\cos^2\theta + b^2] + m_c a^2\cos^2\theta + m_{cw} a^2\sin^2\theta + I_r + I_c \frac{[a\cos\theta\cos\emptyset + b\cos(\theta-\emptyset)]^2}{c^2\cos^2(\emptyset-Q)} + I_g \frac{[b\sin(\theta-Q) - a\sin Q\cos\theta]^2}{d^2\cos^2(\emptyset-Q)}$$

For the solution, during the powder period, we express the powder reaction as a function of the time, then

$$\frac{d\theta}{dt} = \int \frac{AP_b}{D} dt - \left(\frac{BR + C\overline{W}_{cw}}{D}\right)t = \frac{A}{D} M_g - \left(\frac{BR + C\overline{W}_{cw}}{D}\right)t$$

where V_f is the <u>velocity of free recoil of the gun</u>

Integrating again, $\theta = \frac{A}{D} M_g E - \left(\frac{BR + C\overline{W}_{cw}}{2D}\right) t^2$

where E is the <u>displacement of the gun in free recoil</u>. From the solution of these equations, we obtain,

$$\theta_1 = \theta_0 + \frac{A}{D} M_g E - \left(\frac{BR + C\bar{W}_{cw}}{2D}\right)T^2 \quad \text{where } T = \text{powder interval}$$

$$\left(\frac{d\theta}{dt}\right)_1 = \frac{A}{D} M_g V_f - \left(\frac{BR + C\bar{W}_{cw}}{D}\right)T$$

The angular displacement and the angular velocity of the gun at the end of the powder period. Substituting these values in the energy equation, we have

$\bar{W}_B = A_g + A_e + A_R + A_c + A_w + \bar{W}_g + \bar{W}_e + \bar{W}_R + \bar{W}_c - \bar{W}_{cw}$ and therefore can readily determine R the total braking resistance.

(2) <u>Equivalent mass referred to displacement of sliding carriage X:-</u>

In place of a movement and angular acceleration equation, we may consider the inertia and the reactions of the system as reduced to an equivalent translatory mass and force as a function of the displacement of the sliding carriage. Therefore reducing the motion of the system to one of translation along the path of the top carriage.

By direct analysis, we have, if

P_b = powder reaction

X and Y = components of gun trunnion reaction

X' and Y' = components of top carriage on gun lever

X" and Y" = components of reaction on gun lever at crosshead

X''' and Y''' = components of elevating arm reaction on gun

m_g = mass of gun

m_{cw} = mass of counterweight

m_e = mass of elevating arm

m_c = mass of top carriage

m_R = mass of gun lever

Then, for moments about the instantaneous center

$$X(a+b)\cos\theta + Y\,b\sin\theta - Y''\,a\sin\theta - m_R\frac{d^2x}{dt^2}\,a\cos\theta - I_r\frac{d^2\theta}{dt^2} - X'\,a\cos\theta = 0$$

Dividing through by a cos θ, we have

$$X\left(\frac{a+b}{a}\right)Y\,\frac{b}{2}\tan\theta - Y''\tan\theta - m_R\frac{d^2x}{dt^2} - \frac{I_r}{a\cos\theta}\frac{d^2\theta}{dt^2} - X' = 0$$

Since

$$X = P_b\cos\phi - m_g\frac{d^2x_g}{dt^2} - \frac{I_a\frac{d^2Q}{dt^2}\,d\cos\phi - I_g\frac{d^2\phi}{dt^2}\,c\sin Q}{cd\cos(\phi-Q)}$$

$$Y = P_b\sin\phi - m_g\frac{d^2y_g}{dt^2} - \frac{I_c\frac{d^2Q}{dt^2}\,d\sin\phi + I_g\frac{d^2\phi}{dt^2}\,c\cos Q}{cd\cos(\phi-Q)}$$

$$Y'' = m_{cw}\frac{d^2y}{dt^2} + w_{cw}$$

we have on substitution,

$P_b\left(\frac{a+b}{a}\cos\phi + \frac{b}{a}\tan\theta\sin\phi\right) - W_{cw}\tan\theta - X'$ as the equivalent force acting along the top carriage guides and

$$m_g\frac{d^2x_g}{dt^2}\,\frac{a+b}{a} + m_g\frac{d^2y_g}{dt^2}\,\frac{b}{a}\tan\theta + \left[\frac{I_c\frac{d^2Q}{dt^2}\,d\cos\phi - I_g\frac{d^2\phi}{dt^2}\,c\sin Q}{cd\cos(\phi-Q)}\right.$$

$$\frac{a+b}{a} + \left[\frac{I_c\frac{d^2Q}{dt^2}\,d\sin\phi + I_g\frac{d^2Q}{dt^2}\,c\cos Q}{cd\cos(\phi-Q)}\right]\frac{b}{a}\tan\theta + m_{cw}\frac{d^2y}{dt^2}$$

$+\dfrac{I_r}{a\cos\theta}\dfrac{d^2\theta}{dt^2}$ as the equivalent inertia resistance offered by the mass of the total system reduced to the path along the top carriage guides.

THROTTLING CALCULATIONS WITH AND WITHOUT A FILLING IN BUFFER.

4.7 Gun Trailer Mount with U. S. Variable Recoil Valve.

w = weight of projectile 45 lbs.

v = muzzle velocity 2400 ft/sec.

v = 166.13 (in.)

$\bar{w}$ = weight of powder charge 11 lbs.

$P_{b\ max}$ = 34000 lbs/sq.in.

b = 36" (0° to 45°)

4.7 (0° to 45°)

X = total resistance = 17806.8706 lbs.

W_r = weight of recoiling parts = 7550 lbs.

S_1 = spring load =

$P_{vf} = S_f$ = spring load at end of recoil = 16140 lbs.

$F_{vi} = S_o$ = spring load at assembled height =

$W_r \times 1.3 = 9800$ lbs.

$$St = S_o = \frac{16140-9800}{36} = \frac{6340}{36} = 176.111 \text{ lbs} =$$

increase of spring load per inch of recoil.

α = maximum angle of elevation = 45°

$W_r \sin \alpha$ = weight component = 7550 × .70711 = 5338.lbs. 6805

R_s = stuffing box friction = 2.25 × 100 = 225

R_g = guide friction = $W_r u \cos \alpha$

u = coefficient of friction = .15

R_g = 7550×.70711×.15 = 800.8021

Effective area of recoil piston = 9.337 (sq.in)

METHOD OF PLOTTING VELOCITY CURVE - VARIABLE RESISTANCE TO RECOIL.

$$\left.\begin{aligned} V_r &= V_{fo} - \frac{Kt_o}{M_r}\ ; \ (\text{ft/sec}) \\ X_o' &= X_{fo} - \frac{Kt_o^2}{2M_r} \quad (\text{ft}) \end{aligned}\right\} \text{When projectile leaves the muzzle.}$$

$$\left.\begin{aligned} V_m &= V_{fm} - \frac{Kt_m}{M_r} \quad (\text{ft/sec}) \\ X_m &= X_{fm} - \frac{Kt_m^2}{2M_r} \quad (\text{ft}) \end{aligned}\right\} \text{The maximum restrained recoil velocity and corresponding recoil.}$$

where $V_{fm} = V_{fo} + \frac{P_{ob}}{M_r}(t_m - t_o)\left[1 - \frac{P_{ob}(t_m - t_o)}{4M_r(V_f - V_o)}\right]$ (ft/sec)

$$X_{fm} = X_{fo} + \left[V_{fo} + \frac{P_{ob}}{M_r}(t_m - t_o) - \frac{(t_m - t_o)^2}{6M_r(V_f - V_o)}\right](t_m - t_o) \quad (\text{ft})$$

and

$$t_m = T - \frac{K(T - t_o)}{P_{ob}} \quad (\text{sec})$$

$$\left.\begin{aligned} V_r &= V_f - \frac{Kt}{M_r} \quad (\text{ft/sec}) \\ E_r &= E - \frac{Kt^2}{2M_r} \quad (\text{ft}) \end{aligned}\right\} \text{At the end of the powder period}$$

During the retardation period,

$$V_x = -\sqrt{\frac{2\left[K - \frac{m}{2}(b + x - 2E_r)\right](b - x)}{M_r}}$$

and therefore

$$w_x = \frac{cA^{\frac{3}{2}}\sqrt{\dfrac{2\left[K - \frac{m}{2}(b - x) - 2E_r\right](b - x)}{M_r}}}{13.2\sqrt{K - p_a - R_t - W_2 \sin\emptyset}} \quad (\text{sq.in})$$

which gives the required throttling area with a variable resistance to recoil during the retardation period.

$$P_h = K + W_r \sin\emptyset - R_t - \left(S_o + \frac{S_f - S_o x}{b}\right)$$ for spring return recuperators.

<u>Equivalent throttling area</u> : 4.7 A.A.Trailer, Model 1918

$$\frac{1}{W_e^2} = \frac{1}{W_{x_1}^2} + \frac{1}{W_{x_2}^2}$$

Area of one hole = .0113 sq. in.

In battery - W_{xb} = 20 holes = .226 sq.in. $W_{x_2}^2$ = .0510

W_{x_1} = 103 holes = 1.1639 sq.in. $W_{x_1}^2$ = 1.3546

4" Recoil - $W_{x_2}^2$ = 81 holes = .9153 sq.in. $W_{x_2}^2$ = .8377

$W_{x_1}^2$ = 88 holes = .9944 sq.in. $W_{x_1}^2$ = .9888

8" Recoil - W_{x_2} = 86 holes = .9781 sq.in. $W_{x_2}^2$ = .9566

$W_{x_1}^2$ = 77 holes = .8701 sq.in. $W_{x_1}^2$ = .7570

12" Recoil - W_{x_2} = 98 holes = 1.1074 sq.in. $W_{x_2}^2$ = 1.2263

W_{x_1} = 67 holes = .7571 sq.in. $W_{x_1}^2$ = .5732

16" Recoil - W_{x_2} = 107 holes = 1.2091 sq.in. $W_{x_2}^2$ = 1.4619

W_{x_1} = 59 holes = .6667 sq.in. $W_{x_1}^2$ = .4444

20" Recoil - W_{x_2} = 115 holes = 1.2995 sq.in. $W_{x_2}^2$ = 1.6887

W_{x_1} = 50 holes = .5650 sq.in. $W_{x_1}^2$ = .3192

24" Recoil - W_{x_2} = 125 holes = 1.4125 sq.in. $W_{x_2}^2$ = 1.9951

W_{x_1} = 30 holes = .3390 sq.in. $W_{x_1}^2$ = .1149

28" Recoil - W_{x_2} = 140 holes = 1.5820 sq.in. $W_{x_2}^2$ = 2.5027

W_{x_1} = 30 holes = .3390 sq.in. $W_{x_1}^2$ = .1149

32" Recoil - W_{x_2} = 140 holes = 1.5820 sq.in. $W_{x_2}^2$ = 2.5027

W_{x_1} = 24 holes = .2712 sq.in. $W_{x_1}^2$ = .0735

36" Recoil - W_{x_2} = 152 holes = 1.7176 sq.in. $W_{x_2}^2$ = 2.9501

W_{x_1} = 0 holes = 0 sq.in. $W_{x_1}^2$ = 0

Equivalent throttling areas-4.7 A.A. Trailer,Mpdel 1918

$$\frac{1}{W_e} = y, \quad W_e^2 = \frac{1}{y}, \quad y - W_e^2 - W$$

In battery, $\frac{1}{W_e^2} = \frac{1}{1.3546} + \frac{1}{.0510} = 20.355$, .049127, .221

4" Recoil, $\frac{1}{W_e^2} = \frac{1}{.9888} + \frac{1}{.8377} = 2.204$, .453720 .675

8" Recoil, $\frac{1}{W_e^2} = \frac{1}{.7570} + \frac{1}{.9566} = 2.366$ .422654 .650

12"Recoil, $\frac{1}{W_e^2} = \frac{1}{.5732} + \frac{1}{1.2263} = 2.569$, .389256. .623

16"Recoil $\frac{1}{W_e^2} = \frac{1}{.4444} + \frac{1}{1.4619} = 2.934$ .340831 .583

20" Recoil, $\frac{1}{W_e^2} = \frac{1}{.3192} + \frac{1}{1.6887} = 3.724$ .268528, .518

24" Recoil, $\frac{1}{W_e^2} = \frac{1}{.1149} + \frac{1}{1.9951} = 9.204$ .108648 .329

28"Recoil, $\frac{1}{W_e^2} = \frac{1}{.1149} + \frac{1}{2.5027} = 9.102$, .109865, .331

32"Recoil, $\frac{1}{W_e^2} = \frac{1}{.0735} + \frac{1}{2.5027} = 14.00$, .071428, .267

36" Recoil, $\frac{1}{W_e^2}$ ———

Equivalent throttling area - 4.7 A.A.Trailer,Mqdel 1918

(Calculated)

$$R_h = \frac{K^2 A^3 V_x^2}{175 W_x^2} = W_x^2 = \frac{K^2 A^3 V_x^2}{175 R_h}$$

$K = \frac{1}{.7} = 1.43$, $K^2 = 2.045$, $A = 9.337$ sq.in., $A^3 = 913.994$

$K^2 A^3 = 1869\text{-}117$

$$\text{At 1" Recoil} = W_x^2 = \frac{1869.117\times\overline{15.060}^2}{175\times12072.42} = .200656,\ W_x = .447$$

$$\text{At 4" Recoil} = W_x^2 = \frac{1869.117\times\overline{19.330}^2}{175\times11330.17} = .357273,\ W_x = .597$$

$$\text{At 8" Recoil}, = W_x^2 = \frac{1869.117\times\overline{18.035}^2}{175\times10340.49} = .335961,\ W_x = .579$$

$$\text{At 12" Recoil} = W_x^2 = \frac{1869.117\times\overline{16.614}^2}{175\times9350.81} = .315279,\ W_x = .561$$

$$\text{At 16" Recoil} = W_x^2 = \frac{1869.117\times\overline{15.090}^2}{175\times8361.14} = .290878,\ W_x = .531$$

$$\text{At 20" Recoil} = W_x^2 = \frac{1869.117\times\overline{13.245}^2}{175\times7371.46} = .254184,\ W_x = .504$$

$$\text{At 24" Recoil} = W_x^2 = \frac{1869.117\times\overline{11.569}^2}{175\times6381.79} = .223998,\ W_x = .473$$

$$\text{At 28" Recoil} = W_x^2 = \frac{1869.117\times\overline{9.395}^2}{175\times5392.11} = .174836,\ W_x = .418$$

$$\text{At 32" Recoil} = W_x^2 = \frac{1869.117\times\overline{6.604}^2}{175\times4402.43} = .105806,\ W_x = .325$$

At 36" Recoil ———

<u>Equivalent throttling area</u> - With filling in buffer

$A_b = 1.767$ sq.in. $A_o = .76$ sq.in.

$A = 9.337$ sq.in., $A_q = .69$ sq.in.

$$\frac{1}{W_b^2} = \frac{1}{\overline{.76}^2} + \frac{1}{\overline{.69}^2} = \frac{1}{.577} + \frac{1}{.476} = 1.73 + 2.10 = 3.83$$

$$W_b^2 = \frac{1}{3.83} = .261$$

$$W_b^2 = \frac{K_o^2 (A-A_b)^2 V^2}{1.75\left(P_h - \dfrac{K^2 A_b^2 V^2}{175 W_b^2}\right)}$$

$$\text{At 4" Recoil, } W_e^2 = \frac{\overline{1.43}^2\,(9.337-1.767)^3\times\overline{19.33}^2}{175\left(11330 - \dfrac{\overline{1.33}^2\times\overline{1.767}^3\times\overline{19.33}^2}{175\times.261}\right)}$$

$$W_e^2 = \frac{2.04\times433.76\times373.64}{175\left(11330-\dfrac{1.76\times5.51\times373.64}{45.67}\right)}$$

$$W_e^2 = \frac{331271}{1959925} = .169, \quad W_e = .411$$

$$\text{At 8"Recoil, } W_e^2 = \frac{\overline{1.43}^2\,(9.337\times1.767)^3\times\overline{18.03}^2}{175\left(10340-\dfrac{\overline{1.33}^2\times\overline{1.767}^3\times\overline{18.03}^2}{175\times.251}\right)}$$

$$W_e^2 = \frac{2.04\times433.76\times325.08}{175\left(10340-\dfrac{1.76\times5.51\times325.08}{45.67}\right)}$$

$$W_e^2 = \frac{287653}{1797600} = .160, \quad W_e = .400$$

$$\text{At 12" Recoil, } W_e^2 = \frac{\overline{1.43}^2\,(9.337-1.767)^3\,\overline{16.61}^2}{175\left(9350 - \dfrac{\overline{1.33}^2\times\overline{1.767}^3\times\overline{16.61}^2}{175\times.261}\right)}$$

$$W_e^2 = \frac{2.04\times433.76\times275.89}{175\left(9350 - \dfrac{1.76\times5.51\times275.89}{45.67}\right)}$$

$$W_e^2 = \frac{244126}{1626100} = .150, \quad W_e = .387$$

$$\text{At 16" Recoil, } W_e^2 = \frac{\overline{1.43}^2\,(9.337-1.767)^3\,\overline{15.09}^2}{175\left(8361 - \dfrac{\overline{1.33}^2\times\overline{1.767}^3\times\overline{15.09}^2}{175\times.261}\right)}$$

$$W_e^2 = \frac{2.04\times433.76\times227.7}{175\left(8361 - \dfrac{1.76\times5.51\times227.7}{45.67}\right)}$$

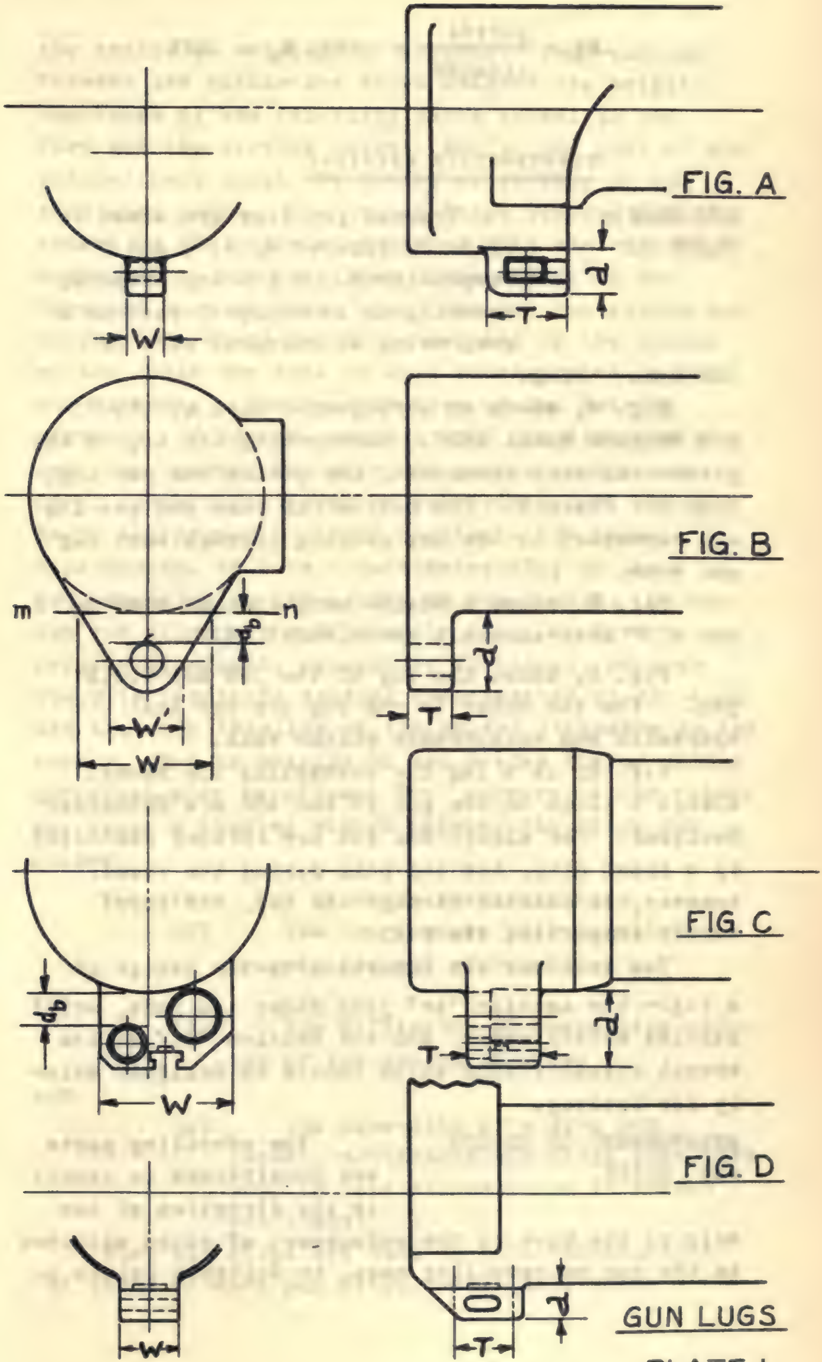


FIG. A

FIG. B

FIG. C

FIG. D

GUN LUGS

PLATE I

$$W_e^2 = \frac{201484}{1454775} = .138,\ W_e = .371$$

CONSTRUCTIVE DETAILS.

GUN LUGS - PLATE I. Typical gun lugs are shown in Plate I, figures A, B, C and D respectively. A gun lug properly speaking is an integral part of a gun, being an integral part of the bree-- ring.

Fig. A, shows an arrangement used on the 75 m/m Puteaux Model 1897. Surrounding the lug is the piston rod yoke connecting the piston and gun lug, fig. H - Plate 3. The piston rod yoke and gun lug are connected by the key passing through both lug and yoke.

Fig. B, shows a simple construction used on the 4.7" anti-aircraft mount,model 1917.

Fig. C, shows the lug of the 155 m/m G.P.F. gun. The two holes in the lug are for the hydraulic and recuperator piston rods.

Fig. D, is a lug for connecting the recoil sleigh n slide to the gun in the 155 m/m Schneider Howitzer. The sleigh and gun are further connected by a front clip, but the pull during the recoil however, is exerted through the lug, the front merely supporting the gun.

Two sections are important in the design of a lug:- the section "ab" just above the rods, which carries mainly shear, and the section "mn" at the breech circumference which should be designed mainly for bending.

ARRANGEMENT OF GUIDES AND CLIPS. The recoiling parts are constrained to recoil in the direction of the axis of the bore by the engagement of clips attached to the gun or recoiling mass, in suitable guides on

the cradle or recuperator forging. The reaction between the guides and clips balance the weight component of the recoiling parts normal to the bore and the turning moment, due to the pull of the various rods about the center of gravity of the recoiling parts. Due to the large turning moment caused by the pulls as compared with the weight component of the recoiling parts normal to the bore and more or less "play" between the guides and clips, the normal reactions exerted by the guides on the clips are more or less concentrated at the end contacts. The distribution of the bearing pressure, of course, depends upon the elasticity and play between the clip and guides, and therefore, assumptions based on experience must be made as to the proper surfaces required. In older type mounts, we have a continuous clip on the gun, engaging in the guides of the cradle. Unless the gun clips are sufficiently long, we have a varying, (gradually decreasing distance), between the clip reactions assumed concentrated at the ends and thus the friction of the guides increases in the recoil. Due to heating of the guides firing unless sufficient play is allowed for, warping of the guides may cause a binding action between the clips and guides.

Therefore, due to these considerations,

(1) the increase in clip reaction towards the end of recoil,

and

(2) the difficulty of preventing warping of the guides or clips

and

(3) the necessity of a long gun jacket, continuous gun clips have been more or less discontinued in modern artillery.

When gun clips are used we have combinations of three or more gun clips. When only three

clips are used it is possible to maintain practically only two clips in contact with the guides throughout the greater part of recoil. This is an advantage since any warping of the guides, etc., does not materially effect the operation of the recoil. With four or more gun clips, we have one or more intermediate clips, thus necessitating a more careful lining up of the gun clips and design of the guides to prevent warping unless considerable play is to be allowed.

Referring to fig.(3) Plate we have an arrangement of three clips A,B and C, which recoil to an intermediate position A',B',C', where the rear clip leaves the guide and the front clip enters the guide. If the clips are equally spaced as they should be, this intermediate position is one-half the length of recoil. In the final position the clips are in the position A" B" C" at the end of recoil. If "l" is the distance between clips, since A should not leave the guide until C enters the guide and at the end of recoil B must be still in contact with the guide, the length of guide should be:

Min. length of guides = 2L = b, (3 gun clips) and therefore,

$$\text{Distance between clips } l = \frac{b}{2}$$

With three clips, during the first half of the recoil, the coordinates with respect to the center of gravity of the recoiling parts of the front and rear clips respectively, become those of B and A while during the latter half, they become those of C and B.

With four clips we have an intermediate clips always in contact with the guides; hence a careful alignment is necessary with more or less to prevent any binding action of the middle clip and throw the greater part of the clip load on the extreme

front and rear clips respectively . Referring to fig.(3) Plate (the clips A,B,C and D move from the battery position, to the midway intermediary position, that is when clip A leaves the guide and clip D just enters the guide. If "l" is the distance between the extreme clips in battery, i.e. between A and C, or between B and D when clips are equally spaced as they should be, we have $l = b$, that is the distance between clips equals the length of recoil. Further the minimum length of guide $= \frac{3}{2} l = \frac{3}{2} b$.

With four clips, the coordinates of the front and rear clip reactions with respect to the center of gravity of the recoiling parts during the first half of recoil become those C and A respectively, while during the latter half they become those of B and D respectively.

Let us now consider the front and rear clip reactions between the guides and clips of the gun.

The clip reactions, become, for the front clip,

$$Q_1 = \frac{P_b e + Bd_b - W_r \cos\phi (x_2 - ny_2)}{x_1 + x_2 - n(y_2 + y_1)}$$

for the rear clip,

$$Q_2 = \frac{P_b e + Bd_b + W_r \cos\phi (x_1 - ny_1)}{x_1 + x_2 - n(y_1 + y_2)}$$

where P_b = the max. total powder reaction on the breech (lbs)

e = the distance from the axis of the bore to the center of gravity of the recoiling parts (inches)

B = the total braking pull excluding the guide friction (lbs)

d_b = the distance down from the center of gravity of the recoiling parts to the line of action of the center of pulls (inches)

W_r = weight of recoiling parts (lbs)

x_1 and y_1 = coordinates of front clip reaction measured from the center of gravity of the recoiling parts.

x_2 and y_2 = coordinates of rear clip reaction measured from the center of gravity of the recoiling parts.

n = coefficient of guide friction = 0.15 approx.

$\emptyset$ = the angle of elevation.

$l = x_1 + x_2$ = the distance between clip reactions.

Since ny_1 and ny_2 are small as compared with x_1 and x_2 respectively, we have for a close approximation,

$$Q_1 = \frac{P_b e + Bd_b - W_r \cos\emptyset . x_2}{l - 2nd_r} = \frac{Bd_b - W_r \cos\emptyset \; x_2}{l} \text{ (approx)}$$

$$Q_2 = \frac{P_b e + Bd_b + W_r \cos\emptyset \; x_1}{l - 2nd_r} = \frac{Bd_b + W_r \cos \emptyset . x_1}{l} \text{ (approx)}$$

where d_r = mean distance from center of gravity of recoiling parts to guide. The guide friction, becomes,

$$R_g = n(Q_1 + Q_2) = \frac{2nBd_b + nW_r \cos\emptyset (x_1 - x_2)}{l - 2nd_r} \text{ (lbs)}$$

The following table is useful in the layout arrangement for the gun clips and proper length of guides, as well as showing the change in clip reaction and guide friction for the two combinations.

No. of Clips	Distance between clips in terms of recoil b	Distance between front and rear clip reactions	Min. length of guides required	Front Clip Reaction (lbs)	Rear Clip Reaction (lbs)	Total Guide Friction (lbs)
3	$\frac{b}{2}$	$\frac{b}{2}$	b	$\frac{2(Bd_b - W_r \cos Ø x_2)}{b}$	$\frac{2(Bd_b + W_r \cos Ø x_2)}{g}$	$\frac{4nBd_b\ 2nW_r \cos Ø (x_1 - x_2)}{b - 2nd_r}$
4	$\frac{b}{2}$	b	$\frac{3}{2}$ b	$\frac{Bd_b - W_r \cos Ø x_2}{b}$	$\frac{Bd_b + W_r \cos Ø x_2}{b}$	$\frac{2nBd_b + nW_r \cos Ø (x_1 - x_2)}{b - 2nd_r}$

DESIGN AND STRENGTH OF GUN CLIPS AND GUIDES. In the design of gun clips and guides, the following points should be considered: (1) General considerations as to layout, protection from dust, etc; (2) the arrangement of clips and guides as outlined in the previous paragraph; (3) the computation of the maximum clip reactions; (4) the design of the clip or guide for allowable bearing pressure; (5) the strength of the clip or guides at their various critical sections, to resist bending, direct stress and shear.

(1) The location of guides in the direction normal to the axis of the bore should be based on the following considerations:-

(a) From a cross section of the gun and recuperator forging, the best position of guides and gun clips can be located with consideration for minimum stress in gun clips. This requires that the guides be located as near the axis of the bore as possible.

(b) For constructive reasons, it is good design to keep the various parts connected with the recoiling parts as near the axis of bore as possible.

(c) The reactions of the guides, however, are quite independent of the position of the guides in a normal direction to the bore, but since the resisting section of the cradle or recuperator forging is very large

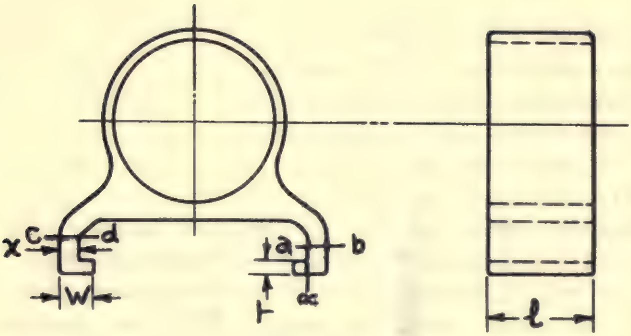


FIG. E

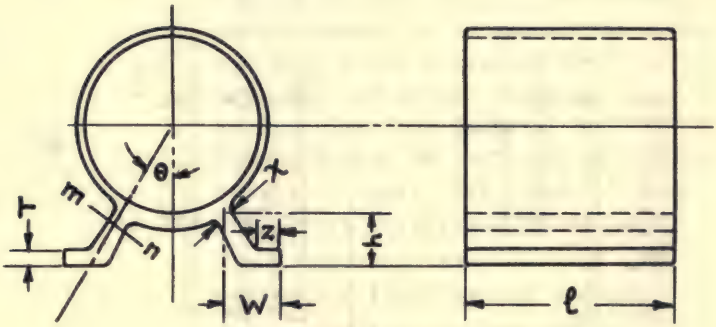


FIG. F

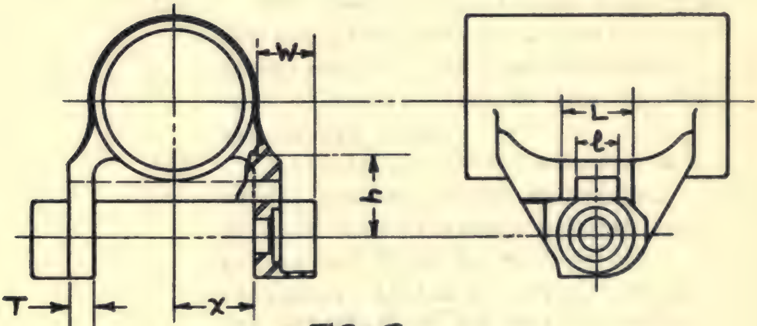


FIG. G

GUN CLIPS

PLATE 2

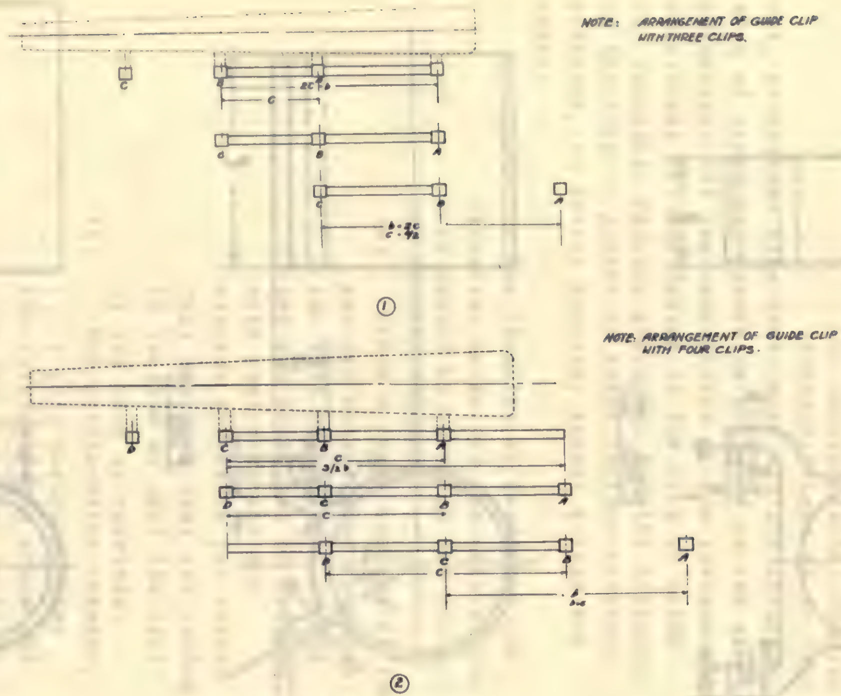

NOTE: ARRANGEMENT OF GUIDE CLIP
WITH THREE CLIPS.
NOTE: ARRANGEMENT OF GUIDE CLIP
WITH FOUR CLIPS.

PLATE 3

as compared with those of the gun clips; gun clips with long projections downward from the gun clip jacket due to guides too far below the axis of the bore are undesirable.

Hence the location of the guides depends upon construction and fabrication features with due consideration to the strength of the gun clips. These features in general demand that the guides be located as close to the axis of the bore as possible.

(2) For small guns, three clips equally spaced as described in the previous paragraph should be used. The front and rear clips should be bevelled off, so that smooth entrance may be made into the guides. Bronze liners either in the clips or guides should be used. For larger caliber guns, more clips should be used since the clip reactions and corresponding friction are reduced. Considerable tolerance should be allowed but very careful alignment made in order to prevent possible binding.

(3) The computation of clip reactions has been tabulated in a previous paragraph, for the common arrangement of either three or four equally spaced gun clips.

(4) The bearing contact during the recoil between guides and clips, depends upon tolerance between the guides and clips as well as the elasticity of the material, and on the magnitude of the wear between

the clips and guides. Therefore, we see the distribution of bearing pressure and the length of contact is completely indeterminate. From practice, however, the following assumption will be made:

(a) Length of gun clip $l = 1.8d$ (in.) approx. where d = diam. of bore.

(b) Constant length $l' = 1.5\,d$ (in.)

(c) Distribution of pressure assumed triangular.

Therefore, if b' = contact width of clip and guide, (inches) we have for the maximum bearing pressure due to the clip reaction Q (lbs).

$$p_{gm} = \frac{2Q}{b'l'} = 1.33 \frac{Q}{b'd} \text{ lbs. per sq.in.}$$

Now the max. allowable bearing pressure steel on bronze, becomes, p_{gm} = 600 to 800 lbs. per sq.in. Hence $b' = .0017$ to $.0022 \frac{Q}{d}$ (inches)

The distance $l-l'$ should be the bevelled length of clip distributed on either end.

With eccentric pulls the side thrust between clips and guides causes a bearing reaction Q' and if b" is the depth of guides in contact with clip, we have, $b' = .0017$ to $.0022 \frac{Q'}{d}$ (inches)

(5) The strength of gun clips depends upon the form or type of gun clip used. In fig.E, plate 2, we have the minimum bearing contact (w-x). The required thickness of the toe T is based on bending at section (a-b). Since the front clip reaction causes this bending, and the load is divided between two

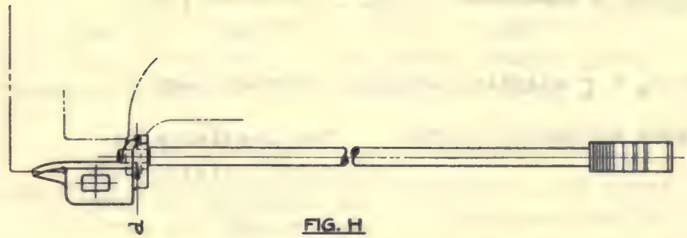


FIG. H

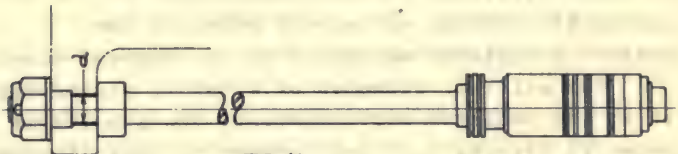


FIG. K

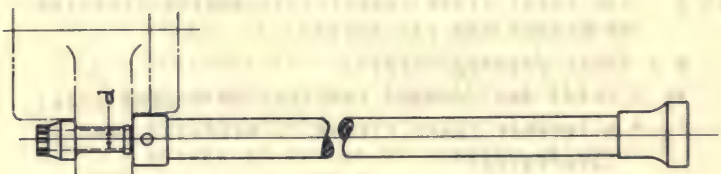


FIG. L

PISTON ROD CONNECTION TO GUN LUG

PLATE 4

front clips on either side. We have,

$$T = 1.225 \sqrt{\frac{Q_1(W-x)}{l\ f_m}} = 0.912 \sqrt{\frac{Q_1(w-x)}{d\ f_m}} \text{ (in)}$$

where $f_m = \frac{1}{2}$ elastic limit of material used.

STRENGTH OF RECOIL PISTON RODS. The greater part of recoil piston rods are subjected to tension during the recoil, and compression during the counter recoil due to the counter recoil buffer reaction. In a few types of recoil systems, we have compression in the rod during the recoil, an example being in the pneumatic cylinder of the 16" U. S. Railway mount.

The critical diameter of a recoil piston rod is at the smallest section within the gun lug as shown in figures H, K and L, Plate 4. This diameter should be based on the recoil pull at maximum elevation and the inertia load at maximum acceleration This load is the same that occurs for the gun lug.

Let P = the total fluid reaction + packing friction on piston and rod (lbs)

B = total braking (lbs)

P_b = total max. powder reaction on breech (lbs)

f_m = allowable fibre stress of material used (lbs/sq.in)

w_p = weight of rod and piston (lbs)

w_r = weight of recoiling parts (lbs)

d = diameter of smallest free section at gun lug.

Then

$$d = \sqrt{\frac{P + \frac{w_p}{w_r}(P_b - B)}{0.7854\ f_m}}$$

For hollow piston rods, with a "filling in" or spear buffer chamber, we must consider a section the greatest distance from the piston but passing through the buffer for maximum inertia and minimum

thickness of the rod. Let w_p' = weight of piston+ rod to section (lbs)

d_{ro} = outside diam. of buffer rod (in)

d_{ri} = inside diam. of buffer chamber (in)

Then using the previous symbols, we have,

$$d_{ro}^2 - d_{ri}^2 = \frac{P + \frac{w_p'}{w_r}(P_b - B')}{0.785\, f_m}$$

usually d_{ri} is fixed in consideration of the buffer design, hence d_{ro} is determined from the above formula.

When piston rods are subjected to compression, during the counter recoil or with a pneumatic recuperator during the recoil, the rod should be treated as a column loaded and constrained at both ends.

The maximum column load on the rod equals the maximum counter recoil buffer load, which may be roughly estimated on the basis of counter recoil stability at horizontal elevation. If

C_s' = constant of counter recoil stability = 0.85 to 0.9

W_s = weight of total gun + carriage (lbs)

l_s' = distance from wheel contact to line of action of W_s, recoiling parts in battery (in)

h = height of center of gravity of recoiling parts above ground (in)

B_x' = counter recoil buffer reaction (lbs)

F_{vi} = recuperator reaction in battery (lbs)

R'= approximate total friction (lbs) = 0.3 W_r

then $B_x' + R' - F_{vi} = C_s' \dfrac{W_s l_s'}{h}$ from which $B_x' = F_{vi} + C' \dfrac{W_s l_s'}{h} - R'$ (lbs)

thus giving the maximum compression load on the rod.

With pneumatic recuperators if the rod is under compression, the maximum compression is

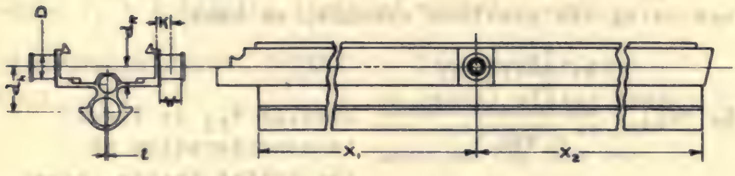


FIG. M

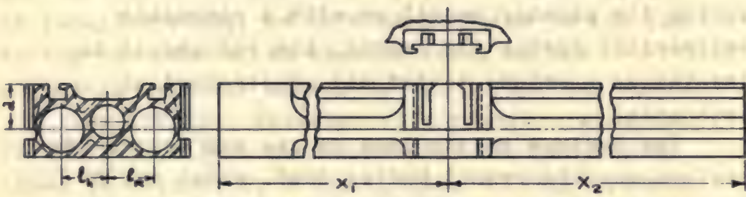


FIG. N

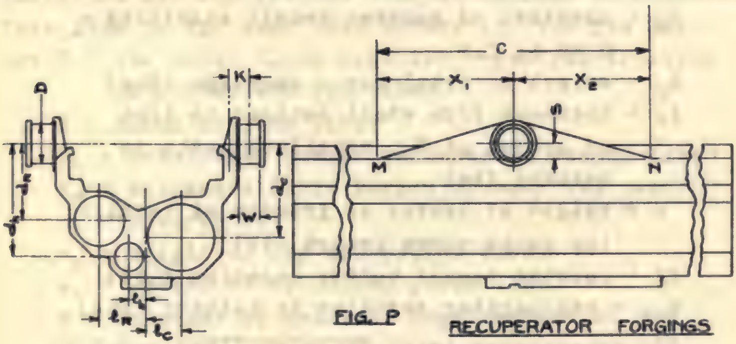


FIG. P

RECUPERATOR FORGINGS

PLATE S

liable to be either at the beginning or end of recoil. At the beginning we have the initial recuperator reaction + the inertia load of the rod, and at the end of recoil the maximum recuperator reaction.

TRUNNIONS AND SUPPORTING BRACKETS. In older mounts, the trunnions were an integral part of the gun, the gun setting directly in the top carriage. With mounts using a recoil system between the gun and top carriage, the trunnions are usually bolted by a supporting bracket to the cradle, though when the recuperator becomes a guide support replacing the necessity of a cradle, the trunnions often are an integral part of the recuperator forging.

Plate 4 shows recuperator forgings with trunnions an integral part of the forging, figures M and P, while fig. N shows a recuperator forging with a trunnion bracketed on.

Plate 6 shows typical trunnions and their supporting brackets which are bolted to cradle .

In fig. M, consideration only of the design of the trunnion itself is necessary, while in fig. P the strength of section m y should be considered as well. Section mn is subjected to bending and shear combined with direct stress.

DESIGN OF TRUNNIONS:

Let w = bearing length of trunnion

D_f = outside diam. of trunnion

d_f = inside diam. of trunnion at section "mn"

f = max. fibre stress, - lbs. per sq.in.

f_b = allowable bearing pressure - lbs.per sq. in.

Let w = width of section "ab" just above the rods

w' = width of section "mn" at the contact of breech circumference and lug.

d_b = the distance down from "mn" to center of gravity of pulls

d = depth of lugs

T = longitudinal length of lug

p_b = max. total powder pressure on breech

w_c = weight of recoiling parts attached to lug.

w_r = total weight of recoiling parts

Then

$$wT = \frac{B + \frac{w_c}{w_r}(p_b - B)}{f_s} \text{ for section "ob"}$$

$$w'T^2 = \frac{6[B + \frac{w_c}{w_r}(p_b - B)]d_b}{f} \text{ for section "mn"}$$

If $kw = w'$, then $T = \dfrac{6d_b f_s}{kf}$ where $w = w'$ as in figures A, C and D,

$k = 1$ and $T = \dfrac{6d_b f_s}{f}$

Very often $d = 2\,d_b$, figures A, C and D, hence

$T = \dfrac{3df_s}{f}$ where T is given.

$$w = \frac{B + \frac{w_c}{w_r}(p_b - B)}{f_s}\ ; \quad w' = \frac{6f_s^2 w^2\, d_b}{f[B + \frac{w_c}{w_r}\,(p_b - B)]}$$

X and Y = the component reactions of the trunnion (See Chapter V)

When no rocker is used, the entire trunnion of width "w" usually has bearing contact in the top carriage trunnion bearing. The design should be based on a consideration of both the allowable bearing pressure f_b and the strength at section

"mn" where the trunnion meets the cradle or trunnion bracket. We have,

$$f_b w\ D_t = \sqrt{X^2+Y^2} \text{ for bearing pressure}$$

$$\frac{f\pi(D_t^4-d_t^4)}{16\ wD_t} = \sqrt{X^2+Y^2} \text{ for strength at section mn}$$

Combining,

$$D_t^4-d_t^4 = \frac{16(X^2+Y^2)}{\pi f\ f_b}$$

Therefore, assuming d_t, we immediately obtain D_t

When, however, a rocker is used the dimensions depend upon the rocker bearing length. Let

w_r = length along trunnion for rocker bearing

w_c = length along trunnion for top carriage bearing

X and Y = top carriage component bearing reactions

X_r and Y_r = component rocker reactions

a= distance from mn to the center of top carriage bearing

b = distance from mn to the center of rocker bearing

M_x = the bending moment at section mn in the plane of the X component reactions.

M_y = the bending moment at section mn in the plane of the Y component reactions.

Then $w = w_r+w_c \qquad a = w_r+\frac{w_c}{2} \qquad b = \frac{w_r}{2}$

$$M_x=Xa+X_rb \qquad M_y = Ya + Y_rb \text{ and } M = \sqrt{M_x^2+M_y^2}$$

then $f_bw_cD_t = \sqrt{X^2+Y^2}$ at the top carriage bearing

$f_bw_rD_t = \sqrt{X_r^2+Y_r^2}$ at the rocker bearing

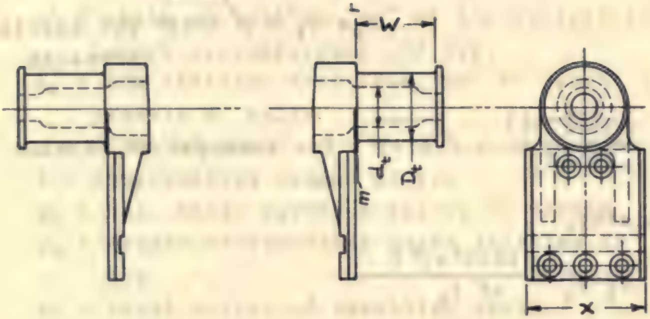


FIG. Q

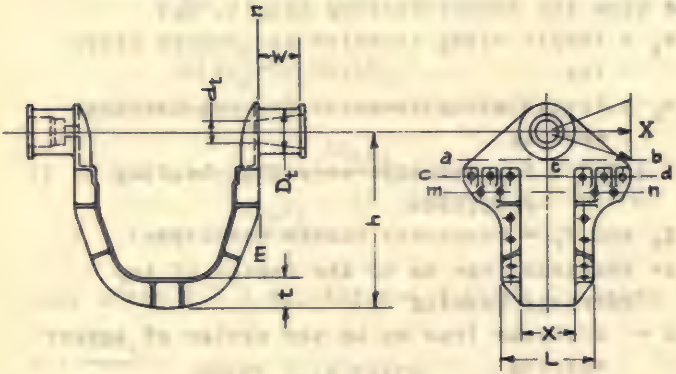


FIG. R

TRUNNIONS

PLATE 6

$$\text{and } f = \frac{32\ M\ D_t}{\pi(D_t^4 - d_t^4)} \quad \text{Further } f = \frac{32M\sqrt{X^2+Y^2}}{f_b w_c (D^4 - d^4)}$$

$$= \frac{32M\sqrt{X_r^2+Y_r^2}}{f_b' w_r (D_t^4 - d_t^4)}$$

Since a direct solution for D_t is complicated a trail solution is preferable. A reasonable procedure would be to solve for D_t from the bending equation assuming arbitrarily values for W_v and W_r. Then knowing D_t approximately we may solve W_v and W_r in consideration of the allowable bearing stress, and then recalculate D_t.

TRUNNION BRACKETS:

In the design of trunnion brackets, we have one of two types:

(1) Where the bracket is secured to a recuperator forging, the bracket merely transmitting the trunnion load to the forging, the Latter of which is stiff enough to carry the bending stresses. Plate VI, fig. Q.

(2) Where the bracket is secured to light built up cradle, as in Plate VI fig. R.

In the latter case the bracket acts as a stiffener and takes up the cross wise bending, the longitudinal shear reaction being transmitted to the cradle only.

For brackets of type (2), assuming the cradle merely to take up the shear reaction of the rivets only, we find a critical section "mn" at the bottom of the bracket, Fig. R, Plate VI

Section "mn" is subjected to:

(1) Cross-wise bending = $Y \frac{d}{2}$

(2) Longitudinal bending = $X \frac{d}{2}$

(3) A shear stress = $\sqrt{X^2+Y^2}$

In brackets that are bolted to a recuperator forging as in fig. Q, Plate VI we have grooves or guides, which engage in corresponding guides or grooves in the recuperator forging. The projections and grooves must be designed to withstand the allowable total bearing stress and total shear as well, both of which equal the X component of the trunnion reaction. The bolts which secure the brackets to the forging, merely take up the tension, due the moment caused by the overhang of the trunnions and the trunnion reaction.

In the design of trunnion brackets, we have other critical sections, as ab of fig. Q and cd of fig. R, that is just above or at the first row of rivets.

In the design of a trunnion bracket, the critical section is near the first row of rivets,as sections ab, cd or mn respectively in fig. R.

The straining action at either one of these sections consists of:

(a) A direct pressure (or tension) due to the Y component of the trunnion reaction.

(b) A shear stress equal to the X component.

(c) A bending moment in the longitudinal plane (due to the moment of the X component = X_r.

(d) A bending moment in a cross sectional plane due to the Y component = Y g.

(e) A torsional or twisting moment due to the X component = X m.

If now I_x and I_y are the moments of inertia of critical section and d_x and d_y are the distances to

the extreme fibres in the longitudinal and crosswise directions, respectively, we have,

$$f = \frac{X_r d_x}{I_x} \pm \frac{Y_g d_y}{I_y} \pm \frac{Y}{A}$$ for the maximum fibre stress.

To design for the proper distribution and required strength of rivets, for brackets of type 2, fig.R, we assume a differential rotation about the bottom of the bracket. Obviously the shear strain for the upper rivets is a maximum.

If now, the vertical distance from the bottom to the top row is r_o, for the next lower row r_1 and so on, then for the shear in the various rows of rivets, we have, $S_o = c\ r_o$, $S_1 = c\ r_1$, $S_2 = c\ r_2$ etc. Further if we have n_o rivets for the top row, n_1 for the next row and so on, then taking moments about the bottom, we have

$c(n_o r_o^2 + n_1 r_1^2 + n_2 r_2^2 --- n_n r_2^2) = X\ r$ where r is the vertical distance from the center line of trunnions to the bottom of the bracket. Hence

$$C = \frac{X_r}{n_o r_o^2 + n_1 r_1^2 -- n_n r_n^2}$$ therefore, assuming $n_o n_1 --- n_n$ respectively and the spacing of the rows $r_o r_1 --- r_n$ respectively, we obtain C.

The shear stress for any one rivet, becomes,

$$S_o = \frac{X\ r\ r_o}{n_o r_o^2 + n_1 r_1^2 - n\ r_n^2}$$ for the top rivets,

$$S_1 = \frac{X\ r\ r_1}{n_o r_o^2 + n_1 r_1^2 --- n_n r_n^2}$$ for the next row,

----

$$S_n = \frac{X\ r\ r_n}{n_o r_o^2 + n_1 r_1^2 --- n_n r_n^2}$$ for the bottom row,

The tensions in the rivets or in the bolts as in fig. Q, are obtained by an exactly similar deflection method. - (See design of bolts for pedestal

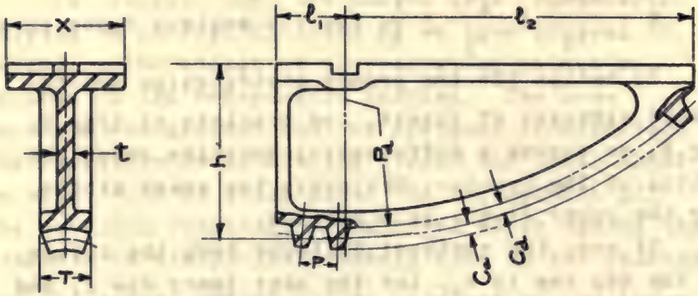


FIG. S

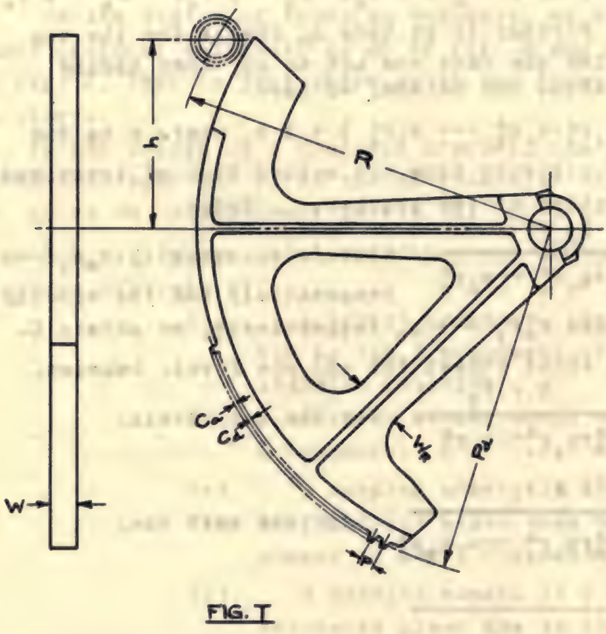


FIG. T

ELEVATING ARC

PLATE 7

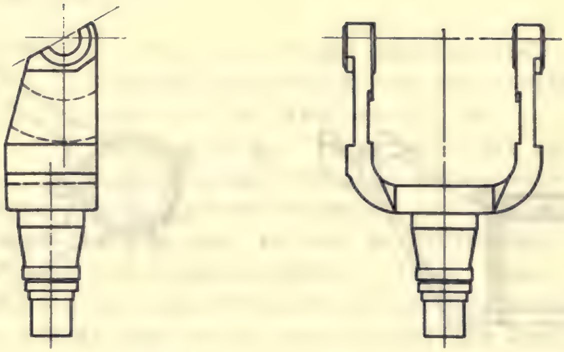


FIG. W

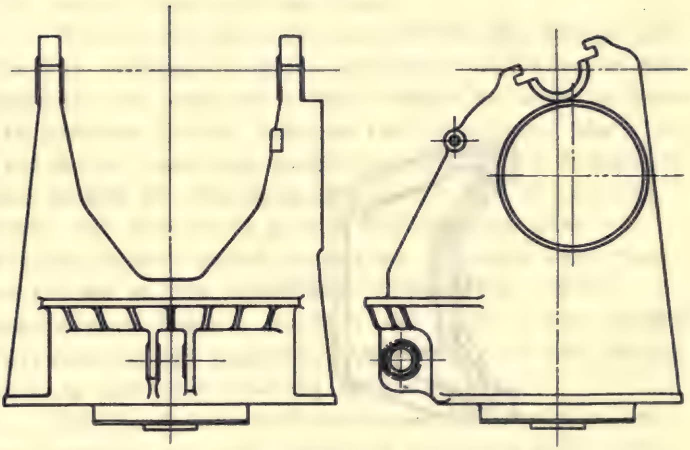
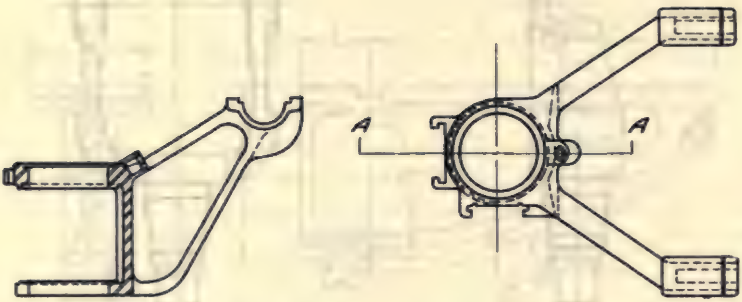


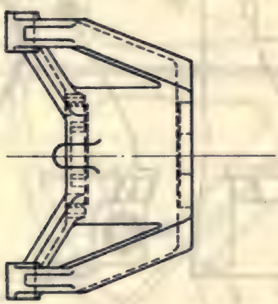
FIG X

TOP CARRIAGE

PLATE 8



SECTION A-A



TOP CARRIAGE

PLATE 9

mounts - External Forces).

TOP CARRIAGES. The top carriage sustains the tipping parts and during the firing takes up the reactions of the tipping parts. These loads are applied at the trunnions and elevating arc respectively. With balancing gear introduced for high angle firing guns, we have an additional reaction due to the balancing gear. These loads are balanced by the supporting forces at the traversing pintle and at some other contact with the base plate or bottom carriage, the arrangement and position of which determine to a considerable degree the type of top carriage used.

With large caliber guns, where the design of the top carriage depends primarily on strength considerations, special effort should be made to throw the greater firing load on the trunnions, the elevating arc reaction merely balancing the moment of the weight of the recoiling parts out of battery. Then, the elevating gear and balancing gear reactions become minor forces as compared with that sustained at the trunnions. Therefore, in a preliminary layout, we have the load at the trunnions balanced by the supporting reactions at the pintle bearing and clip bearing respectively.

Having determined the external reaction, we may examine critical sections and determine their respective strengths. By classifying the various types of top carriages used, certain important sections and the loadings on them may be pointed out.

We have the following types of top carriages:-

WITH MOBILE ARTILLERY.

(1) Top carriages with side frames and connected together by transoms or cross beams supporting the pintle bearing for traversing the top carriage, (Plate 8,fig.1)

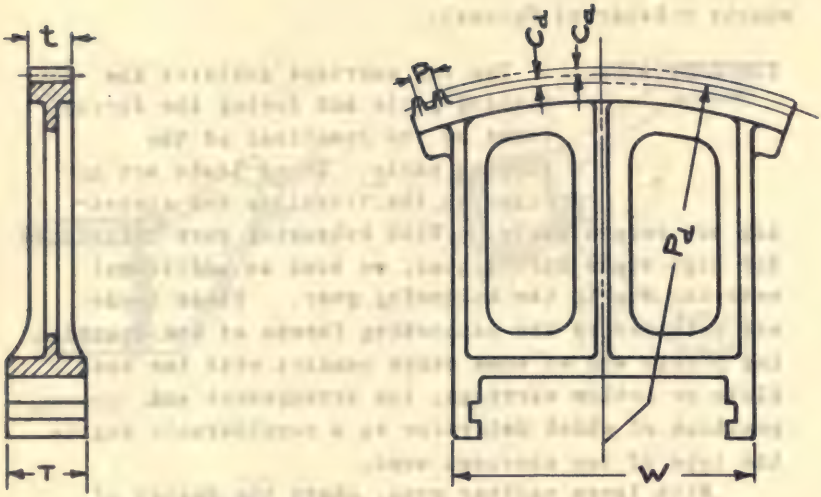


FIG. U

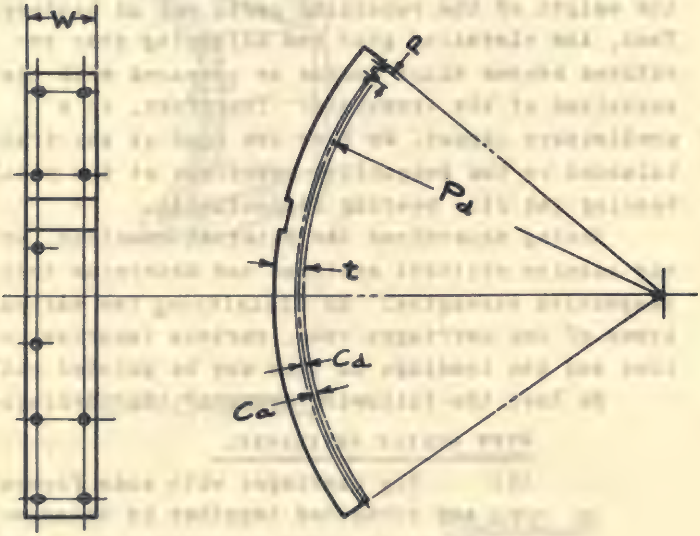


FIG. V

TRAVERSING GEAR

PLATE 10

(2) Pivot yoke type of top carriage, the pintle bearing fitting in the wheel axle and prevented from overturning by a bottom pin fitting in an equalizer bar the latter being connected to the trail, Pivot yoke type of top carriage, when used in a pedestal mount, is prevented from overturning by a sufficient shoulder at the top of the bearing.

(3) Cantilever top carriage used when balancing gear is introduced. The trunnions being at the rear, the pintle bearing at the center and front clip bearing at the front, gives a cantilever loading on the top carriage.

WITH FIXED STATIONARY MOUNTS.

(1) Pivot yoke type of top carriage with small pedestal mounts.

(2) Side frame top carriages with barbette mounts.

WITH RAILWAY ARTILLERY.

(1) Pivot yoke or side frame top carriages with pedestal or barbette mounts, seated on the car frame.

(2) Side frame girders, supporting the trunnions of the tipping parts, directly on the girders, and the girders in turn being supported by the truck reactions, or by a distributed support of special rails.

(1) Side Frame Top Carriages:

Side frame top carriages consist of two side frames either of cast steel or of built up structural steel. The frames are connected together by a heavy transom or cross beam which contains the pintle bearing for traversing. The pintle bearing and transom

are usually located either directly below or to the front of the trunnions. The pintle bearing is designed only to take up a part of the vertical and the entire horizontal component of the reaction of the tipping parts, the overturning moment being balanced by the reactions of either front or rear circular clips. Either a rack or pinion gear is introduced at a given radius on the top carriage for traversing about the pintle. A pinion or worm wheel bearing, for the pinion or worm engaging in the elevating arc is located at a given radius from the trunnion axis. In the design of large guns, special effort should be made to throw the greater load on the trunnions, the elevating bearing merely sustaining the moment of the weight of the recoiling parts out of battery.

External Reactions: See Plate 11 fig.(1)

As a first approximation, assuming the entire firing load to be applied at the trunnions, we have,

$$\left.\begin{array}{l} 2H = K\cos\phi \\ 2V = K\sin\phi + W_t \end{array}\right\} \text{(lbs) approx.}$$

where H and V = the horizontal and vertical load applied to either trunnion

K = the resistance to recoil $= \frac{0.47}{b}\frac{w_r}{g}V_f^2$ approx.

$$V_f = \frac{wv + 4700\bar{w}}{w_r};$$

w = weight of shell

$\bar{w}$ = weight of powder charge

w_r = weight of recoiling parts

b = length of recoil (ft)

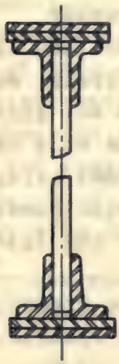
W_t = weight of tipping parts

The pintle reactions, becomes,

$$H_a = 2H = K\cos\phi \text{ (lbs)}$$

$$V_a = \frac{2V\,l_t - 2H\,h_t}{l} = (K\sin\phi + W_t)\frac{l_t}{l} - K\cos\phi.\frac{h_t}{l} \text{ (lbs)}$$

SECTION A-B
TYPICAL SECTIONS OF GIRDERS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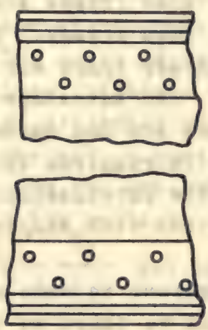
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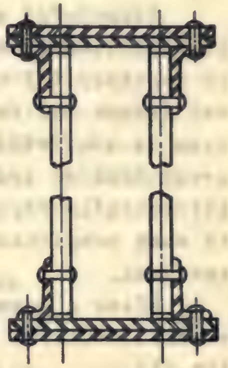
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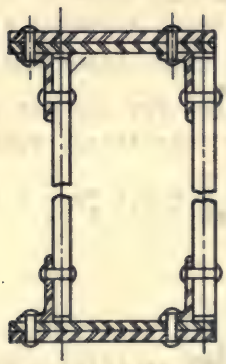
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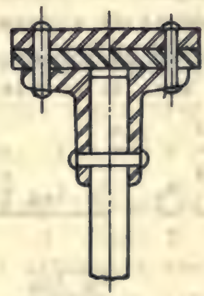
5

PLATE 11

and the rear clip reaction, becomes,

$2V_b = K\sin\theta + W_t - V_a$ (lbs) for the clip reaction on either girder.

Structural Steel Sections of Side Frames:

Structural steel side frames are becoming standardized types of side frames. We have two types of section, (1) box girder types and (2) simple web and flange section . The advantage of a box girder type is that stiffeners are not needed and only one cross beam or transom is required the frames being sufficiently rigid. The fabrication of such however is more complicated, than with simple web and flange sections.

After a layout contour is made of the frame several sections should be taken as (m-n) Plate 11, fig.(1).

A typical box girder shown in Plate 11, fig.(2) and a typical flanged web section is shown in Plate 11 fig.(3). As a first approximation it will be assumed that the flanges take all the bending stress and the webs the entire shear. If

I = moment of inertia of section
d = depth of flange (in)
A = area of flange (sq.in)
f_m = max. allowable fibre stress

then

$$2A\frac{d^2}{4} = \frac{Ad^2}{2} = I \quad \text{approx.}$$

$$\text{Hence } f_m = \frac{M_y}{I} = \frac{V_b\, l_{mn}\, \frac{d}{2}}{I} \quad \text{(lbs/sq.in)}$$

$$= \frac{V_b\, l_{mn}}{A\, d}$$

$$\text{hence } A = \frac{V_b\, l_{mn}}{f_m d} \text{ (sq.in)} \qquad d = \frac{V_b\, l_{mn}}{f_m A} \text{ (in)}$$

Thus with a constant flange section, we must increase the depth of the girder as the distance from

the reaction V_b.

If on the other hand for construction considerations and approximately constant depth of girder is required, the flange area must be increased with the distance l_{mn}. These factors determine the cross section of girder, the area of the flange and the depth of girder. The depth of girder should be sufficient with a given thickness of web "t", not to exceed the maximum allowable shear stress f_s. Since the shear on the web is practically uniform, we have, $f_s\, dt = V_b$ for one web and $2f_s\, dt = V_b$ for two webs as in a box section, then

$$d = \frac{V_b}{f_s t} \text{ one web,} \quad d = \frac{V_b}{2f_s t} \text{ box section}$$

Pitch of Rivets in built up girder:

Let p = pitch of rivets (in)
R = allowable total shearing stress on rivet
d = depth of web
F = total shearing force

Then considering a portion of the web of length p of a compound I section, we have, $Fp = Rd$, where $F=V_b$ or the total shear on the section, hence $p = \frac{Rd}{F}$ (in)

Now for one web and two angle irons connecting the flange plates with web, we have

$$R = 2\,\frac{\pi}{4}\,d_r^2\, f_{sr} = \frac{\pi}{2}\,d_v\, f_{sr}$$ where d_r = diam. of rivet (in)
f_{sr} = allowable shear stress on rivet (lbs/sq.in)

With a built up box section as in Plate 11 fig.4,

$$R = 2\,\frac{\pi}{4}\,d_r^2\, f_{sr} = \frac{\pi}{2}\,d_r^2\, f_{sr}$$

fig.(5) $R = 4\frac{\pi}{4}\,d_r^2\, f_{sr} = \pi\, d_r^2\, f_{sr}$

GENERAL DESIGN PROCEDURE.

Type of gun "Howitzer or Gun"	=	155 M/M Howitzer
Type of Mount "Field Carriage" "Platform Mount" "Caterpillar Mount"	=	Field Carriage
Diam. of bore d (inches)	=	6.1
Muzzle Velocity v (ft/sec)	=	1850
Weight of Projectile w (lbs)	=	95
Weight of powder charge $\bar{w}$ (lbs)	=	14.25
Weight of Recoiling parts W_r	=	4200
Max. pressure on breech p_{bm} (lbs/sq.in)	=	30000
Length of Recoil Max. Elevation b_s (ft)	=	3
Max. angle of elevation $\emptyset_m$ (degrees)	=	65°
Assumed length of horizontal recoil b_h (ft)	=	4

Min. angle of elevation
ϕ_i (degrees) = $-5°$

Travel of projectile up bore
u (inches) = 117.5

Area of bore
$A = 0.785\ d^2$ (sq.in) = 29.75

INTERIOR BALLISTICS.

Area of bore of gun
$A = 0.785\ d^2$ (sq.in) =

Total max. pressure on
breech of gun
$P_{bm} = P_{bm}A$ (lbs) =

Mean constant pressure on
projectile
$P_e = \dfrac{wv^2 \times 12}{644u}$ (lbs) =

Time Abscissa of max.
pressure

$$e = u[(\frac{27}{16}\frac{P_{bm}}{P_e} - 1)$$

$$\pm \sqrt{(1 - \frac{27}{16}\frac{P_{bm}}{P_e}) - 1}] \text{ (in)}$$ =

Pressure on base of breech
when shot leaves muzzle:
$P_{ob} = \dfrac{27}{4} e^2 \dfrac{u}{(e+u)^3} P_{bm}$ (lbs) =

Max.Velocity of free recoil:
$V_f = \dfrac{wv + 4700\ \bar{w}}{w_r}$ (ft/sec) =

Velocity of free recoil when shot leaves muzzle:

$$V_{fo} = \frac{(w+0.5\,\bar{w})v}{w_r} \text{ (ft/sec)} =$$

Time of travel of shot to muzzle:

$$t_o = \frac{3}{2}\,\frac{u}{12v} \text{ sec.} =$$

Time of free expansion of gases:

$$T_{\iota o} = \frac{2(V_f - V_{fo})}{P_{ob}}\;\frac{W_r}{32.2} =$$

Free movement of gun while shot travels to muzzle

$$X_{fo} = \frac{w+0.5\bar{w}}{w_r}\;\frac{u}{12} \text{ (ft)} =$$

Free movement of gun during powder expansion

$$X_{f'o} = \frac{P_{ob}}{W_r}\, g\, \frac{t^2 o}{3} + V_{fo} t_{\iota o} \text{ (ft)} =$$

Total free movement of gun during powder pressure period

$$E = X_{fo} + X_{f'o} \text{ (ft)} =$$

Total time of powder pressure period

$$T = t_\iota + t_{\iota o} =$$

STABILITY: TOTAL RESISTANCE TO RECOIL AT MAXIMUM AND MINIMUM ELEVATION.

Weight of system (gun and carriage) W_s (lbs)	=
Distance from spade point to line of action of W_s (from preliminary layout) l_s (ft)	=
Height of trunnion from ground (assume) h_t (ft)	=
Horizontal distance from spade point to trunnion center (assume) l_t (ft)	=
Distance from center of gravity of recoiling parts to trunnion (assume) s (ft)	=
Moment arm of resistance to recoil for angle of elevation Ø $d = h_t \cos Ø + s - l_t \sin Ø$ (ft)	=
Height to center of gravity of recoiling parts for horizontal recoil $h = h_t + s$ (ft)	=

APPROXIMATE CALCULATION: (E and T not computed)

Velocity of free recoil $V_f = \frac{wv + 4700\bar{w}}{w_r}$ (ft/sec)	=

Travel up bore u (inches) =

Initial recoil constrained energy (approx)

$A_r = \frac{1}{2} \frac{W_r}{g} V_r^2$ (ft/lbs) =

where $V_r = 0.92\ V_f$ (approx) long recoil

$= 0.88\ V_f$ short recoil

Displacement of gun during powder period

$E_r = \left(\frac{w+0.5\bar{w}}{W_r}\right) \frac{u}{12}$ (ft) =

where a = 2.25 for long recoil

= 2.22 for short recoil

---

(1) Constant resistance throughout Recoil.

Constant of horizontal stability

$C_s = \frac{\text{Overturning moment}}{\text{Stabilizing moment}}$ =

(Usually assume 0.85)

Min. length of recoil consistent with stability at minimum elevation

$$b_{min} = \frac{W_s l_s + W_r E_r \cos\emptyset - \sqrt{(W_s l_s + W_r E_r \cos\emptyset)^2 - 4 W_r \cos\emptyset \left(W_s l_s E_r + \frac{d A_r}{c}\right)}}{2\ W_r \cos\emptyset} =$$

(ft)

At 0° Elev. cos Ø = 1 and d = h

Max. allowable recoil at horizontal elevation

$b_{h_{max}} = .035\ V_f\sqrt{h}$ (ft) =

Assumed length of horizontal recoil at min. elevation

b_h (ft) =

Total resistance to recoil at horizontal or minimum elevation

$K_h = \dfrac{A_r}{b_m - E_r}$ (lbs) =

use A_r for long recoil

Assumed length of recoil at max. elevation consistent with clearance b_s (ft) =

Total resistance to recoil at max. elevation ($\emptyset_m$ =

$K_s = \dfrac{A_r}{b_s - E_r}$ (lbs) =

Use A_r for short recoil

---

(2) Variable Resistance to recoil.

Constant of horizontal stability C_s =

Min. length of recoil consistent with stability at min. elevation

$$b_{min} = \frac{1}{W_r\cos\emptyset}\left[W_s l_s - \sqrt{(W_s l_s)^2 - 2W_r\cos\emptyset\left(\frac{A_r}{c}d + W_s l_s E_r - \frac{W_r\cos\emptyset}{2}E_r^2\right)}\right]$$

(ft) at 0° elev. $\cos\emptyset = 1$ and $d = h$

Max. allowable recoil at horizontal elevation

$b_{h_{max}} = .035\ V_f \sqrt{h}$ (ft) =

Assumed length of recoil at horizontal or min. elevation b_h (ft) =

Mean resistance to recoil during retardation period

$K_m = \frac{A_r}{b-E_r}$ (lbs) =

Stability slope

$m = C_s \frac{W_r \cos \emptyset}{d}$ (lbs/ft) =

Mean resistance to recoil in battery

$K = K_m + -(b-E_r)$ (lbs) =

Mean resistance to recoil out of battery

$k = K_m - \frac{m}{2}(b-E_r)$ (lbs) =

Exact calculation E and T computed (See Interior Ballistics).

(1) Constant resistance to recoil.

Constant of stability (assumed) C_s =
(C_s=0.85 usually)

$A = W_r \cos \emptyset_{min}$ =

$B = W_r \cos\phi_{min}(V_f T - E) - W_s l_s$ =

$C = W_s l_s (V_f T - E) + \frac{1}{2} \frac{W_r}{g} V^2 \frac{d}{C_s}$ =

Min. length of recoil consistent with stability at min. elevation

$b_{min} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ (ft) =

Allowable recoil at horizontal elevation

$b_{h_{min.}} = .035 \sqrt{h}$ (ft) =

Assumed length of recoil at minimum elevation

b_h (ft) =

Total resistance to recoil at min. elevation

$K_h = \frac{\frac{1}{2} m_r V_f^2}{b_h - E + V_f T}$ (lbs) =

Max. elevation consistent with clearance b_s (ft) =

Total resistance to recoil at max. elevation

$K_s = \frac{\frac{1}{2} m_r V_f^2}{b_s - E + V_f T}$ (lbs) =

---

(2) Variable resistance to recoil.

Constant of stability (assumed) C_s =

Stability slope $m = C_s \dfrac{W_r \cos \emptyset}{d}$ (lbs/ft) =

Total resistance to recoil during powder period consistent with stability

$$K = \frac{C_s (W_s l_s - W_r E \cos \emptyset)}{d - C_s \dfrac{W_r T^2}{2 m_r} \cos \emptyset} \text{ (lbs)} =$$

$A = m$ =

$$B = \frac{m K T^2}{m_r} - 2K - 2mE =$$

$$C = \left(2E - \frac{m E T^2}{m_r} - 2 V_f T\right) K + \frac{K^2 m T^2}{4 m_r^2} + m E^2 + m_r V_f^2 \text{ ---- } =$$

Min. length of recoil consistent with stability at min. elevation

$$b_{min} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \text{ (ft)} =$$

Allowable recoil at horizontal elevation

$b_{h_{max.}} = .035\, V_f \sqrt{h}$ (ft) =

Assumed length of recoil at minimum elevation b_h (ft) =

Total resistance to recoil during powder period with assumed length of recoil at min. elevation

$$K_h = \frac{m_r V_f^2 + m(b-E)^2}{2[b_h - E + V_f T - \frac{m}{2}\frac{T^2}{m_r}(b-E)]} =$$

Total resistance to recoil in out of battery position with assumed length of recoil at min. elev.

$$k_h = K_h - m(b - E + \frac{K_h T^2}{2m_r}) =$$

Margin of stability at minimum elevation for the assumed long recoil in and out of battery respectively.

Mean constant pressure on breech of gun

$$P_{bc} = 1.12\ (\frac{wv^2 \times 12}{64.4\ u})\ \text{lbs.} =$$

Max. overturning force in battery (stability limit)

$$K_h' = W_s l_s - P_{bc} e (1 + \frac{2nd}{1-2nd_r}) =$$

n = 0.15 to 0.25

$l = \frac{b}{2}$, 3 clips = b, 4 clips

d_r = mean distance to guide friction from bore.

Max. overturning force out of battery

$$k_h' = \frac{W_s l_s - W_r\ b_h \cos \emptyset}{d} =$$

Margin of stability in battery
$K_h' - K_h$ (lbs) =

Margin of stability out of battery $k' = k$ (lbs) =

Estimated Jump of Carriage at Horizontal elevation.

Distance from spade to center of gravity of W_s — d_s (ft) =

Time of recoil (approx.)

$$t_1 = \frac{w_r}{g}\,\frac{V_f}{K_h} \quad \text{(sec)} =$$

Ang. vel. about spade at end of time T_1

$$w_1 = \frac{g(K_h d - W_s l_s) t_1}{w_s d_s^2} \quad \text{(rad/sec)} =$$

Time to max. lift of carriage from end of time t_1

$$t_2 = \frac{d_s^2}{l_s g}\,w_1 \quad \text{(sec)} =$$

Total angular displacement about spade to max. lift.

$$\theta = \frac{1}{2}\,w_2(t_1 + t_2) \quad \text{(rad)} =$$

Lift of wheel from ground

$S_w = l_w\theta = l_s\,\theta$ (approx) (ft) =

where l_w = distance from spade to wheel base (ft)

Potential energy at max. lift

$E_s = W_s l_s\,\theta$ (ft/lbs) =

VARYING THE RECOIL ON ELEVATION:

In general assume the length of recoil at horizontal recoil constant from ϕ_i to ϕ_1 degrees, (usually from 0° to 20° elevation), then, decrease the recoil proportionally with the elevation, or consistent with clearance.

Length of intermediate recoil (ft) from ϕ_1 to ϕ_m degrees.

$$b = \frac{b_h - b_s}{\phi_i - \phi_1}(\phi - \phi_m) + b_s$$

b_h = long recoil

b_s = short recoil

Resistance to recoil: from ϕ_i to ϕ_1

$$\text{Variable } K_h = \frac{m_r V_f^2 + m(b-E)^2}{2\left[b_h - E + V_f T - \frac{m}{2}\frac{T^2}{m_r}(b-E)\right]}$$

$$\text{Constant } K_h = \frac{\frac{1}{2} m_r V_f^2}{b - E + V_f T}$$

from ϕ_2 to ϕ_m

$$\text{assumed constant} = K = \frac{\frac{1}{2} m_r V_f^2}{b - E + V_f T} \quad \text{exact}$$

$$= \frac{0.47\ m_r V_f^2}{b}$$

Moment arm of resistance to recoil about spade $d = h_t \cos\emptyset + S - l_t \sin\emptyset$ (ft)

Angle of Elev. $\emptyset$	Length of recoil b	Stability slope Variable Resist. m	Stability slope Const. resist. m	Total resist. to recoil K	Moment arm of resist. to recoil about spade d	
$\emptyset_i$	b_h	$\frac{C_s W_r \cos\emptyset_i}{d_i}$	0	K_h	d_i	Usually from 0° to 20°
.	.	.	.	.	.	
.	.	.	.	.	.	
.	.	.	.	.	.	
$\emptyset_l$	b_h	$\frac{C_s W_r \cos\emptyset_i}{d_i}$	0	K_h	d_i	
$\emptyset_i$	b_h	0	0	K_h	d_1	From 20° to max. elev.
.	.	.	.	.	.	
.	.	.	.	.	.	
.	.	.	.	.	.	
$\emptyset$	b	0	0	K	d	
.	.	.	.	.	.	
.	.	.	.	.	.	
.	.	.	.	.	.	
$\emptyset_m$	b_s	0	0	K_s	d_m	

RECUPERATOR LAYOUT

Initial recuperator reaction (approx.) (Max. elev. $=\emptyset_m$)

$F_{vi} = 1.3\ W_r(\sin\emptyset_m + 0.3\cos\emptyset_m)$ (lbs) = 5623.8

Min. mean recuperator reaction (Max. elev. $\emptyset_m$)

$F_{vm} = 2W_r[\sin\emptyset_m - 0.3(1-\cos\emptyset_m) + 0.3W_r]$ (lbs) = 7418

Min. allowable ratio of compression

$$M_{min} = \frac{F_{vf}}{F_{vj}} = \frac{1.5(1.665F_{vm} - F_{vi})}{F_{vi}} = 1.79$$

Max. allowable ratio of compression (stability limit)

$$M_{max} = \frac{F_{vf}}{F_{vi}} = \frac{0.8[K_h + W_r(\sin\emptyset_m - 0.3\cos\emptyset)]}{F_{vi}} = 9.$$

Max. allowable ratio of compression (heating limit)

m = 2 to 2.5 =

Mean temperature in recuperator (assumed)

T_m = 20° to 30° C (centigrade) =

Max. temperature due to compression

$$T = T_m\ M\ \frac{k-1}{r} =$$

where k = 1.3 with floating piston

= 1.1 air contact with oil

Ratio of compression used. M	=
Max. allowable air pressure $p_{afm} = 2000$ to 2500 (lbs/sq.in)	=
Final air pressure p_{af} (lbs/sq.in)	
Initial air pressure $p_{ai} = \dfrac{p_{af}}{m}$ (lbs/sq.in)	=
Initial recuperator air volume required: $V_o = \dfrac{F_{vi}}{p_{ai}} b_h \left(\dfrac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}\right)$ (cu.in) b_h = length of horizontal recoil (inches)	=
Effective area of recuperator piston $A_v = \dfrac{F_{vi}}{p_{ai}}$ (lbs/sq.in)	=
Length of air column in terms of recoil stroke $j = \dfrac{l}{b_h}$ $j = 0.8$ to 1.2 usually	=
Actual length of air column $l = j\, b_h$ (in)	=

$$\text{Ratio of } \frac{\text{Air cyl.cross section}}{\text{Effect.area of recuperator}} = \frac{A_a}{A_v}$$

$$r = \frac{A_a}{A_v} = \frac{1}{j}\left(\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}\right) = 2.8$$

Area of cross section of air cylinder

$A_a = r\ A_v$ (sq.in) =

Max. allowable fibre stress in the recuperator piston rod

f_m (lbs/sq.in) =

$f_m = \frac{2}{3}$ to $\frac{1}{2}$ elastic limit usually

Required area of cross section of recuperator piston rod

$a_v' = 1.2\ \frac{F_{vf}}{f_m}$ (sq.in) =

Required diam.of recuperator piston rod

$d_v' = \sqrt{\frac{a_v'}{0.7854}}$ (in) =

Assumed diam. of recuperator piston rod

d_v (in) =

Area of cross section of recuperator piston rod

$a_v = 0.7854\ d_v^2$ =

Required area of recuperator cylinder

$A_{vo}' = A_v' + a_v$ (sq.in) =

Required diam. of recuperator cylinder

$$d'_{vo} = \sqrt{\frac{A'_{vo}}{0.785}} \quad \text{(in)} \qquad =$$

Assumed diam. of recuperator cylinder d_{vo} (in.) =

Area of recuperator cylinder

A_{vo} (sq.in) =

Effective area of recuperator piston

$A_v = A_{vo} - a_v$ (sq.in) =

Initial recuperator pressure

$$p'_{ai} = \frac{F_{vi}}{A_v} \text{ (lbs/sq.in)} \qquad =$$

Final recuperator pressure

$p'_{af} = m\, p'_{ai}$ (lbs/sq.in) =

Initial air volume (exact)

$$V_o = \frac{F_{vi} b_h}{p'_{ai}} \left(\frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} \right) \qquad =$$

k=1.3 to 1.1

Length of air column(exact)

$$l = \frac{V_o}{A_a} \quad \text{(in)}$$

RECOIL BRAKE LAYOUT.

Max.hydraulic pull (at max. elev.)

$P_{hm} = K_s + W_r(\sin\emptyset_m - 0.3\cos\emptyset_m) - F_{vi}$ (lbs)

Max. hydraulic pull(at 0° elev.)

$P_{hc} = K_h - 0.3W_r - F_{vi}$ (lbs)

Max. allowable brake pressure

$p_{h\,max.}$(lbs/sq.in)

$p_{h\,max} = 4000$ to 5000 (lbs/sq.in)

Required effective area of recoil piston

$$A = \frac{P_{hm}}{p_{h\,max}} \text{ (sq.in)} \qquad =$$

Reciprocal of contraction factor of orifice assumed C =

Min. recoil throttling area

$$W_{h\,min} = \frac{C\ A\ V_r}{13.2\sqrt{p_{h\,max}}} \text{ (sq.in)} \qquad =$$

where $V_r = 0.9\ V_f$ (approx.)

Hydraulic brake pressure (at 0° elev.)

$$p_{ho} = \frac{P_{ho}}{A} \text{ (lbs/sq.in)} \qquad =$$

Max. recoil throttling area (at 0° elev.)

$$W_{h\,max} = \frac{C\ A\ V_r}{13.2\sqrt{p_{ho}}} \quad \text{where } V_r = 0.92\ V_f \text{ (approx.)}$$

The battery stabilizing moment of counter recoil

$W_s L_s' = 150$ to $250\ L_a$ (inch lbs.)

L_s' = distance from wheel base to W_s

L_a = distance from spade to wheel contact

Max. buffer reaction of counter recoil

$$B' = F_{vi} + C_s' \frac{W_s L_s'}{b} - 0.3\ W_r \text{ (lbs)} \quad =$$

DIMENSIONS OF HOLLOW PISTON RODS.

Max. allowable buffer pressure

p_{bm}' (lbs/sq.in) =

Assume from 1500 to 2500 (lbs/sq.in)

Area of buffer chamber

$$A_b = \frac{B'}{p_{bm}} \text{ (sq.in)} \quad =$$

Required inside diam.of piston rod

$$d_i = \sqrt{\frac{A_b}{0.7834}} \text{ (in)} \quad =$$

Area of inside cross section of rod

Filloux recoil system

$$A_b' = 3\ W_{bm} \qquad \text{(sq.in)}$$

Required inside diam. of piston rod

Filloux recoil system

$$d_i = \sqrt{\frac{A_b'}{0.7834}} \text{ (in)} \quad =$$

Assumed inside diam. of piston rod

d_i (in) =

Max. allowable fibre stress brake piston rods f_m (lbs/sq.in)

$\frac{3}{8}$ to $\frac{1}{2}$ elastic limit (lbs/sq.in)

Outside diam. of piston rod based on max. allowable tension

$$d_o = \sqrt{d_i^2 + 1.6 \frac{P_h}{f_m}} \quad \text{(in)}$$

Outside diam.of piston rod based on max. hoop compression

$$d_o = \frac{d_i}{\left(1 - \frac{1.3P_{bm}}{f_m}\right)} \quad \text{(in)} \quad =$$

Assumed outside diam. of rod d_o (in)

Area of total cross section of rod

$a_r = 0.7854\, d_o^2$ (sq.in) =

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(2) Dimensions of Solid piston rods

Max. allowable fibre stress

f_m (lbs/sq.in)

$\frac{3}{8}$ to $\frac{1}{2}$ elastic limit (lbs/sq.in)

Required area of piston rods

$$a_r' = 1.3 \frac{P_{hm}}{f_m} \text{(sq.in)} \quad =$$

Corresponding diam. rod

$$d_r' \sqrt{\frac{A_r'}{0.7854}} \quad \text{(in)} \quad =$$

Assumed diam. of rod
d_r (in) =

Area of rod $a_r = 0.7854\ d_r^2$
(sq.in) =

AREA OF DIAM. OF RECOIL CYLINDER

Required area of recoil cylinder

$A_r' = A' + a_r$ (sq.in) =

Corresponding diam. of recoil cylinder

$d' = \sqrt{\dfrac{A_r'}{0.7854}}$ (in) =

Assumed diam. of recoil cylinder
d (in) =

Area of recoil cylinder
$A_r = 0.7854\ d^2$ (sq.in) =

Effective area of recoil piston
$A = A_r - a_r$ (sq.in) =

Max. pressure in recoil cylinder

$P_{h\ max} = \dfrac{P_{hm}}{A}$ (lbs/sq.in) =

PRINCIPLE REACTIONS AND STRESSES THROUGHOUT MOUNT.

Total resistance to recoil at max. elevation

$K_s = \dfrac{\frac{1}{2} m_r V_r^2}{b_s - E + V_f T}$ (lbs) =

Initial recuperator reaction

$F_{vi}=1.3W_r(\sin\phi_m+0.3\cos\phi_m)$ (lbs) =

Max. hydraulic pull (approx.)
max. elev.

$P'_{hm}=K_s+W_r(\sin\phi_m-0.3\cos\phi_m)-F_{vi}$ (lbs)=

FROM RECUPERATOR AND RECOIL BRAKE LAYOUT

DETERMINE:

Distance from axis of bore to line of action of P'_h

d_h (in) =

Distance from axis of bore to line of action of F_v

d_v (in) =

Distance from axis of bore to line of action of resultant braking B

$d_b = \dfrac{P'_h d_h + F_{vi} d_v}{P'_h + F_{vi}}$ (in) =

Distance between guide friction l (in) =

$l = \dfrac{b_h}{2}$ for 3 clips (in)

$l = b_h$ for 4 clips (in)

Coordinates along bore of front and rear clip reactions with respect to center of gravity of recoiling parts

x_1 (in) =

x_2 (in) =

Distance from axis of bore to line of action of mean guide friction (from layout) d_r (in) =

Total braking at max. elevation

$$B = \frac{K_s + W_r(\sin\emptyset - n\cos\emptyset\ \frac{[x_1 - x_2]}{l}}{1 + \frac{2\ nd_b}{l}}\ (\text{lbs}) =$$

where n = 0.1 to 0.2

Recuperator piston friction

$$R_{pv} = .04\ \pi\ D_{vo}\ \frac{F_{vi}}{A_v}\ w_p =$$

Assume W_p = width of packing (in)

Recuperator stuffing box friction

$$R_{pv} = .04\ \pi\ d_v\ \frac{F_{vi}}{A_v}\ W_s\quad (\text{lbs}) =$$

Assume w_s = width of packing (in)

Total recuperator packing friction

$R_{(s+p)}v$ (lbs) =

Hydraulic piston friction

$$R_{ph} = .04\ \pi\ D\ \frac{K_s}{A}\ w_p\ (\text{lbs}) =$$

Hydraulic stuffing box friction

$$R_{sh} = .04\pi\ d_r\ \frac{Ks}{A}\ w_s\ (\text{lbs}) =$$

Total hydraulic packing friction

$R_{(s+p)} = R_{ph} + R_{sh}$ (lbs) =

Total hydraulic pull (max. elev.)

$P'_{hm} = B - F_{vi} - R_{(s+p)}{}^{v}$ =

Total hydraulic reaction
(max. elev.)

$P_{hm} = P'_{hm} - R_{(s+p)}{}^{h}$ (lbs) =

Max. hydraulic pressure

$p_{hm} = \frac{P_{hm}}{A}$ (lbs/sq.in) =

Max. recuperator reaction
$F_{vf} = m\ F_{vi}$ (lbs)
where m = ratio of compression
